Collecting like terms

Expressions can be simplified by collecting together any terms that are made up of the same letters.

5 Simplifying expressions

Like terms are terms that have the same letter(s) with the same powers but can have different numerical coefficients.

Terms with + or − in front of them can be simplified by collecting like terms.

$t + 3t$ can be combined to give $4t$

$t^2 + 3t^2$ can be combined to give $4t^2$

$t + 3t^2$ cannot be combined as t and t^2 are not like terms even though they have the same letter. They have different powers.

Combine the like terms. There are four *t*s so write this as $4t$.

Add or subtract the coefficients on like terms.
$2 + 4 - 3 = 3$

There are two different sets of like terms in this expression. Collect all the *d* terms together and all the *c* terms together.

Combine x^2 terms in the same way you would combine x. Add or subtract the coefficients on the like terms.

2 Expressions with different terms

Sometimes expressions have two or more terms which are not like terms. Arrange the expression so that all of the like terms are next to each other. Then you can collect together all the like terms to give a simplified expression.

For example, to simplify the expression $r + 4p + 2r - 9p$:

1. Rearrange the expression so the like terms are together:
   $r + 2r + 4p - 9p$
2. Add or subtract the coefficients to combine the terms.
   r terms: $1 + 2 = 3$
   p terms: $4 - 9 = -5$
3. Collect the like terms: $3r - 5p$

1 Checklist

- Each term will either have a + or − in front of it.
- Like terms have the same letter(s) and the same power.
- x by itself is same as $1x$.

10 Worked example — Grades 1–2

1 Simplify

(a) $t + t + t + t$

$= 4t$

(b) $u + u + u - u$

$= 2u$

2 Simplify

(a) $2a + 4a - 3a$

$= 3a$

(b) $d + 6d - 4c$

$= 7d - 4c$

Collect the like terms and then combine them. The like terms for y^2 should be combined, and then z added to the expression.

3 Simplify

(a) $4x^2 + 2x^2 + 6x^2$

$= 12x^2$

(b) $7y^2 - 9y^2 + z - y^2$

$= -3y^2 + z$

4 Simplify

(a) $5e + 7f - 6e + 4f$

$= 5e - 6e + 7f + 4f$

$= -e + 11f$

(b) $x + 4x^2 + 3x - 5x^2$

$= x + 3x + 4x^2 - 5x^2$

$= 4x - x^2$

Collect the x and the x^2 terms separately. x and $4x^2$ are not like terms because the powers of x are different.

10 Exam-style practice — Grades 1–2

1 Simplify

(a) $m + m + m - m + m + m - m$ **[1 mark]**

(b) $5x - 3y + 4x - 2y$ **[2 marks]**

2 Simplify

(a) $3c^2 + 5c^2 - c^2$ **[1 mark]**

(b) $9x - 3y - 6x - 7y$ **[2 marks]**

3 Simplify $6t - 3 - 8t + 7$ **[2 marks]**

4 Simplify $7a + 5b - 2a - 9b$ **[2 marks]**

Simplifying expressions

You need to be able to simplify algebraic expressions that include multiplication signs and division signs.

Multiplying expressions

To simplify an algebraic expression that includes a multiplication sign, follow these rules.

1. Multiply all the numbers, including coefficients.
2. Use the laws of indices to simplify the powers of the letters. Go to page 25 to revise laws of indices.

A **coefficient** is the number in front of a letter in an algebraic expression. It means how many lots of that letter there are. For example: 3c means $3 \times c$

a is multiplied by itself 6 times so, using the laws of indices ($a^m \times a^n = a^{m+n}$), you can write it as a^6

5*x* means $5 \times x$. You can multiply in any order and get the same answer, so multiply the numbers and then the letters.

Write the division as a fraction.

Divide the numbers as much as possible, then cancel any common letters on the top and bottom of the fractions.

Letters in algebra can be simplified so that they are written next to each other in alphabetical order.

