Edexcel GCSE (9-1) Mathematics
Foundation Student Book

Confidence • Fluency • Problem-solving • Reasoning

We are seeking endorsement for use with the Edexcel GCSE (9-1) Mathematics specification.
The engine performance of cars can be compared by looking at the power-to-weight ratio. Cars with a high power-to-weight ratio accelerate well. The times taken for cars to accelerate from 0 to 100 km/h are given in the table.

<table>
<thead>
<tr>
<th>Car make and model</th>
<th>Time (0–100 km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferrari 458 Italia</td>
<td>3.3 s</td>
</tr>
<tr>
<td>Lamborghini Aventador</td>
<td>2.7 s</td>
</tr>
<tr>
<td>Porsche 918 Spyder</td>
<td>2.4 s</td>
</tr>
</tbody>
</table>

Which car accelerated the fastest?

11 Numerical fluency

1 Copy and complete
   a $10 \times \square = 60$
   b $\square \times 9 = 900$

2 Find the highest common factor (HCF) of each pair of numbers
   a 25 and 35
   b 24 and 40

3 Work out
   a $24 \text{kg} \div 3$
   b $72 \text{mm} \div 8$
   c £2.80 $\div 4$
   d 300 g $\div 5$

4 Work out
   a $17.5 \div 5$
   b $5.4 \times 6$

5 Work out
   a $40 \div 5 \times 3$
   b $36 \div 4 \times 8$

6 What fraction of each diagram is shaded?
   a
   b

7 Copy and complete
   a $\frac{3}{5} \div \square = \frac{2}{3}$
   b $\frac{24}{32} \div \square = \frac{3}{8}$

Fluency with measures

8 How many
   a g in 1 kg
   b ml in 1 litre?

Geometrical fluency

9 What is the scale factor of this enlargement?
Graphical fluency

10 Plot a line graph for the values in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

a What is the gradient of the line?
b What is the equation of the line?

Challenge

11 Design a set of 20 loop cards. The first two have been done for you.

The last card must end with $\frac{3}{4}$ or 0.75. Explain why.

<table>
<thead>
<tr>
<th>$\frac{3}{4}$</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

11.1 Writing ratios

Objectives

- Use ratio notation.
- Write a ratio in its simplest form.
- Solve simple problems using ratios.

Why learn this?

Outdoor pursuits need to have the correct supervision ratios.

Fluency

- Work out 28 ÷ 4, 45 ÷ 5
- Chloe has 3 red beads and twice as many blue beads. How many blue beads does she have?

1 The bar chart shows the number of teenagers doing each activity at an outdoor pursuits centre on one day.

a How many teenagers are taking part in activities at the centre altogether?
b Which activity is the most popular?

Key point 1

A ratio is a way to compare two or more quantities.

Questions in this unit are targeted at the steps indicated.

2 Write down each ratio of red tins to yellow tins.

a 

b

Q2a hint There is 1 red tin and 2 yellow tins. Write the ratio ‘red : yellow’ using the numbers.

3 Draw tins to show these ratios of red to yellow.

a 4 : 3
b 3 : 4

Discussion Is the ratio 4 : 3 the same as the ratio 3 : 4?

4 A necklace has 30 beads. There is 1 purple bead for every 4 blue beads.

How many beads are

a purple
b blue?

Q4 hint How many sets of ‘1 purple, 4 blue’ are these?

Key point 2

You simplify a ratio by making the numbers as small as possible.

Divide the numbers in the ratio by their highest common factor (HCF).

5 Write each ratio in its simplest form.

a 4 : 12  b 16 : 8

Q5a hint $\frac{4}{12} = \frac{1}{3}$

b 27 : 9  c 7 : 42

d 15 : 20  e 63 : 28

f 18 : 48  g 36 : 45

h 49 : 28

Q5e hint $\frac{49}{28} = \frac{7}{4}$

6 Problem-solving / Reasoning The bar charts show the numbers of gold medals and other medals won at a competition by each group from a gym club.

a What is the ratio of gold medals won to other medals won?

Write your answer in its simplest form.

b The club’s coach says that they won twice as many other medals as gold medals.

Is the coach correct? Explain your answer.

7 Real An after-school club at a primary school is attended by 32 children. It is run by 4 adults. The guidelines say that the adult-to-child ratio should be 1 : 8.

Does the club have enough adults?

8 Problem-solving Which of these ratios are equivalent?

A 36 : 16  B 15 : 60

C 28 : 16  D 126 : 56

E 49 : 28

Q8 communication hint Ratios are equivalent if they have the same simplest form.

Q9a hint The highest common factor of 20, 25 and 15 is 5, so divide all the parts by 5.

9 Write each ratio in its simplest form.

a 20 : 25 : 15  b 36 : 24 : 30

c 56 : 42 : 35  d 16 : 40 : 56
10  **Communication**  Show that these ratios are equivalent.
   10 : 15 : 25  12 : 18 : 30  18 : 27 : 45

11  **STEM**  A recipe for shortbread uses 125 g of butter, 55 g of sugar and 180 g of flour.
   Write the ratio of butter : sugar : flour in its simplest form.

12  **Exam-style question**
   There are 80 marbles in a bag. Of these, 25 are china and the rest are glass.
   Write the ratio of china marbles to glass marbles in its simplest form.

---

### 11.2 Using ratios 1

**Objective**
- Solve simple problems using ratios.

**Why learn this?**
You need to mix paints in the same ratio to get the same colour each time.

**Fluency**
- 1 m = [ ] cm
- 1 cm = [ ] mm
- 1 m = [ ] mm
- 2500 mm = [ ] m

1  **Copy and complete**
   a 2 × [ ] = 100  
   b 3 × [ ] = 75  
   c 2 × [ ] = 150  
   d 6 × [ ] = 300

2  **Work out**
   a 1.5 × 10  
   b 2.45 × 100  
   c 3.71 × 10  
   d 9.37 × 100

3  **Find the HCF of each pair of numbers.**
   a 24 and 30  
   b 35 and 49  
   c 18 and 45  
   d 64 and 80

---

### Example 1

To make orange paint Maria mixes yellow paint with red paint in the ratio 3 : 1.
She uses 4 tins of red paint. How many tins of yellow paint does she use?
Write down the ratio. Use Y for yellow and R for red.

\[ \frac{Y}{R} = \frac{3}{1} \]

Multiply each part by the same number to get an equivalent ratio.

\[ \frac{3 \times 4}{1 \times 4} \]

Maria uses 12 tins of yellow paint.

