There are many situations which can be modelled as motion in a straight line with constant acceleration.

- The acceleration due to gravity is constant. You can find a diver’s vertical speed using the constant acceleration formulae.
- You can calculate the depth of a well by dropping a stone and timing how long it takes to reach the bottom.
- If you are modelling motion under gravity you often ignore air resistance. This skydiver isn’t accelerating. The force of the air resistance equals the force of gravity. The speed at which this happens is called terminal velocity.

After completing this chapter you should be able to

- solve problems involving motion in a straight line with constant acceleration
- model an object moving vertically under gravity
- understand distance–time graphs and speed–time graphs.
2.1 You can use the formulae \( v = u + at \) and \( s = \left( \frac{u + v}{2} \right) t \) for a particle moving in a straight line with constant acceleration.

- You need to learn this list of symbols and what they represent.

| \( s \) | displacement (distance) |
| \( u \) | starting (initial) velocity |
| \( v \) | final velocity |
| \( a \) | acceleration |
| \( t \) | time |

Displacements, velocities and accelerations have directions as well as sizes (or magnitudes). You can derive the formulae for motion in a straight line with constant acceleration.

- acceleration = \( \frac{\text{change in velocity}}{\text{change in time}} \)
  \[ a = \frac{v - u}{t} \]

- \( v = u + at \)

For an object moving horizontally, the positive direction is usually taken as left to right. The starting point of an object is usually taken as the origin from which displacements are measured.

For \( P \): \( s = -4 \text{ m} \) and \( v = 2.5 \text{ m s}^{-1} \)
For \( Q \): \( s = 3 \text{ m} \) and \( v = -6 \text{ m s}^{-1} \)

**Example 1**

A particle is moving in a straight line from \( A \) to \( B \) with constant acceleration \( 3 \text{ m s}^{-2} \). Its speed at \( A \) is \( 2 \text{ m s}^{-1} \) and it takes 8 seconds to move from \( A \) to \( B \). Find \( a \) the speed of the particle at \( B \), \( b \) the distance from \( A \) to \( B \).

\[ a = 3, \ u = 2, \ t = 8, \ v = ?, \ s = ? \]

\[ a \]

\[ v = u + at \]

\[ = 2 + (3 \times 8) \]

\[ = 26 \]

The speed of the particle at \( B \) is \( 26 \text{ m s}^{-1} \).
The distance from A to B is 112 m.

**Example 2**

A cyclist is travelling along a straight road. She accelerates at a constant rate from a speed of 4 m s\(^{-1}\) to a speed of 7.5 m s\(^{-1}\) in 40 seconds. Find 

- **a** the distance she travels in these 40 seconds,
- **b** her acceleration in these 40 seconds.

**a**

\[
 s = \left( \frac{u + v}{2} \right) t
\]

\[
 = \left( \frac{4 + 7.5}{2} \right) \times 40
\]

\[
 = 230
\]

The distance the cyclist travels is 230 m.

**b**

\[
 v = u + at
\]

\[
 7.5 = 4 + 40a
\]

\[
 a = \frac{7.5 - 4}{40} = 0.0875
\]

The acceleration of the cyclist is 0.0875 m s\(^{-2}\).

Choose the right formula then substitute in the values you know.

Model the cyclist as a particle.

You need \(a\) and you know \(v, u\) and \(t\) so you can use \(v = u + at\).

Substitute the values you know into the formula. You can solve this equation to find \(a\).

You could rearrange the formula before you substitute the values:

\[
a = \frac{v - u}{t}
\]

In real-life situations values for the acceleration are often quite small. Large accelerations feel unpleasant and may be dangerous.
If a particle is slowing down it has a negative acceleration. This is called deceleration or retardation.

**Example 3**

A particle moves in a straight line from a point \(A\) to a point \(B\) with constant deceleration 1.5 m s\(^{-2}\). The speed of the particle at \(A\) is 8 m s\(^{-1}\) and the speed of the particle at \(B\) is 2 m s\(^{-1}\). Find \(a\) the time taken for the particle to move from \(A\) to \(B\), \(b\) the distance from \(A\) to \(B\).

After reaching \(B\) the particle continues to move along the straight line with constant deceleration 1.5 m s\(^{-2}\). The particle is at the point \(C\) 6 seconds after passing through the point \(A\). Find \(c\) the velocity of the particle at \(C\), \(d\) the distance from \(A\) to \(C\).

\[u = 8, v = 2, a = -1.5, t = ?, s = ?\]

\[\text{a} \quad v = u + at\]
\[2 = 8 - 1.5t\]
\[1.5t = 8 - 2\]
\[t = \frac{8 - 2}{1.5} = 4\]

The time taken to move from \(A\) to \(B\) is 4 s.

\[\text{b} \quad s = \left(\frac{u + v}{2}\right)t\]
\[= \left(\frac{8 + 2}{2}\right) \times 4 = 20\]

The distance from \(A\) to \(B\) is 20 m.

\[\text{c} \quad u = 8, a = -1.5, t = 6, v = ?\]
\[v = u + at\]
\[= 8 + (-1.5) \times 6\]
\[= 8 - 9 = -1\]

The velocity of the particle is 1 m s\(^{-1}\) in the direction \(BA\).

\[\text{d} \quad s = \left(\frac{u + v}{2}\right)t\]
\[= \left(\frac{8 + (-1)}{2}\right) \times 6\]

The distance from \(A\) to \(C\) is 21 m.
Convert all your measurements into base SI units before substituting values into the formulae.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>SI unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (t)</td>
<td>seconds (s)</td>
</tr>
<tr>
<td>displacement (s)</td>
<td>metres (m)</td>
</tr>
<tr>
<td>velocity (v or u)</td>
<td>metres per second (m s(^{-1}))</td>
</tr>
<tr>
<td>acceleration (a)</td>
<td>metres per second per second (m s(^{-2}))</td>
</tr>
</tbody>
</table>

Example 4

A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 seconds later the car passes a speed-trap where it is registered as travelling at 45 km h\(^{-1}\). Find a the acceleration of the car, b the distance between the traffic lights and the speed-trap.

\[
45 \text{ km h}^{-1} = 45 \times \frac{1000}{3600} \text{ m s}^{-1} = 12.5 \text{ m s}^{-1}
\]

\[
\begin{array}{c|c}
\text{Lights} & \text{Trap} \\
0 \text{ m s}^{-1} & 12.5 \text{ m s}^{-1} \\
\end{array}
\]

\[u = 0, \ v = 12.5, \ t = 30, \ a = ?, \ s = ?\]

a. \[v = u + at\]
\[
12.5 = 0 + 30a
\]
\[
a = \frac{12.5}{30} = \frac{5}{12}
\]
The acceleration of the car is \(\frac{5}{12}\) m s\(^{-2}\).

b. \[s = \left(\frac{u + v}{2}\right)t\]
\[
= \left(\frac{0 + 12.5}{2}\right) \times 30
\]
\[
= 187.5
\]
The distance between the traffic lights and the speed trap is 187.5 m.
Exercise 2A

1. A particle is moving in a straight line with constant acceleration 3 m s$^{-2}$. At time $t = 0$, the speed of the particle is 2 m s$^{-1}$. Find the speed of the particle at time $t = 6$ s.

2. A particle is moving in a straight line with constant acceleration. The particle passes a point with speed 1.2 m s$^{-1}$. Four seconds later the particle has speed 7.6 m s$^{-1}$. Find the acceleration of the particle.

3. A car is approaching traffic lights. The car is travelling with speed 10 m s$^{-1}$. The driver applies the brakes to the car and the car comes to rest with constant deceleration in 16 s. Modelling the car as a particle, find the deceleration of the car.

