Paper 1: Pure Mathematics 1

Answer all questions. Write your answers in the spaces provided.

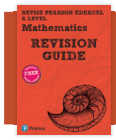
1 Find the set of values for which

$$f(x) = x^3 - 5x^2 + 3x + 4$$

is a decreasing function.

(4)

(Total for Question 1 is 4 marks)



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Hint

Start by differentiating.

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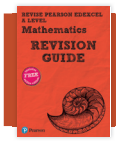
$f(x)$ is decreasing on an interval $[a, b]$ if $f'(x) \leqslant 0$ for all values of x in that interval.

Hint

You can give your answers as an inequality or using set notation.



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Hint Q2a

Evaluate f(4) then use the answer as your input for f.

Hint Q2b

Use partial fraction techniques and compare coefficients.

Hint Q2c

Consider the values of f(x) when $x = 2$ and when $x \to \infty$. You can use your answer to part (b) to work out the value of f(x) as $x \to \infty$.

Hint Q2d

The domain of the inverse function is the same as the range of the original function.

2 $f(x) = \dfrac{5x + 2}{2x - 1}, \quad x \geqslant 2$

(a) Find ff(4)

(2)

(b) Show that f(x) can be written in the form $A + \dfrac{B}{2x - 1}$, where A and B are constants to be found.

(2)

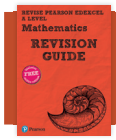
(c) Hence, or otherwise, state the range of f.

(1)

(d) Find $f^{-1}(x)$, stating its domain.

(3)

(Total for Question 2 is 8 marks)



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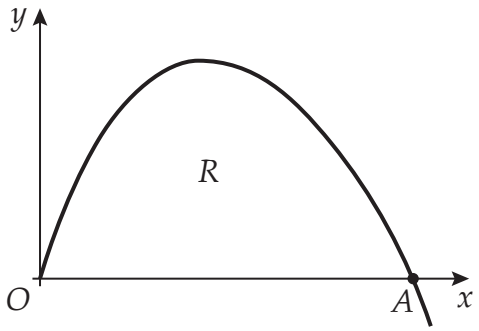
3 The diagram shows the graph of

$$y = 24x - 8x^{\frac{3}{2}}$$



The graph crosses the x-axis at A.

(a) Find the value of the x-coordinate at A.

(2)

(b) Find the area, R, bounded by the curve and the x-axis between the origin, O, and point A.

(4)

(Total for Question 3 is 6 marks)

Hint Q3a

Solve $24x - 8x^{\frac{3}{2}} = 0$

Hint Q3b

Area $= \int_0^a (24x - 8x^{\frac{3}{2}})\,dx$

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Hint Q4b

The stationary points occur when $\frac{dy}{dx} = 0$

Hint Q4c

At a point of inflexion, $\frac{d^2y}{dx^2} = 0$

Problem solving

$\frac{d^2y}{dx^2} = 0$ doesn't guarantee a point of inflexion. You also need the sign of $\frac{d^2y}{d^2x}$ to change on either side of that point.

4 A curve, C, has equation

$$y = x^3 + 6x^2 + 9x + 5$$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **(3)**

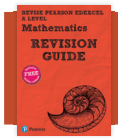
(b) Verify that C has a stationary point when $x = -3$. **(2)**

(c) Determine the nature of this stationary point, giving a reason for your answer. **(2)**

(d) Find the coordinates of the point of inflexion on C. **(2)**

(e) State, with a reason, whether C is concave or convex in the interval $[0, 1]$ **(1)**

(Total for Question 4 is 10 marks)



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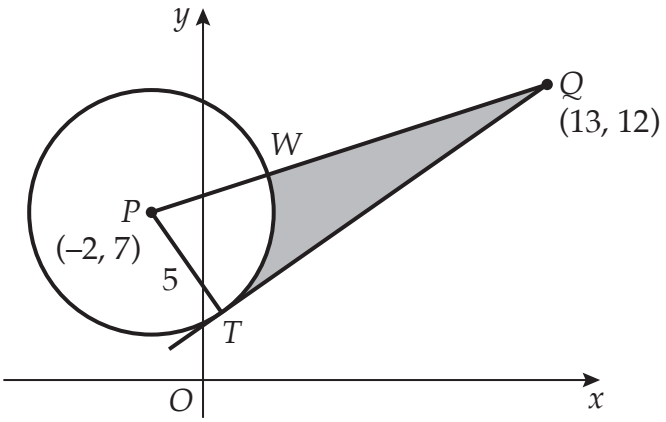
5 A circle has centre, P, at $(-2, 7)$ and radius of 5. The tangent to the circle at T passes through the point Q (13, 12). The straight line PQ cuts the circle at W.



(a) Find the size of angle TPW in radians, giving your answer to 3 decimal places.

(4)

(b) Find the area of the shaded region TWQ, giving your answer to 2 decimal places.

(5)

(Total for Question 5 is 9 marks)

Hint Q5a

Make sure your calculator is in radians mode.

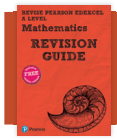
LEARN IT!

Area of sector $= \frac{1}{2}r^2\theta$

Area of triangle $= \frac{1}{2}ab\sin C$

Hint Q5b

Don't round any intermediate values. Use the memory functions on your calculator and write down at least 4 decimal places in your working.



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Hint

Find expressions for the vectors $\overrightarrow{AC}$, $\overrightarrow{BC}$ and $\overrightarrow{AB}$, and then find their magnitudes.

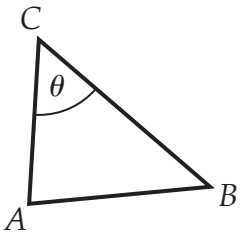
Hint

Use the cosine rule to find θ.

LEARN IT!

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

6



The diagram shows a sketch of triangle ABC. Given that, with reference to a fixed origin, O,

$$\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$$

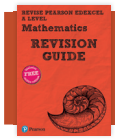
$$\overrightarrow{OB} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

$$\overrightarrow{OC} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

find the size of angle ACB, marked θ on the diagram.

(6)

(Total for Question 6 is 6 marks)



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7 $f(x) = x^3 + x + 8$

(a) Show that $f(x) = 0$ has a root α in the interval $[-2, -1]$

(2)

(b) Find $f'(x)$

(1)

(c) Taking $x_0 = -2$ as a first approximation, apply the Newton–Raphson method three times to obtain an approximate value of α. Give your answer to 3 decimal places.

(4)

(Total for Question 7 is 7 marks)

Hint Q7a

You need to show that $f(x)$ **changes sign** in the given interval. Make sure you state this fact in your answer.

Hint Q7c

The formula for the Newton–Raphson method is given in the formulae booklet:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Hint Q7c

You should find that x_2 and x_3 round to the same value for α when written to 3 d.p.