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4  **STEM**  The ratio of oyster sauce : fish sauce in a stir fry recipe is 2 : 1.
   Clare uses 5 tablespoons of fish sauce.
   How many tablespoons of oyster sauce does she use?

---

### Real

Thomas makes a model of the Eiffel Tower using a ratio of 1 : 1280.
The height of his model is 250 mm.
What is the height of the Eiffel Tower in metres?

---

### Key point 3

Ratios in their **simplest form** only have whole numbers.

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### Example 2

Write 1.5 : 8 as a whole number ratio in its simplest form.

1.5 has 1 decimal place so multiply both sides of the ratio by 10 to get a whole number.

\[ 1.5 \times 10 : 8 \times 10 \]

The HCF is 5 so divide both sides by 5.

---

5  **Real**
   Thomas makes a model of the Eiffel Tower using a ratio of 1 : 1280.
The height of his model is 250 mm.
   What is the height of the Eiffel Tower in metres?

---

### Q5 hint

1 : 1280 means that 1 mm in the model represents 1280 mm in real life.

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### Q12 strategy hint

Start by writing C : G and the information you know.

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### Discussion

What should you multiply by if a number in a ratio has 2 decimal places?

---

6  **Write each ratio as a whole number ratio in its simplest form.**
   a 0.4 : 6  
   b 3.5 : 4.2  
   c 45 : 13.5  
   d 25.6 : 46.4

---

### Discussion

What should you multiply by if a number in a ratio has 2 decimal places?

---

7  **Write each ratio as a whole number ratio in its simplest form.**
   a 0.25 : 3.1  
   b 1.4 : 0.28  
   c 1.62 : 1.8  
   d 4.8 : 11.2

---

### Discussion

What should you multiply by if a number in a ratio has 2 decimal places?

---

8  **STEM / Problem-solving**  Old computer monitors had a width : height ratio of 4 : 3.
   New computer monitors have a width : height ratio of 16 : 9.
   Is each of these monitors old or new?
   a 16.8 inches  
   b 12.6 inches  
   c 18.4 inches  
   d 10.35 inches

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### STEM
   Sterling silver is made from silver and copper in the ratio 92.5 : 7.5.
   Write this ratio in its simplest form.

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### STEM
   The ratio of chick peas : broad beans in a hummus recipe is 2 : 9.
   Jack uses 150 g of chick peas.
   How many grams of broad beans should he use?

---

### STEM / Reasoning
   The ratio of okra : sweet potato in a korma recipe is 15 : 8.
   Raj uses 160 g of sweet potato.
   He has 350 g of okra.
   Will he use all of the okra?
12 **Finance**  Anna splits her monthly net pay into rent money and money left in the ratio 7 : 13. Her rent is £420. How much money does she have left each month?

13 **Exam-style question**

5 schools sent some students to a conference. One of the schools sent both boys and girls. This school sent 16 boys. The ratio of the number of boys it sent to the number of girls it sent was 1 : 2. The other 4 schools sent only girls. Each of the 5 schools sent the same number of students. Work out the total number of students sent to the conference by these 5 schools.

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**11.3 Ratios and measures**

**Objectives**

- Use ratios to convert between units.
- Write and use ratios for shapes and their enlargements.

**Fluency**

- 1 kg = \( \square \) g, 1 m = \( \square \) cm, 1 litre = \( \square \) ml, 1 cm = \( \square \) mm, 1 day = \( \square \) hours
- \( \square \) g, \( \square \) km, \( \square \) litres, \( \square \) mm

1 Faiz has 7 red marbles and 2 blue marbles.

a How many marbles does he have altogether?

b What fraction are i red ii blue?

2 Work out the area of this shape.

3 Work out the volume of this cube.

4 Write these ratios in their simplest form.

a 15 minutes : 1 hour
b 400 g : 1 kg
c 2 hours : 30 seconds

5 A mug contains 250 ml of coffee and a jug contains 1 litre of coffee. Write down the ratio of the amount of coffee in the mug to the amount of coffee in the jug. Give your answer in its simplest form.

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**Key point 4**

You can use ratios to convert between units.

**Example 3**

Convert 8 m to cm.

\[
\begin{align*}
1 \text{ m} &= 100 \text{ cm} \\
8 \text{ m} &= 8 \times 100 \text{ cm} \\
&= 800 \text{ cm}
\end{align*}
\]

The ratio of m : cm is 1 : 100.

---

**Q3 hint** Start by working out how many girls the school sent.

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**Q4 hint** Both parts of the ratio need to be in the same units before you simplify.

**Q4a and c hint** There are 60 seconds in 1 minute and 60 minutes in 1 hour.

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**Q6a hint** km : m = \( \square \) : \( \square \)

**Q6b hint** There are 60 seconds in 1 minute and 60 minutes in 1 hour.

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**Q10 hint** £ : $ = \( \square \) : \( \square \) × 500

**Q11 communication hint** The exchange rate is the amount of money in another currency that your currency will buy or sell for.
12 **Real** Use the table in Q11.
   a. How many £ does Sue get for 300 Swiss francs?
   b. Which is worth more, 49 560 rupees or 85 700 yen?

13 The diagram shows three rectangles.

![Diagram of rectangles A, B, and C with dimensions: A: 5 cm x 2 cm, B: 6 cm x 15 cm, C: 8 cm x 20 cm.]

   a. Write these ratios in their simplest form.
      i. The width of rectangle A to rectangle B.
      ii. The length of rectangle A to rectangle B.
      iii. The area of rectangle A to rectangle B.

