Edexcel GCSE (9-1) Mathematics
Higher Student Book

Confidence • Fluency • Problem-solving • Reasoning

Sample unit

Draft, subject to endorsement

We are seeking endorsement for use with the Edexcel GCSE (9-1) Mathematics specification.
1 NUMBER

Prior knowledge check

Numerical fluency

1 Work out
   a 5 × 0.3
   b 97 × 0.02
   c 6 ÷ 0.2
   d 27 ÷ 0.09
   e 4.2 ÷ 0.1
   f 0.4 × 0.6
   g 0.9 × 0.02
   h 0.09 × 0.09
   i 0.4 ÷ 0.2
   j 0.9 ÷ 0.03
   k 0.45 ÷ 0.3
   l 0.88 ÷ 0.04

2 Choose the correct sign, < or >.
   a 2.7 < 2.5
   b 3.04 < 3.3
   c −2.9 < −2.8
   d −5.16 < −5.5

3 a Write down all the factors of 12 and 18.
b Make a list of the common factors.
c Write down the highest common factor.

4 a Copy and complete the Venn diagram to show the factors of 16 and 20.
   b Write down the highest common factor.

5 a Write down the first 10 multiples of 6 and 9.
b Make a list of the common multiples.
c Write down the lowest common multiple.

6 a Copy and complete the Venn diagram to show the first 10 multiples of 4 and 10.
b Write down the lowest common multiple of 4 and 10.

7 Work out
   a 8 − 2 × 3
   b (8 − 2) × 3
   c 7 − (6 − 1) × 6
   d 24 ÷ (8 − 2)
   e 4² + 1
   f (−6)²

8 Insert brackets to make this calculation correct.
   9 + 18 − 3 = 9

9 Estimate
   a 7.3 × 8.94
   b 47 ÷ 2.1
   c 5.2 + 4.9
   d 7.9 − 2.4
1.1 Number problems and reasoning

**Objectives**
- Work out the total number of ways of performing a series of tasks.

**Why learn this?**
SI in maths means ‘five factorial’ and is equal to \(5 \times 4 \times 3 \times 2 \times 1\).

**Fluency**
- Work out: 
  - \(4 \times 4 \times 4\)
  - \(5 \times 4 \times 3\)
  - \(10 \times 9 \times 8\)
  - \(4 \times 3 \times 2 \times 1\)

1. **a** Copy and complete this list of all possible outcomes for rolling a dice and flipping a coin.

- H, 1
- H, 2
- T, 1
- T, 2

  How many outcomes are there altogether?

2. **a** Copy and complete this list of all possible outcomes for spinner A and spinner B.

- \(A: 2, 1\)
- \(B: 4, 1\)
- \(A: 6, 1\)
- \(B: 3, 4\)
- \(A: 2, 5\)

  How many outcomes are there altogether?

**Q4 hint** Use letters for combinations, for example VP for vegetable soup, pizza.

**Q6 communication hint** A factorial is the result of multiplying a sequence of descending integers. For example \(4! = 4 \times 3 \times 2 \times 1\). Find the factorial button on your calculator.

4. A restaurant offers a set menu for birthday parties.

  **a** Write down all possible combinations of starters and main courses.

  **b** **Reflect** How did you order your list to make sure you didn’t miss any starters or mains?

  The restaurant decides to offer fish (F) as a main course.

  **c** How many possible combinations are there now?

  **d** Copy and complete.

  - 3 starters and 4 mains:
  - 3 starters and 5 mains:
  - \(n\) starters and \(m\) mains:

  A different restaurant offers 2 starters, 4 mains and 3 desserts.

  **e** How many possible combinations are there now?

5. Three people, A, B and C, enter a race.

  **a** Write down the different orders in which they can finish first, second and third.

  Harry says that there are 3 possible winners, but then only 2 possibilities for second place and only one person left for third place.

  **Discussion** Is Harry correct? Explain your answer.

  **b** How many different ways can people finish in

  i a 4-person race
  ii a 6-person race
  iii a 10-person race?

**Q5 communication hint** Inclusive means that the end numbers are also included.

**Exam-style question**
Jess has a 4-digit password for her mobile phone.

  a How many choices are possible for each digit of the code?
  b What is the total number of 4-digit passwords that Jess can create?
  c How many different ways are possible now? (5 marks)

6. Eddie needs to choose a 6-digit code for his computer password.

  **a** How many codes can Eddie create using

  i 6 numbers
  ii 4 numbers followed by 2 letters
  iii 1 number followed by 5 letters?

  Eddie decides that he does not want to repeat a digit or a letter.

  **b** How many ways are possible in parts i to iii now?
1.2 Place value and estimating

Objectives

- Estimate an answer.
- Use place value to answer questions.
- Estimate an answer.

Why learn this?

Builders use estimates to give their clients an idea of how much the work will cost.

Fluency

Which two whole numbers does each square root lie between?

\( \sqrt{3} \) \( \sqrt{17} \)

1. Write each number to
   a) 1 significant figure
   b) 2 significant figures.
   a) 873 209
   b) 2019
   c) 0.007 059

2. Work out
   a) \( 9 \times (4 \times 7) \)
   b) \( 5 + 3 \times 8 \)
   c) \( 7 \times 5 - 4 \times 2 \)
   d) \( 30 - 5 \times 8 \)
   e) \( 72 - 9 \)
   f) \( \sqrt{29} - 4 \)

3. Work out the mean of 3, 6, 7, 9, 15 and 20.

Discussion

What do you notice about your answers? Why has this happened?

4. Use place value to answer questions.

5. Use this fact to work out the calculations below.

   \( 3.7 \times 9.86 = 36.482 \)

   Check your answers using an approximate calculation.

   a) \( 37 \times 9.86 \)
   b) \( 3.7 \times 0.0986 \)
   c) 0.0037 \times 98.6
   d) 36.482 \times 9.86
   e) 36.82 - 98.6
   f) 364.82 ÷ 370

6. Reasoning

   54.8 \times 7.29 = 399.492

   a) Write down three more calculations that have the same answer.
   b) Write down a division that has an answer of 54.8.
   c) Write down a division that has an answer of 0.729.
   d) Charlie says that 54.8 \times 72.9 = 3989.44. Explain why Charlie must be wrong.

7. a) Write down the value of \( \sqrt{4} \) and \( \sqrt{9} \).
   b) Estimate the value of \( \sqrt{5} \), \( \sqrt{6} \), \( \sqrt{7} \) and \( \sqrt{8} \).
   c) Round each estimate to 1 decimal place.
   d) Use a calculator to check your answer to part b.

8. Estimate the value to the nearest tenth.
   a) \( \sqrt{17} \)
   b) \( \sqrt{22} \)
   c) \( \sqrt{84} \)
   d) \( \sqrt{127} \)
   e) \( \sqrt{10} \)
   f) \( \sqrt{40} \)

9. Problem-solving

   A mosaic uses 150 square tiles.
   a) The total area is 3000 cm\(^2\).
   b) Use a calculator to check your answer.

   a) Write down the value of \( 8^2 \) and \( 9^2 \).
   b) Estimate the value of \( 8.3^2 \) and \( 8.8^2 \). Round each estimate to the nearest whole number.
   c) Use a calculator to check your answer to part b.

