Index laws

1 Express $25^{-\frac{3}{2}}$ in the form 5^n **(2)**

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Guided

2 Express $\sqrt{3}(27^{\frac{2}{3}})$ in the form 3^x **(2)**

$\sqrt{3}(27^{\frac{2}{3}}) = 3^{\cdots} \times (27^{\cdots})^2 =$ ..... = ...

3 Simplify $3x(2x^{-\frac{3}{4}})$ **(2)**

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4 Simplify $\dfrac{20x^{\frac{5}{3}}}{4x}$ **(2)**

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Guided

5 Simplify fully $\dfrac{(3x^{\frac{1}{2}})^3}{9x^3}$ **(3)**

$\dfrac{(3x^{\frac{1}{2}})^3}{9x^3} = \dfrac{\cdots\cdots}{9x^3} =$ ..... = ...

First simplify the numerator.

6 Write $\dfrac{3 - x^{\frac{3}{2}}}{\sqrt{x}}$ in the form $3x^p - x^q$ where p and q are constants. **(2)**

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7 Solve $3^{2x+1} \times 9^x = 27$ **(2)**

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Write both sides of the equation as powers of 3 to find x.

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8 Solve $2^{2x-3} \times 4^{x+2} = 8$ **(4)**

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9 Write $\dfrac{6\sqrt{x} + 4x^{-\frac{3}{2}}}{2x^3}$ in the form $3x^p + 2x^q$ where p and q are constants. **(2)**

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Expanding and factorising

Guided **1** Expand $(x - 1)(x + 2)^2$ **(2)**

$(x - 1)(x + 2)^2 = (x - 1)(x^2$ .....$)$

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First expand $(x + 2)^2$.

Multiply out and collect like terms, then multiply by $(x - 1)$.

Guided **2** Factorise completely $x^3 - 9x$ **(2)**

$x^3 - 9x = x(x^2$ .....$)$

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First take out the common factor. Then factorise the expression inside the brackets.

3 Expand $(x - 4)(x + 2)(x - 1)$ **(2)**

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4 Factorise completely $x^3 + 4x^2 - 5x$ **(3)**

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5 Show that $(2 - 3\sqrt{x})^2$ can be written as $4 - k\sqrt{x} + 9x$ where k is a constant. **(2)**

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6 Given that $f(x) = (x^2 - 4x)(x + 3) + 6x$

(a) express $f(x)$ in the form $x(ax^2 + bx + c)$ where a, b and c are constants. **(3)**

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(b) Hence factorise $f(x)$ completely. **(2)**

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Surds

1 Write $\sqrt{72}$ in the form $a\sqrt{2}$ where a is an integer. **(2)**

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Guided **2** Simplify $\sqrt{18} + \sqrt{50}$, giving your answer in the form $a\sqrt{b}$ where a and b are integers. **(2)**

$\sqrt{18} + \sqrt{50} = \sqrt{9 \times 2} + \sqrt{25 \times 2}$

$= 3 \times \sqrt{2} +$ .....

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Guided **3** Express $\dfrac{\sqrt{5}+3}{\sqrt{5}-2}$ in the form $a + b\sqrt{5}$ where a and b are integers. **(4)**

Insert brackets and rationalise the denominator.

Multiply out the brackets in the numerator and the denominator.

$\dfrac{\sqrt{5}+3}{\sqrt{5}-2} = \dfrac{(\sqrt{5}+3)(\sqrt{5}\ .....)}{(\sqrt{5}-2)(\sqrt{5}\ .....)}$

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4 Express $\sqrt{75} + \dfrac{21}{\sqrt{3}}$ in the form $a\sqrt{3}$ where a is an integer. **(3)**

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Rationalise the denominator in the second term.

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5 $(9 + a\sqrt{2})(3 - \sqrt{2}) = 23 - b\sqrt{2}$ where a and b are integers.

Find the values of a and b. **(4)**

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6 $(c - \sqrt{3})^2 = d - 14\sqrt{3}$ where c and d are integers.

Find the values of c and d. **(3)**

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7 Write $\dfrac{3(2-\sqrt{5})}{2+\sqrt{5}}$ in the form $a\sqrt{5} + b$ where a and b are integers. **(4)**

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Lengths and areas

1 A is the point $(-1, 6)$ and B is the point $(3, -2)$.
The length of AB is $p\sqrt{5}$, where p is an integer.
Find the value of p. **(3)**

Draw a sketch showing the positions of points A and B and use Pythagoras' theorem.