   **Discussion** What do you notice about the results?

14 The diagram shows two cubes.

![Diagram of cubes A and B with dimensions: A: 3 cm x 3 cm x 5 cm, B: 5 cm x 5 cm x 5 cm.]

   a. Write these ratios in their simplest form.
      i. Height of A to height of B.
      ii. Area of a face of A to area of a face of B.
      iii. Volume of A to volume of B.

   **Discussion** What do you notice about your answers?

15 A bag contains 11 red counters and 3 white counters.

   a. What is the ratio of red counters : white counters?
   b. What fraction of the counters are red?
   c. What fraction of the counters are white?

   **Discussion** What do you notice about your answers?

16 In a box of mints and toffees, $\frac{2}{3}$ of the sweets are mints and $\frac{1}{3}$ are toffees.

   a. What is the ratio of mints : toffees?
   b. There are 8 toffees in the box. How many mints are there?
   c. How many sweets are in the box altogether?

   **Reflect** Is there more than one way to work out the answer to part c?

17 In a band, $\frac{3}{5}$ of the members are male and the rest are female.

   a. What is the ratio of males : females in the band?
   b. Maya thinks there are 18 people in the band. Explain why she must be wrong.

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**11.4 Using ratios 2**

**Objectives**
- Divide a quantity into 2 parts in a given ratio.
- Divide a quantity into 3 parts in a given ratio.
- Solve word problems using ratios.

**Why learn this?**
The ratio of gold to other metals in a piece of jewellery is used to work out the value of the item.

**Fluency**
Write down the ratio of green : blue : yellow counters on the dish.

1. **Work out**
   a. $35 \div 7$
   b. $10 \div 4$
   c. $4.5 \div 3$
   d. $2.8 \div 4$

2. **Round these figures.**
   a. 12.468 to 2 decimal places
   b. 4.973 to 2 decimal places
   c. 3.1584 to 3 decimal places
   d. 0.012 439 to 3 decimal places

3. **Write these ratios in their simplest form.**
   a. $25 : 10$
   b. $100 : 350$
   c. $6000 : 2000$
   d. $4.5 : 6$

4. **Share these amounts in the ratios given.**
   a. £18 in the ratio 2 : 1
   b. £42 in the ratio 1 : 6
   c. £27 in the ratio 4 : 5
   d. 35 kg in the ratio 2 : 3
   e. 60 m in the ratio 5 : 7
   f. 7.5 litres in the ratio 2 : 3

5. **STEM / Real** Before 2012, 10p coins were made from copper and nickel in the ratio 3 : 1.
   The coins had a mass of 6.5 g. What was the mass of
   a. copper
   b. nickel?

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**ActiveLearn** Homework, practice and support: Foundation 11.4
6 Purple paint is mixed from red paint and blue paint in the ratio 5 : 3.
A painter needs 20 litres of purple paint.
   a How many litres of red paint should he use?
   b How many litres of blue paint should he use?
Discussion How did you check your answers?

7 A fruit drink is made from orange, pineapple and apple juice in the ratio 1 : 2 : 4.
Rita wants to make 35 litres of fruit drink. How much of each type of juice does she need?

8 Exam-style question
   Talil is going to make some concrete mix.
   He needs to mix cement, sand and gravel in the ratio 1 : 3 : 5 by weight.
   Talil has
   • 15 kg of cement
   • 85 kg of sand
   • 100 kg of gravel
   Does Talil have enough cement, sand and gravel to make the concrete mix?
   (4 marks)
   Nov 2012, Q29, 1MA0/1F

9 Share these amounts in the ratios given.
   a £72 in the ratio 2 : 3 : 4
   b 100 g in the ratio 2 : 3 : 5
   c 360 ml in the ratio 3 : 4 : 5
   d £25 in the ratio 1 : 3 : 4
Discussion How should you round if the ratio is money? What about litres? Why?

10 Share these amounts in the ratios given. Round your answers sensibly.
   a £80 in the ratio 2 : 5
   b 70 litres in the ratio 2 : 7

11 Bob and Phil buy a greyhound for £4500.
   Bob pays £3000 and Phil pays £1500.
The greyhound wins an £18 000 prize.
   a Write the amounts they each pay as a ratio.
   b Write the ratio in its simplest form.
   c Divide the prize money in this ratio.
   How much does each of them get?
Discussion Is this a fair way to share the prize money?

12 Andrea and Penny buy a statue for £350.
   Andrea pays £140 and Penny pays £210.
   They sell the statue 3 years later for £475.
   How should they share the money fairly?

13 Problem-solving / Reasoning Two numbers are in the ratio 1 : 3 and their difference is 12.
   What are the numbers?

14 Rob and Simon share £50 in the ratio 7 : 3.
   a How much do they each receive?
   b How much more does Rob get than Simon?
Discussion How could you check your answer?

15 Exam-style question
   Pat and Julie share some money in the ratio 2 : 5
   Julie gets £45 more than Pat.
   How much money did Pat get?
   (3 marks)
   June 2012, Q22, 1MA0/2F

Reflect How can drawing a diagram help you to answer ratio questions?

11.5 Comparing using ratios

Objectives
- Use ratios involving decimals.
- Compare ratios.
- Solve ratio and proportion problems.

Why learn this?
You can use this to calculate and compare the performance of different cars.

Fluency
A bracelet has gold and silver links in the ratio 2 : 7. Which type of link is used most?

1 What are the missing numbers?
   a \( \frac{3}{5} \) \( \square \) \( \frac{10}{\square} \)
   b \( \frac{1}{\square} \) 1 whole = \( \frac{\square}{5} \)
   c \( \frac{1}{\square} + \frac{\square}{6} + \frac{\square}{6} \)
   d \( \frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square} \)

2 Work out
   a \( \frac{2}{3} \) + 3.5
   b \( 1 - \frac{\square}{9} \)
   c \( \frac{3}{4} \times 100 \)
   d \( \frac{\square}{\square} \times 20 \)

3 Simplify these ratios.
   a \( \frac{2}{2.5} : \frac{1.5}{\square} \)
   b \( \frac{5.5}{\square} : \frac{4.5}{\square} \)
   c \( \frac{3}{\text{litres}} : \frac{500}{\text{ml}} \)
   d \( \frac{70}{\text{cm}} : \frac{1}{\text{m}} \)

Key point 5
A proportion compares a part with the whole.

