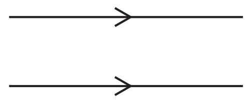
2.1 Angles

Recognise alternate and corresponding angles

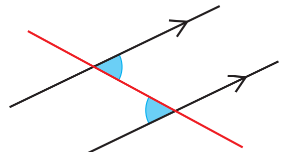
Key words
parallel
alternate
corresponding
vertically opposite

Remember, **parallel** lines are straight lines which never meet or cross.

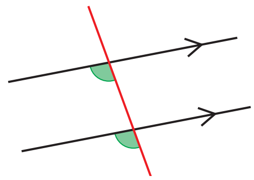
The arrows show that the lines are parallel



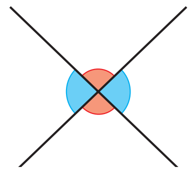
Alternate angles on parallel lines are equal. In this diagram, the **alternate** angles are on *alternate* sides of the red line.



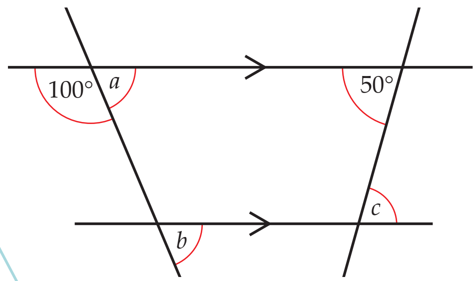
Corresponding angles on parallel lines are equal. In this diagram, the **corresponding** angles are on the *same* side of the red line.



Remember, **vertically opposite** angles are also equal.



Example Calculate the size of the lettered angles in this diagram.
Give reasons for your answers.



$a = 80°$ because $100° + a = 180°$

$b = 80°$ because it is the corresponding angle to a

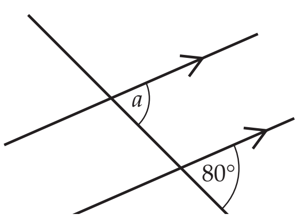
$c = 50°$ because it is the alternate angle to $50°$

Angles on a straight line add up to 180°.

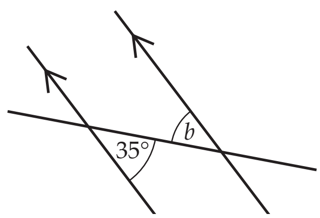
Exercise 2.1

1 Find the size of the lettered angles. Give reasons for your answers.

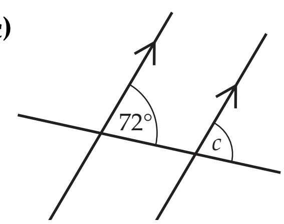
a)



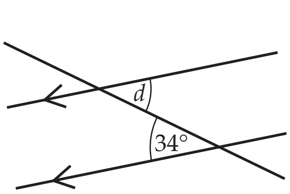
b)



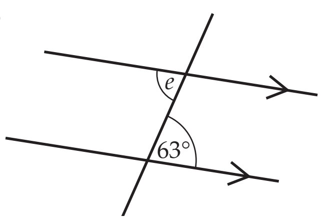
c)



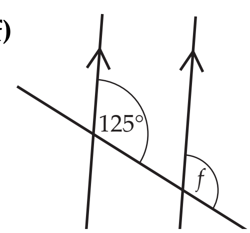
d)



e)

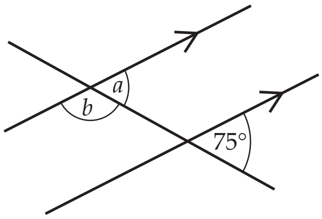


f)



2 Calculate the size of the lettered angles. Give reasons for your answers.

a)



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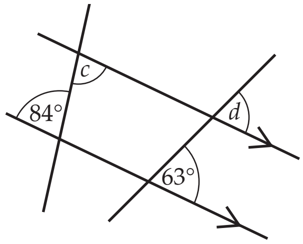
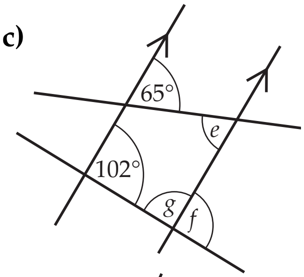
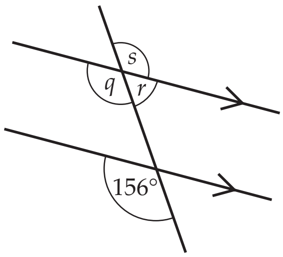


c)



d)



