

Expanding brackets and factorising

This chapter will show you how to

- ✓ expand and simplify expressions with brackets
- ✓ solve equations and inequalities involving brackets
- ✓ factorise by removing a common factor
- ✓ expand two brackets

7.1 Expanding brackets

You will need to know how to

- multiply positive and negative numbers
- add and subtract negative numbers
- collect like terms

When multiplying algebraic terms remember that

$$x \times 3 = 3 \times x = 3x$$

$$y \times y = y^2$$

$$gh = g \times h$$

More complicated multiplications can also be simplified.



EXAMPLE 1

Simplify $3f \times 4g$.

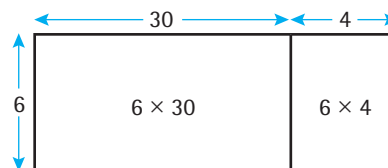
$$\begin{aligned} 3f \times 4g &= 3 \times f \times 4 \times g \\ &= 3 \times 4 \times f \times g \\ &= 12 \times fg \\ &= 12fg \end{aligned}$$

To multiply algebraic terms, multiply the numbers then multiply the letters.

Multiplying a bracket

You can work out 6×34 by thinking of 34 as $30 + 4$.

$$\begin{aligned} 6 \times 34 &= 6 \times (30 + 4) \\ &= 6 \times 30 + 6 \times 4 \\ &= 180 + 24 \\ &= 204 \end{aligned}$$



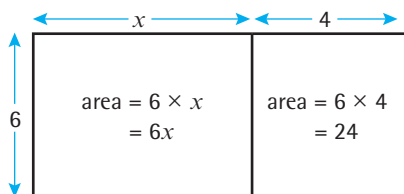
Brackets are often used in algebra.

$$6(x + 4) \text{ means } 6 \times (x + 4)$$

As in the 6×34 example, you have to multiply each term inside the brackets by 6.

$$\begin{aligned} 6(x + 4) &= 6 \times x + 6 \times 4 \\ &= 6x + 24 \end{aligned}$$

It is like working out the area of a rectangle that has length $x + 4$ and width 6



$$\begin{aligned} \text{Total area} &= 6(x + 4) \\ &= 6 \times x + 6 \times 4 \\ &= 6x + 24 \end{aligned}$$

When you do this it is called **expanding the brackets**.

It is also known as **removing the brackets** or **multiplying out the brackets**.

You find the total area by adding the area of the two smaller rectangles.

When you remove the brackets you must multiply each term inside the brackets by the term outside the bracket.



EXAMPLE 2

Simplify these by multiplying out the brackets.

(a) $5(a + 6)$ (b) $2(x - 8)$ (c) $3(2c - d)$

$$\begin{aligned} \text{(a)} \quad 5(a + 6) &= 5 \times a + 5 \times 6 \\ &= 5a + 30 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 2(x - 8) &= 2 \times x - 2 \times 8 \\ &= 2x - 16 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 3(2c - d) &= 3 \times 2c - 3 \times d \\ &= 6c - 3d \end{aligned}$$

You must multiply each term inside the bracket by the term outside the bracket.

$$3 \times 2c = 3 \times 2 \times c = 6 \times c = 6c$$



EXERCISE 7A

- 1 Simplify these expressions.
- (a) $2 \times 5k$ (b) $3 \times 6b$ (c) $4a \times 5$
 (d) $3a \times 2b$ (e) $4c \times 3d$ (f) $x \times 5y$
- 2 Expand the brackets to find the value of these expressions. Check your answers by working out the brackets first.
- (a) $2(50 + 7)$ (b) $5(40 + 6)$ (c) $6(70 + 3)$
 (d) $3(40 - 2)$ (e) $7(50 - 4)$ (f) $8(40 - 3)$
- 3 Remove the brackets from these.
- (a) $5(p + 6)$ (b) $3(x + y)$ (c) $4(u + v + w)$
 (d) $2(y - 8)$ (e) $7(9 - z)$ (f) $8(a - b + 6)$
- 4 Expand the brackets in these expressions.
- (a) $3(2c + 6)$ (b) $5(4t + 3)$ (c) $2(5p + q)$
 (d) $3(2a - b)$ (e) $6(3c - 2d)$ (f) $7(2x + y - 3)$
 (g) $6(3a - 4b + c)$ (h) $2(x^2 + 3x + 2)$ (i) $4(y^2 - 3y - 10)$
- 5 Write down the 6 pairs of cards which show equivalent expressions.