4 In a canoeing lesson \( \frac{3}{5} \) of the group are girls and the rest are boys.
   What is the ratio of girls to boys in the group?

5 Copy and complete the table for different groups of teenagers having canoeing lessons.

<table>
<thead>
<tr>
<th>Fraction of group that are girls</th>
<th>Ratio of girls : boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{5} )</td>
<td>3 : 2</td>
</tr>
<tr>
<td>( \frac{1}{\square} )</td>
<td>( \frac{4}{7} )</td>
</tr>
</tbody>
</table>

ActiveLearn Homework, practice and support: Foundation 11.5
6 STEM In a gluten-free pizza base, \(\frac{7}{12}\) of the flour is rice flour, \(\frac{3}{12}\) is potato flour and the rest is tapioca flour.
Work out the ratio of rice flour : potato flour : tapioca flour.

7 STEM / Real A 5p coin used to be made of copper and nickel in the ratio 3 : 1.
What fraction of the coin was a copper b nickel?

8 Clare and Fiona share a cash prize in the ratio 4 : 3.
What fraction of the prize should a Clare get b Fiona get?

9 STEM / Real A 20p coin is made of copper and nickel in the ratio 21 : 4.
What fraction of the coin is a copper b nickel?

10 The ratio of office staff to manufacturing staff at a company is 2 : 5.
Jo says that \(\frac{2}{5}\) of the people are office staff.
Is she correct? Explain your answer.

11 STEM / Real Red brass is made from a mix of copper, zinc, lead and tin in the ratio 17 : 1 : 1 : 1.
a What fraction of red brass is i copper ii zinc?
b In 100 g of red brass, what are the masses of copper, zinc, lead and tin?
c Lily has plenty of copper and zinc, but only 80 g of lead and 60 g of tin.
What is the maximum amount of red brass she can make?
d Reflect Look back at your working for part c.
Have you clearly shown how you got your answer?

12 STEM A topping for shortbread uses milk, chocolate, butter and syrup in the ratio 4 : 3.5 : 1.5 : 1.
a What fraction of the topping is chocolate?
b In 500 g of the topping what are the masses of milk, chocolate, butter and syrup?
c You have plenty of milk and chocolate, but only 200 g of butter and 100 g of syrup.
What is the maximum amount of topping you can make?

13 Copy and complete to write these as unit ratios.
\(\frac{5}{3}\) \(\frac{2}{7}\)

14 Write each of these in the form \(m : 1\).
Give each answer to a maximum of 2 decimal places.
a 2 : 5 b 7 : 4 c 16 : 9 d 5 : 36 e 9 : 42 f 11 : 56

Example 5
Molly makes a blackcurrant drink by mixing 30 ml of blackcurrant with 450 ml of water.
Hope makes a blackcurrant drink by mixing 40 ml of blackcurrant with 540 ml of water.
Whose drink is the stronger? Explain your answer.
Molly Hope
blackcurrant : water

\(\frac{30}{450}\) \(\frac{40}{540}\)

Simplify to a unit ratio.
Compare the quantity of water per ml of blackcurrant.

Explain your answer.

15 Anna makes orange squash by mixing 50 ml of squash with 850 ml of water.
Jeevan makes orange squash by mixing 60 ml of squash with 1110 ml of water.
Whose squash is the stronger? Explain your answer.

16 Josh makes purple paint by mixing 2 litres of red paint and 500 ml of blue paint.
Dexter makes purple paint by mixing 1.5 litres of red paint and 400 ml of blue paint.
Whose paint is the darker purple? Explain your answer.

17 Raj makes concrete using aggregate to cement in the ratio 1930 : 265.
Sunil makes concrete using aggregate to cement in the ratio 935 : 175.
Whose concrete has the higher proportion of cement?

18 Billy makes mortar by mixing 360 kg of cement with 90 kg of sand.
Sam makes mortar by mixing 340 kg of cement with 85 kg of sand.
Whose mortar has the higher proportion of cement?

19 Exam-style question
STEM Electrum is \(\frac{2}{5}\) gold and \(\frac{7}{5}\) silver.
Jessica says, ‘The amount of silver is one and a half times the amount of gold.’
Is she correct? Explain your answer. (3 marks)

Exam hint Explain your workings and answer by writing your reasons alongside your calculations rather than listing them at the end.
**Unit 11 Ratio and proportion**

### 11.6 Using proportion

**Objectives**
- Use the unitary method to solve proportion problems.
- Solve proportion problems in words.
- Work out which product is better value for money.

**Fluency**
In each pair, which number is bigger?
- 3.28 or 3.7?
- 2.9 or 2.45?
- 1.06 or 1.009?

**Objectives**
- Use the unitary method to solve proportion problems.
- Solve proportion problems in words.
- Work out which product is better value for money.

**Why learn this?**
Working out which product is the better buy helps you get the best value for money.

1. Work out
   - a) $840 \div 7$
   - b) $450 \times 9$
   - c) $2.7 \times 5$
   - d) $4.8 \div 3$
   - e) $10 \times \square = 20$
   - f) $10 \div \square = 5$
   - g) $5 \div \square = 1$
   - h) $7.2 \div \square = 1$

2. Write each of these as a unit ratio.
   - a) $4 : 18$
   - b) $11 : 5$
   - c) $14 : 35$
   - d) $18 : 8$

3. What is the HCF of each pair of numbers?
   - a) 500 and 350
   - b) 600 and 800

4. Which is better value if you are shopping?
   - a) Paying 1.8p per gram or 2.1p per gram?
   - b) Getting 3.25 ml for 1p or 2.85 ml for 1p?

5. A recipe for 4 people uses 6 eggs. How many eggs are needed for
   - a) 8 people
   - b) 2 people
   - c) 6 people
   - d) 10 people?

6. 5 tickets to a theme park cost £125. How much will 18 tickets cost?

