

Chapter 1: Logarithmic functions and indices

1.1 You can simplify expressions by using rules of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Hint: The m th root of a .

Example 1

Simplify these expressions:

a $x^2 \times x^5$

b $2r^2 \times 3r^3$

c $b^4 \div b^4$

d $6x^{-3} \div 3x^{-5}$

e $(a^3)^2 \times 2a^2$

f $(3x^2)^3 \div x^4$

a $x^2 \times x^5$
 $= x^{2+5}$
 $= x^7$

Use the rule $a^m \times a^n = a^{m+n}$ to simplify the index.

b $2r^2 \times 3r^3$
 $= 2 \times 3 \times r^2 \times r^3$
 $= 6 \times r^{2+3}$
 $= 6r^5$

Rewrite the expression with the numbers together and the r terms together.

$$2 \times 3 = 6$$
$$r^2 \times r^3 = r^{2+3}$$

c $b^4 \div b^4$
 $= b^{4-4}$
 $= b^0 = 1$

Use the rule $a^m \times a^n = a^{m+n}$ to simplify the index.

Use the rule $a^m \div a^n = a^{m-n}$

d $6x^{-3} \div 3x^{-5}$
 $= 6 \div 3 \times x^{-3} \div x^{-5}$
 $= 2 \times x^2$
 $= 2x^2$

$$x^{-3} \div x^{-5} = x^{-3 - (-5)} = x^2$$

e $(a^3)^2 \times 2a^2$
 $= a^6 \times 2a^2$
 $= 2 \times a^6 \times a^2$
 $= 2 \times a^{6+2}$
 $= 2a^8$

Use the rule $(a^m)^n = a^{mn}$ to simplify the index.

$$a^6 \times 2a^2 = 1 \times 2 \times a^6 \times a^2 = 2 \times a^{6+2}$$

f $(3x^2)^3 \div x^4$
 $= 27x^6 \div x^4$
 $= 27 \times x^6 \div x^4$
 $= 27 \times x^{6-4}$
 $= 27x^2$

Use the rule $(a^m)^n = a^{mn}$ to simplify the index.

Example 2

Simplify:

a $x^4 \div x^{-3}$ **b** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c** $(x^3)^{\frac{2}{3}}$ **d** $2x^{1.5} \div 4x^{-0.25}$

$$\begin{aligned} \mathbf{a} \quad x^4 \div x^{-3} \\ &= x^{4 - (-3)} \\ &= x^7 \end{aligned}$$

Use the rule $a^m \div a^n = a^{m-n}$.
Remember $-(-) = +$.

$$\begin{aligned} \mathbf{b} \quad x^{\frac{1}{2}} \times x^{\frac{3}{2}} \\ &= x^{\frac{1}{2} + \frac{3}{2}} \\ &= x^2 \end{aligned}$$

This could also be written as \sqrt{x} .
Use the rule $a^m \times a^n = a^{m+n}$.

$$\begin{aligned} \mathbf{c} \quad (x^3)^{\frac{2}{3}} \\ &= x^{3 \times \frac{2}{3}} \\ &= x^2 \end{aligned}$$

Use the rule $(a^m)^n = a^{mn}$.

$$\begin{aligned} \mathbf{d} \quad 2x^{1.5} \div 4x^{-0.25} \\ &= \frac{1}{2}x^{1.5 - (-0.25)} \\ &= \frac{1}{2}x^{1.75} \end{aligned}$$

Use the rule $a^m \div a^n = a^{m-n}$.
 $2 \div 4 = \frac{1}{2}$
 $1.5 - (-0.25) = 1.75$

Example 3

Evaluate:

a $9^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $49^{\frac{2}{3}}$ **d** $25^{-\frac{3}{2}}$

$$\begin{aligned} \mathbf{a} \quad 9^{\frac{1}{2}} \\ &= \sqrt{9} \\ &= \pm 3 \end{aligned}$$