Worked example — Grades 1–2

1. Simplify

   (a) $a \times a \times a \times a \times a \times a$

   $= a^6$

   (b) $5x \times 3x$

   $= 5 \times x \times 3 \times x$

   $= 5 \times 3 \times x \times x = 15x^2$

2. Simplify

   (a) $10x \div 2$

   $= \frac{10 \times x}{2} = 5x$

   (b) $20xy \div y$

   $= \frac{20 \times x \times \cancel{y}}{\cancel{y}} = 20x$

3. Simplify

   (a) $3b \times 4b \times 2b$

   $= 3 \times b \times 4 \times b \times 2 \times b$

   $= 3 \times 4 \times 2 \times b \times b \times b = 24b^3$

   (b) $4x \times 5y$

   $= 4 \times x \times 5 \times y$

   $= 4 \times 5 \times x \times y = 20xy$

4. Simplify $25x^3 \div 5x$

   $= \frac{25 \times x^{\cancel{3}2}}{5 \times \cancel{x}}$

   $= 5 \times x^2 = 5x^2$

Remember that *x* is the same as x^1. Use the rule $a^m \div a^n = a^{m-n}$ to simplify the expression.

Dividing expressions

To simplify an algebraic expression that includes a division sign, follow these rules.

1. Write the expression as a fraction.
2. Cancel the numbers. Write any numbers that are not whole as fractions instead of decimals.
3. Use index rules (page 19) to simplify the powers of the letters.

Worked example — Grade 2

Simplify $\frac{5x^2 \times 4x^4}{6x^3}$

$= \frac{5 \times x^2 \times 4 \times x^4}{6 \times x^3} = \frac{5 \times 4 \times x^2 \times x^4}{6 \times x^3}$

$= \frac{20x^6}{6x^3}$

$= \frac{10x^3}{3}$

Find the HCF of the number at the top and the number at the bottom. Divide by that number to simplify the expression as much as possible, and give your answer as a fraction.

Checklist

- $a \times a = a^2$ (not 2*a*)
- $a \times a \times a = a^3$ (not 3*a*)
- $a \times b = ab$ or ba
- $1a = a$

Exam-style practice — Grades 1–2

1. Simplify $c \times d \times 5$ **[1 mark]**
2. Simplify $3 \times w \times 2$ **[1 mark]**
3. Simplify $3g \times 5h$ **[1 mark]**
4. Simplify $24x \div 6$ **[1 mark]**
5. Simplify $48xy \div 8y$ **[1 mark]**

Writing expressions

It is important to be able to interpret information and then write it in terms of algebraic expressions.

5 Interpreting information

Instructions or rules can be written as algebraic expressions.

Jess wants to put an advert for her school play in the local paper.

The cost is £2 for each line of text, plus a £10 fee.

To write this as an expression, use a letter to represent the number of lines of text.

For example:

The cost is £2 × number of lines + £10

Or $2n + 10$

where n is the number of lines of text.

10 Worked example — Grade 3

1 Crayons are sold in cartons and in tubs.
There are 5 crayons in a carton.
There are 25 crayons in a tub.
Asha buys one tub of crayons.
She takes p crayons out of the tub.

(a) Write down an expression, in terms of p, for the number of crayons left in the tub.

$25 - p$

If there are 25 crayons in the tub to start with, then the number left must be p crayons less than this.

Poppy buys x cartons of crayons and y tubs of crayons.

(b) Write down an expression, in terms of x and y, for the total number of crayons Poppy buys.

$5x + 25y$

Remember that $5 \times x$ is written as $5x$.

2 The cost of hiring a bike for d days can be worked out using this rule.

Add 4 to the number of days' hire.

Multiply your answer by 15

Write down an expression, in terms of d, for the total cost for hiring a bike for d days.

$(d + 4) \times 15$

$15(d + 4)$

The order of operations means that multiplication comes before addition.

In order for this expression to be correct, $d + 4$ must happen before × 15. Place brackets around the $d + 4$ expression to make sure this part of the formula is calculated first.

Exam focus

When you have finished working out an expression, make sure that you have collected all of the like terms and cancelled the number parts or the indices to simplify the equation.

5 Worked example — Grade 3

Sandeep, Pavan and Jake sell toy cars.