4. A particle moves in a straight line from a point $A$ to point $B$ with constant acceleration. The particle passes $A$ with speed 2.4 m s$^{-1}$. The particle passes $B$ with speed 8 m s$^{-1}$, five seconds after it passed $A$. Find the distance between $A$ and $B$.

5. A car accelerates uniformly while travelling on a straight road. The car passes two signposts 360 m apart. The car takes 15 s to travel from one signpost to the other. When passing the second signpost, it has speed 28 m s$^{-1}$. Find the speed of the car at the first signpost.

6. A particle is moving along a straight line with constant deceleration. The points $X$ and $Y$ are on the line and $XY = 120$ m. At time $t = 0$, the particle passes $X$ and is moving towards $Y$ with speed 18 m s$^{-1}$. At time $t = 10$ s, the particle is at $Y$. Find the velocity of the particle at time $t = 10$ s.

7. A cyclist is moving along a straight road from $A$ to $B$ with constant acceleration 0.5 m s$^{-2}$. Her speed at $A$ is 3 m s$^{-1}$ and it takes her 12 seconds to cycle from $A$ to $B$. Find $a$ her speed at $B$, $b$ the distance from $A$ to $B$.

8. A particle is moving along a straight line with constant acceleration from a point $A$ to a point $B$, where $AB = 24$ m. The particle takes 6 s to move from $A$ to $B$ and the speed of the particle at $B$ is 5 m s$^{-1}$. Find $a$ the speed of the particle at $A$, $b$ the acceleration of the particle.

9. A particle moves in a straight line from a point $A$ to a point $B$ with constant deceleration 1.2 m s$^{-2}$. The particle takes 6 s to move from $A$ to $B$. The speed of the particle at $B$ is 2 m s$^{-1}$ and the direction of motion of the particle has not changed. Find $a$ the speed of the particle at $A$, $b$ the distance from $A$ to $B$.

10. A train, travelling on a straight track, is slowing down with constant deceleration 0.6 m s$^{-2}$. The train passes one signal with speed 72 km h$^{-1}$ and a second signal 25 s later. Find $a$ the speed, in km h$^{-1}$, of the train as it passes the second signal, $b$ the distance between the signals.

11. A particle moves in a straight line from a point $A$ to a point $B$ with a constant deceleration of 4 m s$^{-2}$. At $A$ the particle has speed 32 m s$^{-1}$ and the particle comes to rest at $B$. Find $a$ the time taken for the particle to travel from $A$ to $B$, $b$ the distance between $A$ and $B$. 
12 A skier travelling in a straight line up a hill experiences a constant deceleration. At the bottom of the hill, the skier has a speed of 16 m s\(^{-1}\) and, after moving up the hill for 40 s, he comes to rest. Find \(a\) the deceleration of the skier, \(b\) the distance from the bottom of the hill to the point where the skier comes to rest.

13 A particle is moving in a straight line with constant acceleration. The points \(A\), \(B\) and \(C\) lie on this line. The particle moves from \(A\) through \(B\) to \(C\). The speed of the particle at \(A\) is 2 m s\(^{-1}\) and the speed of the particle at \(B\) is 7 m s\(^{-1}\). The particle takes 20 s to move from \(A\) to \(B\).
   \(a\) Find the acceleration of the particle.
   The speed of the particle is \(C\) is 11 m s\(^{-1}\). Find
   \(b\) the time taken for the particle to move from \(B\) to \(C\),
   \(c\) the distance between \(A\) and \(C\).

14 A particle moves in a straight line from \(A\) to \(B\) with constant acceleration 1.5 m s\(^{-2}\). It then moves, along the same straight line, from \(B\) to \(C\) with a different acceleration. The speed of the particle at \(A\) is 1 m s\(^{-1}\) and the speed of the particle at \(C\) is 43 m s\(^{-1}\). The particle takes 12 s to move from \(A\) to \(B\) and 10 s to move from \(B\) to \(C\). Find
   \(a\) the speed of the particle at \(B\),
   \(b\) the acceleration of the particle as it moves from \(B\) to \(C\),
   \(c\) the distance from \(A\) to \(C\).

15 A cyclist travels with constant acceleration \(x\) m s\(^{-2}\), in a straight line, from rest to 5 m s\(^{-1}\) in 20 s. She then decelerates from 5 m s\(^{-1}\) to rest with constant deceleration \(\frac{1}{2}x\) m s\(^{-2}\). Find \(a\) the value of \(x\), \(b\) the total distance she travelled.

16 A particle is moving with constant acceleration in a straight line. It passes through three points, \(A\), \(B\) and \(C\) with speeds 20 m s\(^{-1}\), 30 m s\(^{-1}\) and 45 m s\(^{-1}\) respectively. The time taken to move from \(A\) to \(B\) is \(t_1\) seconds and the time taken to move from \(B\) to \(C\) is \(t_2\) seconds.
   \(a\) Show that \(\frac{t_1}{t_2} = \frac{2}{3}\).
   Given also that the total time taken for the particle to move from \(A\) to \(C\) is 50 s,
   \(b\) find the distance between \(A\) and \(B\).

2.2 You can use the formulae \(v^2 = u^2 + 2as\), \(s = ut + \frac{1}{2}at^2\) and \(s = vt - \frac{1}{2}at^2\) for a particle moving in a straight line with constant acceleration.

You can eliminate \(t\) from the formulae for constant acceleration.

\[
t = \frac{v - u}{a}
\]

\[
s = \left(\frac{u + v}{2}\right)\left(\frac{v - u}{a}\right)
\]

\[2as = v^2 - u^2\]

\(\blacksquare\) \(v^2 = u^2 + 2as\)

Rearrange the formula \(v = u + at\) to make \(t\) the subject.

Substitute this expression for \(t\) into \(s = \left(\frac{u + v}{2}\right)t\).

Multiply out the brackets and rearrange.
Kinematics of a particle moving in a straight line

You can also eliminate \( v \) from the formulae for constant acceleration.

\[ s = \left( \frac{u + u + at}{2} \right) t \]
\[ = \left( \frac{2u + at}{2} \right) t \]
\[ = \left( u + \frac{1}{2} at \right) t \]

**\( s = ut + \frac{1}{2} at^2 \)**

Finally, you can eliminate \( u \) by substituting into this formula:

\[ s = (v - at)t + \frac{1}{2} at^2 \]

**\( s = vt - \frac{1}{2} at^2 \)**

- You need to remember the five formulae for solving problems about particles moving in a straight lines with constant acceleration.
  - \( v = u + at \)
  - \( s = \left( \frac{u + v}{2} \right) t \)
  - \( v^2 = u^2 + 2as \)
  - \( s = ut + \frac{1}{2} at^2 \)
  - \( s = vt - \frac{1}{2} at^2 \)

**Example 5**

A particle is moving along a straight line from \( A \) to \( B \) with constant acceleration 5 m s\(^{-2}\). The velocity of the particle at \( A \) is 3 m s\(^{-1}\) in the direction \( \vec{AB} \). The velocity of the particle at \( B \) is 18 m s\(^{-1}\) in the same direction. Find the distance from \( A \) to \( B \).

\[ a = 5, \ u = 3, \ v = 18, \ s = ? \]
\[ v^2 = u^2 + 2as \]
\[ 18^2 = 3^2 + 2 \times 5 \times s \]
\[ 324 = 9 + 10s \]
\[ s = \frac{324 - 9}{10} = 31.5 \]
\[ AB = 31.5 \text{ m} \]
Example 6

A car is travelling along a straight horizontal road with a constant acceleration of 0.75 \( \text{m s}^{-2} \). The car is travelling at 8 \( \text{m s}^{-1} \) when it passes a pillar box. 12 seconds later it passes a lamp post. Find (a) the distance between the pillar box and the lamp post, (b) the speed with which the car passes the lamp post.

\[ a = 0.75, \ u = 8, \ t = 12, \ s = ? \]

\[ s = ut + \frac{1}{2}at^2 \]
\[ = 8 \times 12 + \frac{1}{2} \times 0.75 \times 12^2 \]
\[ = 96 + 54 = 150 \]

The distance between the pillar box and the lamp post is 150 m.

\[ b = 0.75, \ u = 8, \ t = 12, \ v = ? \]

\[ v = u + at \]
\[ = 8 + 0.75 \times 12 \]
\[ = 17 \text{ m s}^{-1} \]

The speed of the car at the lamp post is 17 \( \text{m s}^{-1} \).

