11 RATIO AND PROPORTION

11.1 Writing ratios

1 Write down each ratio of blue cubes to yellow cubes.
   a  
   b  

2 Draw cubes to show these ratios of blue to yellow.
   a  5 : 2  
   b  2 : 3  

3 A box of chocolates has 20 chocolates. There is 1 nut chocolate for every 4 caramel chocolates.
   How many chocolates are
   a  nut  
   b  caramel?  

4 Write each ratio in its simplest form.
   a  3 : 15  
   b  12 : 6  
   c  32 : 8  
   d  6 : 48  
   e  15 : 25  
   f  72 : 48  
   g  28 : 49  
   h  36 : 42  

5 P / R The bar charts show the numbers of rosettes won in a horse show by teams from a club.

   Number of red rosettes won  
   Number of other rosettes won  

6 A primary school class teacher is planning a class trip. There are 30 children in the class and 5 adults are available to go on the trip. The guidelines say that the adult-to-child ratio should be 1 : 6.
   Are there enough adults for the trip?  

7 Which of these ratios are equivalent?
   A 56 : 49  
   B 120 : 105  
   C 108 : 84  
   D 128 : 112  
   E 63 : 49  

8 Write each ratio in its simplest form.
   a  6 : 21 : 15  
   b  15 : 25 : 30  
   c  32 : 40 : 48  
   d  45 : 72 : 54  

9 Show that these ratios are equivalent.
   15 : 10 : 35  
   18 : 12 : 42  
   36 : 24 : 84  

10 A recipe for sweet pastry uses 200 g of flour, 100 g of butter and 75 g of sugar.
   Write the ratio of flour : butter : sugar in its simplest form.  

11 Exam-style question

   There are 60 sweets in a bag. Of these, 18 are toffee and the rest are fruit flavoured.
   Write the ratio of toffee to fruit-flavoured sweets in its simplest form.  

(2 marks)  

11.2 Using ratios 1

1 The ratio of sand : cement needed to make mortar is 5 : 1.
   Sarah uses 6 kg of cement.
   How many kilograms of sand does she use?
2 Mark makes a model of the Burj Khalifa tower using a ratio of 1 : 2400.
The height of his model is 345 mm. What is the height of the Burj Khalifa tower in metres?

Q2 hint 1 : 2400 means that 1 mm in the model represents 2400 mm in real life.

3 Write each ratio as a whole number ratio in its simplest form.
   a 0.3 : 6
   b 3.2 : 4.8
   c 36 : 1.5
   d 14.4 : 25.6

4 Write each ratio as a whole number ratio in its simplest form.
   a 0.3 : 1.25
   b 37.8 : 6.3
   c 12.5 : 13.75
   d 28.8 : 61.2

5 P Windows are made in standard ratios. 
   Type A are made in the width : height ratio of 5 : 6, and Type B are made in the ratio 5 : 8. 
   Is each of these windows Type A or Type B?
   a
   b

6 Brass screws are made from copper and zinc in the ratio 84.5 : 45.5. 
   Write this ratio in its simplest form.

7 The ratio of fat : flour in a recipe is 3 : 8. 
   Aziz uses 75 g of fat. How many grams of flour should he use?

8 R The ratio of potato : cauliflower in a potato and cauliflower curry recipe is 9 : 15. 
   Sanjay uses 420 g of cauliflower. He has 300 g of potato. 
   Will he use all of the potato?

9 Huw splits his weekly pocket money into spending money and money saved to buy a laptop in the ratio 17 : 13. 
   He gets £15 each week. How much money does he save each week?
A farm shop has 30 hundredweight of potatoes.
They need to be put into 5 kg bags to be sold.
The farm shop manager knows that
• 1 hundredweight = 112 pounds
• 1 pound ≈ 0.454 kilogram
How many bags of potatoes will the farm shop have for sale? (4 marks)

On the day that David buys some euros for his holiday, £1 buys €1.27.
How many euros does he get for £400?