**Example 6**
A recipe for 6 people uses 900 g of mince. How much mince is needed for
- a) 12 people
- b) 3 people
- c) 9 people?

**Discussion**
How would you work out the amounts for 18 people and 15 people?

**Key point 7**
In the unitary method you find the value of one item before finding the value of more.

7. 20 litres of fuel cost £25.38. What do 35 litres cost?

8. Maria gets £31.55 for working 5 hours. How much will she get for working for
   - a) 9 hours
   - b) 26 hours
   - c) 30 hours?

**Reflect**
Did you do part c the same way as parts a and b? Can you show a quicker way?

9. A gym coach orders a tracksuit for each of the 25 gymnasts in the club. The total cost is £1247.50. 2 more gymnasts join the club so he orders an extra 2 tracksuits. What is the total value of the order now?

**Key point 8**
You can use the unitary method to work out which product gives better value for money.

10. Washing powder comes in two sizes.
    - a) Copy and complete to find the unit ratios.
        - i) $1.3 \text{ kg : } £4.40$
        - ii) $2.6 \text{ kg : } £6$
    - b) Which size is the better buy?

11. Jenna can buy her favourite blend of coffee in two sizes: 500 g for £14.90 or 300 g for £8.85. Which is better value for money?

**Discussion**
How did you work it out?

12. David can buy blackcurrant squash in two sizes:
    - 2 litres for £4.99 or 600 ml for £1.98.
    - Which is better value for money?

13. Mary gets paid £31.50 for 5 hours of work. Alice gets paid £50 for 8 hours of work. They both work 36 hours a week. Who earns more money?

14. **Exam-style question**
    Brenda works in an office. She finds out the prices of folders from two companies, Office Deals and Paper World.
    Brenda needs to buy exactly 60 folders. She wants to buy the folders as cheaply as possible.
    Which company should Brenda buy the folders from?
    You must explain your answer.

**Q6 hint** Find the cost of 1 ticket first.

**Exam hint** Highlight or underline key information in a question. What information not given in the boxes should you highlight here?
11.7 Proportion and graphs

Objectives
- Recognise and use direct proportion on a graph.
- Understand the link between the unit ratio and the gradient.

Why learn this?
Knowing simple relationships allows us to make predictions.

Fluency
- What is the equation of a straight line through the origin (0, 0) with gradient 5?
- What is the gradient of the line y = 7x?

1 The graph changes between gallons and pints. Use the graph to work out these conversions.
   a 5 gallons = \( \underline{\text{pints}} \)
   b 9 gallons = \( \underline{\text{pints}} \)
   c 120 pints = \( \underline{\text{gallons}} \)

2 Plot a line graph for the values in the table.
   a What is the gradient of the line?
   b What is the equation of the line?

3 Look at the graph in Q1. Are gallons and pints in direct proportion? Explain your answer.

4 Modelling The table shows some temperatures in both Celsius and Fahrenheit.

<table>
<thead>
<tr>
<th>Temperatures</th>
<th>Celsius</th>
<th>Fahrenheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(^\circ)</td>
<td>12(^\circ)</td>
<td>12(^\circ)</td>
</tr>
<tr>
<td>10(^\circ)</td>
<td>50(^\circ)</td>
<td>50(^\circ)</td>
</tr>
<tr>
<td>20(^\circ)</td>
<td>90(^\circ)</td>
<td>90(^\circ)</td>
</tr>
<tr>
<td>25(^\circ)</td>
<td>140(^\circ)</td>
<td>140(^\circ)</td>
</tr>
</tbody>
</table>

a Plot a line graph on the horizontal axis and Fahrenheit on the vertical axis. Use sensible scales.

b Are Celsius and Fahrenheit in direct proportion? Explain your answer.

c Ice melts at 0\(^\circ\)C. What is this temperature in Fahrenheit?

Key Point 9
When two values are in direct proportion, if one value is zero so is the other. When one value doubles, so does the other.

Key Point 10
When two quantities are in direct proportion, plotting them as a graph gives a straight line through the origin. The origin is the point (0, 0) on a graph.

5 Modelling The table shows the price of pears at a market.

<table>
<thead>
<tr>
<th>Mass of pears (kg)</th>
<th>0.5</th>
<th>1</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of pears (£)</td>
<td>1</td>
<td>2</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

a Plot a line graph for these values.
b Is the price of the pears in direct proportion to the mass? Explain your answer.

6 Modelling This line graph shows the price of grapes by mass.

a Are price and mass in direct proportion?
b Work out the gradient of the line.
c How much does 1 kg of grapes cost?

Discussion How does your answer to b help you answer c?

7 Reasoning Which of these are in direct proportion? Give reasons for your answers.
   a Inches and centimetres
   b Pounds (lb) and kilograms
   c Age and favourite music type
   d Number of teens in swimming pool and time of day
   e Euros and pounds

8 A plumber charges a callout fee of £50 plus £35 per hour he works. Is his total charge \( C \) in direct proportion to the number of hours \( h \) he works?

9 Which of these sketch graphs show one variable in direct proportion to another? Explain your answer.

10 Modelling The table shows the amount paid for different numbers of litres of fuel on one day.

<table>
<thead>
<tr>
<th>Number of litres (( n ))</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (( C ))</td>
<td>£6.60</td>
<td>£13.20</td>
<td>£26.40</td>
<td>£39.60</td>
<td>£66</td>
</tr>
</tbody>
</table>

a Draw the graph.
b Work out the equation of the line.
c Are \( C \) and \( n \) in direct proportion?
d Write a formula linking the number of litres (\( n \)) and the cost (\( C \)).
e Use the formula to work out the cost of 82 litres of fuel on that day.

Reflect Look at the equation of the line and the formula for \( C \) and \( n \). What is the same? What is different?
11 Modelling  Look back at the graph in Q1.
   a  Write a formula that links gallons (G) and pints (P).
   b  Use your formula to change 16.5 gallons to pints.