Sandeep sells x cars. Pavan sells 6 more cars than Sandeep. Jake sells twice as many cars as Sandeep.

Write an expression, in terms of x, for the total number of toy cars sold by Sandeep, Pavan and Jake.

Sandeep	Pavan	Jake
x	$x + 6$	$2x$

The total number of toy cars is $x + x + 6 + 2x$

$= x + x + 2x + 6$

$= 4x + 6$

'6 more than' means adding 6 and 'twice as many' means multiply by 2, or double it.

10 Exam-style practice — Grade 3

1 Ben has x cats.
Jenny has twice as many cats as Ben.
Kathy has 2 more cats than Ben.
Write an expression, in terms of x, for the total number of pets that Ben, Jenny and Kathy have. **[2 marks]**

2 Blank revision cards are sold in packets and in boxes.
There are 8 revision cards in a packet.
There are 27 revision cards in a box.
Avi buys p packets of revision cards and b boxes of revision cards.
Write an expression for the number of revision cards Avi buys, in terms of p and b. **[3 marks]**

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Algebraic formulae

A formula is a mathematical rule. You use algebra to write a formula (the plural of formula is formulae). A formula is similar to an algebraic expression, but it has an equals sign, and more than one variable. You need to be able to substitute numbers into formulae to solve them.

Writing a formula

Peter advertises his business in the local magazine.

To write this as an algebraic formula, substitute the variables for letters.

For example, if the total cost is £T, and the number of hours is n, then the formula Peter can use is:

total cost = (number of hours × £3) + £5

$$T = 3n + 5$$

When you define your variables, you must give their units. If you have a value for n you can now solve this formula. If Peter takes his neighbour's dog on a two-hour walk, how much will he charge?

$T = 3 \times 2 + 5$

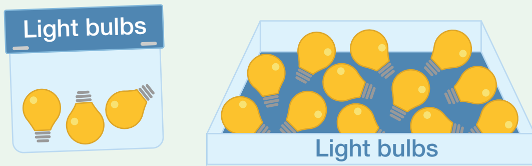
$\quad = 6 + 5 = 11$

He will charge £11

Worked example — Grade 3

Bulbs are sold in packets and in boxes. There are 3 bulbs in a packet. There are 12 bulbs in a box.

Kamran buys x packets of bulbs and y boxes of bulbs.



(a) Write down a formula, in terms of x and y, for the total number, N, of bulbs Kamran buys.

$N = 3 \times x + 12 \times y$

$\quad = 3x + 12y$

The variables are N (the total number of bulbs), x (the number of packets) and y (the number of boxes).

(b) Kamran buys 4 packets and 2 boxes of bulbs. How many bulbs does he buy?

$N = 3 \times 4 + 12 \times 2$

$\quad = 12 + 24$

$\quad = 36$ He buys 36 bulbs.

Substitute the values given in the question into the formula you worked out in part **(a)**.

Worked example — Grade 5

This formula gives you the distance, s metres, travelled by an object in t seconds.

$s = 10t + 5t^2$

Work out the value of s when $t = 3$

$s = 10 \times 3 + 5 \times 3^2$

$\quad = 30 + 5 \times 9$

$\quad = 30 + 45$

$\quad = 75$

Substitute the value of t into the formula.

Order of operations is very important when you are evaluating formulae. Remember to use BIDMAS.

When substituting, you might use brackets. You could write $10t$ as $10(t)$ or $10(3)$. If there are numbers or letters outside brackets, without an operation in between, this means that you multiply the term outside the brackets with whatever is inside the brackets.

For example: $10(3)$ means 10×3

Exam-style practice — Grade 4

1 $L = \dfrac{2x + 3y}{x}$

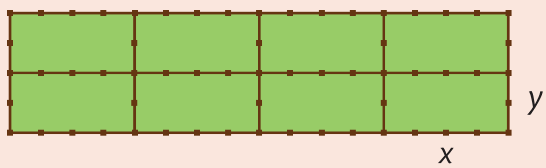
Work out the value of L when

$x = 8$ and $y = 12$

Give your answer as a fraction in its simplest form.