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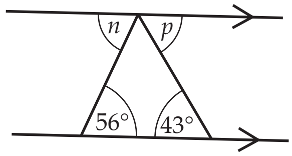

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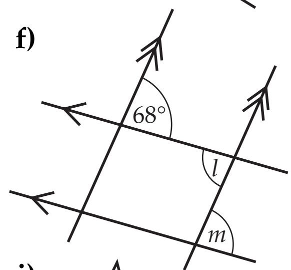
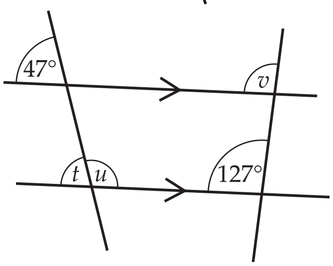
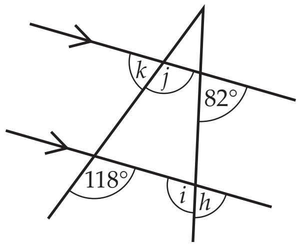


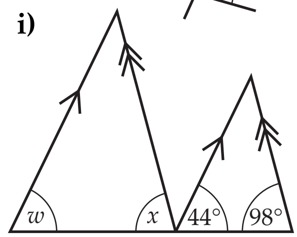
g)



h)

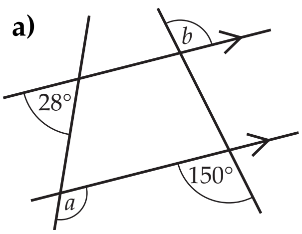


i)

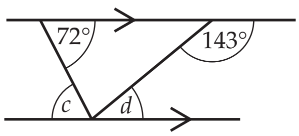


3 Calculate the size of the lettered angles. Give reasons for your answers and write down each step of your working.

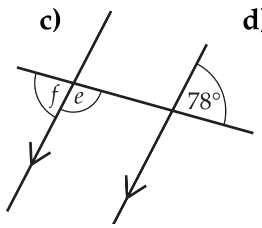
a)



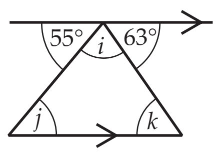
b)



c)



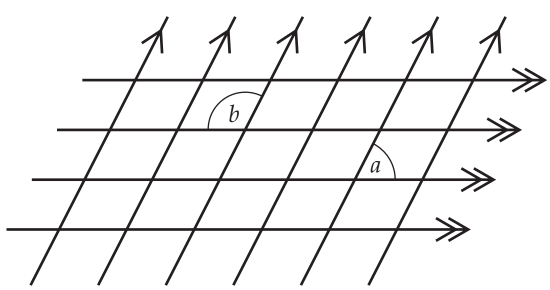
d)



It may be helpful to sketch the diagram and work out some of the other angles first.

Investigation

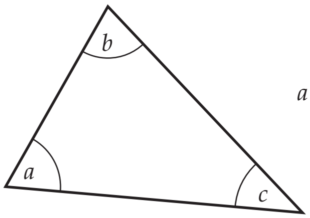
4 **a)** Copy this diagram.
b) Mark all the angles that are equal to a.
c) Mark all the angles that are equal to b.
d) What do you notice?
e) What do you notice about the pattern of angles in a parallelogram?



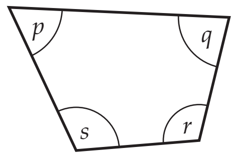
2.2 Calculating angles

- Know and use the fact that the sum of the angles in a triangle is 180°
- Know and use the fact that the sum of the angles in a quadrilateral is 360°
- Understand that the exterior angle of a triangle is equal to the sum of the two interior opposite angles

Key words
proof
interior
exterior



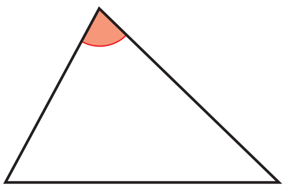
$a + b + c = 180°$

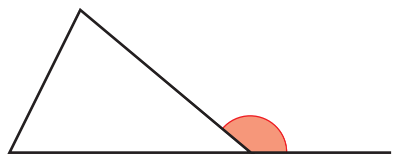


$p + q + r + s = 360°$

The sum of the angles in any triangle is 180°.

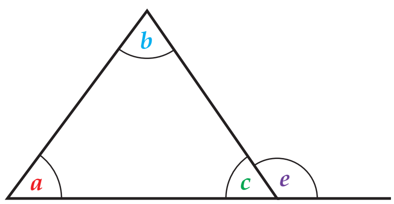
The sum of the angles in any quadrilateral is 360°.