$4(x + 2y)$	$4x + 2y$	$2(4x + y)$	$4(2x - y)$
A	B	C	D
$8x - 8y$	$4x + 8y$	$8(x - y)$	$2x - 8y$
E	F	G	H
$8x + 2y$	$2(x - 4y)$	$2(2x + y)$	$8x - 4y$
I	J	K	L

You can use the same method for expressions that have an *algebraic* term instead of a *number* term outside the bracket.



EXAMPLE 3

Expand the brackets in these expressions.

(a) $a(a + 4)$ (b) $x(2x - y)$ (c) $3t(t^2 + 1)$

(a) $a(a + 4) = a \times a + a \times 4$
 $= a^2 + 4a$

(b) $x(2x - y) = x \times 2x - x \times y$
 $= 2x^2 - xy$

(c) $3t(t^2 + 1) = 3t \times t^2 + 3t \times 1$
 $= 3t^3 + 3t$

Remember, you must multiply each term inside the brackets by the term outside the bracket.

A common mistake is to forget to multiply the second term.

Remember

$$3 \times 2c = 3 \times 2 \times c = 6c$$

Remember $a \times a = a^2$

$$x \times 2x = x \times 2 \times x = 2 \times x \times x = 2x^2$$

Remember $x \times y = xy$

$$3t \times t^2 = 3 \times t \times t \times t = 3t^3$$



EXERCISE 7B

Expand the brackets in these expressions.

- | | | |
|------------------|-------------------|------------------|
| 1 $b(b + 4)$ | 2 $a(5 + a)$ | 3 $k(k - 6)$ |
| 4 $m(9 - m)$ | 5 $a(2a + 3)$ | 6 $g(4g + 1)$ |
| 7 $p(2p + q)$ | 8 $t(t + 5w)$ | 9 $m(m + 3n)$ |
| 10 $x(2x - y)$ | 11 $r(4r - t)$ | 12 $a(a - 4b)$ |
| 13 $2t(t + 5)$ | 14 $3x(x - 8)$ | 15 $5k(k + l)$ |
| 16 $3a(2a + 4)$ | 17 $2g(4g + h)$ | 18 $5p(3p - 2q)$ |
| 19 $3x(2y + 5z)$ | 20 $r(r^2 + 1)$ | 21 $a(a^2 + 3)$ |
| 22 $t(t^2 - 7)$ | 23 $2p(p^2 + 3q)$ | 24 $4x(x^2 + x)$ |

Remember

$$3x \times 4x = 4 \times 3 \times x \times x \\ = 12x^2$$

Adding and subtracting expressions with brackets

Adding

To add expressions with brackets, expand the brackets first, then collect like terms to simplify your answer.

Collecting like terms means adding all the terms in x , all the terms in y and so on.



EXAMPLE 4

Expand then simplify these expressions.

(a) $3(a + 4) + 2a + 10$ (b) $3(2x + 5) + 2(x - 4)$

$$\begin{aligned} \text{(a) } 3(a + 4) + 2a + 10 &= 3a + 12 + 2a + 10 \\ &= 3a + 2a + 12 + 10 \\ &= 5a + 22 \end{aligned}$$

$$\begin{aligned} \text{(b) } 3(2x + 5) + 2(x - 4) &= 6x + 15 + 2x - 8 \\ &= 6x + 2x + 15 - 8 \\ &= 8x + 7 \end{aligned}$$

Expand the brackets first. Then collect like terms.

Expand both sets of brackets first.

Subtracting

If you have an expression like $-3(2x - 5)$, multiply both terms in the brackets by -3 .

$$-3 \times 2x = -6x \quad \text{and} \quad -3 \times -5 = 15$$

$$\text{So } -3(2x - 5) = -3 \times 2x + -3 \times -5 \\ = -6x + 15$$

Multiplying

$$\begin{array}{ccccccc} + & \times & + & = & + \\ + & \times & - & = & - \\ - & \times & + & = & - \\ - & \times & - & = & + \end{array}$$



EXAMPLE 5

Expand these expressions.

(a) $-2(3t + 4)$ (b) $-3(4x - 1)$

$$\begin{aligned} \text{(a) } -2(3t + 4) &= -2 \times 3t + -2 \times 4 \\ &= -6t + -8 \\ &= -6t - 8 \end{aligned}$$

$$\begin{aligned} \text{(b) } -3(4x - 1) &= -3 \times 4x + -3 \times -1 \\ &= -12x + 3 \end{aligned}$$

$$-2 \times 3 = -6 \quad -2 \times 4 = -8$$

$$-3 \times 4 = -12 \quad -3 \times -1 = +3$$



EXAMPLE 6

Expand then simplify these expressions.