[3 marks]

2 A farmer uses 200 metres of fencing to make an enclosure divided into eight equal rectangular pens.



The length of each pen is x metres and the width of each pen is y metres.

(a) Show that $y = 20 - 1.2x$

The total area of the enclosure is A m^2.

(b) Show that $A = 160x - 9.6x^2$ **[3 marks]**

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Algebraic indices

Indices are also called powers. They represent how many times a number has been multiplied by itself. Examples include squaring and cubing numbers.

5 Basic rules of indices

Learn the basic rules of indices.

$a^m \times a^n = a^{m+n}$ | $x^4 \times x^6 = x^{4+6} = x^{10}$

$a^m \div a^n = a^{m-n}$ | $x^4 \div x^6 = x^{4-6} = x^{-2}$

$(a^m)^n = a^{mn}$ | $(x^4)^6 = x^{4\times6} = x^{24}$

$a^{-n} = \frac{1}{a^n}$ | $x^{-4} = \frac{1}{x^4}$

$a^0 = 1$ | $x^0 = 1$

Go to page 15 to revise how indices work.

2 Indices checklist

- Only combine powers (indices) when the base numbers are the same.
- When you multiply, add the powers.
- When you divide, subtract the powers.
- When you raise a power to a power, multiply the powers together.

10 Worked example — Grades 4–5

1 Simplify

(a) $p^7 \times p^4$

$= p^{7+4} = p^{11}$

The base is the same, p, so you can combine the indices.

(b) $p^9 \div p^5$

$= p^{9-5} = p^4$

(c) $(p^2)^4$

$= p^{2\times4} = p^8$

2 Simplify

(a) $\frac{x^5 \times x^7}{x^3}$

$= \frac{x^{5+7}}{x^3} = \frac{x^{12}}{x^3} = x^9$

(b) $\left(\frac{x^8}{x^5}\right)^2$

$= (x^{8-5})^2 = (x^3)^2 = x^6$

Deal with each base letter separately

3 Simplify

(a) $5x^4y^3 \times 2x^3y^2$

$= 5x^4 \times 2x^3 \times y^3 \times y^2 = 10x^7y^5$

(b) $\frac{24x^4y^3}{8x^2y}$

$= 24x^4 \div 8x^2 \times y^3 \div y = 3x^2y^2$

Treat the numbers and each base separately.
$24 \div 8 = 3$
$x^4 \div x^2 = x^2$
$y^3 \div y = y^2$

4 Work out 2^{-3}

$= \frac{1}{2^3} = \frac{1}{8}$

A negative power shows that the value is a reciprocal and can be written as a fraction, $a^{-n} = \frac{1}{a^n}$

Substitute 2^{-3} into $a^{-n} = \frac{1}{a^n}$

5 Worked example — Grade 5

1 Work out the value of n given that $p^4 \times p^n = p^{10}$.

$4 + n = 10$

$n = 6$

Add the indices then set this equal to 10.

2 Work out the value of t given that $(5^4)^t = 5^{12}$

$4t = 12$

$t = 3$

Since the base is the same on both sides, the powers must be equal so form an equation and solve it.

10 Exam-style practice — Grades 4–5

1 Simplify

(a) $p^2 \times p^9$ **[1 mark]**

(b) $\frac{x^4 \times x^6}{x^2}$ **[2 marks]**

(c) $4x^2y^4 \times 3xy$ **[2 marks]**

2 Simplify

(a) $m^8 \div m^2$ **[2 marks]**

(b) $(m^5)^3$ **[2 marks]**

(c) $3w^2y^3 \times 4w^6y$ **[2 marks]**

(d) $\frac{32x^6y^8}{4x^2y}$ **[2 marks]**

3 Work out the value of n given that $x^{15} = x^n \times x^8$ **[1 mark]**

4 $1000^a \times 100^b = 10^x$

Show that $x = 3a + 2b$ **[2 marks]**

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Expanding brackets

Sometimes mathematical expressions include terms written in brackets. You can remove the brackets by expanding them.

Removing brackets

To remove brackets, you **expand** them. This means **multiply**.