Using $a^{\frac{1}{m}} = \sqrt[m]{a}$.
Both 3 and -3 are square roots of 9.
 $\sqrt{9}$ strictly means $+3$ and $\pm\sqrt{9} = \pm 3$ but always check if the negative square root is a required answer.

$$\begin{aligned} \mathbf{b} \quad 64^{\frac{1}{3}} \\ &= \sqrt[3]{64} \\ &= 4 \end{aligned}$$

This means the cube root of 64.
As $4 \times 4 \times 4 = 64$.

$$\begin{aligned} \mathbf{c} \quad 49^{\frac{2}{3}} \\ &= (\sqrt{49})^3 \\ &= \pm 343 \end{aligned}$$

Using $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.
This means the square root of 49, cubed.

$$\begin{aligned} \mathbf{d} \quad 25^{-\frac{3}{2}} \\ &= \frac{1}{25^{\frac{3}{2}}} \\ &= \frac{1}{(\pm\sqrt{25})^3} \\ &= \frac{1}{(\pm 5)^3} \\ &= \pm \frac{1}{215} \end{aligned}$$

Using $a^{-m} = \frac{1}{a^m}$.
 $\sqrt{25} = \pm 5$

Exercise 1A

Simplify these expressions:

- | | |
|---------------------------------------|---|
| 1 a $x^3 \times x^4$ | b $2x^3 \times 3x^2$ |
| c $4p^3 \div 2p$ | d $3x^{-4} \div x^{-2}$ |
| e $k^3 \div k^{-2}$ | f $(y^2)^5$ |
| g $10x^5 \div 2x^{-3}$ | h $(p^3)^2 \div p^4$ |
| i $(2a^3)^2 \div 2a^3$ | j $8p^{-4} \div 4p^3$ |
| k $2a^{-4} \times 3a^{-5}$ | l $21a^3b^2 \div 7ab^4$ |
| m $9x^2 \times 3(x^2)^3$ | n $3x^3 \times 2x^2 \times 4x^6$ |
| o $7a^4 \times (3a^4)^2$ | p $(4y^3)^3 \div 2y^3$ |
| q $2a^3 \div 3a^2 \times 6a^5$ | r $3a^4 \times 2a^5 \times a^3$ |

2 Simplify:

- | | | |
|---|---|---|
| a $x^3 \div x^{-2}$ | b $x^5 \div x^7$ | c $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$ |
| d $(x^2)^{\frac{3}{2}}$ | e $(x^3)^{\frac{5}{3}}$ | f $3x^{0.5} \times 4x^{-0.5}$ |
| g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ | h $5x^{1\frac{2}{5}} \div x^{\frac{2}{5}}$ | i $3x^4 \times 2x^{-5}$ |

3 Evaluate:

- | | | |
|---|-------------------------------|---|
| a $25^{\frac{1}{2}}$ | b $81^{\frac{1}{2}}$ | c $27^{\frac{1}{3}}$ |
| d 4^{-2} | e $9^{-\frac{1}{2}}$ | f $(-5)^{-3}$ |
| g $(\frac{3}{4})^0$ | h $1296^{\frac{1}{4}}$ | i $(1\frac{9}{16})^{\frac{3}{2}}$ |
| j $(\frac{27}{8})^{\frac{2}{3}}$ | k $(\frac{6}{5})^{-1}$ | l $(\frac{343}{512})^{-\frac{2}{3}}$ |

1.2 You can write a number exactly using surds, e.g. $\sqrt{2}$, $\sqrt{3} - 5$, $\sqrt{19}$. You cannot evaluate surds exactly because they give never-ending, non-repeating decimal fractions, e.g. $\sqrt{2} = 1.414\ 213\ 562\dots$
The square root of a prime number is a surd.

• You can manipulate surds using these rules:

$$\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

• You can rationalise the denominator of $\frac{1}{\sqrt{a}}$ by multiplying the top and bottom by \sqrt{a} .