Example 7

A particle moves with constant acceleration 1.5 \( \text{m s}^{-2} \) in a straight line from a point \( A \) to a point \( B \), where \( AB = 16 \text{ m} \). At \( A \), the particle has speed 3 \( \text{m s}^{-1} \). Find the speed of the particle at \( B \).

\[ a = 1.5, \ u = 3, \ s = 16, \ v = ? \]

\[ v^2 = u^2 + 2as \]
\[ = 3^2 + 2 \times 1.5 \times 16 \]
\[ v = \sqrt{57} = 7.5498... \]

The speed of the particle at \( B \) is 7.55 \( \text{m s}^{-1} \), to three significant figures.
**Example 8**

A particle is moving in a straight horizontal line with constant deceleration $4 \text{ m s}^{-2}$. At time $t = 0$ the particle passes through a point $O$ with speed $13 \text{ m s}^{-1}$ travelling towards a point $A$ where $OA = 20 \text{ m}$. Find \textbf{a} the times when the particle passes through $A$, \textbf{b} the velocities of the particle when it passes through $A$, \textbf{c} the values of $t$ when the particle returns to $O$.

\[
\begin{align*}
\text{Positive direction} & \\
13 \text{ m s}^{-1} & \\
O \quad \text{20 m} \quad A
\end{align*}
\]

\textbf{a} $\quad a = -4$, $u = 13$, $s = 20$, $t = ?$

\[
s = ut + \frac{1}{2}at^2
\]

\[
20 = 13t - \frac{1}{2} \times 4t^2 = 13t - 2t^2
\]

\[
2t^2 - 13t + 20 = 0
\]

\[
(2t - 5)(t - 4) = 0
\]

\[
t = \frac{5}{2}, 4
\]

The particle moves through $A$ twice, $2\frac{1}{2}$ seconds and 4 seconds after moving through $O$.

\textbf{b} $\quad u = 13$, $a = -4$, $t = \frac{5}{2}$, $v = ?$

\[
v = u + at = 13 - 4 \times \frac{5}{2} = 3
\]

\[
u = 13, a = -4, t = 4, v = ?
\]

\[
v = u + at = 13 - 4 \times 4 = -3
\]

When $t = \frac{5}{2}$, the particle passes through $A$ with velocity $3 \text{ m s}^{-1}$ in the direction $\overrightarrow{OA}$.

When $t = 4$, the particle passes through $A$ with velocity $3 \text{ m s}^{-1}$ in the direction $\overrightarrow{AO}$.

- The particle is decelerating so the value of $a$ is negative.

- You are told the values of $a$, $u$ and $s$ and asked to find $t$. You are given no information about $v$ and are not asked to find it so you choose the formula without $v$.

- This is a quadratic equation. You can solve it using factorisation, or by using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- There are two answers. Both are correct. The particle moves from $O$ to $A$, goes beyond $A$ and then turns round and returns to $A$.

- There are two values of $t$ and you have to find the velocity of both. The formula $v = u + at$ is the simplest one to use.

- This answer is positive, so the particle is moving in the positive direction (away from $O$).

- The value of $v$ is negative when $t = 4$, so the particle is moving in the negative direction (towards $O$).

- Remember a velocity has a direction as well as a magnitude. Velocity is a vector quantity. When you are asked for a velocity, your answer must contain a direction as well as a magnitude.
c The particle returns to O when $s = 0$.

$s = 0, u = 13, a = -4, t = \, ?$

$s = ut + \frac{1}{2}at^2$

$0 = 13t - 2t^2$

$= t(13 - 2t)$

$t = 0, \frac{13}{2}$

The particle returns to O 6.5 seconds after it first passed through O.

When the particle returns to O, its displacement (distance) from O is zero.

The first solution ($t = 0$) represents the starting position of the particle. The other solution ($t = \frac{13}{2}$) tells you when the particle returns to O.

Example 9

A cyclist is moving along a straight road with constant acceleration. She first passes a shop and 10 seconds later, travelling at 8 m s$^{-1}$, she passes a street sign. The distance between the shop and the street sign is 60 m. Find a the acceleration of the cyclist, b the speed with which she passed the shop.

a $t = 10, v = 8, s = 60, a = \, ?$

$s = vt - \frac{1}{2}at^2$

$60 = 8 \times 10 - \frac{1}{2} \times a \times 100$

$60 = 80 - 50a$

$50a = 80 - 60 = 20$

$a = \frac{20}{50} = 0.4$

The acceleration of the cyclist is 0.4 m s$^{-2}$.

There is no $u$, so you choose the formula without $u$.

Substitute into the formula and solve the equation for $a$.

b $v = 8, t = 10, a = 0.4, u = \, ?$

$v = u + at$

$8 = u + 10 \times 0.4$

$u = 8 - 10 \times 0.4 = 4$

The cyclist passes the shop with speed 4 m s$^{-1}$.

$v = u + at$ has been chosen as it is a simple formula. You could also use $s = \left(\frac{u + v}{2}\right)t$ which would avoid using your answer to part a.
Example 10

A particle \( P \) is moving on the \( x \)-axis with constant deceleration 2.5 m s\(^{-2} \). At time \( t = 0 \), the particle \( P \) passes through the origin \( O \), moving in the positive direction of \( x \) with speed 15 m s\(^{-1} \). Find \( a \) the time between the instant when \( P \) first passes through \( O \) and the instant when it returns to \( O \), \( b \) the total distance travelled by \( P \) during this time.

\[ a = -2.5, \ u = 15, \ s = 0, \ t = ? \]
\[ s = ut + \frac{1}{2}at^2 \]
\[ 0 = 15t - \frac{1}{2} \times 2.5 \times t^2 \]
\[ 0 = 60t - 5t^2 \]
\[ = 5t(12 - t) \]
\[ t = 0, \ t = 12 \]

The particle \( P \) returns to \( O \) after 12 s.

\[ b = -2.5, \ u = 15, \ v = 0, \ s = ? \]
\[ v^2 = u^2 + 2as \]
\[ 0^2 = 15^2 - 2 \times 2.5 \times s \]
\[ 5s = 225 \]
\[ s = \frac{225}{5} = 45 \]

The distance \( OA = 45 \) m.

The total distance travelled by \( P \) is
\[ 2 \times 45 \text{ m} = 90 \text{ m}. \]

Exercise 2B

1 A particle is moving in a straight line with constant acceleration 2.5 m s\(^{-2} \). It passes a point \( A \) with speed 3 m s\(^{-1} \) and later passes through a point \( B \), where \( AB = 8 \) m. Find the speed of the particle as it passes through \( B \).

2 A car is accelerating at a constant rate along a straight horizontal road. Travelling at 8 m s\(^{-1} \), it passes a pillar box and 6 s later it passes a sign. The distance between the pillar box and the sign is 60 m. Find the acceleration of the car.
3 A cyclist travelling at 12 m s\(^{-1}\) applies her brakes and comes to rest after travelling 36 m in a straight line. Assuming that the brakes cause the cyclist to decelerate uniformly, find the deceleration.