10. Estimate to the nearest whole number.
    a) \( 3.2^2 \)
    b) \( 4.7^2 \)
    c) \( 1.7^2 \)
    d) \( 7.1^2 \)
    e) \( 6.3^2 \)
    f) \( 9.8^2 \)

11. Estimate answers to these.
    a) \( 11.2 - \sqrt{50.3} \times 4.08 \)
    b) \( 1.98 \times 3.14 \times 2 - 8.85 \)
    c) \( \frac{88.72 - 21.9}{\sqrt{35.5}} \)
    d) \( \sqrt{27.3} - 3.85 \)

12. a) Use your calculator to work out each answer.
    b) Use your calculator to check your answer to part b.

13. The sum of the values on these cards is 12.
    Work out the missing number.

14. Problem-solving

   A large cubic dice has a side length of 9.2 cm.
   Estimate the surface area of the dice.

15. Problem-solving

   The area of a square is 80 cm\(^2\).
   Estimate the perimeter of the square.

16. Problem-solving

   Pieces of turf are 1 m long by 0.5 m wide. Each piece costs £3.79.
   a) Estimate the cost of turf required to cover these spaces.
      i) 9.6 m by 2.4 m
      ii) 6.2 m by 1.9 m
      iii) 4.4 m by 2.1 m
   b) Use a calculator to work out each answer.

17. STEM

   Robert uses a spreadsheet to record his runs for 10 innings.
   a) Use estimates to show that Robert's average is wrong.
   b) Work out Robert's correct average to the nearest tenth.

Active Learn

Homework, practice and support: Higher 1.2
1.3 HCF and LCM

Objectives
• Write a number as the product of its prime factors.
• Find the HCF and LCM of two numbers.

Why learn this?
Astronomers use the lowest common multiple of patterns in the orbits of the Sun and the Moon to predict solar eclipses.

Fluency
Work out
• \(2 \times 3 \times 5^2 \times 7 = 420\)
• \(2^3 \times 3^2 \times 5 = 270\)
• \(7^4 \times 5^3 = 2401000\)

Objectives
• Write a number as the product of its prime factors.
• Find the HCF and LCM of two numbers.

Why learn this?
Astronomers use the lowest common multiple of patterns in the orbits of the Sun and the Moon to predict solar eclipses.

1 Write down all the factors of 20.
   a
   b Which of these factors are prime numbers?
2 Write down all the prime numbers between 1 and 20.
   a
   b Write down all the factors of 24.
   c Copy and complete this Venn diagram.

3 a Copy and complete this factor tree for 40.
   b Write 40 as a product of its prime factors.
   40 = \(2 \times 2 \times 2 \times 5\)
   c Write 75 as a product of its prime factors.
4 Write each number as a product of its prime factors in index form.
   a 18
   b 42
   c 25
   d 36
   e 24
   f 80

7 Write each number as a product of its prime factors in index form.
   a \(2^1\times 3^2\)
   b \(2^1\times 3^2\)
   c \(2^2\times 3^1\)
   d \(2^1\times 3^2\)
   e \(2^2\times 3^2\)
   f \(2^3\times 3^2\)

8 120 can be written as a product of its prime factors in the form \(2^m \times n \times p\).
   Work out \(m\), \(n\) and \(p\).

Example 1
Find the highest common factor and lowest common multiple of 24 and 60.

\(24 = 2 \times 2 \times 3 = 12\)
\(60 = 2 \times 2 \times 3 \times 5 = 120\)

Write each number as a product of prime factors

The highest common factor (HCF) of 24 and 60
\(= 2 \times 2 \times 3 = 12\)
The lowest common multiple (LCM) of 24 and 60
\(= 2 \times 2 \times 3 \times 5 = 120\)

9 Find the highest common factor (HCF) and lowest common multiple (LCM) of
   a 24 and 30
   b 20 and 42
   c 8 and 18
   d 15 and 45
   e 27 and 36
   f 33 and 66

10 Real / Problem-solving One bus leaves the bus station every 15 minutes. Another bus leaves every 12 minutes. At 2:30 pm both buses leave the bus station. At what time will this next happen?

11 Real / Problem-solving Amber wants to tile her bathroom. It measures 1.2 m by 2.16 m. She finds square tiles with a side length of 10 cm, 12 cm or 18 cm. Which of these tiles will fit the wall exactly?

Discussion How do you know whether to find the HCF or LCM for Q10 and Q11?

12 Problem-solving The HCF of two numbers is 2.
Write down three possible pairs of numbers.

13 Problem-solving The LCM of two numbers is 18.
One of the numbers is 18.
   a Write down all the possibilities for the other number.
   b Describe the set of numbers you have created.

14 \(48 = 2^4 \times 3\) and \(36 = 2^2 \times 3^2\)
Write down, as a product of its prime factors,
   a the HCF of 48 and 36
   b the LCM of 48 and 36.

Q7 communication hint In index form means to write a number to a power or an index. \(2^3\) is written in index form. \(3\) is the power or index.
Q9 hint Draw a Venn diagram for each question to help you.
Q10 strategy hint Work out the LCM first.
Q12 hint First choose two numbers where 2 is a factor. Is 2 the highest common factor of these numbers?
Q14 hint You could draw a Venn diagram.
15 Exam-style question
Given that $A = 2^3 \times 3^4 \times 5^2$ and $B = 2^2 \times 3^6 \times 5$
Write down, as a product of its prime factors,
- the HCF of $A$ and $B$
- the LCM of $A$ and $B$.

(2 marks)

16 Write 80 as a product of its prime factors.

Discussion How can you use the prime factor decomposition of 80 to quickly work out the prime factor decomposition of 160? What about 40?

17 Problem-solving The prime factor decomposition of 2100 is $2^2 \times 3 \times 5^2 \times 7$.
Write down the prime factor decompositions of
- 75
- 24
- 12
- 30

18 a Harry says the prime factors of 75 appear in the prime factor decomposition of 2100, so 2100 is divisible by 75.
   Is 2100 divisible by 24, 12 or 30?
b Use prime factor decomposition to show that 792 is divisible by 12.
c Is 792 divisible by 132? Explain your answer.
d Is 792 divisible by 27? Explain your answer.

19 In prime factor form, 700 = $2^2 \times 5^2 \times 7$ and 1960 = $2^3 \times 5 \times 7^2$
a What is the HCF of 700 and 1960?
   Give your answer in prime factor decomposition.
b What is the LCM of 700 and 1960?
   Give your answer in prime factor decomposition.
c Which of these are factors of 350 and 1960?
   - $2 \times 5 \times 7$
   - 49
   - 20
   - $2^2 \times 5 \times 7^2$
d Which of these are multiples of 350 and 1960?
   - $2^2 \times 5 \times 7^3$
   - $2^5 \times 5^2 \times 7^2$
   - $2^3 \times 5 \times 7$

14 Calculating with powers (indices)

Objectives
- Use powers and roots in calculations.
- Multiply and divide using index laws.
- Work out a power raised to a power.

Why learn this?
A googol is a 1 followed by 100 zeros. It can be written as $10^{100}$.

Fluency
Work out
- $6^2$
- $(-4)^2$
- $2^4$
- $1^5$

15 Strategy hint Look at the common prime factors.

16 Work out
- $3^1$
- $(-1)^3$
- $4 \times 4^2$
- $3^2 \times 5$

17 Calculating with powers (indices)

2 Work out
- $2^1 \times 10^2$
- $0.2^3$
- $3 \times \sqrt{16}$
- $\sqrt{81} \times \sqrt{64}$

3 Copy and complete.
   - $a = 16$
   - $b = 64$
   - $c = 27$
   - $d = 64$

Key point 2
The inverse of a cube is the cube root.
$2^3 = 8$, so the cube root of 8, $\sqrt[3]{27} = 3$.