An expression such as $2(x + 4)$ can be expanded by multiplying 2 and x, and 2 and 4. There is an invisible multiplication sign between the 2 and $(x + 4)$.

$$2(x + 4) = 2 \times (x + 4)$$
$$= 2 \times x + 2 \times 4$$
$$= 2x + 8$$

Negative terms

When the term outside is negative, you have to multiply the terms inside the bracket by a negative number.

For example:

$$-5(a - 2) = -5 \times (a - 2)$$
$$= -5 \times a + -5 \times -2$$
$$= -5a + 10$$

A negative multiplied by another negative gives a positive number: $-5 \times -2 = +10$

Go to page 2 to revise multiplying negative numbers.

Problem solving

You can be asked to use multiple skills in one question. If you are asked to expand and simplify, you need to expand all the brackets and then collect like terms and combine them.

For example, to expand and simplify $5e(e + 1) - 3e(4 - 6e)$:

$$5(e + 1) - 3e(4 - 6e)$$
$$= 5 \times e + 5 \times 1 - 3e \times 4 - 3e \times -6e$$
$$= 5e + 5 - 12e + 18e^2$$
$$= 18e^2 - 7e + 5$$

Worked example — Grade 2

Expand

(a) $4(x + 5)$

$= 4 \times x + 4 \times 5$

$= 4x + 20$

(b) $3(x - 7)$

$= 3 \times x - 3 \times 7$

$= 3x - 21$

When multiplying out brackets, always multiply every term inside the bracket by the term outside.

Worked example — Grades 3–4

1 Expand

(a) $-x(x - 3)$

$= -x \times x - x \times -3$

$= -x^2 + 3x$

(b) $-3x(x + 1)$

$= -3x \times x - 3x \times 1$

$= -3x^2 - 3x$

Draw arrows from the term outside the brackets to each term inside, so you know which terms you need to multiply.

After multiplying out the brackets, collect the like terms and combine them.

2 Expand and simplify

(a) $5(x + 7) + 3(x - 2)$

$= 5 \times x + 5 \times 7 + 3 \times x - 3 \times 2$

$= 5x + 35 + 3x - 6$

$= 5x + 3x + 35 - 6$

$= 8x + 29$

(b) $3m(m + 4) - 2m(4m + 1)$

$= 3m \times m + 3m \times 4 - 2m \times 4m - 2m \times 1$

$= 3m^2 + 12m - 8m^2 - 2m$

$= 3m^2 - 8m^2 + 12m - 2m$

$= -5m^2 + 10m$ or $10m - 5m^2$

Exam focus

Write out the expression with each separate operation in it. Include any negative numbers.

This will help you make sure you haven't missed any terms or signs out.

Exam-style practice — Grades 2–4

1 Expand

(a) $5(m + 2)$ **[1 mark]**

(b) $-3(n + 6)$ **[1 mark]**

(c) $x(x - 5)$ **[1 mark]**

(d) $3x(x - 2)$ **[1 mark]**

(e) $-4x(x - 1)$ **[1 mark]**

2 Expand and simplify

(a) $7a + 4(a - 2b)$ **[2 marks]**

(b) $4(3 + 2g) + 2(5 - 3g)$ **[2 marks]**

(c) $4r(3 + 4p) + 3p(8 - r)$ **[2 marks]**

(d) $t(3t + 4) + 3t(3 + 2t)$ **[2 marks]**

Expanding double brackets

Sometimes you will need to multiply out two sets of brackets and then simplify the result to find the correct expression.

10 Double brackets

You can often multiply out two sets of brackets to obtain a simplified expression. There are three different methods for doing this.

Box method

The box method is a visual representation of multiplying out brackets.

$\times$	x	$+4$
x	x^2	$4x$
$+2$	$2x$	8

Collect and simplify the terms:

$x^2 + 2x + 4x + 8 = x^2 + 6x + 8$

Expansion method

Multiply each term in the first bracket by the whole of the second bracket:

$$(x + 4)(x + 2) = x(x + 2) + 4(x + 2)$$
$$= x^2 + 2x + 4x + 8$$
$$= x^2 + 6x + 8$$

Simplify the final expression by collecting like terms.

