This topic deals with some of the macroscopic (life size) scale properties of solids, liquids and gases, and how these properties make them interact with the rest of the world around us. This is important if you want to build structures and machines that are strong enough to do their jobs.

**What are the theories?**

The strength of a material can be defined in many different ways, and we will look at several of these. There is a difference in the strength of a material when being crushed, compared to it being stretched to breaking point. And how does a reluctance to bend compare with a reluctance to be dented? The properties of a particular material make it suitable for some applications, but not others – so knowledge of these properties is essential for designers.

The movement of liquids and gases (fluids) is different in different conditions. Factors such as the temperature and speed of movement have a huge impact on the way that a fluid will move.

**What is the evidence?**

Materials testing can be done on a small scale in a school laboratory. Many of these experiments are reproduced on a larger scale for industrial applications. You will carry out some investigations of materials strength and learn how to describe the behaviour of materials. You may also have opportunities to observe the movement of fluids in simple laboratory situations.

**What are the implications?**

You may be able to make some strength measurements of your own – you will certainly have to make some calculations of strengths for different materials. Under various circumstances, these values can affect the design of things from tennis balls to prams to nuclear power stations.

How fluids flow is a result of properties of the liquid or gas in question. We will consider the flow of a fluid over a solid, as well as of a fluid flowing through another fluid or through itself. Understanding fluid flow is clearly important in the design of aircraft, but you may never have realised that aeronautical engineers are also employed by manufacturers of tomato ketchup.

The map opposite shows you all the knowledge and skills you need to have by the end of this topic. The colour in each box shows which chapter they are covered in and the numbers refer to the sections in the Edexcel specification.
Chapter 2.1

- Understand and use the terms density, laminar flow, streamline flow, terminal velocity, turbulent flow, upthrust and viscous drag (18)
- Recognise and use the expression for Stokes’ law, \( F = 6\pi \eta \text{av} \) and upthrust = weight of fluid displaced (20)
- Recall and use primary or secondary data to show that the rate of flow of a liquid is related to its viscosity (19)

Chapter 2.2

- Investigate and use Hooke’s law, \( F = k\Delta x \), and know that it applies only to some materials (23)
- Investigate elastic and plastic deformation of a material and distinguish between them (25)
- Calculate the elastic strain energy \( E_{el} \) in a deformed material sample, using the expression \( E_{el} = \frac{1}{2}F\Delta x \), and from the area under its force–extension graph (27)
- Obtain and draw force–extension, force–compression and tensile/compressive stress–strain graphs. Identify the limit of proportionality, elastic limit and yield point (22)
- Obtain and draw force–extension, force–compression and tensile/compressive stress–strain graphs. Identify the limit of proportionality, elastic limit and yield point (22)
- Explain the meaning and use of, and calculate tensile/compressive stress, tensile/compressive strain, strength, breaking stress, stiffness and Young modulus. Obtain the Young modulus for a material (24)
- Explore and explain what is meant by the terms brittle, ductile, hard, malleable, stiff and tough. Use these terms, give examples of materials exhibiting such properties and explain how these properties are used in a variety of applications (26)
2.1 Fluid flow

Fluids

Have you ever wondered why it is sometimes so difficult to get tomato ketchup out of the bottle? The answer is that the manufacturers make it thick on purpose. Market research shows that consumers enjoy a certain consistency of ketchup on their chips, and producing it that thick makes the sauce flow very slowly.

This chapter will explain various aspects of the movements of fluids, including some of the ways in which fluid properties are measured. A fluid is defined as any substance that can flow. Normally this means any gas or liquid, but solids made up of tiny particles can sometimes behave as fluids. An example is the flow of sand through an hourglass.

Density

One of the key properties of a fluid is its density. Density is a measure of the mass per unit volume of a substance. Its value depends on the mass of the particles from which the substance is made, and how closely those particles are packed:

\[
\rho = \frac{m}{V}
\]

The equation for calculating density works for mixtures and pure substances, and for all states of matter. Thus, fluid density is also mass per unit volume.