(a) $3(2t + 1) - 2(2t + 4)$ (b) $8(x + 1) - 3(2x - 5)$

$$\begin{aligned} \text{(a) } 3(2t + 1) - 2(2t + 4) &= 6t + 3 - 4t - 8 \\ &= 6t - 4t + 3 - 8 \\ &= 2t - 5 \end{aligned}$$

$$\begin{aligned} \text{(b) } 8(x + 1) - 3(2x - 5) &= 8x + 8 - 6x + 15 \\ &= 8x - 6x + 8 + 15 \\ &= 2x + 23 \end{aligned}$$

Remember to multiply both terms in the second bracket by -2 .Expand the brackets first. Remember that $-3 \times -5 = +15$. Then collect like terms.

EXERCISE 7C

Expand these expressions.

1 $-2(2k + 4)$ 2 $-3(2x + 6)$ 3 $-5(3n - 1)$

4 $-4(3t + 5)$ 5 $-3(4p - 1)$ 6 $-2(3x - 7)$

Expand then simplify these expressions.

7 $3(y + 4) + 2y + 10$ 8 $2(k + 6) + 3k + 9$

9 $4(a + 3) - 2a + 6$ 10 $3(t - 2) + 4t - 10$

11 $3(2y + 3) + 2(y + 5)$ 12 $4(x + 7) + 3(x + 4)$

13 $3(2x + 5) + 2(x - 4)$ 14 $2(4n + 5) + 5(n - 3)$

15 $3(x - 5) + 2(x - 3)$ 16 $4(2x - 1) + 2(3x - 2)$

17 $3(2b + 1) - 2(2b + 4)$ 18 $4(2m + 3) - 2(2m + 5)$

19 $2(5t + 3) - 2(3t + 1)$ 20 $5(2k + 2) - 4(2k + 6)$

21 $8(a + 1) - 3(2a - 5)$ 22 $2(4p + 1) - 4(p - 3)$

23 $5(2g - 4) - 2(4g - 6)$ 24 $2(w - 4) - 3(2w - 1)$

25 $x(x + 3) + 4(x + 2)$ 26 $x(2x + 1) - 3(x - 4)$

7.2 Solving equations involving brackets

Equations sometimes involve brackets.

When dealing with equations involving brackets, you usually expand the brackets first.



EXAMPLE 7

Solve $4(c + 3) = 20$.

Method A

$$4(c + 3) = 20$$

$$4c + 12 = 20$$

$$4c + 12 - 12 = 20 - 12$$

$$4c = 8$$

$$\frac{4c}{4} = \frac{8}{4}$$

$$c = 2$$

Method B

$$4(c + 3) = 20$$

$$c + 3 = \frac{20}{4}$$

$$c + 3 = 5$$

$$c = 5 - 3$$

$$c = 2$$

Expand the bracket by multiplying both terms inside the bracket by the term outside the bracket.

You must subtract 12 from both sides before dividing both sides by 4.

Since 4 divides exactly into 20 you can divide both sides by 4 first.



EXAMPLE 8

Solve $2(3p - 4) = 7$.

$$2(3p - 4) = 7$$

$$6p - 8 = 7$$

$$6p - 8 + 8 = 7 + 8$$

$$6p = 15$$

$$\frac{6p}{6} = \frac{15}{6}$$

$$p = 2.5$$

Expand the bracket.

You must add 8 to both sides before dividing both sides by 6.



EXERCISE 7D

1 Solve these equations.

(a) $4(g + 6) = 32$ (b) $7(k + 1) = 21$ (c) $5(s + 10) = 65$

(d) $2(n - 4) = 6$ (e) $3(f - 2) = 24$ (f) $6(v - 3) = 42$

(g) $4(m - 3) = 14$ (h) $2(w + 7) = 19$

2 Solve these equations.

(a) $4(5t + 2) = 48$ (b) $3(2r + 4) = 30$ (c) $2(2b + 2) = 22$

(d) $2(3w - 6) = 27$ (e) $3(4x - 2) = 24$ (f) $5(2y + 11) = 40$

(g) $6(2k - 1) = 36$ (h) $3(2a - 13) = 18$

When two brackets are involved, expand both brackets then collect like terms before solving.



