1.1 Accuracy and bounds

Objectives

- Identify upper and lower bounds of continuous data measured to whole number values and decimal values.
- Solve problems involving upper and lower bounds.

Key point 1

Measurements rounded to the nearest unit could be up to half a unit smaller or larger than the rounded value. The possible values of \( x \) that round to 43.8 grams, to 1 d.p., are 43.75 \( \leq x < 43.85 \). 43.75 and 43.85 are known as the lower bound (lb) and upper bound (ub) respectively.

Key point 2

For a sum or a difference of two quantities, use these rules:

- Lower bound \((A + B) = lb(A) + lb(B)\)
- Upper bound \((A + B) = ub(A) + ub(B)\)
- Lower bound \((A - B) = lb(A) - ub(B)\)
- Upper bound \((A - B) = ub(A) - lb(B)\)

For a product or a quotient of two quantities, use these rules:

- Lower bound \((A \times B) = lb(A) \times lb(B)\)
- Upper bound \((A \times B) = ub(A) \times ub(B)\)
- Lower bound \((A \div B) = lb(A) \div ub(B)\)
- Upper bound \((A \div B) = ub(A) \div lb(B)\)

1. Each of these measurements has been rounded to the accuracy stated. Write an inequality to show its smallest and largest values. Use \( x \) for the measurement.
   a. 24 cm (to the nearest centimetre)
   b. 125 m (to the nearest 5 metres)
   c. 9.8 seconds (to 1 d.p.)
   d. 8.46 kg (to 3 s.f.)
2 Write the lower and upper bounds for each of these measurements.
   a 6 m (to the nearest metre)
   b 70 mm (to the nearest 10 mm)
   c 4.9 litres (to 1 d.p.)
   d 19.76 seconds (to 2 d.p.)

3 A plank of wood measures 2 metres, to the nearest cm.
   A piece of length 83.5 cm, correct to 1 d.p. is sawn off.
   What is the minimum length of the piece of wood that is left?
   **Q3 hint** Minimum length remaining = minimum original length – maximum length sawn off

4 **Reasoning** A football pitch measures 106 m by 78 m, with both lengths measured to the nearest metre.
   Work out the maximum perimeter of the pitch.

5 **Reasoning** A rectangle has a length of 8.5 cm and a width of 5.2 cm, both given to the nearest millimetre.
   Work out the lower and upper bounds for the area of the rectangle.
   **Q5 hint** Use the rules for working out the lower and upper bounds of a product.

6 **Reasoning** A vehicle is travelling at a constant speed of 23.6 m/s, correct to 1 d.p.,
   for 18 seconds, correct to the nearest second.
   Work out the upper bound for the distance it will travel.

7 **Reasoning** A cube has a side length of 7.2 cm, correct to 1 d.p.
   What is the smallest possible volume of the cube?

8 **Problem-solving** There are 12 identical marbles in a bag.
   The bag weighs 16 g, correct to the nearest gram.
   Each marble weighs 67 g, correct to the nearest gram.
   What is the difference between the minimum and maximum possible weights of the bag of marbles?

9 **Reasoning** \(x = 23.6, \ y = 9.4\) and \(z = 18.7\), all correct to 1 d.p.
   a Work out the minimum value of \(2x - 3y\)
   b Work out the maximum value of \(z^2 - xy\)

10 **Reasoning** A gold bar has a volume of 12.4 cm\(^3\),
    correct to 1 d.p. and a mass of 239 g, correct to 3 s.f.
    Write an inequality to show the minimum and maximum values of the density. Use \(d\) for the density.
    Give your answer correct to 2 d.p.
   **Q10 hint** This is a division. Check the rules for maximum and minimum values.

11 **Problem-solving** A triangle has an area of 48 cm\(^2\), to the nearest whole number, and a base of 14.3 cm, to 1 d.p.
   Work out the minimum value of the height of the triangle.
   Give your answer correct to 2 d.p.
   **Q11 hint** Rearrange the formula for the area of a triangle before you substitute.

12 **Reasoning** \(x = 8.1\) (1 d.p.), \(y = 230\) (2 s.f.) and \(z = 13.9\) (1 d.p.).
   Work out the minimum and maximum values of \(\frac{y}{x^2 - z}\)
   Give your answers correct to 2 d.p.
1.2 Number problems and reasoning

Objective
• Work out the total number of ways of performing a series of tasks.

Key point 3
When choice A can be made in \( m \) ways and choice B can be made in \( n \) ways, the total number of ways of choosing A then B is \( m \times n \)

A factorial is the result of multiplying a sequence of descending integers.
The \( x! \) button on your calculator is the ‘factorial’ button. For example, \( 4! = 4 \times 3 \times 2 \times 1 = 24 \)

1 Problem-solving A key pad has the digits 0 to 9 on it.
   a How many ways can you select a 4-digit code if repetitions are allowed?
   b How many ways can you select a 4-digit code if repetitions are not allowed?