Choose an amount in £ from the cloud. Choose an exchange rate from the table.

74

<table>
<thead>
<tr>
<th>Currency</th>
<th>£1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euros</td>
<td>1.27</td>
</tr>
<tr>
<td>Turkish lira</td>
<td>3.56</td>
</tr>
<tr>
<td>Croatian kuna</td>
<td>9.71</td>
</tr>
<tr>
<td>Albanian lek</td>
<td>176.22</td>
</tr>
<tr>
<td>Tunisian dinar</td>
<td>2.90</td>
</tr>
<tr>
<td>Moroccan dirham</td>
<td>14.06</td>
</tr>
</tbody>
</table>

Work out how much money you will get. Repeat for two more amounts and currencies.

9 P Use the table in Q8.
   a How many £ does Vicky get for 500 Turkish lira?
   b Which is worth more, 500 Moroccan dirham or 3800 Croatian kuna?

10 P The diagram shows three right-angled triangles.

Write these ratios in their simplest form.
   a The base of triangle A to triangle C.
   b The height of triangle A to triangle C.
   c The area of triangle A to triangle C.

11 The diagram shows two cuboids.

Write these ratios in their simplest form.
   a Height of A to height of B.
   b Area of the shaded face of A to area of the shaded face of B.
   c Volume of A to volume of B.

12 A pencil case contains 4 pens and 9 pencils.
   a What is the ratio of pens : pencils?
   b What fraction of the contents of the pencil case are pens?
   c What fraction of the contents of the pencil case are pencils?

13 In a multipack of crisps, \( \frac{3}{5} \) of the packets of crisps are salt and vinegar and \( \frac{2}{5} \) of the packets are cheese and onion.
   a What is the ratio of salt and vinegar : cheese and onion?
   b There are 6 packets of cheese and onion in the multipack. How many packets of salt and vinegar are there?
   c How many packets of crisps are in the multipack altogether?

14 R In a school drama group, \( \frac{3}{4} \) of the group are girls and the rest are boys.
   a What is the ratio of boys : girls in the group?
   b Rashid thinks there are 27 students in the group. Explain why he must be wrong.

11.4 Using ratios 2

1 Share these amounts in the ratios given.
   a 24 kg in the ratio 3 : 1 
   b 72 m in the ratio 1 : 5 
   c £32 in the ratio 5 : 3 
   d £250 in the ratio 3 : 7 
   e 42 litres in the ratio 5 : 2 
   f 12.5 km in the ratio 3 : 2

2 Solder used for joining metal pipes is made from a mixture of tin and lead in the ratio 3 : 2.
Paul has 425 g of solder.
What is the mass of
   a tin 
   b lead?
3 Green dye is mixed from yellow dye and blue dye in the ratio 3 : 4.
Amir needs 35 litres of green dye.
   a How many litres of yellow dye should he use?
   b How many litres of blue dye should he use?

4 A cereal mix is made from oats, wheat and barley in the ratio 3 : 2 : 1.
Chantal wants to make 600 g of cereal mix. How much of each type of ingredient does she need?

5 **Exam-style question**
Jamila is going to make some shortbread. She needs to mix flour, butter and sugar in the ratio 3 : 2 : 1 by weight.
Jamila wants to make 1.5 kg of shortbread.
   • 450 g of butter
   • 1500 g of flour
   • 500 g of sugar
Does Jamila have enough flour, butter and sugar to make the shortbread? (4 marks)

6 Share these amounts in the ratios given.
   a 750 g in the ratio 2 : 3 : 5
   b 600 ml in the ratio 3 : 2 : 7
   c £84 in the ratio 3 : 5 : 6
   d £30 in the ratio 4 : 5 : 3

7 Share these amounts in the ratios given. Round your answers sensibly.
   a £60 in the ratio 3 : 4
   b 125 g in the ratio 5 : 4

8 R Jennifer and Tom buy a flat to rent out for £125 000.
   Jenifer pays £75 000 and Tom pays £50 000. They rent out the flat for £650 per month.
   a Write the amounts they each pay as a ratio.
   b Write the ratio in its simplest form.
   c Divide the rent in this ratio. How much of the rent does each of them get?