4 Work out
- $\sqrt[3]{27}$
- $\sqrt[10]{100}$
- $\sqrt[1000]{100}$
- $\sqrt[125]{1000}$

Discussion Explain why it is possible for you to find the cube root of a negative number, but not the square root.

5 Work out these. Use a calculator to check your answers.
   - $\sqrt[3]{4^2 + 3^2}$
   - $\sqrt[3]{10^2 + 5^2}$
   - $43 - \sqrt[3]{-27}$
   - $33 - \sqrt[3]{-6} - (4^2)$
   - $\sqrt[3]{5^2 + 3 \times \sqrt[3]{7}}$
   - $5^2 \times \sqrt[3]{-27}$
   - $\sqrt[3]{\frac{8}{9} - \frac{1}{9}}$
   - $\sqrt[3]{\frac{64}{9} - \sqrt[3]{-1}}$
   - $\sqrt[3]{0.2^2 \times \sqrt[3]{125}}$
   - $\sqrt[3]{\frac{1}{8}}$

6 Work out
   - $(3^3 - 5^3) \times 21$
   - $20 - (3 \times 4^2 - (2^2 \times 3))$
   - $(12 - (7 - 5)^3) + 9$

Discussion Do the index laws work for negative indices?

7 Work out
   - $\sqrt{16}$
   - $\sqrt[3]{81}$
   - $\sqrt[10]{100\,000}$

Discussion How can you work out the answers to part a by using the indices of the powers you are multiplying?

8 Work out
   - $10^3 \times 10^2$
   - $10^5$
   - $10^6 \times 10^2$
   - $10^8$

Discussion Do the index laws work for negative indices?

9 Write each product as a single power.
   - $3^2 \times 3^4$
   - $4^2 \times 4^8$
   - $9^{-3} \times 9^{-4}$

Key point 3
To multiply, add the indices.
$x^m \times x^n = x^{m+n}$
10 Find the value of \(a\).
   a \(8^4 \times 8^a = 8^7\)
   b \(6^3 \times 6^0 = 6^3\)
   c \(2^3 \times 2^a = 2^{10}\)
11 Write these calculations as a single power. Give your answers in index form.
   a \(21 \times 3^4 \times 3^x = 3^3\)
   b \(2^4 \times 64\)
   c \(5 \times 125\)
   d \(32 \times 4\)
   e \(8 \times 8 \times 8\)
   f \(9 \times 27 \times 3\)

12 Reasoning
   a i Work out \(5 \times 5 \times 5 \times 5 \times 5 \) by cancelling. Write your answer as a power of 5.
   ii Copy and complete. \(5^5 \times 5^2 = 5^x\)
   b Copy and complete. \(4^3 \times 4^2 = 4^x\)
   c Work out \(6^5 \div 6^4\)
   Discussion How can you quickly find \(7^9 \div 7^3\) without writing all the 7s?
13 Work out
   a \(7^6 \div 7^2\)
   b \(4^{-5} \div 4^3\)
   c \(3^6 \div 3^{-7}\)

14 Find the value of \(a\).
   a \(9^6 - 9^a = 9^6\)
   b \(4^5 - 4^a = 4\)
   c \(7^6 - 7^a = 7^9\)

15 Problem-solving
   a Yu multiplies three powers of 9 together. 
      \(9^4 \times 9^x \times 9^{12}\)
      What could the three powers be when
      i all three powers are different
      ii all three powers are the same?
   b Harvey divides two powers of 5. 
      \(5^3 \div 5^x\)
      What could the two powers be when
      i both numbers are greater than 5^0
      ii the power of one number is double the power of the other number?
16 Work out these. Write each answer as a single power.
   a \(5^3 \times 5^2 \div 5^x\)
   b \(6^3 \div 6^3 \times 6^5\)
   c \(5^4 \times 5^{-3} \div 5^x\)
   d \(8^{-2} \times 8^{-6} \div 8^{-7}\)
17 Real / STEM The hard drive of Tom's computer holds \(2^{18}\) bytes of data.
   He buys a USB memory stick that holds \(2^{14}\) bytes of data.
   a How many memory sticks does he need to back up his computer?
   b What fraction of the external hard drive does he use when backing up his computer?
4. Work out the value of n.
   a) $40 = 5 \times 2^n$
   b) $3^n \times 3^n = 3^8$
   c) $5^{2n} = 5^6$
   d) $\frac{1}{2} = 4^n$

5. a) Work out
   i) $2^{-1}$
   ii) $4^{-1}$
   iii) $5^{-1}$
   iv) $10^{-1}$
   b) Use your answers to part a to write these negative indices as fractions.
   c) Work out
   i) $2^{-2}$
   ii) $4^{-2}$
   iii) $5^{-2}$
   iv) $10^{-2}$
   d) Use your answers to part c to write these negative indices as fractions.
   e) Work out
   i) $\left(\frac{1}{2}\right)^{-1}$
   ii) $\left(\frac{3}{4}\right)^{-2}$

**Discussion** What is the rule for writing negative indices as fractions?

6. Match these negative indices to their fractions.

   \[
   \begin{array}{cccc}
   \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 \\
   8 & 3 & 2 & 16 \\
   \end{array}
   \]

   a) Write a matching tile for the two tiles that are left over.
   b) Copy and complete. $\left(\frac{2}{3}\right)^{-1}$

**Key point 8**

$x^{-n} = \frac{1}{x^n}$ for any $n, x \neq 0$

7. Work out
   a) $3^{-1}$
   b) $2^{-4}$
   c) $10^{-5}$
   d) $\left(\frac{8}{3}\right)^{-1}$
   e) $\left(\frac{4}{5}\right)^{-3}$
   f) $\left(\frac{1}{2}\right)^{-1}$
   g) $\left(0.7\right)^{-1}$
   h) $\left(0.1\right)^{-5}$
   i) $\left(0.4\right)^{-3}$
   j) $\left(5^{-1}\right)^{0}$

**Q7f strategy hint** Convert mixed numbers to improper fractions.

**Q7g strategy hint** Convert decimals to fractions.

8. a) Work out
   i) $49^{\frac{1}{2}}$
   ii) $16^{\frac{1}{2}}$
   iii) $121^{\frac{1}{2}}$
   iv) $\left(\frac{4}{25}\right)^{\frac{1}{2}}$
   b) Copy and complete. $a^2$ is the same as the _______ _______ of $a$.
   c) Work out
   i) $27^{\frac{1}{3}}$
   ii) $1000^{\frac{1}{3}}$
   iii) $-1^{\frac{1}{3}}$
   iv) $\left(\frac{1}{1000}\right)^{\frac{1}{3}}$
   d) Copy and complete. $a^3$ is the same as the _______ _______ of $a$.
   e) Copy and complete.
   i) $625 = 5^4$ so $625^{\frac{1}{4}}$ = [ ]
   ii) $32 = 2^5$ so $32^{\frac{1}{5}}$ = [ ]

9. Work out
   a) $36^{\frac{1}{2}}$
   b) $81^{\frac{1}{2}}$
   c) $\left(\frac{1}{9}\right)^{\frac{1}{2}}$
   d) $\left(\frac{16}{25}\right)^{\frac{1}{2}}$
   e) $\left(\frac{64}{49}\right)^{\frac{1}{2}}$
   f) $-8^{\frac{1}{3}}$
   g) $\left(\frac{1}{27}\right)^{\frac{1}{3}}$
   h) $\left(\frac{16}{125}\right)^{\frac{1}{3}}$