Example 4

Simplify:

a $\sqrt{12}$

b $\frac{\sqrt{20}}{2}$

c $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

a $\sqrt{12}$

$$= \sqrt{(4 \times 3)}$$

$$= \sqrt{4} \times \sqrt{3}$$

$$= 2\sqrt{3}$$

Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.

$$\sqrt{4} = 2$$

b $\frac{\sqrt{20}}{2}$

$$= \frac{\sqrt{4} \times \sqrt{5}}{2}$$

$$= \frac{2 \times \sqrt{5}}{2}$$

$$= \sqrt{5}$$

$$\sqrt{20} = \sqrt{4} \times \sqrt{5}$$

$$\sqrt{4} = 2$$

Cancel by 2.

c $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

$$= 5\sqrt{6} - 2\sqrt{6} \times \sqrt{4} + \sqrt{6} \times \sqrt{49}$$

$$= \sqrt{6}(5 - 2\sqrt{4} + \sqrt{49})$$

$$= 6(5 - 2 \times 2 + 7)$$

$$= \sqrt{6}(8)$$

$$= 8\sqrt{6}$$

 $\sqrt{6}$ is a common factor.Work out the square roots $\sqrt{4}$ and $\sqrt{49}$.

$$5 - 4 + 7 = 8$$

Example 5

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$

b $\frac{12}{\sqrt{2}}$

a $\frac{1}{\sqrt{3}}$

$$= \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

Multiply the top and bottom by $\sqrt{3}$.

$$\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$$

b $\frac{12}{\sqrt{2}} = \frac{12}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$$= \frac{12\sqrt{2}}{2}$$

$$= 6\sqrt{2}$$

Multiply the top and bottom by $\sqrt{2}$.Remember $\sqrt{2} \times \sqrt{2} = 2$

Simplify your answer

Exercise 1B

Simplify:

- | | | |
|---|---|--|
| 1 $\sqrt{28}$ | 2 $\sqrt{72}$ | 3 $\sqrt{50}$ |
| 4 $\sqrt{32}$ | 5 $\sqrt{90}$ | 6 $\frac{\sqrt{12}}{2}$ |
| 7 $\frac{\sqrt{27}}{3}$ | 8 $\sqrt{20} + \sqrt{80}$ | 9 $\sqrt{200} + \sqrt{18} - \sqrt{72}$ |
| 10 $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$ | 11 $1\sqrt{28} - 2\sqrt{63} + \sqrt{7}$ | 12 $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$ |
| 13 $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$ | 14 $\frac{\sqrt{44}}{\sqrt{11}}$ | 15 $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$ |
| 16 $\frac{1}{\sqrt{5}}$ | 17 $\frac{1}{\sqrt{11}}$ | 18 $\frac{1}{\sqrt{2}}$ |
| 19 $\frac{\sqrt{3}}{\sqrt{15}}$ | 20 $\frac{\sqrt{12}}{\sqrt{48}}$ | 21 $\frac{\sqrt{5}}{\sqrt{80}}$ |
| 22 $\frac{\sqrt{12}}{\sqrt{156}}$ | 23 $\frac{\sqrt{7}}{\sqrt{63}}$ | |

1.3 You need to know how to write an expression as a logarithm

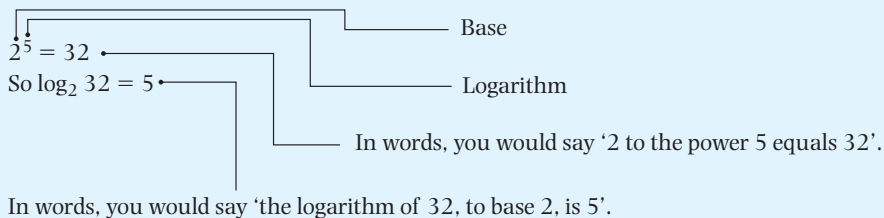
- $\log_a n = x$ means that $a^x = n$, where a is called the base of the logarithm.

In the IGCSE the base of the logarithm will always be a positive integer greater than 1.