9 P Kamal and Aisha buy a car together for £6300.
   Aisha pays £2700 and Kamal pays £3600. They sell the car 4 years later for £4200.
   How should they share the money fairly?

10 P / R Two whole numbers are in the ratio 2 : 5 and their product is a multiple of 3. What are the smallest numbers possible?

11 Sharon and Jo share £90 in the ratio 7 : 5.
   a How much do they each receive?
   b How much more does Sharon get than Jo?

12 **Exam-style question**
Kevin and Lesley share a bag of sweets in the ratio 3 : 4.
Kevin gets 15 fewer sweets than Lesley. How many sweets did Lesley get? (3 marks)

11.5 Comparing using ratios

1 In a karate class \( \frac{4}{9} \) of the group are men and the rest are women. What is the ratio of men to women in the class?

2 Copy and complete the table for different groups of people having karate lessons.

<table>
<thead>
<tr>
<th>Fraction of group that are men</th>
<th>Ratio of men : women</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{7} )</td>
<td>5 : 3</td>
</tr>
<tr>
<td>( \frac{11}{12} )</td>
<td>5 : 6</td>
</tr>
</tbody>
</table>

3 In a particular fruit juice mix, \( \frac{5}{12} \) is orange juice, \( \frac{4}{12} \) is mango juice and the rest is apple juice. Work out the ratio of orange juice : mango juice : apple juice.

4 A car windscreen washer mix can be made from water and concentrate in the ratio 5 : 1. What fraction of the washer mix is
   a water
   b concentrate?
5 Mark and Jon share a pizza in the ratio 5 : 4. What fraction of the pizza should
   a Jon get
   b Mark get?

6 Gilding brass is made of copper and zinc in the ratio 19 : 1. What fraction of the brass is
   a copper
   b zinc?

7 R The ratio of boys to girls in a music club is 3 : 7. Curtis says that \( \frac{3}{7} \) of the club members are boys. Is he correct? Explain your answer.

8 R Fruit cookies are made by mixing flour, butter, sugar and dried fruit in the ratio 3 : 2 : 1 : 1.
   a What fraction of the cookie mix is
      i flour
      ii butter?
   b In 700 g of cookie mix, what are the masses of flour, butter, sugar and dried fruit?
   c Joyce has plenty of flour and dried fruit, but only 150 g of butter and 200 g of sugar. What is the maximum amount of cookie mix she can make? Show your working clearly.

9 R A building mix uses cement, lime, building sand and gravel in the ratio 1.5 : 1.5 : 5 : 2
   a What fraction of the mix is lime?
   b In 50 kg of the mix, what are the masses of cement, lime, building sand and gravel?
   c You have plenty of cement, gravel and lime, but only 22.5 kg of building sand. What is the maximum amount of the building mix you can make?

Q9 hint Write the ratio with whole numbers first.

10 Copy and complete to write these as unit ratios.
   a \( \frac{7}{5} : \frac{1}{5} \)
   b \( \frac{4}{9} : \frac{4}{9} \)

11 Write each of these in the form \( m : 1 \).
   Give each answer to a maximum of 2 decimal places.
   a 11 : 4
   b 17 : 3
   c 56 : 15
   d 3 : 16
   e 7 : 32
   f 23 : 60

12 R Millie and Hannah are dyeing T-shirts. Millie makes an orange dye by mixing 75 g of yellow dye with 25 g of red dye. Hannah mixes 60 g of yellow dye with 24 g of red dye for her orange dye. Whose dye is darker orange? Explain your answer.

13 R Liam makes wallpaper paste by mixing 230 g of glue with 8 litres of water. Ryan makes some more wallpaper paste by mixing 95 g of glue with 3500 ml of water. Who makes the stronger paste? Explain your answer.