12 Modelling  The ratio of miles to km is 5 : 8.
   a  Copy and complete this table of values for miles and kilometres.
      Miles  0  5  10
      Kilometres
   b  Draw a conversion graph for miles to kilometres.
   c  Write a formula linking kilometres (y) and miles (x).
   d  Write 5 : 8 as a unit ratio.
   e  What do you notice about the unit ratio and the gradient of the line?

13 Exam-style question
   STEM / Problem-solving  A formula to change between temperature in kelvins (K) and in degrees Celsius (C) is
   \[ K = C + 273.15. \]
   Are kelvins and degrees Celsius in direct proportion?
   Explain your answer.  (2 marks)

11.8 Proportion problems

Objectives
   • Recognise different types of proportion.
   • Solve word problems involving direct and inverse proportion.

Why learn this?
   A manager needs to understand proportion to allocate the right number of people to a job.

Fluency
   Will 5 men take more or less time to build a wall than 3 men?

1 Work out
   a  £20 – 8  
   b  £36 – 5  
   c  £2.45 × 3  
   d  £5.90 × 3

2 How many hours and minutes in
   a  315 minutes  
   b  200 minutes  
   c  2\frac{1}{2} hours  
   d  4\frac{1}{3} hours?

Key point 11
   When two values are in inverse proportion, one increases at the same rate as the other decreases. For example, as one doubles (+2) the other halves (–2).

Example 7
   It takes 2 painters 7 days to paint a house. How many days does it take 1 painter to paint an identical house?
   2 people take 7 days, so 1 person takes \(2 \times 7 = 14\) days.
   It takes 1 person twice (×2) as long as 2 people.

Explain your answer.  (2 marks)

Q3a hint  Will it take 1 man more or less time than 3 men?
Q4 hint  60 minutes = 1 hour
Q5 hint  Do you expect them to cost more or less?
Q6 strategy hint  How many could 1 person transplant in 7 hours?
Q9 strategy hint  How many chickens does he have to feed?

Q12a hint  Are miles and km in direct proportion?
Q12b hint  Put miles on the x-axis and km on the y-axis.

Q14a hint  Give your question to a classmate. Mark their answer. Did they show their working clearly?

Q15 strategy hint  Start by deciding if you expect it to take more or less time. This will tell you whether it is direct or inverse proportion.

ActiveLearn  Homework, practice and support: Foundation 11.8
Unit 11 Ratio and proportion

11 Problem-solving

Objective  • Use bar models to help you solve problems.

Example 8

Miguel and Jean share a tin of biscuits in the ratio 3 : 5. Jean has 6 more biscuits than Miguel. How many biscuits were in the tin?

- Draw a rectangular bar to represent all the biscuits in the tin.
- Miguel
- Jean

- Split the bar into the ratio 3 : 5 for Miguel and Jean.

- Compare the bars for Miguel and Jean.

- Label difference between the bars as 6 biscuits.

- 2 sections represent 6 biscuits.

- Work out 1 section.

- 8 sections represent all the biscuits.

- Check: Miguel has: 3 × 3 = 9 biscuits

- Jean has: 3 × 5 = 15 biscuits

- Jean has 15 − 9 = 6 more biscuits

- Total number of biscuits = 9 + 15 = 24 biscuits

Example 9

In an orchestra, 10% of the musicians play brass instruments, 15% play woodwind and 5% play percussion. 28 musicians play stringed instruments. How many musicians are in the orchestra altogether?

- Draw a bar to represent all the musicians in the orchestra.

- Split the bar into 10% sections.

- One section = musicians.

- Total number of musicians =

Example 10

The sizes of the angles in a triangle are in the ratio 1 : 3 : 5. Sketch the triangle and mark the angles in degrees.

- Draw a bar to represent the sum of all the angles in a triangle. Split the bar into sections in the ratio 1 : 3 : 5. One section =

Example 11

Hilary and Ruth share a flat with a monthly rent of £885. Hilary’s bedroom is twice the size of Ruth’s, so she pays twice as much rent as Ruth. How much do they each pay?

- £

- Hilary

- Ruth

Example 12

Amjit buys 3 bunches of tulips and 1 bunch of roses. Bunches of roses cost twice as much as bunches of tulips. Amjit hands over a £20 note and receives £5 change. How much is a bunch of roses?

- Tulips

- Tulips

- Tulips

Example 13

Toby and Isy share some money in the ratio 5 : 7. Isy gets £60 more than Toby. How much money does Toby get?

- Draw a bar to show the ratio 5 : 7.

- Label the sections. How many sections represent £60?

- One section =

Example 14

A farmer has sheep, cows and pigs in the ratio 12 : 3 : 2. The farmer has 150 more sheep than pigs. How many cows has she got?

- Draw a bar to represent all the children.

- Split the bar into sections that show the numbers of boys and girls.

- Work out how many lengths the boys swam in total and how many lengths the girls swam in total.

- Work out how many lengths the children swam altogether.

Example 15

8 boys and 12 girls go to swimming lessons. In one lesson, the mean number of lengths swum by the boys is 4.5, and the mean number of lengths swum by the girls is 2. Work out the mean number of lengths swum by all the children in that lesson.

- Reflect How did drawing bar models help you? Is this a strategy you would use again to solve problems?

Check up

Simple proportion and best buys

1 A recipe for 4 people uses 150 g of sugar. How much sugar is needed for

- a 8 people

- b 2 people

- c 6 people?

2 Helen can buy 300 ml of shampoo for £3.50 or 75 ml of shampoo for £1. Which is better value for money?

Ratio and proportion

3 A necklace has 24 beads. There is 1 red bead for every 3 blue beads. How many beads are

- a red

- b blue?

4 Write each ratio in its simplest form.

- a 24 : 32

- b 27 : 18 : 54

- c 2.5 : 3.5

ActiveLearn Homework, practice and support: Foundation 11 Check up  Draft, subject to endorsement
5 Copy and complete these conversions.
   a  4.2 km = \[\text{m}\]
   b  0.05 litres = \[\text{ml}\]
   c  £200 = \[\text{euros}\] (\(£1 = 1.23\text{euros}\))
   d  25 km = \[\text{miles}\] (8 km = 5 miles)

6 Iona makes a scale model of a sailing boat using the ratio 1 : 25.
   Her model is 60 cm long.
   How long is the real sailing boat? Give your answer in metres.