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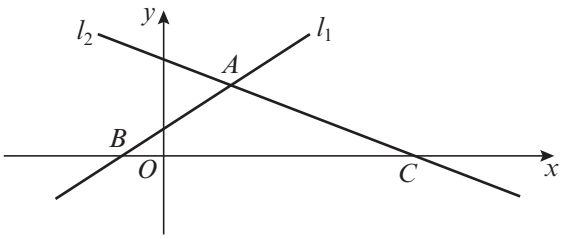
Guided

2 The line l_1 has equation $y = x + 1$
The line l_2 has equation $x + 3y - 15 = 0$
l_1 and l_2 intersect at the point A.

(a) Find the coordinates of A. **(3)**



Start by solving the equations simultaneously.

$x + 3(x + 1) - 15 = 0$

$x + 3x +$ .....

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l_1 crosses the x-axis at the point B.
l_2 crosses the x-axis at the point C.

(b) Find the area of triangle ABC. **(3)**

Substitute $y = 0$ into the equations to find the x-coordinates of B and C, then use Area = $\frac{1}{2}$ × base × height.

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3 The line l_1 has equation $3x - 2y + 18 = 0$
The line l_2 is perpendicular to l_1 and passes through the point $(1, 4)$.
Find the area of the triangle formed by the lines l_1, l_2 and the x-axis. **(9)**

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Equation of a circle

1 A circle C has centre $(4, -1)$ and radius 6.
Write down the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ **(2)**

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Guided **2** The circle C has centre $(2, 3)$ and passes through the point $(-1, 7)$.

(a) Find an equation for C. **(4)**

$r = \sqrt{(-1 - 2)^2 + (\ldots\ldots - \ldots\ldots)^2} = \ldots\ldots$

$(x\ldots\ldots)^2 + (y\ldots\ldots)^2 = r^2$

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First find r^2 using the formula for the distance between two points.

Then find an equation for C using $(x - a)^2 + (y - b)^2 = r^2$ where (a, b) are the coordinates of the centre of C.

(b) Verify that the point $(5, 7)$ lies on C. **(1)**

$(5\ldots\ldots)^2 + (7\ldots\ldots)^2 = \ldots\ldots$

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Substitute $x = 5$ and $y = 7$ into the left-hand side of the equation of the circle. Show all your working to verify that the value of the expression is equal to $25 = 5^2$.

3 The points A and B have coordinates $(-3, 5)$ and $(5, 11)$ respectively.
Given that AB is a diameter of the circle C, find an equation for C. **(5)**

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4. The circle C has equation $x^2 + y^2 + 2x - 6y = 6$

(a) Find the centre and the radius of C. **(5)**

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Problem solving First rearrange the formula, then complete the square to write it in the form $(x - a)^2 + (y - b)^2 = r^2$.

(b) Find the coordinates of the points where C crosses the coordinate axes, giving your answers as simplified surds. **(6)**

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Circle properties

1 The circle C has equation $x^2 + y^2 + 4x - 6y = 12$
The points P (1, 7) and Q (−5, −1) lie on the circle. Show that PQ is a diameter of C. **(2)**

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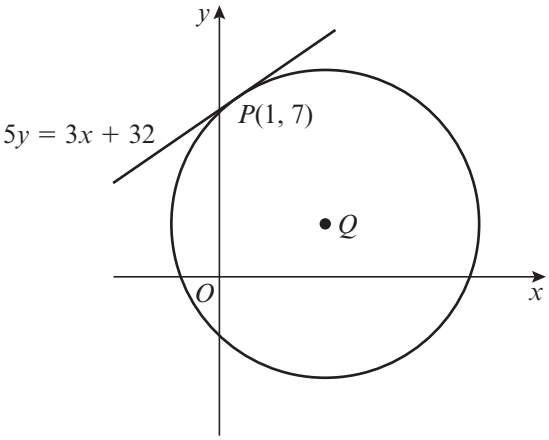
Guided

2 The line $5y = 3x + 32$ is a tangent to the circle C, touching C at the point P (1, 7), as shown in the diagram. The point Q is the centre of C.



(a) Find an equation of the straight line through P and Q in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

The line through P and Q is **perpendicular** to the tangent.

Gradient of tangent = ...

Gradient of line through P and Q = ...

Equation of line is $y - y_1 = m(x - x_1)$

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(b) Given that Q lies on the line $y = 2$, find the coordinates of Q. **(1)**

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3 A circle has equation $(x - 2)^2 + (y + 5)^2 = 180$
The tangent to the circle at the point (8, 7) meets the x-axis at P and the y-axis at Q.

Problem solving Find the gradient from the centre to (8, 7).
Use $y - y_1 = m(x - x_1)$ to find the equation of the tangent.

(a) Find the coordinates of P and Q. **(5)**

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(b) Find the area of the triangle PQC, where C is the centre of the circle. **(4)**

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