14 R Nadya makes a recipe that uses flour to butter in the ratio 500 : 190. Karl makes a different recipe using flour to butter in the ratio 425 : 150. Whose recipe has the higher proportion of butter?

15 R Tina and Gareth are running a marathon. Tina runs for 4 hours 45 minutes and walks for 45 minutes. Gareth runs for 5 hours 15 minutes and walks for 75 minutes. Who walks for a higher proportion of their race time? Explain your answer.

16 Exam-style question
A paint mixture is \( \frac{4}{7} \) blue and \( \frac{3}{7} \) red. Jack says, ‘The amount of blue is one and a third times the amount of red.’ Is he correct? Explain your answer. (3 marks)
11.6 Using proportion

1. 6 apples cost £1.50. What is the cost of
   a) 12 apples
   b) 2 apples
   c) 4 apples
   d) 10 apples?

2. 4 tickets to a show cost £80. How much will 17 tickets cost?

3. 60 litres of compost cost £4.50. What do 84 litres cost?

4. Edward gets £43.05 for 7 hours of work. How much will he get for working for
   a) 12 hours
   b) 25 hours
   c) 35 hours?

5. A group of 14 friends want to go on holiday together. The total cost of the holiday is £2996 and they all pay the same amount. 3 more friends agree to join them. What is the total cost of the holiday now?

6. Ketchup bottles come in two sizes.
   a) Copy and complete to find the unit ratios.
      i) $\frac{342 \text{ g}}{\text{£1.25}} = \frac{x \text{ g}}{\text{£1}}$
      ii) $\frac{460 \text{ g}}{\text{£1.75}} = \frac{x \text{ g}}{\text{£1}}$
   b) Which size is the better buy?
      Q6b hint: Which gives more grams for £1?

7. Kasia can buy bottles of cola in two sizes:
   a) a 1.25-litre bottle for £1.55 or a 1.75-litre bottle for £1.85.
      Which is better value for money?

8. Ajay can buy bags of rice in two sizes:
   1.5 kg for £3.27 or 600 g for £1.32.
   Which is better value for money?

9. Tom and Fred are both house painters.
   Tom charges £146.10 labour for a job that takes 6 hours. Fred charges £262.90 labour for a job that takes 11 hours.
   Who will earn more for a job that takes 17 hours?

10. Exam-style question
    Jill is making cupcakes. She finds out the prices of cupcake cases from two companies, Bake Rite and Kitchen Plus.

    | Company       | Pack of 60 cupcake cases | Pack of 80 cupcake cases |
    |---------------|--------------------------|--------------------------|
    | Kitchen Plus  | £2.25                    | £2.89                    |
    | Bake Rite     | £2.25                    | £2.89                    |

    She needs 240 cupcake cases and wants to buy them as cheaply as possible.
    Which company should Jill buy the cupcake cases from?
    You must explain your answer. (4 marks)

11.7 Proportion and graphs

1. Look at the graph.

   Are kilograms and pounds in direct proportion? Explain your answer.

2. The table shows some amounts of liquid in litres and in pints.

<table>
<thead>
<tr>
<th>Litres</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pints</td>
<td>3.5</td>
<td>7</td>
<td>10.5</td>
<td>17.5</td>
</tr>
</tbody>
</table>

   a) Plot a line graph for these values.
   b) Are litres and pints in direct proportion? Explain your answer.
   c) Car oil is sold in a 5-litre can. How many pints is this?
3 The table shows the price of nails at a DIY store.

<table>
<thead>
<tr>
<th>Mass of nails (kg)</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of nails (£)</td>
<td>1.20</td>
<td>2.40</td>
<td>4.80</td>
<td>12</td>
</tr>
</tbody>
</table>

- a Plot a line graph for these values.
- b Is the price of the nails in direct proportion to the mass? Explain your answer.