Example 6

Write as a logarithm $2^5 = 32$.

Here $a = 2$, $x = 5$, $n = 32$.



Example 7

Rewrite as a logarithm:

a $10^3 = 1000$ **b** $5^4 = 625$ **c** $2^{10} = 1024$

a $\log_{10} 1000 = 3$

b $\log_5 625 = 4$

c $\log_2 1024 = 10$

Example 11Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$

$$\mathbf{a} \quad \log_a (x^2 y z^3) \qquad \mathbf{b} \quad \log_a \left(\frac{x}{y^3} \right) \qquad \mathbf{c} \quad \log_a \left(\frac{x\sqrt{y}}{z} \right) \qquad \mathbf{d} \quad \log_a \left(\frac{x}{a^4} \right)$$

$$\begin{aligned} \mathbf{a} \quad \log_a (x^2 y z^3) &= \log_a (x^2) + \log_a y + \log_a (z^3) \\ &= 2 \log_a x + \log_a y + 3 \log_a z \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad \log_a \left(\frac{x}{y^3} \right) &= \log_a x - \log_a (y^3) \\ &= \log_a x - 3 \log_a y \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad \log_a \left(\frac{x\sqrt{y}}{z} \right) &= \log_a (x\sqrt{y}) - \log_a z \\ &= \log_a x + \log_a \sqrt{y} - \log_a z \\ &= \log_a x + \frac{1}{2} \log_a y - \log_a z \end{aligned} \quad \text{Use the power law } (\sqrt{y} = y^{\frac{1}{2}}).$$

$$\begin{aligned} \mathbf{d} \quad \log_a \left(\frac{x}{a^4} \right) &= \log_a x - \log_a (a^4) \\ &= \log_a x - 4 \log_a a \\ &= \log_a x - 4 \qquad \log_a a = 1. \end{aligned}$$

Exercise 1D**1** Write as a single logarithm:

$\mathbf{a} \quad \log_2 7 + \log_2 3$

$\mathbf{b} \quad \log_2 36 - \log_2 4$

$\mathbf{c} \quad 3 \log_5 2 + \log_5 10$

$\mathbf{d} \quad 2 \log_6 8 - 4 \log_6 3$

$\mathbf{e} \quad \log_{10} 5 + \log_{10} 6 - \log_{10} \left(\frac{1}{4} \right)$

2 Write as a single logarithm, then simplify your answer:

$\mathbf{a} \quad \log_2 40 - \log_2 5$

$\mathbf{b} \quad \log_6 4 + \log_6 9$

$\mathbf{c} \quad 2 \log_{12} 3 + 4 \log_{12} 2$

$\mathbf{d} \quad \log_8 25 + \log_8 10 - 3 \log_8 5$

$\mathbf{e} \quad 2 \log_{10} 20 - (\log_{10} 5 + \log_{10} 8)$

3 Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$:

$\mathbf{a} \quad \log_a (x^3 y^4 z)$

$\mathbf{b} \quad \log_a \left(\frac{x^5}{y^2} \right)$

$\mathbf{c} \quad \log_a (a^2 x^2)$

$\mathbf{d} \quad \log_a \left(\frac{x\sqrt{y}}{z} \right)$

$\mathbf{b} \quad \log_a \sqrt{ax}$

1.5 You can use the change of base formulae to solve equations of the form $a^x = b$

Working in base a , suppose that: $\log_a x = m$

Writing this as a power: $a^m = x$

Taking logs to a different base b : $\log_b (a^m) = \log_b x$

Using the power law: $m \log_b a = \log_b x$

Writing m as $\log_a x$: $\log_b x = \log_a x \times \log_b a$

This can be written as:

- $\log_a x = \frac{\log_b x}{\log_b a}$

This is the change of base rule for logarithms.