EXAMPLE 9

Solve $2(2m + 10) = 12(m - 1)$.

$$\begin{aligned} 2(2m + 10) &= 12(m - 1) \\ 4m + 20 &= 12m - 12 \\ 20 + 12 &= 12m - 4m \\ 32 &= 8m \\ m &= 4 \end{aligned}$$

When an equation involves fractions, it can be transformed into an equation without fractions by multiplying all terms by the LCM of the numbers in the denominators.



EXAMPLE 10

Solve the equation $\frac{x + 17}{4} = x + 2$.

$$\begin{aligned} \frac{x + 17}{4} &= x + 2 \\ 4\left(\frac{x + 17}{4}\right) &= 4(x + 2) \\ x + 17 &= 4x + 8 \\ 17 - 8 &= 4x - x \\ 9 &= 3x \\ x &= 3 \end{aligned}$$

Like terms are terms of the same kind. In Example 9 there are only terms in m and number terms.

Expand the brackets on both sides of the equation and collect like terms.

Collect terms in m on the RHS because $12m$ on the RHS is greater than $4m$ on the LHS. This keeps the m term positive.

LCM means Lowest Common Multiple.

Multiply both sides by 4, collect like terms and then finally divide by 3.

Note the use of brackets.

Collect the terms in x on the RHS and the numbers on the LHS. $4x$ on the RHS is greater than x on the LHS.



EXAMPLE 11

Solve the equation $\frac{x-6}{3} = \frac{x+4}{5}$.

$$\frac{x-6}{3} = \frac{x+4}{5}$$

$$\frac{15(x-6)}{3} = \frac{15(x+4)}{5}$$

$$5(x-6) = 3(x+4)$$

$$5x - 30 = 3x + 12$$

$$5x - 3x = 12 + 30$$

$$2x = 42$$

$$x = 21$$

Look at the denominators. 3 and 5 have a LCM of 15 so multiply both sides of the equation by 15.

Note the use of brackets. Always put them in when you multiply in this way.

Then solve using the method shown in Example 9.



EXAMPLE 12

Solve the equation $\frac{2x+3}{6} + \frac{x-2}{3} = \frac{5}{2}$.

$$\frac{6(2x+3)}{6} + \frac{6(x-2)}{3} = \frac{6(5)}{2}$$

$$2x + 3 + 2(x - 2) = 15$$

$$2x + 3 + 2x - 4 = 15$$

$$4x - 1 = 15$$

$$4x = 16$$

$$x = 4$$

The LCM here is 6.

Note the use of brackets.

The most common mistake is to forget to multiply **all** terms by the LCM.

This means the number on the RHS as well as the terms on the LHS.



EXERCISE 7E

1 Solve these equations.

(a) $2a + 4 = 5(a - 1)$

(b) $3(d - 2) = 2d - 1$

(c) $5(x + 3) = 11x + 3$

(d) $12p + 3 = 3(p + 7)$

(e) $4t + 3 = 3(2t - 3)$

(f) $3b - 4 = 2(2b - 7)$

(g) $8(3g - 1) = 15g + 10$

(h) $3(2k + 6) = 17k + 7$

(i) $2(y + 5) = 3y + 12$

(j) $5r + 3 = 4(2r + 3)$

2 Solve the following equations by expanding both brackets.

(a) $2(b + 1) = 8(2b - 5)$

(b) $5(4a + 7) = 3(8a + 9)$

(c) $6(x - 2) = 3(3x - 8)$

(d) $5(2p + 2) = 6(p + 5)$

(e) $9(3s - 4) = 5(4s - 3)$

(f) $4(10t - 7) = 3(6t - 2)$

(g) $4(2w + 2) = 2(5w + 7)$

(h) $3(3y - 2) = 7(y - 2)$

Use Example 9 to help.