**Worked examples**

**Example 1**

A house brick is 23 cm long, 10 cm wide and 7 cm high. Its mass is 3.38 kg. What is the brick’s density?

\[
\rho = \frac{m}{V} = \frac{3.38 \text{ kg}}{0.23 \times 0.10 \times 0.07 = 1.61 \times 10^{-3} \text{ m}^3} = 2100 \text{ kg m}^{-3}
\]

**Example 2**

At 20 °C, a child’s balloon filled with helium is a sphere with a radius of 20 cm. The mass of helium in the balloon is 6 grams. What is the density of helium at this temperature?

\[
\rho = \frac{m}{V} = \frac{0.006 \text{ kg}}{(4/3)\pi (0.20)^3 = 0.0335 \text{ m}^3} = 0.179 \approx 0.18 \text{ kg m}^{-3}
\]

**Table 2.1.1** Examples of density values for solids, liquids and gases.

<table>
<thead>
<tr>
<th>Material</th>
<th>State</th>
<th>Density/kg m^{-3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>Gas (sea level, 20 °C)</td>
<td>1.2</td>
</tr>
<tr>
<td>Pure water</td>
<td>Liquid (4 °C)</td>
<td>1000</td>
</tr>
<tr>
<td>Sulfuric acid (95% conc)</td>
<td>Liquid (20 °C)</td>
<td>1839</td>
</tr>
<tr>
<td>Cork</td>
<td>Solid</td>
<td>240</td>
</tr>
<tr>
<td>Ice</td>
<td>Solid</td>
<td>919</td>
</tr>
<tr>
<td>Window glass</td>
<td>Solid</td>
<td>2579</td>
</tr>
<tr>
<td>Iron</td>
<td>Solid</td>
<td>7850</td>
</tr>
<tr>
<td>Gold</td>
<td>Solid</td>
<td>19320</td>
</tr>
</tbody>
</table>

**Fig. 2.1.1** Density is very important in determining how heavy an object is.
When an object is submerged in a fluid, it feels an upwards force caused by the fluid pressure – the upthrust. It turns out that the size of this force is equal to the weight of the fluid that has been displaced by the object. This is known as Archimedes’ Principle. Thus, if the object is completely submerged, the mass of fluid displaced is equal to the volume of the object multiplied by the density of the fluid:

\[ m = V \rho \]

The weight of fluid displaced (i.e. upthrust) is then found using the relationship:

\[ W = mg \]

This principle gets its name from the famous legend of the ancient Greek scientist Archimedes running naked from his bath through the streets of Syracuse in about 265 BC, shouting ‘Eureka!’ (‘I’ve found it!’). According to this story, the king of Syracuse thought that his goldsmith may have stolen some gold, by creating a crown out of gold and silver mixed together and claiming it was pure gold. The king asked Archimedes to work out if this suspicion was true, without damaging the intricate wreath-style crown.

When Archimedes climbed into his bath, puzzling over the problem, the water overflowed. Observing this overflow, Archimedes realised that if the crown were submerged, it would displace its own volume of water and would experience an upwards force, or upthrust. Using a balance to weigh the crown when it was suspended in water, Archimedes could find the upthrust, and therefore the weight of water displaced and the volume of the crown. He could then calculate the density of the metal in the crown and compare it with the standard density for gold. He had solved the problem!

**Why does a brick sink?**

If the house brick from the example calculation of density on p. 52 were dropped in a pond, it would experience an upthrust equal to the weight of the volume of water displaced by the brick. As the density of water is 1000 kg m\(^{-3}\), the mass of water displaced by the brick would be:

\[ 1000 \times 1.61 \times 10^{-3} = 1.61 \text{ kg} \]

This has a weight of:

\[ 1.61 \times 9.81 = 15.8 \text{ N} \]

so there is an upward force on the brick of 15.8 N.

If we compare the weight of the brick with the upthrust when it is submerged, the resultant force will be downwards:

- weight = 3.38 \times 9.81 = 33.2 N downwards
- upthrust = 15.8 N upwards
- resultant force = 33.2 – 15.8 = 17.6 N downwards

So, the brick will accelerate downwards within the water until it reaches the bottom of the pond, which then exerts an extra upwards force to balance the weight so the brick rests on the bottom.

**fig. 2.1.2** Scuba diving equipment includes a buoyancy control device which can change volume to displace more or less water. This varies the upthrust and so helps the diver move up or down.