4 This line graph shows the price of petrol by volume.

![Price of petrol graph]

- a Are price and volume in direct proportion?
- b Work out the gradient of the line.
- c How much does 1 litre of petrol cost?

5 R Which of these are in direct proportion? Give reasons for your answers.
- a Miles and kilometres
- b Age and number of pets
- c Litres and millilitres
- d Number of people at a bus stop and time of day

6 P An electrician charges a callout fee of £35 plus £42 per hour she works. Is her total charge \( C \) in direct proportion to the number of hours \( h \) she works?

7 Which of these sketch graphs show one variable in direct proportion to the other?

- A
- B
- C
- D
- E

8 The table shows the amount paid for different size containers of milk.

<table>
<thead>
<tr>
<th>Number of litres, ( n )</th>
<th>0.6</th>
<th>1</th>
<th>1.2</th>
<th>2</th>
<th>2.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost, ( C ) (p)</td>
<td>48</td>
<td>80</td>
<td>96</td>
<td>160</td>
<td>192</td>
</tr>
</tbody>
</table>

- a Draw the graph.
- b Work out the equation of the line.
- c Are \( C \) and \( n \) in direct proportion?
- d Write a formula linking the number of litres \( n \) and the cost \( C \).
- e Use the formula to work out the cost of 3.6 litres of milk.

9 Look back at the graph in Q1.

- a Write the ratio kilograms : pounds in its simplest form.
- b Work out the gradient of the graph.
- c Write a formula that links kilograms \( K \) and pounds \( P \).

10 On a particular day, the ratio of GB pounds to US dollars is 5 : 9.

- a Copy and complete this table of values for pounds and dollars.

<table>
<thead>
<tr>
<th>GB pounds (£)</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>US dollars ($)</td>
<td>0</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

- b Draw a conversion graph for pounds to dollars.
- c Write 5 : 9 as a unit ratio.
- d Write a formula linking dollars \( y \) and pounds \( x \).

11 Exam-style question

P A plumber uses a formula to work out the cost \( C \) of a job according to the number of hours \( H \) it takes. His formula is

\[ C = 35H + 45 \]

Is the plumber’s charge for a job in direct proportion to the amount of time it takes? Explain your answer.

(2 marks)

Exam hint

You won’t get any marks for just ‘Yes’ or ‘No’. You must write ‘Yes’ or ‘No’ because …………….
11.8 Proportion problems

1. 3 children build a giant sandcastle in 4 hours. How long will it take
   a. 1 child
   b. 5 children?

Q1a hint Will it take 1 child more or less time than 3 children?

2. 2 decorators take $\frac{5}{2}$ hours to paint 3 rooms. How long will it take
   a. 3 decorators
   b. 5 decorators?

Give your answers in hours and minutes.

3. 3 school uniform shirts cost £13.50. How much will 5 shirts cost?

4. R A farmer needs to pick his 750 kg crop of strawberries. He wants all the work done in 8 hours or less. He knows that 1 person can pick 8 kg of strawberries in an hour. How many people does he need to do the job?

5. A group of 5 cyclists can complete a circular 20-mile cycle in 2 hours. How long will it take a group of 8 cyclists to complete the same ride?

6. It takes 15 minutes for 24 cupcakes to cook. How long will it take for 32 cupcakes?

7. R A farmer has enough bales of silage to feed 120 cattle for 30 days. He sells 30 cattle. How many days will the silage last for?

8. A print run of 40 exam papers uses 560 sheets of paper. How many sheets of paper would be needed to print 215 copies of the same exam paper?

9. R A caretaker needs to clean 35 classrooms. He knows that 2 people can clean a room in 20 minutes. He needs all the work to be done in 3 hours after school. How many cleaners does he need to employ?

10. R It costs £85.82 to hire a car for 7 days. How many days does Amit hire a car for if he pays £208.42?

11. R It takes 6 people 5 hours to pick 600 kg of apples. The apple grower employs 2 more people. How long will it take to pick 600 kg of apples now?