Using this rule, notice in particular that $\log_a b = \frac{\log_b b}{\log_b a}$, but $\log_b b = 1$, so:

- $\log_a b = \frac{1}{\log_b a}$

Example 12

Solve the following equations, giving your answers to 3 significant figures.

a $3^x = 20$

b $8^x = 11$

c $10^x = 0.7$

a $3^x = 20 \Rightarrow x = \log_3 20$

Use the definition of logarithms from section 1.3.

By change of base formula,
changing to base 10

Some calculators can evaluate $\log_3 20$. If your calculator does not have this facility, you can use the change of base formula and use base 10

$$\log_3 20 = \frac{\log_{10} 20}{\log_{10} 3}$$

The log button on your calculator uses \log_{10} .
Use this to find $\log_{10} 20$ and $\log_{10} 3$.

$$\begin{aligned} \log_3 20 &= \frac{1.3010\dots}{0.4771\dots} \\ &= 2.73 \end{aligned}$$

Give answer to 3 sf.

b $8^x = 11 \Rightarrow x = \log_8 11$

Use the definition from section 1.3.

Changing to base 10

$$\begin{aligned} \log_8 11 &= \frac{\log_{10} 11}{\log_{10} 8} \\ &= 1.15 \end{aligned}$$

Evaluate using calculator and give answer to 3 sf.

c $10^x = 0.7 \Rightarrow x = \log_{10} 0.7$
 $= -0.155$

This can be found directly using the log button on a calculator.

NB A logarithm can give a negative answer:

$$\log_b x < 0 \text{ when } 0 < x < 1$$

Example 13Solve the equation $\log_5 x + 6 \log_x 5 = 5$:

$$\log_5 x + \frac{6}{\log_5 x} = 5$$

Use change of base rule (special case).

Let $\log_5 x = y$

$$y + \frac{6}{y} = 5$$

$$y^2 + 6 = 5y$$

Multiply by y .

$$y^2 - 5y + 6 = 0$$

$$(y - 3)(y - 2) = 0$$

So $y = 3$ or $y = 2$

$$\log_5 x = 3 \text{ or } \log_5 x = 2$$

$$x = 5^3 \text{ or } x = 5^2$$

Write as powers.

$$x = 125 \text{ or } x = 25$$

Exercise 1E

1 Find, to 3 decimal places:

a $\log_7 120$

b $\log_3 45$

c $\log_2 19$

d $\log_{11} 3$

e $\log_6 4$

2 Solve, giving your answer to 3 significant figures:

a $8^x = 14$

b $9^x = 99$

c $12^x = 6$

3 Solve, giving your answer to 3 significant figures:

a $2^x = 75$

b $3^x = 10$

c $5^x = 2$

d $4^{2x} = 100$

4 Solve, giving your answer to 3 significant figures:

a $\log_2 x = 8 + 9 \log_x 2$

b $\log_4 x + 2 \log_x 4 + 3 = 0$

c $\log_2 x + \log_4 x = 2$

1.6 You need to be familiar with the functions $y = a^x$ and $y = \log_b x$ and to know the shapes of their graphsAs an example, look at a table of values for $y = 2^x$:

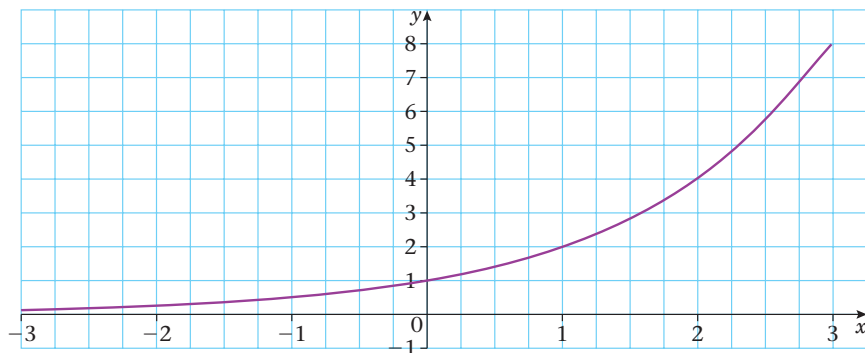
x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Note that

$$2^0 = 1 \text{ (in fact } a^0 = 1 \text{ always if } a > 0)$$

and $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ (a negative index implies the 'reciprocal' of a positive index)**Hint:**A function that involves a variable power such as x is called an exponential function.