**fig. 2.1.3 a** If the upthrust on an object is less than its weight, then the object will sink through a fluid. **b** An object will remain at rest when balanced forces act on it.
Floating

Imagine an object falling into a fluid. When the object is at the surface there is no upthrust, because no fluid has been displaced. As the object sinks deeper into the fluid, it displaces a greater volume of the fluid, so increasing the upthrust acting upon it. When the upthrust and weight are balanced exactly, the object will float. So for an object to float, it will have to sink until it has displaced its own weight of fluid.

Worked examples

A giant garbage barge on New York’s Hudson River is 60 m long and 10 m wide. What depth of the hull will be under water if it and its cargo have a combined mass of \(1.5 \times 10^6\) kg? (Assume density of Hudson river water = 1000 kg m\(^{-3}\).)

To float:

\[
\text{upthrust} = \text{weight}
\]

\[
\text{weight} = mg = 1.5 \times 10^6 \times 9.81 = 1.47 \times 10^7 \text{N}
\]

\[
\therefore \text{upthrust} = 1.47 \times 10^7 \text{N}
\]

The upthrust is equal to the weight of the volume of water displaced by the hull:

\[
\text{upthrust} = \rho \times V \times g
\]

where:

volume, \(V = \text{length of hull, } l \times \text{width of hull, } w \times \text{depth of hull under water, } d\)

So:

\[
\text{upthrust} = 1000 \times 60 \times 10 \times d \times 9.81
\]

\[= 5.89 \times 10^6 \times d
\]

\[
\therefore d = 1.47 \times 10^7
\]

\[= 5.89 \times 10^6
\]

\[d = 2.497 = 2.5 \text{ m}
\]

The hull will be 2.5 m under water.

HSW The Plimsoll Line

During the 1870s the MP Samuel Plimsoll fought a long struggle to pass a law in the English parliament to protect sailors. Merchant seaman had long known that overloaded ships were dangerous and prone to sinking in high seas. In the year 1873–1874 more than 400 ships were lost in the water around the United Kingdom, with a loss of over 500 lives. So-called ‘coffin ships’ became notorious. In some cases ships were so overloaded and in such a poor state of repair that sailors refused to put to sea in them and were imprisoned for desertion. Ship owners made huge profits from selling goods overseas, and if ships were lost at sea they would not lose money because they could claim on their insurance.

Plimsoll was determined to improve safety at sea. After many defeats, a law was passed in 1876 making it compulsory for every ship registered in England to be painted with a ‘Plimsoll Line’. The mark shows the safe waterline on the hull of a loaded ship, and its correct position is worked out using density calculations.

fig. 2.1.4 The Plimsoll Line on a ship indicates the safe loading level. The water surface must not be above the line indicating the temperature and location of the seas to be crossed (for example TF refers to tropical fresh water). As cargo is loaded, the ship will sink lower to displace a greater weight of water and thus balance the new, heavier overall weight of the ship.
The hydrometer

The idea of floating at different depths is the principle behind the hydrometer, used to determine the density of a fluid. The device has a constant weight, so it will sink lower in fluids of lesser density. This is because a greater volume of a less dense fluid must be displaced to balance the weight of the hydrometer. Scale markings on the narrow stem of the hydrometer indicate the density of liquid.

Comparing the density of alcoholic drinks with that of water gives an indication of the proportion of alcohol they contain. The lower the density, the greater the alcohol content, as alcohol has a lower density than the water it is mixed with. This has long been the basis for the taxation of alcohol.

### Table 2.1.2

<table>
<thead>
<tr>
<th>Type of drink</th>
<th>Alcohol content/abv (alcohol by volume, given as the percentage proportion of alcohol)</th>
<th>Taxation rate/£ per hectolitre (100 l) of product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Still cider and perry</td>
<td>1.2% &lt; abv &lt; 7.5%</td>
<td>26.48</td>
</tr>
<tr>
<td>Sparkling cider and perry</td>
<td>5.5% &lt; abv &lt; 8.5%</td>
<td>172.33</td>
</tr>
<tr>
<td>Wine</td>
<td>1.2% &lt; abv &lt; 4%</td>
<td>54.85</td>
</tr>
<tr>
<td>Sparkling wine</td>
<td>5.5% &lt; abv &lt; 8.5%</td>
<td>172.33</td>
</tr>
</tbody>
</table>

The greater the percentage of alcohol, the greater the tax on the drink.