FOIL

In the FOIL method, you multiply out the brackets in a particular order.

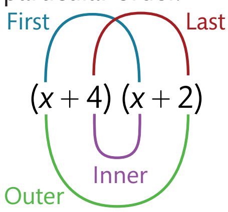
$(x + 4)(x + 2)$

F: $x \times x = x^2$

O: $x \times 2 = 2x$

I: $4 \times x = 4x$

L: $4 \times 2 = 8$



Collect and simplify the terms:

$x^2 + 2x + 4x + 8 = x^2 + 6x + 8$

10 Worked example — Grades 4–5

1 Expand and simplify $(2x - 5)(x - 4)$

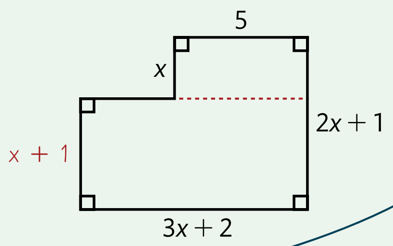
Using the box method

$\times$	$2x$	-5
x	$2x^2$	$-5x$
-4	$-8x$	$+20$

$$(2x - 5)(x - 4) = 2x^2 - 5x - 8x + 20$$
$$= 2x^2 - 13x + 20$$

2 In this shape, all the measurements are in metres. The area of the shape is A m^2.

Find a formula for A in terms of x.



$2x + 1 - x = x + 1$

Area of small rectangle $= 5x$

Area of large rectangle $= (x + 1)(3x + 2)$

$= 3x^2 + 2x + 3x + 2$

$= 3x^2 + 5x + 2$

Total area $= 5x + 3x^2 + 5x + 2$

So $A = 3x^2 + 10x + 2$

$(x + 3)^2 = (x + 3)(x + 3)$

5 Worked example — Grade 4

Expand and simplify $(x + 2)(x - 3)$

Using the FOIL method

F: $x \times x = x^2$

O: $x \times -3 = -3x$

I: $2 \times x = 2x$

L: $2 \times -3 = -6$

$x^2 - 3x + 2x - 6 = x^2 - x - 6$

After you have expanded the brackets, always simplify the expression.

Problem solving

Find the missing length, then split the shape into two parts and find their areas.

10 Exam-style practice — Grade 4

1 Expand and simplify

(a) $(x + 3)^2$ **[2 marks]**

(b) $(x - 4)^2$ **[2 marks]**

(c) $(x - 1)^2$ **[2 marks]**

2 Expand and simplify

(a) $(x + 2)(x - 2)$ **[2 marks]**

(b) $(x - 4)(x + 7)$ **[2 marks]**

(c) $(x - 1)(x - 5)$ **[2 marks]**

(d) $(x - 3)(x - 9)$ **[2 marks]**

Factorising

Factorising is the reverse of expanding brackets. To factorise an expression, you need to find the highest common factor of all the terms in the expression. Go to page 12 to revise highest common factors.

5 Factorising

To factorise an expression first find the highest common factor (HCF) of all the terms in the expression.

$4a + 8$

Both terms are divisible by 2 and 4, so 4 is the HCF. Write 4 on the outside of your brackets.

$4(\quad)$

Then work out the terms inside the bracket by dividing the terms in the original expression by the HCF.

$4a \div 4 = a$ and $8 \div 4 = 2$

$4(a + 2)$

Factorising is the opposite of expanding brackets.

You can check your answer, by expanding the brackets; it should give you the original expression.

Expanding brackets

$4(a + 2) = 4 \times a + 4 \times 2 = 4a + 8$

Factorising

The terms $4x$ and $10xy$ have 2 as a common factor. x appears in both terms, so it is also a common factor. Divide the expression by 2 first, and then by x.

Find the common factors of both terms. 6 and p will divide into each term. t is only in one term of the expression, so it is not a factor.

Expand the brackets to check your answer.