**Q4a hint** $40 = 5 \times 4$

**How do you write this number as 2?**

**Q10 hint** $x^{-n} = \frac{1}{x^n}$

\[
\begin{array}{ll}
\text{Example 2} & \\
\text{Work out the value of} & \\
a) 27^{\frac{1}{3}} & b) 16^{\frac{1}{2}}
\end{array}
\]

\[
\begin{array}{lll}
a) & 27^{\frac{1}{3}} = (27)^{\frac{1}{3}} = 3 & b) 16^{\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{\sqrt{16}} = \frac{1}{4} \\
\end{array}
\]

**Use the rule $(a^n)^m = a^{mn}$. Work out the cube root of 27 first. Then square your answer.**

10. Work out
   a) $25^{-\frac{1}{2}}$
   b) $64^{-\frac{1}{2}}$
   c) $\left(\frac{9}{25}\right)^{-\frac{1}{2}}$

**Q2a hint** $64^{\frac{1}{2}} = \sqrt{64} = \boxed{8}$

11. Work out
   a) $64^{\frac{1}{2}}$
   b) $10000^{\frac{1}{5}}$
   c) $16^{\frac{1}{2}}$
   d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$
   e) $27^{\frac{1}{3}}$
   f) $-81^{-\frac{1}{4}}$

**Q2a hint** $64^{\frac{1}{2}} = \sqrt{64} = \boxed{8}$

12. Work out
   a) $27^{\frac{1}{3}} \times 9^{\frac{1}{3}}$
   b) $\left(\frac{4}{25}\right)^{-\frac{1}{2}} \times \left(\frac{8}{27}\right)^{\frac{1}{3}}$
   c) $\left(\frac{81}{16}\right)^{\frac{1}{2}} \times \left(\frac{9}{25}\right)^{-\frac{1}{2}}$

**Q12a hint** First work out $27^{\frac{1}{3}}$. Then work out $9^{\frac{1}{3}}$. Then multiply these numbers together.

13. Find the value of $n$.
   a) $16 = 2^n$
   b) $\frac{1}{27} = 2^{-n}$
   c) $\frac{1}{100} = 10^n$
   d) $\frac{4}{9} = \left(\frac{2}{3}\right)^n$
   e) $\frac{1}{27} = 3^{-n}$
   f) $\frac{8}{8} = 8^n$

14. **Problem-solving / Reasoning** Will says that $25^{\frac{1}{2}} \times 64^{\frac{1}{2}} = 80$

   a) Show that Will is wrong.
   b) What mistake did he make?

15. **Problem-solving / Reasoning**

Match these indices to their fractions.

\[
\begin{array}{cccccc}
\frac{8}{27} & \frac{81}{16} & \frac{1}{4} & \frac{16}{9} & \frac{4}{9} & \frac{4}{8}
\end{array}
\]
1.6 Powers of 10 and standard form

Objectives
• Write a number in standard form.
• Calculate with numbers in standard form.

Why learn this?
Scientists use standard form to write very small or very large numbers.

Fluency
• Work out
  \(4.5 \times 10^3 \quad 0.0063 \times 10^0 \quad 69.4 \times 0.1 \quad 845.3 \times 0.001\)
• Which of these are the same as \(\div 10\)?
  \(4.5 \times 1000 \quad 0.0063 \times 100 \quad 69.4 \times 0.1 \quad 845.3 \times 0.001\)

1. Copy and complete. If your answer is a fraction, write it as a decimal too.
   a. \(10^5 = \) ___________
   b. \(10^{-1} = \) ___________
   c. \(10^{-2} = \) ___________
   d. \(10^{-3} = \) ___________
   e. \(10^4 = \) ___________
   f. \(10^{-5} = \) ___________

2. Write down the value of \(x\).
   a. \(10^4 = 1000\) ___________
   b. \(10^5 = x\) ___________
   c. \(10^7 = 100000000\) ___________
   d. \(10^{-1} = x\) ___________
   e. \(10^4 = 0.0001\) ___________
   f. \(10^{-6} = x\) ___________

3. Copy and complete.
   a. \(5670000000\) million
   b. \(158000000\) million
   c. \(4908340000\) billion

4. Copy and complete the table of prefixes.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Letter</th>
<th>Power</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
<td>(10^{12})</td>
<td>1 000 000 000 000</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>(10^9)</td>
<td>1 000 000 000</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>(10^6)</td>
<td>1 000 000</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>(10^3)</td>
<td>1 000</td>
</tr>
<tr>
<td>deci</td>
<td>d</td>
<td>(10^{-1})</td>
<td>0.1</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>(10^{-2})</td>
<td>0.01</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>(10^{-3})</td>
<td>0.001</td>
</tr>
<tr>
<td>micro</td>
<td>μ</td>
<td>(10^{-6})</td>
<td>0.000 001</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>(10^{-9})</td>
<td>0.000 000 001</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>(10^{-12})</td>
<td></td>
</tr>
</tbody>
</table>

Key point 11
Some powers of 10 have a name called a prefix. Each prefix is represented by a letter.
For example, kilo means \(10^3\) and is represented by the letter \(k\), as in kg for kilogram.

5. Convert
   a. \(15\) mg into grams
   b. \(7\) nm into metres
   c. \(1.7\) g into kg
   d. \(7.3\) ps into seconds.

6 STEM Write these measurements in metres.
   a. The size of the influenza virus is about \(1.2\) μm.
   b. The radius of a hydrogen atom is \(25\) pm.
   c. A fingernail grows about \(0.9\) nm every second.

Q4 communication hint

Prefix \(\mu\), the letter for the prefix micro, is the Greek letter \(\mu\) or \(\mu\).

Q5a hint
Use a number line.

mg: \(\) ___________

\(1\) ___________

\(0.001\) ___________

\(15\) ___________

Example 3
Work out \((5 \times 10^3) \times (7 \times 10^6)\)
Rewrite the multiplication grouping the numbers and the powers.

\(5 \times 7 \times 10^3 \times 10^6\)

Simplify using multiplication and the index law \(x^m \times x^n = x^{m+n}\)
This is not in standard form because \(35\) is not between 1 and 10.

\(35 = 3.5 \times 10^1\)

Write 35 in standard form.

\(35 \times 10^2 = 3.5 \times 10^1 \times 10^3 = 3.5 \times 10^{10}\)

Work out the final answer.

Q6

Q9 hint
Write the number between 1 and 10 first. Then multiply by a power of 10.

Q10 communication hint
An ordinary number is a whole number or a decimal which is not multiplied by a power of 10.
1.7 Surds

Objectives
- Understand the difference between rational and irrational numbers.
- Simplify a surd.
- Rationalise a denominator.

Why learn this?
Surds are used to express irrational numbers in exact form.

Fluency
- What does the dot above the 1 mean in 0.\(\overline{1}\)?
- What are the missing numbers?
  \[
  147 = 3 \times □ \quad 125 = □ \times 25 \quad 180 = 5 \times □ \quad 96 = □ \times 16
  \]

1. Write each number as a product of its prime factors.
   \[
   \begin{align*}
   a &= 8 & b &= 18 & c &= 24 & d &= 48 \\
   e &= 2 \times 3^2 & f &= 2 \times □
   \end{align*}
   \]

2. Write each number as a fraction in its simplest form.
   \[
   \begin{align*}
   a &= 0.6 & b &= 0.85 & c &= 1.625 \\
   d &= 4.25 & e &= 0.3 & f &= 1.5
   \end{align*}
   \]

Exam-style question
\[
(7 \times 10^3) + (7 \times 10^2) + (7 \times 10^1) = 700070.07
\]
Write down a possible set of values for \(x, y\) and \(z\). (3 marks)

\[
Q13g \text{ hint } (5 \times 10^3)^2 = (5 \times 10^3) \times (5 \times 10^3)
\]

Q13b hint Use your answers from part a to write the answer as an ordinary number. Then convert this to standard form.