### Questions

1. A bottle of whiskey contains 1 litre of the drink. The mass of the liquid in the bottle is 0.915 kg. What is the density of this brand of whiskey? (1000 litres = 1 m³)

2. The radius of a hockey ball is 36 mm and its mass is 140 g. What is its density
   a. in g cm⁻³
   b. in kg m⁻³?

3. Estimate the mass of air in this room.
   (Assume density of air = 1 kg m⁻³.)

4. A golf ball has a diameter of 4.72 cm. If a golfer hits it into a stream, what upthrust does the ball experience when it is completely submerged?
   (Assume density of water = 1000 kg m⁻³.)

5. Explain why a ship’s Plimsoll Line has a mark for fresh water which is higher on the hull than the mark for salt water. (Assume density of salt water = 1100 kg m⁻³.)

6. A ball bearing of mass 180 g is hung on a thread in oil of density 800 kg m⁻³. Calculate the tension in the string, if the density of the ball bearing is 8000 kg m⁻³.
If you ski down a hill, you can go faster by tucking your body into a crouching position. By presenting a smaller area to air resistance, you reduce the force slowing you down. However, speed skiers chasing world record speeds go further in their efforts to increase their speeds.

When a fluid moves, there are two ways this can happen: laminar flow (also called streamline flow) and turbulent flow. In general, laminar flow occurs at lower speeds, and will change to turbulent flow as the fluid velocity increases past a certain value. The velocity at which this changeover occurs will vary depending upon the fluid in question and the shape of the area through which it is flowing.

If we take a simple example like water flowing slowly through a pipe, it will be laminar flow. Think of the water in the pipe as several concentric cylinders from the central axis outwards to the layer of water in contact with the pipe itself (fig. 2.1.7). Friction between the outermost layer and the pipe wall means this layer will only be able to move slowly. The next layer in will experience friction with the slow-moving outermost layer, but this will be less than the friction between the outermost layer and the pipe. Thus this inner layer will move faster than the outermost layer. The next layer in moves faster again, with the velocity of each layer increasing nearer the centre, where the very central cylinder of water is moving the fastest.

As with most areas of scientific investigation, Isaac Newton produced much work on the subject of fluid flow. In particular, he is credited with the development of equations to describe the frictional force between the layers in streamline flow. If a liquid follows his formulae, as most common liquids do, it is known as a Newtonian liquid.

The lines of laminar fluid flow are called streamlines. At any point on any one of these streamlines, the velocity of the flow will be constant over time. In the wind tunnel in fig. 2.1.8, the smoke would flow over the car in exactly the same pattern forever if all the wind tunnel factors were kept constant. Changing the speed of the airflow in the tunnel allows designers to test how the prototype would behave at faster speeds, and at what point laminar flow changes to turbulent flow.

In turbulent flow the fluid velocity in any given place changes over time. The flow becomes chaotic and eddies form, causing unpredictable higher and lower pressure areas. A poorly designed car would cause turbulent flow of air over it. In the wind tunnel the smoke trails over the car would be seen to swirl in ever-changing patterns. Turbulent flow increases the drag on a vehicle and so increases fuel consumption.
Streamline flow produces much lower air resistance than turbulent flow. Thus by altering the aerodynamics of their suits, skiers can raise the velocity at which the air movement past their body will change from laminar flow to turbulent flow. This is the principle behind all ‘streamlined’ designs, such as sports cars and boats.

1. Give three examples of objects which are designed to reduce the amount of turbulent flow of air or water over them.

2. Draw diagrams to illustrate the basic definitions of streamline flow and turbulent flow. Explain how your diagrams show each type of flow.

3. Explain these poetic observations of the flow of a Lake District stream:

   In the gentle time of a late summer, a creek over boulder flowed smooth.

   As autumn fell, floating leaf after leaf skipped round the rock, chasing like giddy schoolgirls playing ‘Follow the leader’.

   In winter’s depth, all frozen stood, ice on stone, stone on ice.