$6p(2p^2t - 3) = 12p^3t - 18p$

10 Worked example — Grades 3–4

Factorise fully

(a) $6y + 12$

$= 6(y + 2)$

2 and 6 are the common factors, so the HCF is 6

(b) $4x + 10xy$

The HCF is $2x$

$= 2(2x + 5xy)$

$= 2x(2 + 5y)$

(c) $12p^3t - 18p$

It can help to write out p^3 as $(p \times p \times p)$

$= 6(2p^3t - 3p)$

$= 6(2(p \times p \times p)t - 3p)$

$= 6p(2(p \times p)t - 3)$

The HCF is $6p$

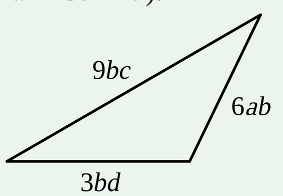
$= 6p(2p^2t - 3)$

Problem solving

1. The perimeter is the sum of all the sides, so add the three sides together.
2. The expression you need to end up with has a multiple on the outside of the brackets, so you know to factorise your expression for the perimeter. Find the HCF of all three terms.
3. Factorise each term fully.
4. Check your answer by expanding the brackets.

5 Worked example — Grade 5

Show that the perimeter of this triangle can be written as $3b(2a + 3c + d)$.



Perimeter $= 6ab + 9bc + 3bd$

To factorise, HCF is $3b$

So $6ab + 9bc + 3bd = 3b(2a + 3c + d)$

Check: $3b \times 2a + 3b \times 3c + 3b \times d$

$= 6ab + 9bc + 3bd$

10 Exam-style practice — Grades 3–5

Factorise fully

(a) $5x + 20$	**[1 mark]**	**(b)** $8a^2 + 12a$	**[2 marks]**
(c) $x^2 - 6x$	**[2 marks]**	**(d)** $3a^2b + 6ab^2$	**[2 marks]**
(e) $6y^2 - 9xy$	**[2 marks]**	**(f)** $8x^2 + 4xy$	**[2 marks]**

Linear equations

When you solve a linear equation your aim is to find the value of the unknown. You can often do this by rearranging the equation and using inverse operations.

5 Unknown on one side

You may be asked to solve an equation that involves an unknown on one side, such as

$6x + 10 = 34$

You will often need to use **inverse operations**, which are 'opposite' calculations.

The term $+10$ can be removed from the equation by subtracting 10 from each side. This gives:

$6x + 10 - 10 = 34 - 10$

$6x = 24$

To get x by itself, divide both sides by 6

$\frac{6x}{6} = \frac{24}{6}$

$x = 4$

Every line of working should include an equals (=) sign. You should line the equals signs underneath each other.

5 Unknown on both sides

Sometimes an equation has unknowns on both sides. To solve such an equation, rearrange it so that all the unknowns are on one side.

$3 - 4x = 15 - x$

$3 - 4x + 4x = 15 - x + 4x$

$3 = 15 + 3x$

$3 - 15 = 15 - 15 + 3x$

$-12 = 3x$

$\frac{-12}{3} = \frac{3x}{3}$

$-4 = x$ or $x = -4$

Remove the term $-4x$ from the equation by adding $4x$ to each side.

To get x by itself, divide both sides by 3

10 Worked example — Grades 4–5

1 Solve

$5x + 7 = 11$

$5x + 7 - 7 = 11 - 7$

$5x = 4$

$x = \frac{4}{5}$

2 Solve

$7x + 8 = 2x - 3$

$7x - 2x + 8 = 2x - 2x - 3$

$5x + 8 - 8 = -3 - 8$

$5x = -11$

$x = -\frac{11}{5}$

3 Andy, Tom and Chris play hockey. Andy has scored 9 more goals than Chris, Tom has scored 6 more goals than Andy. The total number of goals scored by the three players is 90

How many goals did they each score?