Exam hint
Don’t just write down the possible values – give your working to show how you worked out the values.

Q13d hint Your calculator’s display shows 8.485281374.

Trish presses the \(S\) button.

\[
\sqrt{3} = \sqrt{3} \quad \sqrt{5} = \sqrt{5} \
\]

\[
\sqrt{3} \times \sqrt{5} = \sqrt{15}
\]

\[
\sqrt{15} = \sqrt{15}
\]

Q13e hint Workout these. Use a calculator to check your answers.

Key point 13
\[
\sqrt{MN} = \sqrt{M} \times \sqrt{N}
\]

1. Write out these. Use a calculator to check your answers.
   \[
   a \quad b \quad c
   \]
   \[
   \begin{align*}
   a &= \sqrt{2} \times \sqrt{3} & b &= \sqrt{6} \\
   c &= \sqrt{3} \times \sqrt{5} & d &= \sqrt{15} \\
   e &= \sqrt{2} \times \sqrt{10} & f &= \sqrt{3} \times \sqrt{5}
   \end{align*}
   \]

Key point 14
A surd is a number that cannot be simplified by removing the square root sign. For example, \(\sqrt{5}\) is a surd, but \(\sqrt{4}\) is not a surd, because \(\sqrt{4} = 2\). All surds are irrational numbers.

5. Simplify these surds.
   \[
   a \quad b \quad c \quad d \quad e \quad f
   \]
   \[
   \begin{align*}
   a &= \sqrt{20} & b &= \sqrt{300} & c &= \sqrt{44} & Q5 \text{ hint } \text{Find a factor that is also a square number.} \\
   d &= \sqrt{50} & e &= 4 \sqrt{5} & f &= 6 \sqrt{5}
   \end{align*}
   \]

6. Reasoning / Problem-solving
   Trish types \(\sqrt{72}\) into her calculator. Her display shows 6.2.
   a. Show how the calculator worked this out.
   b. What does the \(S\rightarrow D\) button do?
   c. What is the original surd?
   d. Reflect How did you find the surd?

Key point 15
Rational numbers can be written as a fraction in the form \(\frac{a}{b}\), where \(a\) and \(b\) are integers and \(b \neq 0\).

2. \(\sqrt{8}\) is rational as it can be written as \(\frac{2}{2}\).

0.\(\overline{2}\) is rational as it can be written as \(\frac{2}{9}\).

\(\frac{\sqrt{2}}{2}\) is irrational.

7. The table shows the numbers below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Rational</th>
<th>Irrational</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{6})</td>
<td>(\frac{3}{\sqrt{2}})</td>
<td>(\frac{6}{8})</td>
</tr>
<tr>
<td>(-\sqrt{6})</td>
<td>(-\sqrt{8})</td>
<td>(-\sqrt{17})</td>
</tr>
<tr>
<td>(\sqrt{17})</td>
<td>(1.4)</td>
<td>(\sqrt{4})</td>
</tr>
<tr>
<td>(\sqrt{49})</td>
<td>(0.3)</td>
<td>(0.3)</td>
</tr>
</tbody>
</table>

8. Copy and complete the table using the numbers below.

\[
\begin{align*}
\text{Rational} & \quad \text{Irrational} \\
\sqrt{6} & \quad \frac{3}{\sqrt{2}} & \quad \frac{6}{8} \\
-\sqrt{6} & \quad -\sqrt{8} & \quad -\sqrt{17} \\
\sqrt{17} & \quad 1.4 & \quad \sqrt{4} \\
\sqrt{49} & \quad 0.3 & \quad 0.3 \\
\end{align*}
\]
**Key point 16**

\[ \frac{m}{n} = \frac{\sqrt{m}}{\sqrt{n}} \]

9 Simplify
   \[ a \frac{\sqrt{7}}{\sqrt{4}} = b \frac{\sqrt{5}}{\sqrt{9}} = c \frac{\sqrt{12}}{\sqrt{49}} = d \frac{\sqrt{12}}{\sqrt{49}} = \frac{\sqrt{5}}{\sqrt{9}} \]

10 Solve the equation \( x^2 = 90 \), giving your answer as a surd in its simplest form.
   **Discussion** Can you solve the equation \( x^2 = 90 \) in the same way? Explain your answer.

11 Solve these equations, giving your answer as a surd in its simplest form.
   a) \( 4x^2 = 200 \)
   b) \( \frac{1}{3}x^2 = 80 \)
   c) \( 3x^2 = 36 \)
   d) \( 2x^2 - 14 = 42 \)

12 The area of a square is 60 cm\(^2\).
   Find the length of one side of the square. Give your answer as a surd in its simplest form.

13 a) Work out
   i) \( 5\sqrt{2} \times 4\sqrt{27} \)
   ii) \( 4\sqrt{5} \times 6\sqrt{12} \)
   iii) \( 9\sqrt{10} \times 4\sqrt{5} \)
   iv) \( 8\sqrt{6} \times 3\sqrt{3} \)
   b) Use a calculator to check your answers to parts i to iv.

**Key point 17**

To **rationalise the denominator**, the denominator should be a rational number and all the square roots should have no factors that are square numbers.

---

**Example 4**

Rationalise the denominator.
   a) \( \frac{1}{\sqrt{2}} \)
   b) \( \frac{5}{\sqrt{15}} \)
   a) \( \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \)
   b) \( \sqrt{15} = \sqrt{3} \times 5 = \frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{3}}{5} \)

14 Rationalise the denominators. Simplify your answers if possible.
   a) \( \frac{1}{\sqrt{7}} \)
   b) \( \frac{1}{\sqrt{3}} \)
   c) \( \frac{\sqrt{3}}{\sqrt{5}} \)
   d) \( \frac{\sqrt{11}}{\sqrt{10}} \)
   e) \( \frac{2}{\sqrt{8}} \)
   f) \( \frac{1}{\sqrt{15}} \)
   g) \( \frac{32}{\sqrt{40}} \)
   h) \( \frac{11}{\sqrt{11}} \)

---

**Example 5**

A furniture maker orders 22 metal legs. He uses the legs to make three-legged stools and four-legged chairs. Describe the different ways he can use all the legs.

<table>
<thead>
<tr>
<th>Stools</th>
<th>Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 stool</td>
<td>4 chairs</td>
</tr>
<tr>
<td>2 stools</td>
<td>4 chairs</td>
</tr>
<tr>
<td>6 stools</td>
<td>1 chair</td>
</tr>
</tbody>
</table>

He can use 22 legs to make 2 stools and 4 chairs or 6 stools and 1 chair.

An alternative method is to use a list.

<table>
<thead>
<tr>
<th>Number of stools</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of chairs</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Legs left over</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

15 **Reasoning / Problem-solving**

Ben types \( \frac{1}{\sqrt{7}} \) into his calculator. His display shows \( \frac{\sqrt{7}}{7} \).
   a) Show that \( \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7} \).
   b) Use your calculator to check your answers from Q14.

16 The area of a rectangle is 20 cm\(^2\).
   The length of one side is \( \sqrt{5} \) cm.
   Work out the length of the other side. Give your answer as a surd in its simplest form.

17 Work out the area of these shapes.
   a) \( \frac{3}{\sqrt{2}} \)
   b) \( \frac{4}{\sqrt{7}} \)
   c) \( \frac{3}{\sqrt{8}} \)

Give your answer as a surd in its simplest form.