12. Example

Exam-style question
It takes 7 men 4 hours to dig a ditch. How long would it take 5 men? (2 marks)

11 Problem-solving

Solve problems using these strategies where appropriate:
• Use pictures
• Use smaller numbers
• Use bar models
• Use $x$ for the unknown
• Use a flow diagram
• Use more bar models.

1. Jay, Alix and Cam have dinner. Jay has only a starter. He pays one tenth of the bill.
   The total bill is £40. Cam has extra drinks. She pays half of the bill.
   a. How much does Jay pay?
   b. How much does Alix pay?
   c. What is the ratio of Alix’s bill to Cam’s bill?
      Write your answer in its simplest form.

2. Ajesh has 8 box sets of his favourite drama. He stacks them in a pile next to 20 DVDs. Both piles are the same height.
   a. What is the ratio of the number of box sets to DVDs?
      Write your answer in its simplest form.
   b. One DVD is about 1.4 cm thick. How thick is each box set?

3. A group of 6 students takes 3 days to paint a classroom. A group of 4 students takes 4 days to paint an identical classroom. Does a student in the first group work faster than a student in the second group?
4 **R** Ms Chapman wants to hire minibuses for a Year 8 trip to the museum. The teacher : student ratio must be at least 1 : 12 on each minibus. 60 students are going on the trip. Each minibus seats 25 people.
   a. What is the lowest number of teachers needed?
   b. How many minibuses will be needed?
   c. How many spare seats will there be?

5 **R** 3 bags, A, B and C, contain different amounts of money. The amount in bag B is \( \frac{3}{5} \) of the amount in bag A. The amount in bag C is \( \frac{1}{4} \) of the amount in bag B.
   a. There is £60 in bag A. How much money is in bags B and C?
   b. What is the ratio A : C in its simplest form?
   c. Would the ratio be different if bag A contained £80? Explain your answer.

6 **R** All the sweets in a packet are either cubes or spheres. The ratio of cubes to spheres is 3 : 5.
   a. How many sweets could be in the bag? What is the smallest number possible?
   b. The sweets have either blue wrappers or yellow wrappers. The ratio of blue to yellow wrappers is 5 : 7.
   c. What is the smallest number of sweets possible now?
   d. With the number of sweets found in part b, how many of each type are there?

   Q6b hint You need a number that works for both ratios. Draw bar models to help you.

7 **R** Two of the angles in a quadrilateral are 60° and 90°. The other two angles are in the ratio 3 : 4.
   a. What are the missing angles?
   b. There are two different types of quadrilateral with these angles. What are they?

8 **P** The first term in a sequence is \( n \).
   The term-to-term rule is \( 4n - 2 \).
   The fourth term is 22. What is \( n \)?

9 **R** Kirti places a 1 kg bag of sugar on her faulty scales. It reads 1.2 kg. She then places a 5 kg bag of rice on it. It reads 6 kg.
   a. What is the ratio of real weight to faulty reading of the scales?
   b. A recipe requires 800 g of flour. What do the scales need to show for Kirti to weigh out the correct amount?

10 Last year, Devan, Nikhil and Jasdeep shared their company profits in the following way. Devan got \( \frac{9}{20} \), Nikhil got \( \frac{3}{10} \) and Jasdeep got the rest.
    This year they are splitting their profits in the ratio 5 : 4 : 3.
    In both years the company’s profit was £180 840.
    a. Nikhil is happy with the change. Explain why.
    b. Using fractions, show that the change did not make a difference to Jasdeep.

11 **Exam-style question**
   At St Andrew’s High there are 160 Year 11 students. 65% of them will be staying on to attend the sixth form college.
   Of those staying on, the ratio of boys to girls is 6 : 7.
   Of those leaving at the end of the year, the ratio of boys to girls is 5 : 2.
   How many more boys are staying on than leaving? (5 marks)