4 A particle moves along a straight line from P to Q with constant acceleration 1.5 m s\(^{-2}\). The particle takes 4 s to pass from P to Q and PQ = 22 m. Find the speed of the particle at Q.

5 A particle is moving along a straight line OA with constant acceleration 2 m s\(^{-2}\). At O the particle is moving towards A with speed 5.5 m s\(^{-1}\). The distance OA is 20 m. Find the time the particle takes to move from O to A.

6 A train is moving along a straight horizontal track with constant acceleration. The train passes a signal at 54 km h\(^{-1}\) and a second signal at 72 km h\(^{-1}\). The distance between the two signals is 500 m. Find, in m s\(^{-2}\), the acceleration of the train.

7 A particle moves along a straight line, with constant acceleration, from a point A to a point B where AB = 48 m. At A the particle has speed 4 m s\(^{-1}\) and at B it has speed 16 m s\(^{-1}\). Find a the acceleration of the particle, b the time the particle takes to move from A to B.

8 A particle moves along a straight line with constant acceleration 3 m s\(^{-2}\). The particle moves 38 m in 4 s. Find a the initial speed of the particle, b the final speed of the particle.

9 The driver of a car is travelling at 18 m s\(^{-1}\) along a straight road when she sees an obstruction ahead. She applies the brakes and the brakes cause the car to slow down to rest with a constant deceleration of 3 m s\(^{-2}\). Find a the distance travelled as the car decelerates, b the time it takes for the car to decelerate from 18 m s\(^{-1}\) to rest.

10 A stone is sliding across a frozen lake in a straight line. The initial speed of the stone is 12 m s\(^{-1}\). The friction between the stone and the ice causes the stone to slow down at a constant rate of 0.8 m s\(^{-2}\). Find a the distance moved by the stone before coming to rest, b the speed of the stone at the instant when it has travelled half of this distance.

11 A particle is moving along a straight line OA with constant acceleration 2.5 m s\(^{-2}\). At time \(t = 0\), the particle passes through O with speed 8 m s\(^{-1}\) and is moving in the direction OA. The distance OA is 40 m. Find a the time taken for the particle to move from O to A, b the speed of the particle at A. Give your answers to one decimal place.

12 A particle travels with uniform deceleration 2 m s\(^{-2}\) in a horizontal line. The points A and B lie on the line and AB = 32 m. At time \(t = 0\), the particle passes through A with velocity 12 m s\(^{-1}\) in the direction \(\overrightarrow{AB}\). Find a the values of \(t\) when the particle is at B, b the velocity of the particle for each of these values of \(t\).

13 A particle is moving along the x-axis with constant deceleration 5 m s\(^{-2}\). At time \(t = 0\), the particle passes through the origin O with velocity 12 m s\(^{-1}\) in the positive direction. At time \(t\) seconds the particle passes through the point A with x-coordinate 8. Find a the values of \(t\), b the velocity of the particle as it passes through the point with x-coordinate \(-8\).
14 A particle $P$ is moving on the $x$-axis with constant deceleration $4 \text{ m s}^{-2}$. At time $t = 0$, $P$ passes through the origin $O$ with velocity $14 \text{ m s}^{-1}$ in the positive direction. The point $A$ lies on the axis and $OA = 22.5 \text{ m}$. Find $a$ the difference between the times when $P$ passes through $A$, $b$ the total distance travelled by $P$ during the interval between these times.

15 A car is travelling along a straight horizontal road with constant acceleration. The car passes over three consecutive points $A$, $B$ and $C$ where $AB = 100 \text{ m}$ and $BC = 300 \text{ m}$. The speed of the car at $B$ is $14 \text{ m s}^{-1}$ and the speed of the car at $C$ is $20 \text{ m s}^{-1}$. Find $a$ the acceleration of the car, $b$ the time take for the car to travel from $A$ to $C$.

16 Two particles $P$ and $Q$ are moving along the same straight horizontal line with constant accelerations $2 \text{ m s}^{-2}$ and $3.6 \text{ m s}^{-2}$ respectively. At time $t = 0$, $P$ passes through a point $A$ with speed $4 \text{ m s}^{-1}$. One second later $Q$ passes through $A$ with speed $3 \text{ m s}^{-1}$, moving in the same direction as $P$. 

$a$ Write down expressions for the displacements of $P$ and $Q$ from $A$, in terms of $t$, where $t$ seconds is the time after $P$ has passed through $A$.

$b$ Find the value of $t$ where the particles meet.

$c$ Find the distance of $A$ from the point where the particles meet.

2.3 You can use the formulae for constant acceleration to model an object moving vertically in a straight line under gravity.

- The force of gravity causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object. This means that in a vacuum an apple and a feather would both accelerate downwards at the same rate.

- On earth, the acceleration due to gravity is represented by the letter $g$ and is approximately $9.8 \text{ m s}^{-2}$. The actual value of the acceleration can vary by very small amounts in different places due to the changing radius of the Earth and height above sea level.

In M1 you will always use $g = 9.8 \text{ m s}^{-2}$. This is an approximation to two significant figures. If you use this value in your working you should give your answer to the same degree of accuracy.

- An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8 \text{ m s}^{-2}$

- When solving problems about vertical motion you can choose the positive direction to be either upwards or downwards. Acceleration due to gravity is always downwards, so if the positive direction is upwards then $a = -9.8 \text{ m s}^{-2}$.

- The total time that an object is in motion from the time it is projected (thrown) upwards to the time it hits the ground is called the time of flight. The initial speed is sometimes called the speed of projection.
Example 11

A ball \( B \) is projected vertically upwards from a point \( O \) with speed 12 m s\(^{-1} \). Find \( a \) the greatest height above \( O \) reached by \( B \), \( b \) the total time before \( B \) returns to \( O \).

\[ u = 12 \]
\[ v = 0 \]
\[ a = -9.8 \]
\[ s = h \]
\[ v^2 = u^2 + 2as \]
\[ 0^2 = 12^2 - 2 \times 9.8 \times h \]
\[ h = \frac{12^2}{2 \times 9.8} = \frac{144}{19.6} = 7.346... \]

The greatest height above \( O \) reached by \( B \) is 7.4 m, to two significant figures.

\[ s = 0 \]
\[ u = 12 \]
\[ a = -9.8 \]
\[ t = ? \]
\[ s = ut + \frac{1}{2}at^2 \]
\[ 0 = 12t - \frac{1}{2} \times 9.8 \times t = t(12 - 4.9t) \]
\[ t = \frac{12}{4.9} = 2.448... \]

The time taken for \( B \) to return to \( O \) is 2.4 s, to two significant figures.
Example 12

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find a the time the book takes to reach the floor, b the speed with which the book strikes the floor.

Model the book as a particle moving in a straight line with a constant acceleration of magnitude 9.8 m s\(^{-2}\).

As the book is moving downwards throughout its motion, it is sensible to take the downwards direction as positive.

You have taken the downwards direction as positive and gravity acts downwards. Here the acceleration is positive.

Assume the book has an initial speed of zero.

Choose the formula without \(v\).

Solve the equation for \(t^2\) and use your calculator to find the positive square root.

Remember to give the answer to two significant figures.

Choose the formula without \(t\).

Remember to show your working to at least three significant figures. You can use unrounded values in your calculations by using the Ans button on your calculator.

\(s = 1.4\)
\(a = +9.8\)
\(u = 0\)
\(t = ?\)
\(s = ut + \frac{1}{2}at^2\)
\(1.4 = 0 + \frac{1}{2} \times 9.8 \times t^2\)
\(t^2 = \frac{1.4}{4.9} = 0.2857\ldots\)
\(t = \sqrt{0.2857\ldots} = 0.5345\ldots\)
The time taken for the book to reach the floor is 0.53 s, to two significant figures.