Andy	Tom	Chris
$x + 9$	$x + 9 + 6$	x

$x + 9 + x + 9 + 6 + x = 90$

$3x + 24 = 90$

$3x = 90 - 24$

$3x = 66$

$x = \frac{66}{3} = 22$

Andy	Tom	Chris
$22 + 9 = 31$	$22 + 9 + 6 = 37$	22

2 Equations with brackets

Always multiply out brackets first, then collect like terms. You will be expected to solve an equation such as:

$2(2x + 5) - (3x + 4) = 9(2x + 5)$

$4x + 10 - 3x - 4 = 18x + 45$

$x + 6 = 18x + 45$

$x - x + 6 = 18x - x + 45$

$6 - 45 = 17x + 45 - 45$

$-39 = 17x$

$-\frac{39}{17} = x$

$x = -\frac{39}{17}$

Multiply out the brackets.

Collect like terms.

Problem solving

Assign a letter to the unknown value and create an algebraic equation using this letter.

Set up the equation by adding all the expressions and equating them to the total number of goals.

10 Exam-style practice — Grades 4–5

1 Solve

(a) $6x + 3 = 24$ **[2 marks]**

(b) $3(2x - 1) = 6$ **[2 marks]**

(c) $3x + 7 = 5x - 1$ **[3 marks]**

2 Ann is x years old. Ben is twice as old as Ann. Carl is 4 years younger than Ann. The total of their ages is 92 years.

Work out the age of each person. **[3 marks]**

Rearranging formulae

Formulae show the relationship between two or more variables. These formulae can be rearranged to make a different letter the subject.

5 Finding the subject

The subject of a formula is the letter on its own on one side of the equals sign.

This is often on the left-hand side (LHS) of the equals sign. For example, this is the formula for the area of a circle:

$$A = \pi r^2$$

In this formula, A is the subject.

Rearranging formulae is very similar to solving equations where inverse operations are used. If there is another letter or number on the same side of the equals sign as the subject, then you need to use inverse operations to remove it.

This example involves just one inverse operation. The inverse operation of + is −, so subtract 5 from each side of the formula to get h on its own.

The inverse operation of × is ÷, so divide both sides by 4 to make m the subject.

5 Worked example — Grades 2–3

1 Make h the subject of the formula $P = h + 5$

$P = h + 5 \quad (-5)$

$P - 5 = h$

$h = P - 5$

2 Make m the subject of the formula $n = 4m$

$n = 4m \quad (\div 4)$

$\frac{n}{4} = m$

$m = \frac{n}{4}$

3 Make y the subject of the formula $G = 3y - 7$

$G = 3y - 7 \quad (+7)$

$G + 7 = 3y \quad (\div 3)$

$\frac{G+7}{3} = y$

$y = \frac{G+7}{3}$

There are two separate inverse operations here: +7 to each side, and then divide both sides by 3

5 Two or more inverse operations

Sometimes finding the subject of a formula involves two or more operations. For example, to make t the subject of the formula $v = u + 10t$:

$v = u + 10t$

$-u$ from both sides

$v - u = 10t$

÷10 for both sides

$\frac{v-u}{10} = t$

Move the subject to the LHS

$t = \frac{v-u}{10}$

Make sure you carry out each inverse operation step by step.

For most formulae, start by adding or taking away any numbers or letters that are on the same side of the formula as the subject. Then complete any multiplication or division you need to do.

5 Worked example — Grades 4–5

1 Make W the subject of the formula $P = 2L + 2W$

$P = 2L + 2W \quad (-2L)$

$P - 2L = 2W \quad (\div 2)$

$\frac{P-2L}{2} = W$

$W = \frac{P-2L}{2}$

Here you can take $2L$ across to the LHS as part of the first inverse operation.

2 Make X the subject of the formula $Y = 3X^2 - 10$

$Y = 3X^2 - 10 \quad (+10)$

$Y + 10 = 3X^2 \quad (\div 3)$

$\frac{Y+10}{3} = X^2$

$X = \sqrt{\frac{Y+10}{3}}$

The inverse operation of 'squared' is 'square root'.

10 Exam-style practice — Grades 2–4

1 Make h the subject of the formula $x = 5h + 8$ **[2 marks]**

2 Make g the subject of the formula $t = \frac{gh}{10}$ **[2 marks]**

3 Make a the subject of the formula $v^2 = u^2 + 2as$ **[2 marks]**

4 Rearrange $y = \frac{1}{2}x + 1$ to make x the subject. **[2 marks]**