---

**1 Problem solving**

**Objective**

- Use pictures or lists to help you solve problems.
1 There are 20 chairs in a conference room. The conference organiser can put 4, 5 or 6 chairs at a table.
   a Describe the different ways the room can be arranged so that all the chairs are used.
   b What is the maximum number of tables required in the room?

2 A play park is 18 m wide and 31.5 m long. The council plans to enclose it with a fence, using a supporting post every 2.25 m. How many posts does the council need?

3 When two plant stakes are placed end to end, their total length is 1.45 m. When the two stakes are placed side by side, one is 0.15 m longer than the other. What lengths are the stakes? Give your answer in cm.

4 **Finance** In a canteen, a starter costs £0.80, a main costs £2.40 and a dessert costs £1.20. Three friends bought lunch and paid £10 in total. They each had at least two courses. How many starters, mains and desserts did they buy?

5 **Finance** A bicycle shop hires road bikes for £25 per day and tandems for £40 per day. One family pays £155. Which type of bicycles did they hire?
   a Which type of bicycles did they hire?
   b How many people are in the family?

6 A tour company offers three different walking tours. The landmark tour leaves every 15 minutes. The parks tour leaves every 20 minutes. The museum tour leaves every 45 minutes. All walking tours start at 9 am. When do the landmark, parks and museum tours next leave at the same time?

7 **Reflect** How can you solve Q6 without making a list?

**Discussion** Does it matter how you solve a maths problem?

---

**Check up**

---

**Calculations, factors and multiples**

1 16.7 × 9.2 = 153.64
   Use this fact to work out the calculations below.
   Check your answers using an approximate calculation.
   a 167 × 9.2
   b 15364 – 1.67

2 Estimate the value of $\sqrt{54}$ to the nearest tenth.

3 a Estimate $\frac{1}{3} (65.1 – 6.17) \times 1.98$
   b Use your calculator to work out each answer.
   Give your answers correct to 1 decimal place.

4 Write 90 as a product of its prime factors in index form.

5 Find the highest common factor (HCF) and lowest common multiple (LCM) of 14 and 18.

6 In prime factor form, $2450 = 2 \times 5^2 \times 7^2$ and $68600 = 2^3 \times 5^2 \times 7^3$.
   a What is the HCF of 2450 and 68600? Give your answer in prime factor form.
   b What is the LCM of 2450 and 68600? Give your answer in prime factor form.

---

**Indices and surds**

7 Copy and complete.
   a $\sqrt{9} = 1000$
   b $4^3 = \square$
   c $\sqrt{25} = 16$

8 Work out
   a $\frac{9^2 - 1^2}{10}$
   b $\left(\sqrt{6^2} - 2^2\right) \times 3^2$
   c $\sqrt{\sqrt{81} + (-2)^3}$

9 Write each product as a single power.
   a $9^3 \times 9^2$
   b $27 \times 9 \times 27$
   c $5^2 \times 5^2$
   d $2^8 \times 16$
   e $(4^6)^3$
   f $(4^3)^{-1}$

10 Work out
   a $2^{-4}$
   b $25^{1/2}$
   c $\frac{16}{81}^{1/2}$
   d $16^{-\frac{1}{2}}$

11 Simplify
   a $\sqrt{54}$
   b $\sqrt{1000}$

12 Rationalise the denominators. Simplify your answers if possible.
   a $\frac{1}{\sqrt{10}}$
   b $\frac{4}{\sqrt{8}}$

---

**Standard form**

13 Write these numbers in standard form.
   a $32 040 000$
   b $0.0007$

14 Write these as ordinary numbers.
   a $5.6 \times 10^4$
   b $1.09 \times 10^{-3}$

15 Work out
   a $(5 \times 10^5) \times (9 \times 10^3)$
   b $(3 \times 10^9) - (6 \times 10^5)$
   c $(7 \times 10^8)^2$

16 **Reflect** How sure are you of your answers? Were you mostly just guessing ☹️ Feeling doubtful 😕 Confident 😊
   What next? Use your results to decide whether to strengthen or extend your learning.

---

**ActiveLearn** Homework, practice and support: Higher 1 Check up
1 Strengthen

Calculations, factors and multiples

1 Copy and complete these number patterns.
   a $0.38 \times 29.4 = 11.172$
   b $601 \times 54 = 94.67 = 63.5$
   $3.8 \times 29.4 = \phantom{11.172}$
   $601 \times 54 = 94.67 = \phantom{63.5}$
   $38 \times 29.4 = \phantom{11.172}$
   $601 \times 54 = 94.67 = \phantom{63.5}$
   $380 \times 29.4 = \phantom{11.172}$
   $6011 \times 54 = 94.67 = \phantom{63.5}$
   $3800 \times 29.4 = \phantom{11.172}$
   $60115 \times 54 = 94.67 = \phantom{63.5}$

2 $8.9 \times 7.21 = 64.169$
   Use this fact to work out the calculations below.
   a $8.9 \times 7.21$
   b $0.89 \times 0.721$
   c $8.9 \times 72.1$

   b
   c
   d
   e
   f

3 Copy and complete this square root number line.

a Copy and complete these calculations in index form.
   b Check your answers using an approximate calculation.
   c Use this fact to work out the calculations below.
   d Estimate the value to the nearest tenth.
   e Use a calculator to work out each answer.
   f Write 60 as a product of its prime factors.
   g Write 18 as a product of its prime factors.
   h Write 45 as a product of its prime factors.

4 Write each number as a product of its prime factors in index form.
   a $24$
   b $80$
   c $45$
   d $30$
   e $16$
   f $72$

5 a Estimate
   b $9.4 \times 7.21$
   c $0.94 \times 0.721$
   d $0.094 \times 0.0721$
   e $0.094 \times 0.0721$
   f $0.0094 \times 0.00721$

6 a Estimate
   b $26.64 + \sqrt{80.7}$
   c $\sqrt{6.91} \times 9.23$
   d $3.95^2 + 2.03^2$
   e $\sqrt{7.91} \times 9.89$
   f $2.98^3 + 3.21^3$

7 a Copy and complete this factor tree for 60 until you end up with just prime factors.
   b Write 60 as a product of its prime factors.
   c Write your answer to part b in index form.

8 a Copy and complete this square root number line.
   b Write out a number pattern to help you.
   c Round each answer correct to 1 decimal place.
   d Write your answer to part a in index form.

9 a Write 18 as a product of its prime factors.
   b Write 45 as a product of its prime factors.
   c Copy and complete this diagram.