\(s = 1.4\)
\(a = 9.8\)
\(u = 0\)
\(v = ?\)
\(v^2 = u^2 + 2as\)
\(= 0^2 + 2 \times 9.8 \times 1.4 = 27.44\)
\(v = \sqrt{27.44} = 5.238\ldots \approx 5.2\)
The book hits the floor with speed 5.2 m s\(^{-1}\), to two significant figures.
Example 13

A ball is projected vertically upwards, from a point $X$ which is 7 m above the ground, with speed $21 \text{ m s}^{-1}$. Find $a$ the greatest height above the ground reached by the ball, $b$ the time of flight of the ball.

**a**

$u = 21$
$v = 0$
$a = -9.8$
$s = ?$
$v^2 = u^2 + 2as$
$0^2 = 21^2 + 2 \times (-9.8) \times s$
$s = \frac{441}{19.6} = 22.5$

$(22.5 + 7) \text{ m} = 29.5 \text{ m}$

The greatest height reached by the ball above the ground is 30 m, to two significant figures.

**b**

$s = -7$
$u = 21$
$a = -9.8$
$t = ?$
$s = ut + \frac{1}{2}at^2$
$-7 = 21t - 4.9t^2$
$4.9t^2 - 21t - 7 = 0$
$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$
$t = \frac{-(-21) \pm \sqrt{((-21)^2 - 4 \times 4.9 \times (-7))}}{2 \times 4.9}$
$t = 21 \pm \frac{\sqrt{578.2}}{9.8} \approx 21 \pm 24.046$
$t \approx 4.5965, -0.3108$

The time of flight of the ball is 4.6 s, to two significant figures.
A particle is projected vertically upwards from a point $O$ with speed $u$ m s$^{-1}$. The greatest height reached by the particle is 62.5 m above $O$. Find $a$ the value of $u$, $b$ the total time for which the particle is 50 m or more above $O$.

### Example 14

**a**

- $v = 0$
- $s = 62.5$
- $a = -9.8$
- $u = ?$

\[ v^2 = u^2 + 2as \]
\[ 0^2 = u^2 - 2 \times 9.8 \times 62.5 \]
\[ u^2 = 1225 \]
\[ u = \sqrt{1225} = 35 \]

**b**

- $s = 50$
- $u = 35$
- $a = -9.8$
- $t = ?$

\[ s = ut + \frac{1}{2}at^2 \]
\[ 50 = 35t - 4.9t^2 \]
\[ 4.9t^2 - 35t + 50 = 0 \]

\[ t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \]
\[ = \frac{35 \pm \sqrt{(35^2 - 4 \times 4.9 \times 50)}}{9.8} \]
\[ = \frac{35 \pm \sqrt{245}}{9.8} = \frac{35 \pm 15.6525}{9.8} \]
\[ = 5.1686\ldots, 1.9742\ldots \]
\[ (5.1686\ldots) - (1.9742\ldots) \approx 3.194 \]

The total time for which the particle is 50 m or more above $O$ is 3.2 s, to two significant figures.
Example 15

A ball $A$ falls vertically from rest from the top of a tower 63 m high. At the same time as $A$ begins to fall, another ball $B$ is projected vertically upwards from the bottom of the tower with speed $21 \text{ m s}^{-1}$. The balls collide. Find the distance of the point where the balls collide from the bottom of the tower.

For $A$, the motion is downwards
- $u = 0$
- $a = 9.8$
- $s = ut + \frac{1}{2}at^2$
- $s_1 = 4.9t^2$

For $B$, the motion is upwards
- $u = 21$
- $a = -9.8$
- $s = ut + \frac{1}{2}at^2$
- $s_2 = 21t - 4.9t^2$

The height of the tower is 63 m.
- $s_1 + s_2 = 63$
- $4.9t^2 + (21t - 4.9t^2) = 63$
- $21t = 63$
- $t = 3$
- $s_2 = 21t - 4.9t^2$
- $= 21 \times 3 - 4.9 \times 3^2 = 18.9$

The balls collide 19 m from the bottom of the tower, to two significant figures.
Exercise 2C

1. A ball is projected vertically upwards from a point $O$ with speed $14 \text{ m s}^{-1}$. Find the greatest height above $O$ reached by the ball.

2. A well is 50 m deep. A stone is released from rest at the top of the well. Find how long the stone takes to reach the bottom of the well.

3. A book falls from the top shelf of a bookcase. It takes 0.6 s to reach the floor. Find how far it is from the top shelf to the floor.

4. A particle is projected vertically upwards with speed $20 \text{ m s}^{-1}$ from a point on the ground. Find the time of flight of the particle.

5. A ball is thrown vertically downward from the top of a tower with speed $18 \text{ m s}^{-1}$. It reaches the ground in 1.6 s. Find the height of the tower.

6. A pebble is catapulted vertically upwards with speed $24 \text{ m s}^{-1}$. Find $a$ the greatest height above the point of projection reached by the pebble, $b$ the time taken to reach this height.

7. A ball is projected upwards from a point which is 4 m above the ground with speed $18 \text{ m s}^{-1}$. Find $a$ the speed of the ball when it is 15 m above its point of projection, $b$ the speed with which the ball hits the ground.

8. A particle $P$ is projected vertically downwards from a point 80 m above the ground with speed $4 \text{ m s}^{-1}$. Find $a$ the speed with which $P$ hits the ground, $b$ the time $P$ takes to reach the ground.

9. A particle $P$ is projected vertically upwards from a point $X$. Five seconds later $P$ is moving downwards with speed $10 \text{ m s}^{-1}$. Find $a$ the speed of projection of $P$, $b$ the greatest height above $X$ attained by $P$ during its motion.

10. A ball is thrown vertically upwards with speed $21 \text{ m s}^{-1}$. It hits the ground 4.5 s later. Find the height above the ground from which the ball was thrown.

11. A stone is thrown vertically upward from a point which is 3 m above the ground, with speed $16 \text{ m s}^{-1}$. Find $a$ the time of flight of the stone, $b$ the total distance travelled by the stone.

12. A particle is projected vertically upwards with speed $24.5 \text{ m s}^{-1}$. Find the total time for which it is $21 \text{ m or more above its point of projection}$.

13. A particle is projected vertically upwards from a point $O$ with speed $u \text{ m s}^{-1}$. Two seconds later it is still moving upwards and its speed is $\frac{1}{3}u \text{ m s}^{-1}$. Find $a$ the value of $u$, $b$ the time from the instant that the particle leaves $O$ to the instant that it returns to $O$.

14. A ball $A$ is thrown vertically downwards with speed $5 \text{ m s}^{-1}$ from the top of a tower block 46 m above the ground. At the same time as $A$ is thrown downwards, another ball $B$ is thrown vertically upwards from the ground with speed $18 \text{ m s}^{-1}$. The balls collide. Find the distance of the point where $A$ and $B$ collide from the point where $A$ was thrown.
15 A ball is released from rest at a point which is 10 m above a wooden floor. Each time the ball strikes the floor, it rebounds with three-quarters of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball a the first time it rebounds from the floor, b the second time it rebounds from the floor.

16 A particle $P$ is projected vertically upwards from a point $O$ with speed $12 \text{ m s}^{-1}$. One second after $P$ has been projected from $O$, another particle $Q$ is projected vertically upwards from $O$ with speed $20 \text{ m s}^{-1}$. Find a the time between the instant that $P$ is projected from $O$ and the instant when $P$ and $Q$ collide, b the distance of the point where $P$ and $Q$ collide from $O$.

2.4 You can represent the motion of an object on a speed–time graph or a distance–time graph

- In a speed–time graph speed is always plotted on the vertical axis and time is always plotted on the horizontal axis. This speed–time graph represents the motion of a particle accelerating from speed $u$ at time 0 to speed $v$ at time $t$.