The graph of $y = 2^x$ looks like this:



Other graphs of the type $y = a^x$ are of a similar shape, always passing through $(0, 1)$.

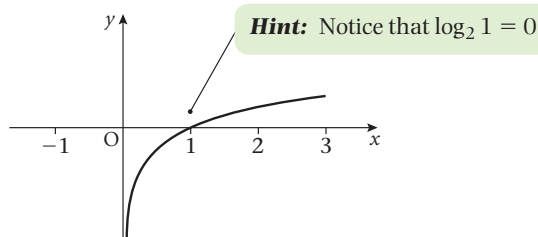
Now look at the table of values of $y = \log_2 x$:

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	-3	-2	-1	0	1	2	3

You should note that the values for x and y have swapped around.

This means that the shape of the curve is simply a reflection in the line $y = x$.

The graph of $y = \log_b x$ will have a similar shape and it will always pass through $(1, 0)$ since $\log_b 1 = 0$ for every value of b .



Hint: The y axis is an asymptote to the curve.

Example 14

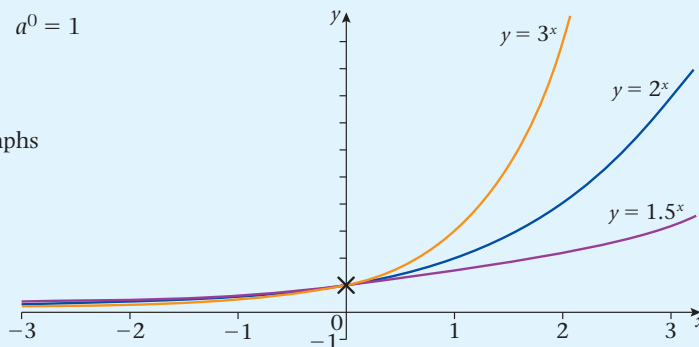
- a** On the same axes sketch the graphs of $y = 3^x$, $y = 2^x$ and $y = 1.5^x$
b On another set of axes sketch the graphs of $y = (\frac{1}{2})^x$ and $y = 2^x$.

- a** For all the three graphs, $y = 1$ when $x = 0$. $a^0 = 1$

$$\text{When } x > 0, \quad 3^x > 2^x > 1.5^x$$

$$\text{When } x < 0, \quad 3^x < 2^x < 1.5^x$$

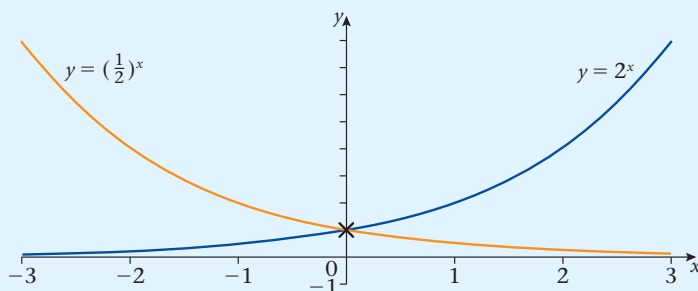
Work out the relative positions of the three graphs



b $\frac{1}{2} = 2^{-1}$

So $y = \left(\frac{1}{2}\right)^x$ is the same as $y = (2^{-1})^x = 2^{-x}$. $(a^m)^n = a^{mn}$

So the graph of $y = \left(\frac{1}{2}\right)^x$ is a reflection in the y -axis of the graph of $y = 2^x$.



Example 15

On the same axes, sketch the graphs of $y = \log_2 x$ and $y = \log_5 x$.