7 Hazel mixes pink paint using red paint and white paint in the ratio 1 : 4.
   How much white paint should she use with 5 tins of red paint?

8 Naadim and Bal share £60 in the ratio 2 : 3.
   a  How much do they each receive?
   b  What fraction of the total amount does Naadim receive?
   c  Show how you checked your answer is correct.

9 Write each of these as a unit ratio. Give your answers to a maximum of 2 decimal places.
   a  8 : 5
   b  7 : 19

10 The ratio of primary school children to secondary school children in a club is 2 : 3.
    There are 12 primary school children in the club.
    a  How many secondary school children are there?
    b  What is the total number of children in the club?

11 Luke makes lemon squash using 40 ml of squash and 380 ml of water.
    Joseph makes lemon squash using 50 ml of squash and 475 ml of water.
    Who has made the stronger squash? Explain your answer.

12 The table shows the number of Barbadian dollars you get for different numbers of US dollars.
    | US dollars | 5 | 10 | 50 |
    | Barbadian dollars | 10 | 20 | 100 |
    a  Draw a conversion graph with US dollars on the x-axis and Barbadian dollars on the y-axis.
    b  Are US dollars and Barbadian dollars in direct proportion?
    c  Write a formula linking US dollars and Barbadian dollars.

13 It takes 4 people 3 days to lay the cables for high-speed fibre broadband.
    How long would it take
    a  1 person
    b  6 people?

14 It takes 3 machines 4 minutes to print 600 pages.
    How long will it take 8 machines to print 600 pages?

15 How sure are you of your answers? Were you mostly
    Just guessing 😞 Feeling doubtful 😞 Confident 😊
    What next? Use your results to decide whether to strengthen or extend your learning.

16 a  How many different ways can you divide this rectangle into two parts in the ratio 1 : 3?

   b  How many of your answers show a symmetrical pattern?

11 Strengthen

Simple proportion and best buys
1 A bus ticket costs £3. Work out the cost of
   a  2 tickets
   b  5 tickets.

2 It costs £32 for 4 teenagers to go to the cinema.
   How much does it cost for
   a  8 teenagers
   b  2 teenagers
   c  10 teenagers?

3 Harry can wash 6 cars in 1 hour.
   How many cars can he wash in 3\(\frac{1}{2}\) hours?

4 Which is better value for money:
   200 g of lotion for £2.50 or
   300 g of lotion for £3.60?

Ratio and proportion
1 Andie makes a bracelet with green beads and yellow beads.
    She uses 1 green bead for every 2 yellow beads.
    The bracelet has 12 beads altogether.
    How many beads are
    a  green
    b  yellow?
2 Write each ratio in its simplest form.
   a  2 : 6
   b  6 : 9
   c  10 : 15
   Q2b hint You cannot divide by 2. Try dividing by 3.
   d  24 : 36 : 60
   e  2.5 : 6
   f  2.4 : 1.6
   Q2e hint Choose a number to multiply by to give whole numbers first.

3 Complete these conversions.
   a  3.8 m = [cm]
   b  2.5 litres = [ml]
   c  £600 = [euros (£1 = 1.23 euros)]
   d  24 pints = [gallons (1 gallon = 8 pints)]
   Q3a hint
   Q3c hint

4 Jim mixes red paint and blue paint in the ratio 1 : 5.
   He uses 3 tins of red paint.
   How much blue paint should he use?

5 Jenny makes a scale model of a train carriage using the ratio 1 : 20.
   Her model is 24 cm long.
   a How long is the real train carriage in cm?
   b Write the real length in metres.

6 The ratio of butter to sugar in a recipe is 2 : 1.
   Ian uses 6 ounces of butter.
   How many ounces of sugar should he use?

7 Isabel and Freya share £20 in the ratio 2 : 3.
   a How much do they each receive?
   b Show how you have checked your answer.
   Q7b hint Add your answers for Isabel and Freya's shares together.

8 A piece of cloth is cut into three pieces in the ratio 1 : 2 : 5.
   The piece of cloth is 240 cm long.
   a How long is each piece?
   b Show how you have checked your answer.

9 A shop sells jam doughnuts and toffee doughnuts.
   The ratio of jam doughnuts to toffee doughnuts sold is 2 : 3.
   The shop sold 18 jam doughnuts.
   a How many toffee doughnuts were sold?
   b How many doughnuts were sold altogether?

10 The ratio of quad bikes to go-karts at an activity centre is 3 : 7.
    a What fraction of the vehicles are go-karts?
    b What fraction of the vehicles are quad bikes?

11 ¾ of the bikes at a cycle shop are road bikes.
    The rest are mountain bikes.
    a What is the ratio of road bikes : mountain bikes?
    b There are 27 road bikes. How many mountain bikes are there?
    c How many bikes are there in the cycle shop altogether?

12 Write each of these as a unit ratio. Give your answers to a maximum of 2 decimal places.
   a  4 : 1
   b  25 : 2
   c  13 : 5
   d  7 : 22

13 Luke makes lemon squash using 20 ml of squash and 180 ml of water.
    Ben makes lemon squash using 50 ml of squash and 500 ml of water.
    Who has made the stronger squash? Explain your answer.

Proportion, graphs and inverse proportion

1 The sketch graph shows an object moving at a constant speed.
   Distance (miles) 0
   Time (minutes)  
   a Is this graph a straight line?
   b Does the graph go though (0, 0)?
   c Is the distance travelled in direct proportion to the time taken?
   Q1c hint Did you answer ‘Yes’ to parts a and b?

2 An electrician charges a callout fee of £60 and £30 per hour she works.
   Hours worked 0 1 2 3
   Total charge (£) 60 90 120 150
   a Draw a graph for the table of her charges.
   b Is her total charge in direct proportion to the hours she works?
   Q2b hint Answer Q1a and Q1b for the graph you have drawn.
3 A shop sells material for bridesmaid dresses at £4 per metre. Is the price of material proportional to the length bought? Answer Q1a and Q1b for the graph you have drawn.