3 Solve these equations.

$$(a) \frac{d+3}{15} = 3 - d$$

$$(c) \frac{3x-1}{8} = x - 2$$

$$(e) \frac{c-8}{4} = c + 1$$

$$(b) \frac{6y-5}{5} = y + 3$$

$$(d) \frac{6+a}{2} = a + 4$$

$$(f) \frac{10-b}{3} = 12 + b$$

4 Solve these equations.

$$(a) \frac{x+1}{3} = \frac{x-1}{4}$$

$$(c) \frac{3x+1}{5} = \frac{2x}{3}$$

$$(e) \frac{x+2}{7} = \frac{3x+6}{5}$$

$$(b) \frac{2x-1}{3} = \frac{x}{2}$$

$$(d) \frac{x+3}{2} = \frac{x-3}{5}$$

$$(f) \frac{8-x}{2} = \frac{2x+2}{5}$$

5 Solve these equations.

$$(a) \frac{x+1}{2} + \frac{x+2}{5} = 3$$

$$(c) \frac{3x+2}{5} + \frac{x+2}{3} = 2$$

$$(e) \frac{x-3}{4} - \frac{x+3}{3} = 1$$

$$(b) \frac{x+2}{4} + \frac{x+1}{7} = 3$$

$$(d) \frac{3x-1}{5} - \frac{x+2}{3} = \frac{1}{5}$$

$$(f) \frac{2x+5}{4} - \frac{x+4}{3} = 2$$

Use Example 10 to help.

Use Example 11 to help.

Use Example 12 to help.

7.3 Solving inequalities involving brackets

Inequalities can also involve brackets.

Remember that there is usually more than one answer when you solve an inequality and you need to state all possible values of the solution set.



EXAMPLE 13

Solve these inequalities.

$$(a) 9 \leq 3(y-1) \quad (b) 3(2x-5) > 2(x+4)$$

$$(a) \quad 9 \leq 3(y-1)$$

$$9 \leq 3y - 3$$

$$9 + 3 \leq 3y$$

$$12 \leq 3y$$

$$4 \leq y$$

$$(b) \quad 3(2x-5) > 2(x+4)$$

$$6x - 15 > 2x + 8$$

$$6x - 2x > 8 + 15$$

$$4x > 23$$

$$x > \frac{23}{4}$$

$$x > 5\frac{3}{4}$$

You must remember to keep the inequality sign in your answer.

For example, if you leave (a) as $4 = y$ you will lose a mark because you have not included *all* possible values of y .

If you are asked for integer solutions to (b) the final answer will be $x \geq 6$.



EXAMPLE 14

n is an integer.

List the values of n such that $-11 < 2(n - 3) < 1$.

$$-11 < 2(n - 3) < 1$$

$$-11 < 2n - 6 < 1$$

$$-11 + 6 < 2n < 1 + 6$$

$$-5 < 2n < 7$$

$$-2.5 < n < 3.5$$

Values of n are $-2, -1, 0, 1, 2, 3$

This is a double inequality.

Expand the bracket.

Add 6 throughout.

Remember to list the **integer** solutions as you were asked to in the question.

Remember to include 0.



EXERCISE 7F

1 Solve these inequalities.

(a) $2(x - 7) \leq 8$

(b) $7 < 2(m + 5)$

(c) $4(3w - 1) > 20$

(d) $3(2y + 1) \leq -15$

(e) $2(p - 3) > 4 + 3p$

(f) $1 - 5k < 2(5 + 2k)$

(g) $5(x - 1) \geq 3(x + 2)$

(h) $2(n + 5) \leq 3(2n - 2)$

2 Solve these inequalities then list the integer solutions.

(a) $-4 \leq 2x \leq 8$

(b) $-6 < 3y < 15$

(c) $-8 \leq 4n < 17$

(d) $-12 < 6m \leq 30$

(e) $-5 < 2(t + 1) < 7$

(f) $-3 < 3(x - 4) < 6$

(g) $-6 \leq 5(y + 1) \leq 11$

(h) $-17 < 2(2x - 3) \leq 10$

7.4 Factorising by removing a common factor

Factorising an algebraic expression is the opposite of expanding brackets. To factorise an expression, look for a **common factor** – that is, a number that divides into all the terms in the expression. To factorise completely, use the HCF of the terms.

For example,

$6x + 10$ can be written as $2(3x + 5)$

because $6x = 2 \times 3x$

and $10 = 2 \times 5$

HCF means highest common factor.

2 is a factor of $6x$.

2 is also a factor of 10.

So 2 is a common factor of $6x$ and 10.

Notice that the common factor is the term outside the bracket.



EXAMPLE 15

Copy and complete these.

(a) $3t + 15 = 3(\square + 5)$

(b) $4n + 12 = \square(n + 3)$

(a) $3t + 15 = 3(t + 5)$

(b) $4n + 12 = 4(n + 3)$

Because $3 \times t = 3t$ (and $3 \times 5 = 15$) $4 \times n = 4n$ and $4 \times 3 = 12$ 

EXAMPLE 16

Factorise these expressions.