   The bright thaw springs a maelstrom, water currents churning and swirling as drunken Maypolers.

Demonstrating types of flow

Turbulent flow was first demonstrated by Osborne Reynolds in 1883 in an experiment showing coloured water flowing in a glass tube. A similar experiment can be set up in a school laboratory to show turbulence caused by faster fluid flow, or by different shapes of obstacles (fig. 2.1.9). At most speeds, a smooth, curved obstacle will produce less turbulence than a squarer one.

A few crystals of potassium manganate(VII) will produce purple trails in the water flow which can then be made to pass around objects made from Blu-tack®. You can alter the flow rate and the obstacle shapes in order to see how the flow changes.
When you wade through a swimming pool, you find it much harder than walking through air. The friction acting against you (called viscous drag) is greater in water than it is in air. This frictional force in fluids is due to viscosity. If the frictional force caused by movement through the fluid is small, we say the viscosity is low.

Newton developed a formula for the friction in liquids which includes several factors. One of these factors relates to the particular liquid in question. Clearly, it would be even harder to wade through a swimming pool of treacle than one full of water. This fluid-dependent factor is called the coefficient of viscosity and has the symbol, \( \eta \), the Greek letter eta.

As viscosity determines the friction force acting within a fluid, it has a direct effect on the rate of flow of the fluid. Consider the differing rates of flow of a river of lava (see table 2.1.3) compared with the viscosity of the lava.

<table>
<thead>
<tr>
<th>Lava type</th>
<th>Viscosity</th>
<th>Approximate flow rate km h(^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basaltic</td>
<td>Least</td>
<td>30–60</td>
</tr>
<tr>
<td>Andesitic</td>
<td>In between</td>
<td>10</td>
</tr>
<tr>
<td>Rhyolitic</td>
<td>Most</td>
<td>1</td>
</tr>
</tbody>
</table>

table 2.1.3 How is the rate of flow related to the viscosity of the fluid?

The rate of flow of a fluid through a pipe is inversely proportional to the viscosity of the fluid. In 1838, Jean Poiseuille, a French doctor and physiologist, investigated the flow of fluids in pipes and proved the connection between flow rate and viscosity. Poiseuille was interested in blood flow through the body, but his law is immensely important in industrial design. For example, the rate of flow of liquid chocolate through pipes in the manufacture of sweets will vary with the chocolate’s viscosity, which will vary depending on the exact recipe used to produce it. More sugar may mean greater viscosity, and thus slower flow through the pipes, and thus less chocolate per sweet.

Poiseuille flow

You can investigate how fluid flow rate depends on its viscosity by doing an experiment very similar to those carried out by Poiseuille in the mid-nineteenth century.

Using a constant pressure, water forced through a narrow pipe will flow at a certain rate, inversely proportional to its viscosity. By varying the height of the water tank, you can record measurements of this ‘head of pressure’, \( h \), against the flow rate and the gradient of the best fit line will allow you to calculate the viscosity of the water.

Using the student’s results shown in fig. 2.1.12, you can plot a graph of the flow rate (\( V \)) against height (\( h \)) and hence calculate the viscosity of water (\( \eta \)).

Poiseuille’s equations tells us that the gradient of the graph:

\[
\frac{\pi \rho g r^4}{8\eta l}
\]

where \( r \) is the internal radius of the capillary tube, \( \rho \) is the density of water and \( g = 9.81 \text{ N kg}^{-1} \).

Compare the result from this experiment with the figures in table 2.1.3.
An even greater variation in viscosity of liquid chocolate is caused by changes in its temperature. If a sweet manufacturer wants to account for variation in a recipe (which might come from something as minor as a change in supplier of cocoa beans), they can adjust the flow rate by altering the temperature. Viscosity is directly related to fluid temperature (Table 2.1.4). In general, liquids have a lower coefficient of viscosity at higher temperatures. For gases, viscosity increases with temperature.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Temperature/°C</th>
<th>Viscosity/Pa s</th>
<th>Fluid</th>
<th>Temperature/°C</th>
<th>Viscosity/Pa s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0</td>
<td>0.000017</td>
<td>Glycerine</td>
<td>40</td>
<td>6700</td>
</tr>
<tr>
<td>Air</td>
<td>20</td>
<td>0.000018</td>
<td>Glycerine</td>
<td>20</td>
<td>1.5</td>
</tr>
<tr>
<td>Air</td>
<td>100</td>
<td>0.000022</td>
<td>Glycerine</td>
<td>30</td>
<td>0.63</td>
</tr>
<tr>
<td>Water</td>
<td>0</td>
<td>0.0018</td>
<td>Chocolate</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Water</td>
<td>20</td>
<td>0.0010</td>
<td>Chocolate</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>Water</td>
<td>100</td>
<td>0.0003</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1.4 Viscosities of various fluids.