4 The ratio of gallons to pints is 1 : 8. Write a formula that shows the relationship between gallons (g) and pints (p).

5 It takes 3 people 4 hours to lay some pipes. Copy and complete the table.

<table>
<thead>
<tr>
<th>Number of people</th>
<th>Longer or shorter?</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6 It takes 4 washing machines 12 hours to wash the hotel laundry. How long will it take 6 machines?

7 The lengths of the sides of a triangle are in the ratio 3 : 4 : 5. The perimeter of the triangle is 18 cm. What is the length of the longest side?

8 Real Potting compost is made using loam, peat and sand in the ratio 7 : 3 : 2. Molly uses $\frac{3}{2}$ litres of sand.

9 Sunil and Karl shared some money in the ratio 3 : 5. Sunil gave $\frac{1}{4}$ of his money to Lucas. Lucas got £12.50. How much money was shared originally by Sunil and Karl?
10 Modelling  3 boys did some gardening. They earned £129.60. They shared the money in the ratio of the number of hours they each worked. Ollie worked for $\frac{9}{2}$ hours, Sam worked for $\frac{8}{4}$ hours and Peter worked for $6 \frac{1}{4}$ hours. How much did each boy receive?

11 Modelling  Adam is tiling a room. He can fit 80 tiles in 1 hour. He takes a 15-minute morning break and $\frac{3}{4}$ hour for lunch. He has to fit 540 tiles. He starts work at 8:30 am. At what time will he finish work?

12 A box contains 480 counters. There are twice as many red counters as yellow counters. There are three times as many blue counters as red counters. The rest of the counters are green. How many green counters are in the box?

13 The sizes of the angles in a quadrilateral are in the ratio 2 : 4 : 5 : 7. What size is the largest angle?

14 Reasoning / Problem-solving  Sasha wants to buy a new camera. She can either buy it from a local dealer or order it from the USA online.

<table>
<thead>
<tr>
<th>Option</th>
<th>Cost (in £)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local dealer</td>
<td>£399</td>
</tr>
<tr>
<td>Imported from USA</td>
<td>£529 + taxes and duties of 20%</td>
</tr>
</tbody>
</table>

Both prices include postage. The exchange rate is £1 = $1.68. She wants to pay the cheaper total cost. Which option should she choose?

15 The chart shows the proportions of sand and cement needed to mix mortar for brickwork. A builder uses 78 kg of cement to make his mortar. How much sand should he use?

16 50 people go on a trip to the cinema. The ratio of adults to teenagers on the trip is 2 : 3. 30 per cent of the adults are senior citizens. The table shows the ticket prices. What is the total cost for all 50 people to visit the cinema?

11 Knowledge check

- A ratio is a way to compare two or more quantities. .............................................. Mastery lesson 11.1
- You simplify a ratio by making the numbers as small as possible. Divide the numbers in the ratio by their highest common factor (HCF). ................................................................. Mastery lesson 11.1
5  At Mini Mart, 4 tubs of ice cream cost £9.40.  
At Dave’s Deli, 3 tubs of the same ice cream cost £7.50.  
At which shop is ice cream better value for money? (3 marks)

6  A school with 900 students has 150 computers. Write the ratio of the number of students to the number of computers in the form $m : 1$. (1 mark)

7  There are 250 marbles in a tin.  
40 marbles are metal, 60 are stoneware and the rest are glass or china.  
There are twice as many glass marbles as china marbles.  
How many are  
\( a \) china  
\( b \) glass? (4 marks)

8  Sam, Jack and Ali share £45 in the ratio 2 : 3 : 4.  
a  What fraction does Sam get?  
b  How much does Ali get? (4 marks)

9  James, Isaac and Lucas share £30 in the ratio of their ages.  
James is 10 years old, Isaac is 8 years old and Lucas is 7 years old.  
Isaac gives a third of his share to his Dad.  
How much money does Isaac have now? (4 marks)

10  Here are the ingredients needed to make a beef pie for 6 people.  
a  How much beef is needed to make a beef pie for 15 people? (3 marks)  
b  If you have 5 eggs and plenty of the other ingredients, can you make a beef pie for 8 people? (3 marks)

11  A company makes fruit drinks.  
A machine fills crates of 12 bottles with drink. In 1 hour the machine can fill 18 000 bottles.  
How many seconds does the machine take to fill a crate of 12 bottles? (4 marks)

12  The graph shows a car moving at a constant speed.  
\[ \text{Distance–time graph} \]

\[ \text{a} \) Does the graph show distance and time in direct proportion? Explain your answer. (1 mark)  
\[ \text{b} \) Write down the equation of the line. (1 mark)

13  3 machines can print a batch of leaflets in 2 hours.  
How long would it take 4 machines to print the same batch of leaflets? (3 marks)

14  **Exam-style question**  
Each day a company posts some small letters and some large letters.  
The company posts all the letters by first class post.  
The tables show information about the cost of sending a small letter by first class post and the cost of sending a large letter by first class post.

\[ \begin{array}{|c|c|} \hline \text{Weight} & \text{First class post} \\ \hline 0–100 g & 60p \\ 101–250 g & £1.00 \\ 251–500 g & £1.50 \\ 501–750 g & £2.50 \\ \hline \end{array} \]

One day the company wants to post 200 letters.  
The ratio of the number of small letters to the number of large letters is 3 : 2  
70% of the large letters weigh 0–100 g.  
The rest of the large letters weigh 101–250 g.  
Work out the total cost of posting the 200 letters by first class post. (5 marks)

---

**Sample student answers**

**Exam-style question**

Potatoes cost £9 for a 12.5 kg bag at a farm shop.  
The same type of potatoes cost £1.83 for a 2.5 kg bag at a supermarket.  
Where are the potatoes the better value, at the farm shop or at the supermarket? You must show your working. (4 marks)

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**Exam hint**

Remember that there are marks for showing your method and for communicating your answer clearly, as well as for getting the answer right.
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