(a) $5a + 20$

(b) $4x - 12$

(c) $x^2 + 7x$

(d) $6p^2q^2 - 9pq^3$

$$\begin{aligned} \text{(a)} \quad 5a + 20 &= 5 \times a + 5 \times 4 \\ &= 5(a + 4) \\ &= 5(a + 4) \end{aligned}$$

$$\begin{aligned} \text{Check } 5(a + 4) &= 5 \times a + 5 \times 4 \\ &= 5a + 20 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 4x - 12 &= 4 \times x - 4 \times 3 \\ &= 4(x - 3) \\ &= 4(x - 3) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad x^2 + 7x &= x \times x + x \times 7 \\ &= x(x + 7) \\ &= x(x + 7) \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 6p^2q^2 - 9pq^3 &= 3 \times 2 \times p \times p \times q \times q - 3 \times 3 \times p \times q \times q \times q \\ &= 3pq^2(2p - 3q) \\ &= 3pq^2(2p - 3q) \end{aligned}$$

5 is a factor of $5a$ $5a = 5 \times a$
 5 is also a factor of 20 $20 = 5 \times 4$
 So 5 is a common factor of $5a$ and 20 and is the term outside the bracket.

Check your answer by removing the brackets.

2 is a factor of $4x$ and 12.
 4 is also a common factor of $4x$ and 12. Use 4 because it is the highest common factor (HCF) of $4x$ and 12. Always look for the HCF.

x is a common factor of x^2 and $7x$.

The HCF is $3pq^2$.



EXERCISE 7G

1 Copy and complete these.

(a) $3x + 15 = 3(\square + 5)$

(b) $5a + 10 = 5(\square + 2)$

(c) $2x - 12 = 2(x - \square)$

(d) $4m - 16 = 4(m - \square)$

(e) $4t + 12 = \square(t + 3)$

(f) $3n + 18 = \square(n + 6)$

(g) $2b - 14 = \square(b - 7)$

(h) $4t - 20 = \square(t - 5)$

Use Example 15 to help.

Don't forget to check your answers by removing the brackets.

2 Factorise these expressions.

- (a) $5p + 20$ (b) $2a + 12$ (c) $3y + 15$
 (d) $7b + 21$ (e) $4q + 12p$ (f) $6k + 24l$

3 Factorise these expressions.

- (a) $4t - 12$ (b) $3x - 9$ (c) $5n - 20$
 (d) $2b - 8$ (e) $6a - 18b$ (f) $7k - 7$

4 Factorise these expressions.

- (a) $y^2 + 7y$ (b) $x^2 + 5x$ (c) $n^2 + n$
 (d) $x^2 - 7x$ (e) $p^2 - 8p$ (f) $a^2 - ab$

5 Factorise these expressions.

- (a) $6p + 4$ (b) $4a + 10$ (c) $6 - 4t$
 (d) $12m - 8n$ (e) $25x + 15y$ (f) $12y - 9z$

6 Factorise completely.

- (a) $3x^2 - 6x$ (b) $8x^2 - xy$ (c) $8a + 4ab$
 (d) $p^3 - 5p^2$ (e) $3t^3 + 6t^2$ (f) $10yz - 15y^2$
 (g) $18a^2 + 12ab$ (h) $16p^2 - 12pq$

7 Factorise these expressions.

- (a) $4ab^2 + 6ab^3$ (b) $10xy - 5x^2$
 (c) $3p^2q - 6p^3q^2$ (d) $8mn^3 + 4n^2 - 6m^2n$
 (e) $6h^2k - 12hk^3 - 18h^2k^2$

Use Example 16(a) to help.
Remember $5 = 5 \times 1$

Use Example 16(c) to help.
Remember $n = n \times 1$

Remember $6p = 2 \times 3p$

Use Example 16(a) to help. Look for the common factors in the terms.

Expanding two brackets

You can use a grid method to multiply two numbers.

For example,

$$34 \times 57 = (30 + 4) \times (50 + 7)$$

\times	50	7
30	1500	210
4	200	28

$$= 30 \times 50 + 30 \times 7 + 4 \times 50 + 4 \times 7$$

$$= 1500 + 210 + 200 + 28$$

$$= 1938$$

You can also use a grid method when you multiply two brackets together. You have to **multiply each term in one bracket by each term in the other bracket**.