The change in viscosity of a liquid with temperature can be observed in a school laboratory using a re-sealable tin or bottle half full of a test fluid (like golden syrup). The temperature of the liquid is varied using a water bath. The viscosity of the liquid will affect the rate at which the tin or bottle rolls down a fixed ramp.

Questions

1. Why is the world record for 100 m swimming slower than for 100 m sprinting?
2. Describe how temperature affects viscosity for liquids and gases.
3. How and why would holding a swimming competition in a warmer pool affect the times achieved by swimmers?
4. Why might a chocolate manufacturer alter their machinery so it functioned at a higher temperature?
You have previously learned that the acceleration due to gravity, near the surface of the Earth, is 9.81 m/s$^{-2}$. An object falling in a vacuum does indeed accelerate at this rate. However, it is unusual for objects to be dropped near the surface of the Earth in a vacuum (in nearly all such cases a physics teacher is likely to be demonstrating to a class!). In reality, in order to calculate an object’s actual acceleration when falling, we need to go back to Newton’s second law. We know that $a = \sum F/m$. If we can take account of all the forces acting on an object, and combine these to find a resultant force, we can calculate the resulting acceleration.

For a falling object like a skydiver, we need to include the weight, the upthrust caused by the object being in the fluid air, and the viscous drag force caused by the movement. The tricky part is that the viscous drag varies with speed through the fluid, and that is constantly changing as a result of the acceleration. Usually, we consider the equilibrium situation, in which the weight exactly balances the sum of upthrust and drag, meaning that the falling velocity remains constant. This constant velocity is the terminal velocity.

**Viscous drag**

You would find it difficult to wade through a swimming pool filled with treacle because of the treacle’s viscous drag. This is the friction force between a solid and a fluid. Calculating this fluid friction force can be relatively simple. On the other hand, it can be very complicated for large objects, fast objects, and irregularly shaped objects, as the turbulent flow creates an unpredictable situation.

**HSW Stokes’ Law**

In the mid-nineteenth century, Sir George Gabriel Stokes, an Irish mathematician and physicist at Cambridge University, investigated fluid dynamics and derived an equation for the viscous drag ($F$) on a small sphere at low speeds. This formula is now called Stokes’ Law:

$$F = 6\pi \eta r v$$

where $r$ is the radius of sphere (m), $v$ is the velocity of sphere (m s$^{-1}$), and $\eta$ is the coefficient of viscosity of the fluid (Pa s).

Thus in such a simple situation, the drag force is directly proportional to the radius of the sphere, and directly proportional to the velocity, neither of which is necessarily an obvious outcome.

Stokes’ publication of this law was delayed slightly while he considered the news that similar conclusions had previously been made by scientists in other parts of Europe, notably Navier and Poisson. At that time, communication between scientists was much slower and more limited than it is now, and it was common for the same results to be discovered independently and simultaneously. In this case, Stokes decided that his work was sufficiently different from that of the others to justify publishing it.

For simplicity, we will only consider simple situations, like a solid sphere moving slowly in a fluid. Imagine a ball bearing dropping through a column of oil, for example.
If you consider the terminal velocity of the ball bearing in terms of the forces in detail, then:

weight = upthrust + Stokes force

\[ m_s g = \text{weight of fluid displaced} + 6\pi\eta v_{\text{term}} \]

where \( m_s \) is the mass of the sphere and \( v_{\text{term}} \) is its terminal velocity.