![Speed-time graph](image)

**Gradient of line** = \[
\frac{\text{change of velocity}}{\text{time}} = \frac{v - u}{t} = a
\]

So the gradient of the speed–time graph is the acceleration of the particle. If the line is straight the acceleration is constant.

**Using the formula for the area of a trapezium**:

Shaded area = \[
\left(\frac{u + v}{2}\right)t = s
\]

So the area under the speed–time graph is the distance travelled by the particle.

- The gradient of a speed–time graph is the acceleration.
- The area under a speed–time graph is the distance travelled.

You can also draw acceleration–time graphs and distance–time graphs for the motion of a particle. Time is always plotted on the horizontal axis.

If a particle is moving with constant speed its distance–time graph will be a straight line. If it is accelerating or decelerating then its distance–time graph will be a curve.
Example 16

A car accelerates uniformly from rest for 20 seconds. It travels at a constant speed for the next 40 seconds, then decelerates uniformly for 20 seconds until it is stationary. Sketch a an acceleration–time graph, b a distance–time graph for the motion of the car.

Example 17

The figure shows a speed–time graph illustrating the motion of a cyclist moving along a straight road for a period of 12 s. For the first 8 s, she moves at a constant speed of 6 m s$^{-1}$. She then decelerates at a constant rate, stopping after a further 4 s.

Find a the distance travelled by the cyclist during this 12 s period, b the rate at which the cyclist decelerates.

Model the cyclist as a particle moving in a straight line.
a The distance travelled is given by

\[ s = \frac{1}{2}(a + b)h \]

\[ = \frac{1}{2}(8 + 12) \times 6 \]

\[ = 10 \times 6 = 60 \]

The distance travelled by the cyclist is 60 m.
CHAPTER 2

b The acceleration is the gradient of the slope.
\[ a = \frac{-6}{4} = -1.5 \]
The deceleration is 1.5 m s\(^{-2}\).

Example 18

A car is waiting at traffic lights. When the lights turn green, the car accelerates uniformly from rest to a speed of 10 m s\(^{-1}\) in 20 s. This speed is then maintained until the car passes a road sign 50 s after leaving the traffic lights.

a Sketch a speed–time graph to illustrate the motion of the car.
b Find the distance between the traffic lights and the road sign.

Model the car as a particle moving in a straight line.

a

b The distance travelled is given by
\[ s = \frac{1}{2} (a + b)h \]
\[ = \frac{1}{2} (30 + 50) \times 10 \]
\[ = 40 \times 10 = 400 \]
The distance between the traffic lights and the road sign is 400 m.
Example 19

A particle moves along a straight line. The particle accelerates uniformly from rest to a speed of 8 m s\(^{-1}\) in \(T\) seconds. The particle then travels at a constant speed of 8 m s\(^{-1}\) for 5\(T\) seconds. The particle then decelerates uniformly to rest in a further 40 s.

a Sketch a speed–time graph to illustrate the motion of the particle.

Given that the total distance travelled by the particle is 600 m, b find the value of \(T\), c sketch an acceleration–time graph illustrating the motion of the particle.

\[\text{The area of the trapezium is:} \]
\[s = \frac{1}{2}(a + b)h\]
\[= \frac{1}{2}(5T + 6T + 40) \times 8\]
\[= 4(11T + 40)\]

The distance moved is 600 m.

\[4(11T + 40) = 600\]
\[44T + 160 = 600\]
\[T = \frac{600 - 160}{44} = 10\]

For the first ten seconds the \(v\)-coordinate increases by 8 as the \(t\)-coordinate increases by 10. This gives a positive answer.

The acceleration in the first 10 s is given by
\[a = \frac{8}{10} = 0.8.\]

The acceleration in the last 40 s is given by
\[a = \frac{-8}{40} = -0.2\]

In the last forty seconds the \(v\)-coordinate decreases by 8 as the \(t\)-coordinate increases by 40. This gives a negative answer.

The length of the shorter of the two parallel sides is 5\(T\). The length of the longer side is \(T + 5T + 40 = 6T + 40\).
Example 20

A car C is moving along a straight road with constant speed 17.5 m s\(^{-1}\). At time \(t = 0\), C passes a lay-by. At time \(t = 0\), a second car D leaves the lay-by. Car D accelerates from rest to a speed of 20 m s\(^{-1}\) in 15 s and then maintains a constant speed of 20 m s\(^{-1}\). Car D passes car C at a road sign.

a On the same diagram, sketch speed–time graphs to illustrate the motion of the two cars.

b Find the distance between the lay-by and the road sign.

\[
\text{You should label the lines so that it is clear which represents the motion of C and which represents the motion of D.}
\]

\[
\text{It is difficult to find the distance travelled directly. You can find the time the cars pass first. It does not matter what letter you choose for the time. T has been used here.}
\]

\[
\text{As C is travelling at a constant speed you use the formula distance = speed \times time.}
\]

\[
\text{The longer of the parallel sides of the trapezium is T. The shorter of the parallel sides is } (T - 15).
\]

\[
\text{As the cars were at the lay-by at the same time and the road sign at the same time, the distance travelled by both of them is the same. You equate the distances to get an equation in } T \text{ and solve it.}
\]

\[
\text{To find the distance travelled you can substitute into the expression for the distance travelled by either C or by D.}
\]

\[v(m \text{ s}^{-1})\]

\[t(s)\]

\[D\]

\[C\]

\[O\]

\[15\]

\[T\]

\[20\]

\[17.5\]

\[\text{Let D pass C at time } t = T.\]

\[\text{The distance travelled by C is given by}\]

\[s = 17.5T\]

\[\text{The distance travelled by D is given by}\]

\[s = \frac{1}{2}(a + b)h\]

\[= \frac{1}{2}(T - 15 + T) \times 20\]

\[= 10(2T - 15)\]

\[\text{The distances travelled by C and D are the same.}\]

\[10(2T - 15) = 17.5T\]

\[20T - 150 = 17.5T\]

\[2.5T = 150\]

\[T = \frac{150}{2.5} = 60\]

\[s = 17.5T = 17.5 \times 60 = 1050\]

\[\text{The distance from the lay-by to the road sign is 1050 m.}\]
**Example 21**

A particle is moving along a horizontal axis Ox. At time \( t = 0 \), the particle is at rest at O. The particle then accelerates at a constant rate, reaching a speed of \( u \text{ m s}^{-1} \) in 16 s. The particle maintains the speed of \( u \text{ m s}^{-1} \) for a further 32 s. After 48 s, the particle is at a point A, where \( OA = 320 \text{ m} \).

**a** Sketch a speed–time graph to illustrate the motion of the particle.

**b** Find the value of \( u \).

**c** Sketch a distance–time graph for the particle.

---

**Diagram:**

- **v(m s\(^{-1}\))**
  - \( O \)
  - \( u \)
  - \( 16 \)
  - \( 48 \)
  - \( t(s) \)

- **s(m)**
  - \( O \)
  - 320
  - 64
  - \( 16 \)
  - \( 48 \)
  - \( t(s) \)

---

**Solution:**

\[ \frac{1}{2}(32 + 48) \times u = 320 \]

\[ 40u = 320 \]

\[ u = 8 \]

**c** The gradient of the line in the first 16 s is the acceleration and is given by

\[ a = \frac{u}{16} = \frac{8}{16} = \frac{1}{2} \]

\[ s = ut + \frac{1}{2}at^2 \]

\[ = \frac{1}{4}t^2 \]

When \( t = 16 \), \( s = \frac{1}{4} \times 16^2 = 64 \).
Exercise 2D

1

The diagram shows the speed–time graph of the motion of an athlete running along a straight track. For the first 4 s, he accelerates uniformly from rest to a speed of 9 m s\(^{-1}\). This speed is then maintained for a further 8 s. Find

a the rate at which the athlete accelerates,

b the total distance travelled by the athlete in 12 s.