An **interior** angle is *inside* the shape.

An **exterior** angle is *outside* the shape. It is made by extending one of the lines.



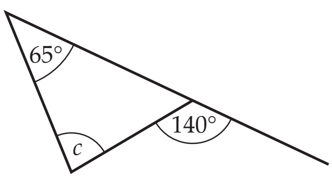
$c + e = 180°$
(angles on a straight line)
$a + b + c = 180°$
(angles in a triangle)
so $a + b = e$

The **exterior** angle of a triangle is equal to the sum of the two **interior** opposite angles.

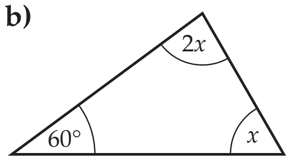
Example

Calculate the size of the lettered angles, giving reasons for your answers.

a)



b)



a) $c + 65° = 140°$
$c = 140° - 65°$
$= 75°$

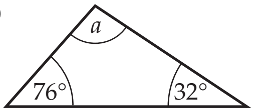
b) $2x + x + 60° = 180°$
$3x + 60° = 180°$
$3x = 120°$
$x = 40°$

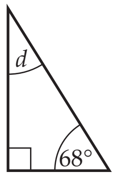
The sum of angles in a triangle is 180°.

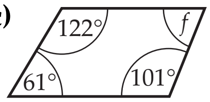
The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

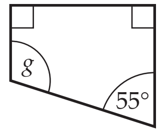
Exercise 2.2

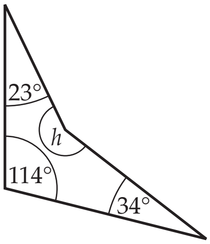
1. Calculate the size of the lettered angles, giving reasons for your answers.

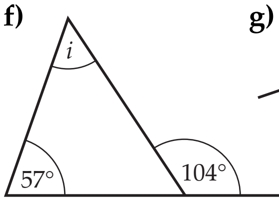
a)


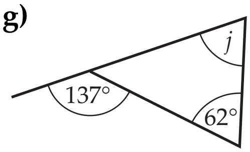
b)


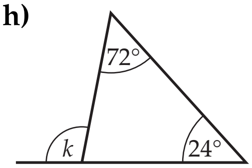
c)


d)


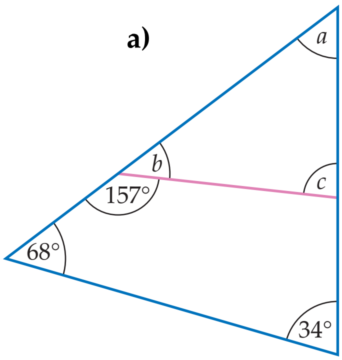
e)


f)


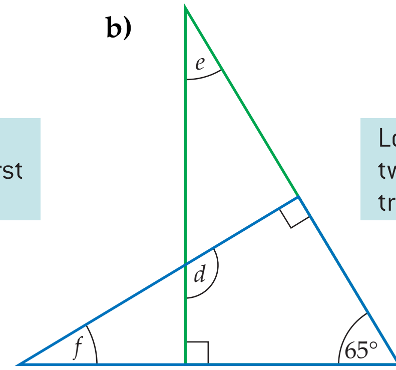
g)


h)


2. Calculate the size of the lettered angles, giving reasons for your answers.

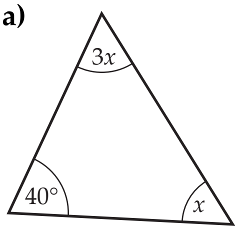
a)


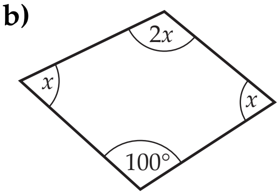
Look at the blue triangle first and find *a*.

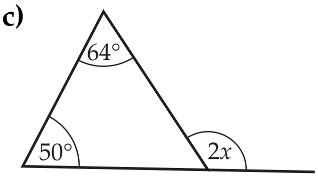
b)


Look for the two large triangles.