For example,

To expand and simplify $(x + 2)(x + 5)$

\times	x	5
x	x^2	$5x$
2	$2x$	10

$$(x + 2)(x + 5) = x \times x + x \times 5 + 2 \times x + 2 \times 5$$

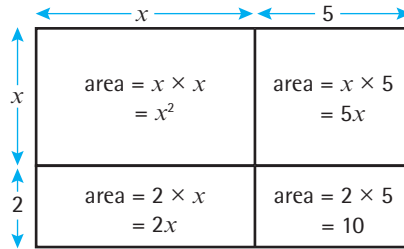
$$= x^2 + 5x + 2x + 10$$

$$= x^2 + 7x + 10$$

You simplify the final expression by collecting the like terms.
 $5x + 2x = 7x$.

It is like working out the area of a rectangle of length $x + 5$ and width $x + 2$.

$$\begin{aligned}\text{Total area} &= (x + 2)(x + 5) \\ &= x^2 + 5x + 2x + 10 \\ &= x^2 + 7x + 10\end{aligned}$$



EXAMPLE 17

Expand and simplify these.

(a) $(a + 4)(a + 10)$

(b) $(t + 6)(t - 2)$

(a)

×	a	10
a	a^2	$10a$
4	$4a$	40

$$\begin{aligned}(a + 4)(a + 10) &= a \times a + a \times 10 + 4 \times a + 4 \times 10 \\ &= a^2 + 10a + 4a + 40 \\ &= a^2 + 14a + 40\end{aligned}$$

(b)

×	t	-2
t	t^2	$-2t$
6	$6t$	-12

$$\begin{aligned}(t + 6)(t - 2) &= t \times t + t \times (-2) + 6 \times t + 6 \times (-2) \\ &= t^2 - 2t + 6t - 12 \\ &= t^2 + 4t - 12\end{aligned}$$

Remember you can use a grid to help.

Remember to multiply each term in the first bracket by each term in the second bracket.

Remember you are multiplying by -2 .
+ve \times -ve = -ve.

Look again at the last example.

$$\begin{aligned}(t + 6)(t - 2) &= t^2 - 2t + 6t - 12 \\ &= t^2 + 4t - 12\end{aligned}$$

The **F**irst terms in each bracket multiply to give t^2 .
The **O**utside pair of terms multiply to give $-2t$.
The **I**nside pair of terms multiply to give $+6t$.
The **L**ast terms in each bracket multiply to give -12 .

This method is often known as **FOIL** and is another way of expanding brackets.



EXERCISE 7H

Expand and simplify.

- | | | |
|---------------------|----------------------|----------------------|
| 1 $(a + 2)(a + 7)$ | 2 $(x + 3)(x + 1)$ | 3 $(x + 5)(x + 5)$ |
| 4 $(t + 5)(t - 2)$ | 5 $(x + 7)(x - 4)$ | 6 $(n - 5)(n + 8)$ |
| 7 $(x - 4)(x + 5)$ | 8 $(p - 4)(p + 4)$ | 9 $(x - 9)(x - 4)$ |
| 10 $(h - 3)(h - 8)$ | 11 $(y - 3)(y - 3)$ | 12 $(4 + a)(a + 7)$ |
| 13 $(m - 7)(8 + m)$ | 14 $(6 + q)(7 + q)$ | 15 $(d + 5)(4 - d)$ |
| 16 $(8 - x)(3 - x)$ | 17 $(x - 12)(x - 7)$ | 18 $(y - 16)(y + 6)$ |

Be careful when there are negative signs – this is where a lot of mistakes are made.

Squaring an expression

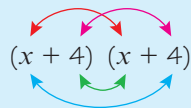
You can use the same method of expanding two brackets for examples involving the square of an expression.

To square an expression, write out the bracket twice and expand.



EXAMPLE 18

Expand and simplify $(x + 4)^2$.



$$\begin{aligned}
 (x + 4)^2 &= (x + 4)(x + 4) \\
 &= x^2 + 4x + 4x + 16 \\
 &= x^2 + 8x + 16
 \end{aligned}$$

You need to multiply the expression $(x + 4)$ by itself so write down the bracket twice and expand as you did in Example 17 or use FOIL as in this example.

Notice that you do not just square the x and the 4, there are two other terms in the expansion.