2

A car is moving along a straight road. When \( t = 0 \) s, the car passes a point \( A \) with speed 10 m s\(^{-1}\) and this speed is maintained until \( t = 30 \) s. The driver then applies the brakes and the car decelerates uniformly, coming to rest at the point \( B \) when \( t = 42 \) s.

a Sketch a speed–time graph to illustrate the motion of the car.

b Find the distance from \( A \) to \( B \).

3

The diagram shows the speed–time graph of the motion of a cyclist riding along a straight road. She accelerates uniformly from rest to 8 m s\(^{-1}\) in 20 s. She then travels at a constant speed of 8 m s\(^{-1}\) for 40 s. She then decelerates uniformly to rest in 15 s. Find

a the acceleration of the cyclist in the first 20 s of motion,

b the deceleration of the cyclist in the last 15 s of motion,

c the total distance travelled in 75 s.

4

A car accelerates at a constant rate, starting from rest at a point \( A \) and reaching a speed of 45 km h\(^{-1}\) in 20 s. This speed is then maintained and the car passes a point \( B \) 3 minutes after leaving \( A \).

a Sketch a speed–time graph to illustrate the motion of the car.

b Find the distance from \( A \) to \( B \).

5

A motorcyclist starts from rest at a point \( S \) on a straight race track. He moves with constant acceleration for 15 s, reaching a speed of 30 m s\(^{-1}\). He then travels at a constant speed of 30 m s\(^{-1}\) for \( T \) seconds. Finally he deCELERates at a constant rate coming to rest at a point \( F \), 25 s after he begins to decelerate.

a Sketch a speed–time graph to illustrate the motion.

Given that the distance between \( S \) and \( F \) is 2.4 km,

b calculate the time the motorcyclist takes to travel from \( S \) to \( F \).
6 A train is travelling along a straight track. To obey a speed restriction, the brakes of the train are applied for 30 s reducing the speed of the train from 40 m s\(^{-1}\) to 16 m s\(^{-1}\). The train then continues at a constant speed of 16 m s\(^{-1}\) for a further 70 s. The diagram shows a speed–time graph illustrating the motion of the train for the total period of 100 s. Find
   a the retardation of the train in the first 30 s.
   b the total distance travelled by the train in 100 s.

7 A train starts from a station X and moves with constant acceleration 0.6 m s\(^{-2}\) for 20 s. The speed it has reached after 20 s is then maintained for \(T\) seconds. The train then decelerates from this speed to rest in a further 40 s, stopping at a station Y.
   a Sketch a speed–time graph to illustrate the motion of the train.
   Given that the distance between the stations is 4.2 km, find
   b the value of \(T\),
   c the distance travelled by the train while it is moving with constant speed.

8 A particle moves along a straight line. The particle accelerates from rest to a speed of 10 m s\(^{-1}\) in 15 s. The particle then moves at a constant speed of 10 m s\(^{-1}\) for a period of time. The particle then decelerates uniformly to rest. The period of time for which the particle is travelling at a constant speed is 4 times the period of time for which it is decelerating.
   a Sketch a speed–time graph to illustrate the motion of the particle.
   Given that the total distance travelled by the particle is 480 m,
   b find the total time for which the particle is moving,
   c sketch an acceleration–time graph illustrating the motion of the particle.

9 A particle moves 100 m in a straight line. The diagram is a sketch of a speed–time graph of the motion of the particle. The particle starts with speed \(u\) m s\(^{-1}\) and accelerates to a speed 10 m s\(^{-1}\) in 3 s. The speed of 10 m s\(^{-1}\) is maintained for 7 s and then the particle decelerates to rest in a further 2 s. Find
   a the value of \(u\),
   b the acceleration of the particle in the first 3 s of motion.
10 The diagram is an acceleration–time graph to show the motion of a particle. At time \( t = 0 \) s, the particle is at rest. Sketch a speed–time graph for the motion of the particle.

11 A motorcyclist \( M \) leaves a road junction at time \( t = 0 \) s. She accelerates at a rate of \( 3 \text{ m s}^{-2} \) for 8 s and then maintains the speed she has reached. A car \( C \) leaves the same road junction as \( M \) at time \( t = 0 \) s. The car accelerates from rest to \( 30 \text{ m s}^{-1} \) in 20 s and then maintains the speed of \( 30 \text{ m s}^{-1} \). \( C \) passes \( M \) as they both pass a pedestrian.
   a On the same diagram, sketch speed–time graphs to illustrate the motion of \( M \) and \( C \).
   b Find the distance of the pedestrian from the road junction.

12 A particle is moving on an axis \( Ox \). From time \( t = 0 \) s to time \( t = 32 \) s, the particle is travelling with constant speed \( 15 \text{ m s}^{-1} \). The particle then decelerates from \( 15 \text{ m s}^{-1} \) to rest in \( T \) seconds.
   a Sketch a speed–time graph to illustrate the motion of the particle.
   b Find the value of \( T \).
   c Sketch a distance–time graph illustrating the motion of the particle.

Mixed exercise 2E

1 A car travelling along a straight road at \( 14 \text{ m s}^{-1} \) is approaching traffic lights. The driver applies the brakes and the car comes to rest with constant deceleration. The distance from the point where the brakes are applied to the point where the car comes to rest is 49 m. Find the deceleration of the car.

2 A ball is thrown vertically downward from the top of a tower with speed \( 6 \text{ m s}^{-1} \). The ball strikes the ground with speed \( 25 \text{ m s}^{-1} \). Find the time the ball takes to move from the top of the tower to the ground.

3 The diagram is a speed–time graph representing the motion of a cyclist along a straight road. At time \( t = 0 \) s, the cyclist is moving with speed \( u \text{ m s}^{-1} \). The speed is maintained until time \( t = 15 \) s, when she slows down with constant deceleration, coming to rest when \( t = 23 \) s. The total distance she travels in 23 s is 152 m. Find the value of \( u \).
4 A stone is projected vertically upwards with speed $21 \text{ m s}^{-1}$. Find
   a the greatest height above the point of projection reached by the stone,
   b the time between the instant that the stone is projected and the instant that it reaches its greatest height.

5 A train is travelling with constant acceleration along a straight track. At time $t = 0 \text{ s}$, the train passes a point $O$ travelling with speed $18 \text{ m s}^{-1}$. At time $t = 12 \text{ s}$, the train passes a point $P$ travelling with speed $24 \text{ m s}^{-1}$. At time $t = 20 \text{ s}$, the train passes a point $Q$. Find
   a the speed of the train at $Q$,
   b the distance from $P$ to $Q$.

6 A car travelling on a straight road slows down with constant deceleration. The car passes a road sign with speed $40 \text{ km h}^{-1}$ and a post box with speed of $24 \text{ km h}^{-1}$. The distance between the road sign and the post box is $240 \text{ m}$. Find, in $\text{m s}^{-2}$, the deceleration of the car.

7 A skier is travelling downhill along a straight path with constant acceleration. At time $t = 0 \text{ s}$, she passes a point $A$ with speed $6 \text{ m s}^{-1}$. She continues with the same acceleration until she reaches a point $B$ with speed $15 \text{ m s}^{-1}$. At $B$, the path flattens out and she travels from $B$ to a point $C$ at the constant speed of $15 \text{ m s}^{-1}$. It takes $20 \text{ s}$ for the skier to travel from $B$ to $C$ and the distance from $A$ to $C$ is $615 \text{ m}$. Find
   a Sketch a speed–time graph to illustrate the motion of the skier.
   b Find the distance from $A$ to $B$.
   c Find the time the skier took to travel from $A$ to $B$.