10 Find the highest common factor (HCF) and lowest common multiple (LCM) of
   a $20$ and $30$
   b $21$ and $28$
   c $15$ and $25$
   d $44$ and $36$

Indices and surds

1 Copy and complete.
   a $2 \times 3 \times 2 \times 5 = \phantom{30}$
   b $5 \times 5 = \phantom{25}$
   c $-3 \times -3 \times -3 = (-3) \times (-3) \times (-3)$

2 Work out
   a $2^3 = \phantom{8}$
   b $5^3 = \phantom{125}$
   c $4^2 = \phantom{64}$
   d $10^2 = \phantom{1000}$

3 Work out
   a $\sqrt{61}$
   b $\sqrt{16}$
   c $\sqrt{27} \times \sqrt{64}$
   d $10 \times \sqrt{25}$
   e $\sqrt{-3} \times \sqrt{49}$
   f $\sqrt{9} \times \sqrt{-125} \times \sqrt{1000}$

4 Work out
   a $5 \times (4^2 - 3^2) - 2^3$
   b $[9^3 - 3^3] ^ \frac{1}{3}$
   c $60 - (4^2 - 18^2)$
   d $\sqrt{25} + \sqrt{125} + \sqrt{16}$
   e $\sqrt{2} \times \sqrt{3}$
   f $\sqrt{2} \times \sqrt{49} + \sqrt{8}$

Q1a hint
Q1b hint
Q2 hint
Q3a i hint
Q3a ii hint
Q3a iii hint
Q3b hint
Q3c hint
Q3d hint
Q4a hint
Q4b hint
Q4c hint
Q4d hint
Q5a i hint
Q5a ii hint
Q5a iii hint
Q5b hint
Q5c hint
Q6 a
Q6 b
Q6 c
Q7b hint
Q8 hint
Q9a hint
Q9b hint
Q9c hint
Q9d hint
Q9e hint
Q9f hint
Q9g hint
Q9h hint
Q9i hint
Q10 hint

Homework, practice and support: Higher 1 Strengthen
5 Copy and complete.
   a \((3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3) = \ldots\)
   b \((4 \times 4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4 \times 4) = \ldots\)
   c \(\frac{6 \times 6 \times 6}{6 \times 6} = \ldots\)
   d \(7 \times 7 \times 7 \times 7 \times 7 \times 7 = \ldots\)
   e To multiply powers, the indices. To divide powers, the indices.

6 Work out
   a \(5^2 \times 5^3 = \ldots\)
   b \(7^2 \times 7^9 = \ldots\)
   c \(5^3 \times 3^2 = \ldots\)
   d \(9^6 \div 9^2 = \ldots\)
   e \(8^3 \times 8^6 = \ldots\)

7 Write these as a single power of a prime number.
   a \(16 \times 8 = \ldots\)
   b \(25 \times 125 \times 25 = \ldots\)
   c \(16 \times 64 \times 8 = \ldots\)
   d \(27 \times 27 \times 9 = \ldots\)

8 Copy and complete.
   a \((4^2)^3 = 4\times 4 \times 4 \times 4 = \ldots\)
   b \((6^3)^4 \times 6^3 \times 6^3 \times 6^3 \times 6^3 = \ldots\)
   c \((7^3)^2 = \ldots\)
   d \((8^3)^7 = \ldots\)
   e To work out a power raised to a power, the indices.

9 a Work out using a calculator
   i \(\sqrt[3]{169} = \ldots\)
   ii \(169^\frac{1}{3} = \ldots\)
   b Use your answer to part a to work these out without a calculator.
   i \(64^\frac{1}{2} = \ldots\)
   ii \(25^\frac{1}{2} = \ldots\)
   iii \(81^\frac{1}{2} = \ldots\)
   iv \(144^\frac{1}{2} = \ldots\)
   c Work out using a calculator
   i \(\sqrt[3]{512} = \ldots\)
   ii \(512^\frac{1}{3} = \ldots\)
   d Use your answer to part c to work these out without a calculator.
   i \(125^\frac{1}{3} = \ldots\)
   ii \(27^\frac{1}{3} = \ldots\)
   iii \(1000^\frac{1}{3} = \ldots\)
   iv \(8^\frac{1}{3} = \ldots\)

10 a Work out
   i \(64^\frac{1}{2} = \ldots\)
   ii \(64^\frac{1}{3} = \ldots\)
   b Use your answer to Q9d to work out
   i \(125^\frac{1}{3} = \ldots\)
   ii \(27^\frac{1}{3} = \ldots\)
   iii \(1000^\frac{1}{3} = \ldots\)
   iv \(8^\frac{1}{3} = \ldots\)
   c Use your answer to Q9e to work out 16\(^\frac{1}{3}\).

11 a Copy and complete.
   i \(4^3 = \ldots\)
   ii \(\frac{1}{10^2} = \ldots\)
   iii \(\frac{1}{3} = \ldots\)
   iv \(3^\frac{1}{2} = \ldots\)
   v \(\frac{7}{6} = \ldots\)
   vi \(a^{\frac{1}{n}} = \ldots\)
   b Work out
   i \(4^1 = \ldots\)
   ii \(10^2 = \ldots\)
   iii \(36^\frac{1}{2} = \ldots\)
   iv \(729^\frac{1}{3} = \ldots\)

12 Copy and complete.
   a \(50 = \ldots\)  \(\sqrt[3]{50} = \ldots\)
   b \(84 = \ldots\)  \(\sqrt[3]{84} = \ldots\)
   c \(\sqrt{96} = \ldots\)
   d \(\sqrt[3]{175} = \ldots\)
   e \(\sqrt[3]{128} = \ldots\)

13 Work out
   a \(\sqrt[3]{4} \times \sqrt[3]{4} = \ldots\)
   b \(\sqrt[3]{25} \times \sqrt[3]{25} = \ldots\)
   c \(\sqrt[3]{17} \times \sqrt[3]{17} = \ldots\)
   d \(\sqrt[3]{21} \times \sqrt[3]{21} = \ldots\)

14 Rationalise the denominator.
   Simplify your answer if possible.
   a \(\frac{1}{\sqrt[3]{17}} = \ldots\)
   b \(\frac{3}{\sqrt[3]{21}} = \ldots\)
   c \(\frac{1}{\sqrt[3]{6}} = \ldots\)
   d \(\frac{6}{\sqrt[3]{20}} = \ldots\)

Standard form
1 A number written in standard form looks like this.

\(A \times 10^n\)

number
  as
times
between
1
and
10

Are these numbers in standard form?
If not, give reasons why.
   a \(9.004 \times 10^{-3} = \ldots\)
   b \(32 \times 10^5 = \ldots\)
   c \(7.3 \text{ million} = 7.3 \times 10^6 = \ldots\)
   d \(0.8 \times 10^7 = \ldots\)

2 Write each number using standard form.
   a \(68 000 = 6.8 \times 10^4 = \ldots\)
   b \(94 000 000 = 9.4 \times 10^7 = \ldots\)
   c \(801 000 = 8.01 \times 10^5 = \ldots\)
   d \(0.000 004 = 4 \times 10^{-5} = \ldots\)
   e \(0.000 0053 = 5.3 \times 10^{-6} = \ldots\)

3 Work out
   a \((4 \times 10^3) \times (2 \times 10^7) = \ldots\)
   b \((3 \times 10^2) \times (2 \times 10^9) = \ldots\)
   c \((6 \times 10^5) \times (1 \times 10^3) = \ldots\)
   d \((6 \times 10^6) \times (8 \times 10^4) = \ldots\)
   e \((7 \times 10^3) \times (8 \times 10^2) = \ldots\)
   f \((8 \times 10^{-2}) \times (6 \times 10^{-3}) = \ldots\)

4 Real A gymnastics floor is a square with side length \(1.2 \times 10^3\) cm.
Work out the area of the floor.

1 Extend

1 Problem-solving Square A has a side length of 9.2 cm.
Square B has a perimeter of 34.4 cm.
Square C has an area of 80 cm\(^2\).
   a Which square has the greatest perimeter?
   b Which square has the smallest area?
2 Show that $27^2 = 9^3 = 3^6$.

3 **Exam-style question**

Here are some properties of a number.
- It is a common factor of 216 and 540.
- It is a common multiple of 9 and 12.

Write two numbers with these properties. (6 marks)

4 a Write 48, 90 and 150 as products of their prime factors.
   b Use a Venn diagram to work out the HCF and LCM of 48, 90 and 150.
   Discussion Explain how the diagram can be used to find the HCF and LCM of any two of the numbers.