For the sphere, the mass \( m_s \) is given by:

\[ m_s = \text{volume} \times \text{density of sphere} = \frac{4}{3}\pi r^3 \rho_s \]

so the weight of the sphere \( W_s \) is given by:

\[ W_s = m_s g = \frac{4}{3}\pi r^3 \rho_s g \]

For the sphere, the upthrust is equal to the weight of fluid displaced. The mass \( m_f \) of fluid displaced is given by:

\[ m_f = \text{volume} \times \text{density of fluid} = \frac{4}{3}\pi r^3 \rho_f \]

so the weight of fluid displaced \( W_f \) is given by:

\[ W_f = m_f g = \frac{4}{3}\pi r^3 \rho_f g \]

Overall then:

\[ \frac{4}{3}\pi r^3 \rho_s g = \frac{4}{3}\pi r^3 \rho_f g + 6\pi\eta v_{\text{term}} \]

We can rearrange the equation to find the terminal velocity:

\[ v_{\text{term}} = \frac{\frac{4}{3}\pi r^3 g (\rho_s - \rho_f)}{6\pi\eta} \]

Cancelling the \( \pi \) and the radius term:

\[ v_{\text{term}} = \frac{2r^2 g (\rho_s - \rho_f)}{9\eta} \]

So terminal velocity is proportional to the square of the radius. This means that a larger sphere falls faster. And because the radius is squared, it falls much faster! Don’t forget that this is based on a slow-moving small sphere – more complex situations have more complex equations.

It must be remembered that the simple slow-falling sphere is not a common situation and in most real applications the terminal velocity value is a result of more complex calculations. However, the principle that larger objects generally fall faster holds true for most objects without a parachute.

<table>
<thead>
<tr>
<th>Falling object</th>
<th>Terminal velocity/m s(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skydiver</td>
<td>60</td>
</tr>
<tr>
<td>Golf ball</td>
<td>32</td>
</tr>
<tr>
<td>Hail stone (0.5 cm radius)</td>
<td>14</td>
</tr>
<tr>
<td>Raindrop (0.2 cm radius)</td>
<td>9</td>
</tr>
</tbody>
</table>

*table 2.1.5 Some typical real terminal velocities in air.*

To work out the terminal velocity of two different sizes of steel ball bearing falling through glycerine in a measuring cylinder, we need to know the densities of steel and glycerine, along with the viscosity of glycerine and the radii of the two ball bearings.

The viscosity of glycerine is highly temperature dependent: at 20°C we can take \( \eta = 1.5 \text{ Pa} \cdot \text{s} \)

Density of steel = 7800 kg m\(^{-3}\)

Density of glycerine = 1200 kg m\(^{-3}\)

\( g = 9.81 \text{ m} \cdot \text{s}^{-2} \)

a For a 1 mm radius ball bearing:

\[ v_{\text{term}} = \frac{2r^2 g (\rho_s - \rho_f)}{9\eta} \]

\[ v_{\text{term}} = \frac{2 (1 \times 10^{-3})^2 \times 9.81 \times (7800 - 1200)}{9 \times 1.5} \]

\[ v_{\text{term}} = 9.6 \times 10^{-3} \text{ m} \cdot \text{s}^{-1} \]

b For a 2 mm radius ball bearing:

\[ v_{\text{term}} = \frac{2r^2 g (\rho_s - \rho_f)}{9\eta} \]

\[ v_{\text{term}} = \frac{2 (2 \times 10^{-3})^2 \times 9.81 \times (7800 - 1200)}{9 \times 1.5} \]

\[ v_{\text{term}} = 3.8 \times 10^{-2} \text{ m} \cdot \text{s}^{-1} \]

Comparing the values, we see that doubling the ball’s radius makes its terminal velocity four times as great.

1 Use Stokes’ Law to calculate the viscous drag on a ball bearing with a radius of 1 mm, falling at 1 mm s\(^{-1}\) through liquid chocolate at 30°C.

2 Why is it difficult to calculate the terminal velocity for a cat falling from a high rooftop?

3 A spherical meteorite, of radius 2 m and made of pure iron, falls towards Earth.

   a For its fall through the air, calculate the meteorite’s terminal velocity.

   b It lands in a tropical freshwater lake at 20°C and continues sinking underwater. Calculate its new terminal velocity.

   c What assumptions have you made in order to make these calculations?

(See *table 2.1.1* for density data and *table 2.1.4* for viscosity data.)