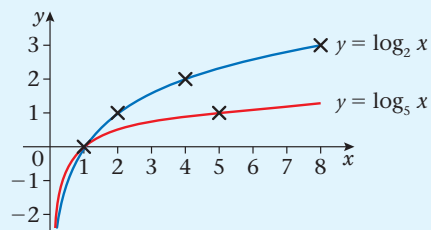
For both graphs $y = 0$ when $x = 1$. Since $\log_b 1 = 0$ for every value of b

But $\log_2 2 = 1$ so $y = \log_2 x$ passes through $(2, 1)$

and $\log_5 5 = 1$ so $y = \log_5 x$ passes through $(5, 1)$.

By considering the shape of the graphs between $y = 0$ and $y = 1$, you can see that $\log_2 x > \log_5 x$ for $x > 1$.

Since the log graphs are reflections of the exponential graphs then from Example 14 you can see that the reverse will apply the other side of $(1, 0)$. So $\log_2 x < \log_5 x$ for $x < 1$.



Exercise 1F

1 On the same axes sketch the graphs of

a $y = 4^x$

b $y = 6^x$

c $y = \left(\frac{1}{4}\right)^x$

2 On the same axes sketch the graphs of

a $y = 3^x$

b $y = \log_3 x$

c $y = \left(\frac{1}{3}\right)^x$

3 On the same axes sketch the graphs of

a $y = \log_4 x$

b $y = \log_6 x$

4 On the same axes sketch the graphs of

a $y = 1^x$

b $y = \log_3 x$

c Write down the coordinates of the point of intersection of these two graphs.

Exercise 1G

1 Simplify:

a $y^3 \times y^5$

b $3x^2 \times 2x^5$

c $(4x^2)^3 \div 2x^5$

d $4b^2 \times 3b^3 \times b^4$

2 Simplify:

a $9x^3 \div 3x^{-3}$

b $(4^2)^{\frac{1}{3}}$

a $3x^{-2} \times 2x^4$

d $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

3 Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$

d $\left(\frac{225}{289}\right)^{\frac{2}{3}}$

4 Simplify:

a $\frac{3}{\sqrt{63}}$

b $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

5 Rationalise:

a $\frac{1}{\sqrt{3}}$

b $\frac{15}{\sqrt{5}}$

6 **a** Express $\log_a(p^2q)$ in terms of $\log_a p$ and $\log_a q$.

b Given that $\log_a(pq) = 5$ and $\log_a(p^2q) = 9$, find the values of $\log_a p$ and $\log_a q$.

7 Solve the following equations giving your answers to 3 significant figures:

a $5^x = 80$

b $7^x = 123$

8 **a** Given that $\log_3 x = 2$, determine the value of x .

b Calculate the value of y for which $2 \log_3 y - \log_3(y + 4) = 2$.

c Calculate the values of z for which $\log_3 z = 4 \log_z 3$.

9 Find the values of x for which $\log_3 x - 2 \log_x 3 = 1$.

10 Solve the equation

$$\log_3(2 - 3x) = \log_9(6x^2 - 19x + 2).$$

Logarithms

- 1** You can simplify expressions by using rules of indices (powers).

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

- 2** You can manipulate surds using the rules:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

- 3** The rule to rationalise surds is:

Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the top and bottom by \sqrt{a} .

- 4** $\log_a n = x$ means that $a^x = n$, where a is called the base of the logarithm.

- 5** $\log_a 1 = 0$

$$\log_a a = 1$$

- 6** $\log_{10} x$ is sometimes written as $\log x$.

- 7** The laws of logarithms are

$$\log_a xy = \log_a x + \log_a y \quad (\text{the multiplication law})$$

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y \quad (\text{the division law})$$

$$\log_a (x)^k = k \log_a x \quad (\text{the power law})$$

- 8** From the power law,

$$\log_a \left(\frac{1}{x}\right) = -\log_a x$$

- 9** The change of base rule for logarithms can be written as $\log_a x = \frac{\log_b x}{\log_b a}$

- 10** From the change of base rule, $\log_a b = \frac{1}{\log_b a}$