EXERCISE 7I

1 Expand and simplify.

- | | | |
|-----------------|-----------------|-----------------|
| (a) $(x + 5)^2$ | (b) $(x + 6)^2$ | (c) $(x - 3)^2$ |
| (d) $(x + 1)^2$ | (e) $(x - 4)^2$ | (f) $(x - 5)^2$ |
| (g) $(x + 7)^2$ | (h) $(x - 8)^2$ | (i) $(3 + x)^2$ |
| (j) $(2 + x)^2$ | (k) $(5 - x)^2$ | (l) $(x + a)^2$ |

2 Copy and complete these by finding the correct number to go in each box.

- (a) $(x + \square)^2 = x^2 + \square x + 36$ (b) $(x - \square)^2 = x^2 - \square x + 49$
 (c) $(x + \square)^2 = x^2 + 18x + \square$ (d) $(x - \square)^2 = x^2 - 20x + \square$

In question 1, see if you can spot the pattern between the terms in the brackets and the final expression.

3 Expand and simplify.

- (a) $(x + 4)(x - 4)$ (b) $(x + 5)(x - 5)$ (c) $(x + 2)(x - 2)$
 (d) $(x - 11)(x + 11)$ (e) $(x - 3)(x + 3)$ (f) $(x - 1)(x + 1)$
 (g) $(x + 9)(x - 9)$ (h) $(x + a)(x - a)$ (i) $(t + x)(t - x)$



EXAMPLE 19

Expand and simplify $(3x - y)(x - 2y)$.

\times	x	$-2y$
$3x$	$3x^2$	$-6xy$
$-y$	$-xy$	$2y^2$

$$\begin{aligned}(3x - y)(x - 2y) &= 3x^2 - 6xy - xy + 2y \\ &= 3x^2 - 7xy + 2y^2\end{aligned}$$

What happens to the x term when you multiply brackets of the form $(x + a)(x - a)$?

Remember to multiply each term in the first bracket by each term in the second bracket.

Be careful when there are negative signs. This is where a lot of mistakes are made.

$$+ve \times -ve = -ve.$$

$$-ve \times -ve = +ve.$$



EXERCISE 7J

Expand and simplify.

- 1 $(3a + 2)(a + 4)$ 2 $(5x + 3)(x + 2)$ 3 $(2t + 3)(3t + 5)$
 4 $(4y + 1)(2y + 7)$ 5 $(6x + 5)(2x + 3)$ 6 $(4x + 3)(x - 1)$
 7 $(2z + 5)(3z - 2)$ 8 $(y + 1)(7y - 8)$ 9 $(3n - 5)(n + 8)$
 10 $(3b - 5)(2b + 1)$ 11 $(p - 4)(7p + 3)$ 12 $(2z - 3)(3z - 4)$
 13 $(5x - 9)(2x - 1)$ 14 $(2y - 3)(2y - 3)$ 15 $(2 + 3a)(4a + 5)$
 16 $(3x + 4)^2$ 17 $(2x - 7)^2$ 18 $(5 - 4x)^2$
 19 $(2x + 1)(2x - 1)$ 20 $(3y + 2)(3y - 2)$ 21 $(5n + 4)(5n - 4)$
 22 $(3x + 5)(3x - 5)$ 23 $(1 + 2x)(1 - 2x)$ 24 $(3t + 2x)(3t - 2x)$

Can you see the connection between questions 19–24 and question 3 in Exercise 7I?

EXAMINATION QUESTIONS

1 Solve the inequality $7 - 5x \geq -17$, given that x is a **positive** integer. [3]

(CIE Paper 2, Nov 2000)

2 Solve the inequality $25 - 3x < 7$. [2]

(CIE Paper 2, Jun 2001)

3 Solve the inequality $3(x + 7) < 5x - 9$. [2]

(CIE Paper 2, Jun 2002)

4 (a) Solve the inequality $5 - \frac{2x}{3} > \frac{1}{2} + \frac{x}{4}$. [3]

(b) List the positive integers which satisfy the inequality $5 - \frac{2x}{3} > \frac{1}{2} + \frac{x}{4}$. [1]

(CIE Paper 2, Nov 2002)

5 Solve the equation $\frac{x}{4} - 8 = -2$. [2]

(CIE Paper 2, Jun 2004)

6 (a) Factorise completely $12x^2 - 3y^2$. [2]

(b) (i) Expand $(x - 3)^2$. [2]

(ii) $x^2 - 6x + 10$ is to be written in the form $(x - p)^2 + q$.
Find the values of p and q . [2]

(CIE Paper 2, Jun 2004)

7 Solve the equation $\frac{3x - 2}{5} = 8$. [2]

(CIE Paper 2, Nov 2004)