5 **Real** A new school is deciding whether their lessons should be 30, 50 or 60 minutes.

Each length of lesson fits exactly into the total teaching time of the school day.

How long is the teaching time of the school day?

Discussion Ryan says there is more than one answer to this question. Is Ryan correct?

6 **Reasoning**

a Use prime factors to explain why numbers ending in a zero must be divisible by 2 and 5.
   b How many zeros are there at the end of $24 \times 37 \times 56 \times 72$?
   c Each length of lesson fits exactly into the total teaching time of the school day.

Explain your answer.

7 Write each of these as a simplified product of powers.
   a $10^3 \times 2^2 \times 5^4 = (2 \times 5)^3 \times 2^2 \times 5^4 = 2^3 \times 5^4 \times 2^2 \times 5^4$
   b $6^3 \times 2^4 \times 3^2$
   c $15^3 \times 10^3 \times 6^2$
   d $30^4 \times 24^2 \times 15^3$

8 **STEM** Write each answer
   i as an ordinary number
   ii in standard form.
   a Saturn has a diameter of 120 536 000 m.
      Convert this to kilometres.
   b The distance from the Sun to Mars is 227 900 000 km.
      Convert this distance to metres.
   c The diameter of a grain of sand is 4 μm.
      Convert this to metres.
   d The wavelength of an X-ray is 0.1 nm.
      Convert this to metres.

9 Every six months, new licence plates are issued in the UK.

A licence plate consists of two letters, then two numbers, then three letters.

The numbers are fixed, but the letters vary.

a If all letters can be used, how many possible combinations are there?
   b If only 21 letters can be used, how may possible combinations are there?

10 **STEM / Problem-solving** A container ship carries $1.8 \times 10^6$ kg.

An aeroplane can carry $3.8 \times 10^5$ kg.

What is the difference in their mass? Write your answer in standard form.

11 **Exam-style question**

Work out
   a $\frac{5}{\sqrt{2}} + \frac{8}{\sqrt{32}}$
   b $\frac{7}{\sqrt{72}} - \frac{3}{\sqrt{8}}$

Write each answer in the form $\alpha/\beta$. (3 marks)

12 A square has an area of $\frac{3}{2}$ cm$^2$.
   a Work out the length of one side.
   b Work out the perimeter.

Give your answers as surds in their simplest forms.

13 Write $3^\frac{2}{3}$ as a surd and rationalise the denominator.

14 **Exam-style question**

A restaurant offers 5 starters, 7 mains and 3 desserts.

A customer can choose
   • just one course
   • any combination of two courses
   • all three courses.

Show that a customer has 191 options altogether. (3 marks)

15 What is the value of $x$?
   a $3^x = \frac{1}{27}$
   b $2 \times 2^x = \frac{1}{8}$
   c $3^{2x} = \frac{1}{9}$

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**1 Knowledge check**

- When there are $m$ ways of doing one task and $n$ ways of doing a different task, the total number of ways the two tasks can be done is $m \times n$. .......... Mastery lesson 1.1
- You can round numbers to 1 or 2 significant figures to estimate the answer to a calculation. .......... Mastery lesson 1.1
- You can use a prime factor tree to write a number as the product of its prime factors. .......... Mastery lesson 1.2
- You can use a Venn diagram of prime factors to work out the highest common factor and lowest common multiple of two numbers. .......... Mastery lesson 1.3
- The prime factor decomposition of a number is the number written as the product of its prime factors. It is usually written in index form. .......... Mastery lesson 1.3
- When multiplying powers, add the indices: $x^m \times x^n = x^{m+n}$
  When dividing powers, subtract the indices: $x^m \div x^n = x^{m-n}$
  To raise a power to another power, multiply the indices.
  $x^{\frac{1}{2}} = \frac{1}{x^n}$
  $x^\frac{1}{2} = \sqrt{x}$
  $x^\frac{1}{3} = (\sqrt[3]{x})^n$ .......... Mastery lesson 1.4, 1.5
- A number in standard form is written in the format $A \times 10^n$, where $A$ is a number between 1 and 10 and $n$ is an integer. .......... Mastery lesson 1.6
To write a number in standard form:
• work out the value of \(A\)
• work out how many times \(A\) must be multiplied or divided by 10.
This is the value of \(n\).

Mastery lesson 1.6
To simplify a surd, identify any factors that are square numbers.

Mastery lesson 1.7
To rationalise a denominator, multiply the numerator and the denominator by the surd in the denominator and simplify.

Mastery lesson 1.7

1 Unit test
Log how you did on your Student Progression Chart.

1 6.23 \times 5.4 = 33.642
   a Write down two more multiplications with an answer of 33.642. (3 marks)
   b Write down a division with an answer of 0.623. (3 marks)

2 Exam-style question
List these numbers in order, starting with the smallest.
Show your working.
\[
\begin{array}{cccc}
1.2^2 & \sqrt{27} & \sqrt{69} & 13.74
\end{array}
\] (3 marks)

3 a Estimate (17.9 – \(\sqrt{\underline{\underline{36.13}}}) \times 3.89
   b Use a calculator to work out the answer. Give your answer correct to 1 decimal place. (2 marks)

4 a Write 42 as a product of its prime factors.
   b Use your answer to write 84\(^2\) as a product of its prime factors in index form. (4 marks)

5 Work out the HCF and LCM of 75 and 30. (3 marks)

6 Real
Ben and Sadie are doing a sponsored walk around a circuit. Ben takes 25 minutes to do one circuit and Sadie takes 45 minutes. They start together at 9:30 am. When will they next cross the start line together? (2 marks)

7 Find the value of \(a\).
   a \(5^1 \times 5^a = 5^b\)
   b \(6^a \div 6^{-5} = 6^8\)
   c \(8^a \times 8^a = 8^4\) (3 marks)

8 Write (3\(^4\))^\(\frac{1}{2}\) as a single power. (1 mark)

9 Write each number in standard form.
   a 0.00000065
   b 0.9 million
   c 320 \times 10^7 (3 marks)

10 \(9^{18} = 27^x\)
   Work out the value of \(x\). (2 marks)

11 Use prime factors to determine whether 2520 is divisible by 18. (2 marks)

12 Write \(\left(\frac{4}{3}\right)^2\) as a fraction in its simplest form. (2 marks)

13 Work out the area of this shape.
   Write your answer as a simplified surd. \(\sqrt{45}\) \(\sqrt{30}\) (3 marks)

Sample student answer
The maths is correct, but the student will only get 2 marks. Why?

Sample student answer
The maths is correct, but the student will only get 2 marks. Why?

Exam-style question
One sheet of paper is 9 \(\times 10^{-3}\) cm thick.
Mark wants to put 500 sheets of paper into the paper tray of his printer. The paper tray is 4 cm deep. Is the paper tray deep enough for 500 sheets of paper? You must explain your answer. (3 marks)

Exam-style question
How many different 4-digit odd numbers are there, where the first digit is not zero? (3 marks)

Exam-style question
Let \(x = 6 \times 10^5\) and \(y = 8 \times 10^4\). Work out
   a \(x + y\)
   b \(x - y\)
   c \(x \times y\)
   d \(\frac{y}{x}\)
   Write your answers in standard form. (2 marks)

Sample student answer
The maths is correct, but the student will only get 2 marks. Why?

Sample student answer
The maths is correct, but the student will only get 2 marks. Why?

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9 \times 10^{-3} \times 500
0.009 \times 500
9 \times 500 = 4500, -1000 = 4.5
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