2 Problem-solving Tom wants to make a code for his tablet.
   He decides to select two letters and three digits. He has a choice of the letters A, B, C and D, together with the digits 1 to 5.
   Repeat letters and/or digits are not allowed.
   a How many possible codes are there?
   b He decides to select two odd digits and one even digit.
      How many codes are now possible?
   c How many codes would be possible if repetitions were allowed?

Q2 hint Work out how many ways the letters can be chosen, then how many ways the digits can be chosen, using the \( m \times n \) rule.

3 Problem-solving Car registration plates used to consist of three letters, followed by three digits, followed by one letter.
   How many different registration plates were possible if repetitions were not allowed in the first three letters or in the three digits?

4 Problem-solving You can use the digits 1, 2, 3, 5, 7 and 8.
   a How many 3-digit numbers can you make if repetitions are not allowed?
   b How many of these 3-digit numbers will be odd?
   c How many of these 3-digit numbers will be less than 200?

Q4b hint Select the last digit first because you need the number to be odd. How many ways can you do this? Then select the other digits.
Q4c hint For the number to be less than 200, what digit must it start with? Then select the other digits.

5 Problem-solving A restaurant offers 4 starters, 6 main courses and 5 desserts.
   a If customers can choose from all three courses, how many ways can this be done?
   b If customers can choose either a starter and a main course or a main course and a dessert, how many different options are possible?

6 Reasoning There are six points, A, B, C, D, E and F, on the circumference of a circle.
   How many chords (for example, AD) can be drawn?

Q6 hint Remember that AB is the same chord as BA.

7 Problem-solving The letters of the word TRIANGLE are rearranged at random.
   In how many of the possible arrangements will the first and last letters be vowels?

8 Problem-solving Tess has to make a 3-digit number using the digits 1, 2, 3, 4, 5, 6 and 7.
   Repetitions are not allowed.
   a How many ways can she make a number between 200 and 500?
   b How many ways can she make an even number greater than 600?
1.3 Indices, roots and surds

**Objectives**

- Use index laws, including negative and fractional indices.
- Apply laws of indices to work out powers and roots of numbers written in index notation.
- Apply laws of indices to work out positive and negative fractional powers of whole numbers.
- Use surds to give exact answers to calculations.
- Simplify expressions involving surds, including expanding brackets and rationalising the denominator.

**Key point 4**

Use these **rules of indices** to multiply, divide and work out a power of a power:

\[ x^m \times x^n = x^{m+n} \quad \frac{x^m}{x^n} = x^{m-n} \quad (x^m)^n = x^{mn} \]

Use these rules of indices to work out zero, negative and fractional powers:

\[ x^0 = 1 \quad x^{-n} = \frac{1}{x^n} \quad x^{\frac{1}{n}} = \sqrt[n]{x} \quad x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m \]

**Key point 5**

Use these rules to multiply and divide **surds**:

\[ \sqrt{mn} = \sqrt{m} \times \sqrt{n} \quad \sqrt[\frac{m}{n}]{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}} \]

Use these methods to **rationalise a denominator** to give the denominator as an integer:

\[ \frac{a}{\sqrt{b}} = \frac{a \sqrt{b}}{b} \quad \frac{1}{a \sqrt{b}} = \frac{\sqrt{b}}{ab} \quad \frac{c}{d + \sqrt{e}} = \frac{c d - \sqrt{e}}{d^2 - e} \]

1. **Warm up**

   **Q1f hint** Rewrite the roots as a fractional index before you simplify.

   \[ \sqrt{\frac{5}{8}} = \left(\frac{5}{8}\right)^{\frac{1}{2}} = \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{2\sqrt{2}} \]

2. **Q2a hint** (x - 3)(x + 8)

   \[ (x - 3)(x + 8) = x^2 + 5x - 24 \]

3. **Work out the value of**

   a) \( 2^{-\frac{1}{2}} \times 2^{\frac{1}{2}} \)
   b) \((3^{-1})^2\)
   c) \(3^4 \div 3^{-1}\)
   d) \((5^{-3})^{-1}\)
   e) \((2^{-1})^2\)
   f) \(\sqrt{5^8}\)
   g) \(\sqrt[3]{10^2}\)
   h) \(\sqrt[4]{32}\)

4. **Work out the value of**

   a) \(16^\frac{1}{2}\)
   b) \((-216)^\frac{1}{3}\)
   c) \(64^\frac{1}{2}\)
   d) \(625^\frac{1}{3}\)

5. **Work out the value of**

   a) \(81^{-\frac{1}{2}}\)
   b) \(25^{-1.5}\)
   c) \(343^{-\frac{1}{3}}\)
   d) \((-64)^\frac{1}{3} \times 3^{-2}\)
   e) \(100^{1.5} \times 5^{-3}\)