8 A child drops a ball from a point at the top of a cliff which is $82 \text{ m}$ above the sea. The ball is initially at rest. Find
   a the time taken for the ball to reach the sea,
   b the speed with which the ball hits the sea.
   c State one physical factor which has been ignored in making your calculation.

9 A particle moves along a straight line, from a point $X$ to a point $Y$, with constant acceleration. The distance from $X$ to $Y$ is $104 \text{ m}$. The particle takes $8 \text{ s}$ to move from $X$ to $Y$ and the speed of the particle at $Y$ is $18 \text{ m s}^{-1}$. Find
   a the speed of the particle at $X$,
   b the acceleration of the particle.
   The particle continues to move with the same acceleration until it reaches a point $Z$. At $Z$ the speed of the particle is three times the speed of the particle at $X$.
   c Find the distance $XZ$.

10 A pebble is projected vertically upwards with speed $21 \text{ m s}^{-1}$ from a point $32 \text{ m}$ above the ground. Find
   a the speed with which the pebble strikes the ground,
   b the total time for which the pebble is more than $40 \text{ m}$ above the ground.
11 A particle $P$ is moving along the $x$-axis with constant deceleration $2.5 \text{ m s}^{-2}$. At time $t = 0$, $P$ passes through the origin with velocity $20 \text{ m s}^{-1}$ in the direction of $x$ increasing. At time $t = 12$ s, $P$ is at the point $A$. Find
   a the distance $OA$,
   b the total distance $P$ travels in 12 s.

12 A train starts from rest at a station $P$ and moves with constant acceleration for 45 s reaching a speed of $25 \text{ m s}^{-1}$. The train then maintains this speed for 4 minutes. The train then uniformly decelerates, coming to rest at a station $Q$.
   a Sketch a speed–time graph illustrating the motion of the train from $P$ to $Q$.
   b The distance between the stations is 7 km.
   c Find the deceleration of the train.
   d Sketch an acceleration–time graph illustrating the motion of the train from $P$ to $Q$.

13 A particle moves 451 m in a straight line. The diagram shows a speed–time graph illustrating the motion of the particle. The particle starts at rest and accelerates at a constant rate for 8 s reaching a speed of $2u \text{ m s}^{-1}$ at time $t = 26$ s. Find
   a the value of $u$,
   b the distance moved by the particle while its speed is less than $u \text{ m s}^{-1}$.

14 A particle is moving in a straight line. The particle starts with speed $5 \text{ m s}^{-1}$ and accelerates at a constant rate of $2 \text{ m s}^{-1}$ for 8 s. It then decelerates at a constant rate coming to rest in a further 12 s.
   a Sketch a speed–time graph illustrating the motion of the particle.
   b Find the total distance moved by the particle during its 20 s of motion.
   c Sketch a distance–time graph illustrating the motion of the particle.

15 A boy projects a ball vertically upwards with speed $10 \text{ m s}^{-1}$ from a point $X$, which is 50 m above the ground. $T$ seconds after the first ball is projected upwards, the boy drops a second ball from $X$. Initially the second ball is at rest. The balls collide 25 m above the ground. Find the value of $T$.

16 A car is moving along a straight road with uniform acceleration. The car passes a check-point $A$ with speed $12 \text{ m s}^{-1}$ and another check-point $C$ with speed $32 \text{ m s}^{-1}$. The distance between $A$ and $C$ is 1100 m.
   a Find the time taken by the car to move from $A$ to $C$.
   Given that $B$ is the mid-point of $AC$,
   b find the speed with which the car passes $B$. 

\[ v(t) = \begin{cases} 
2u & \text{for } 0 \leq t \leq 8 \\
u & \text{for } 8 < t < 20 \\
0 & \text{for } 20 \leq t \leq 26 
\end{cases} \]
17 A particle is projected vertica lly upwards with a speed of 30 m s\(^{-1}\) from a point \(A\). The point \(B\) is \(h\) metres above \(A\). The particle moves freely under gravity and is above \(B\) for a time 2.4 s. Calculate the value of \(h\).

18 Two cars \(A\) and \(B\) are moving in the same direction along a straight horizontal road. At time \(t = 0\), they are side by side, passing a point \(O\) on the road. Car \(A\) travels at a constant speed of 30 m s\(^{-1}\). Car \(B\) passes \(O\) with a speed of 20 m s\(^{-1}\), and has constant acceleration of 4 m s\(^{-2}\). Find
   a) the speed of \(B\) when it has travelled 78 m from \(O\),
   b) the distance from \(O\) of \(A\) when \(B\) is 78 m from \(O\),
   c) the time when \(B\) overtakes \(A\).

19 A car is being driven on a straight stretch of motorway at a constant speed of 34 m s\(^{-1}\), when it passes a speed restriction sign \(S\) warning of road works ahead and requiring speeds to be reduced to 22 m s\(^{-1}\). The driver continues at her speed for 2 s after passing \(S\). She then reduces her speed to 22 m s\(^{-1}\) with constant deceleration of 3 m s\(^{-2}\), and continues at the lower speed.
   a) Draw a speed–time graph to illustrate the motion of the car after it passes \(S\).
   b) Find the shortest distance before the road works that \(S\) should be placed on the road to ensure that a car driven in this way has had its speed reduced to 22 m s\(^{-1}\) by the time it reaches the start of the road works.

20 A train starts from rest at station \(A\) and accelerates uniformly at 3\(x\) m s\(^{-2}\) until it reaches a speed of 30 m s\(^{-1}\). For the next \(T\) seconds the train maintains this constant speed. The train then retards uniformly at \(x\) m s\(^{-2}\) until it comes to rest at a station \(B\). The distance between the stations is 6 km and the time taken from \(A\) to \(B\) is 5 minutes.
   a) Sketch a speed–time graph to illustrate this journey.
   b) Show that \(\frac{40}{x} + T = 300\).
   c) Find the value of \(T\) and the value of \(x\).
   d) Calculate the distance the train travels at constant speed.
   e) Calculate the time taken from leaving \(A\) until reaching the point half-way between the stations.
Summary of key points.

1. You need to know these symbols and what they represent.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>displacement (distance)</td>
</tr>
<tr>
<td>u</td>
<td>starting (initial) velocity</td>
</tr>
<tr>
<td>v</td>
<td>final velocity</td>
</tr>
<tr>
<td>a</td>
<td>acceleration</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
</tbody>
</table>

2. If a particle is slowing down it has a negative acceleration. This is called deceleration or retardation.

3. Convert all your measurements into base SI units before substituting values into the formulae.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>SI unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (t)</td>
<td>seconds (s)</td>
</tr>
<tr>
<td>displacement (s)</td>
<td>metres (m)</td>
</tr>
<tr>
<td>velocity (v or u)</td>
<td>metres per second (m s(^{-1}))</td>
</tr>
<tr>
<td>acceleration (a)</td>
<td>metres per second per second (m s(^{-2}))</td>
</tr>
</tbody>
</table>

4. You need to remember the five formulae for solving problems about particles moving in a straight line with constant acceleration.

- \( v = u + at \)
- \( s = \left( \frac{u + v}{2} \right)t \)
- \( v^2 = u^2 + 2as \)
- \( s = ut + \frac{1}{2}at^2 \)
- \( s = vt - \frac{1}{2}at^2 \)

5. An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of \( g = 9.8 \text{ m s}^{-2} \).

6. The gradient of a speed–time graph illustrating the motion of a particle represents the acceleration of the particle.

7. The area under a speed–time graph illustrating the motion of a particle represents the distance moved by the particle.

8. Area of a trapezium = average of the parallel sides \( \times \) height

\[
\frac{1}{2}(a + b) \times h
\]

9. At constant speed, distance = speed \( \times \) time