3. Work out the value of x in each of the following:

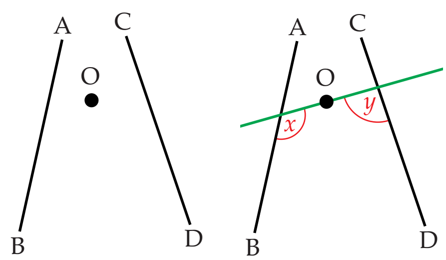
a)


b)


c)


Investigation

4. Draw two straight lines AB and CD.
Mark a point O between them.
Draw a line through O that crosses the two lines.
Mark the two angles, underneath the line, as x and y.
Measure the size of x and y with a protractor.
Draw some more lines through O. Each time measure the angles underneath the line and record your results in a table.
What do you notice? Can you explain why this happens?



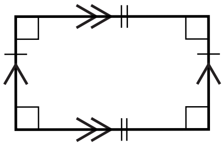
Extend AB and CD so they meet.

2.3 Quadrilaterals

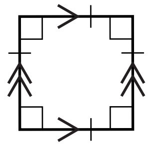
Recognise and classify quadrilaterals by their geometric properties

Key words
parallelogram
rhombus
isosceles trapezium
kite
arrowhead or delta

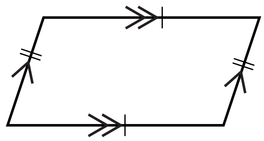
A quadrilateral is a four-sided shape. There are many different quadrilaterals. Some have special names because they have particular geometric properties.



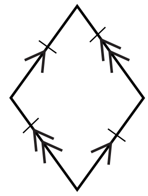
Rectangle
- Opposite sides are equal and parallel
- Diagonals bisect each other
- Rotation symmetry order 2



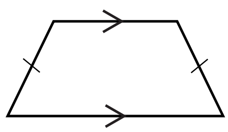
Square
- A rectangle with all four sides the same length
- Diagonals bisect each other at right angles
- Diagonals are lines of symmetry



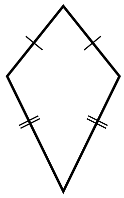
Parallelogram
- Opposite sides are equal and parallel
- Diagonals bisect each other
- Rotation symmetry order 2



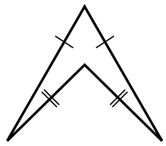
Rhombus
- A parallelogram with all four sides the same length
- Diagonals bisect each other at right angles
- Both diagonals are lines of symmetry



Isosceles trapezium
- A trapezium with two opposite non-parallel sides
- One line of symmetry
- Both diagonals are the same length



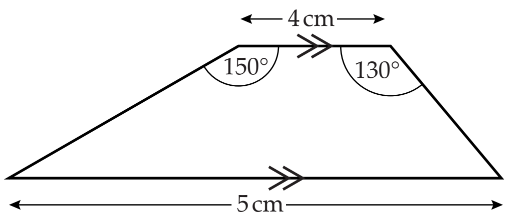
Kite
- Two pairs of adjacent sides that are the same length
- No interior angle is larger than 180°
- One line of symmetry
- Diagonals cross at right angles



Arrowhead or Delta
- Two pairs of adjacent sides that are the same length
- One interior angle is larger than 180°
- One line of symmetry
- Diagonals cross at right angles outside the shape

Example

Julie has to draw a sketch of an isosceles trapezium.
She writes the measurements on the diagram.
Explain why this sketch cannot show an isosceles trapezium.

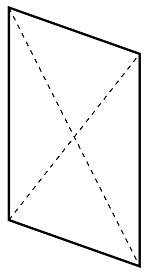


It is not an isosceles trapezium because it does not have any equal angles. This means there is no line of symmetry.

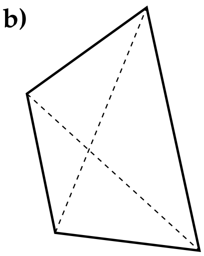
Exercise 2.3

1 Name each shape, using its geometric properties to explain your answers. You will need to measure lines and angles.

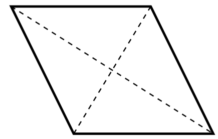
a)



b)



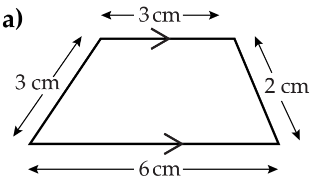
c)



The diagonals are shown as dashed lines.

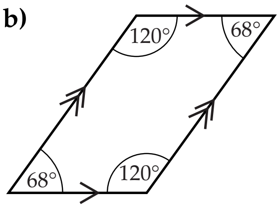
2 Jules draws some sketches of quadrilaterals, marks the size of their sides and angles and names them. Some of the names are incorrect. Find which sketches have incorrect names and explain why they are incorrect.

a)



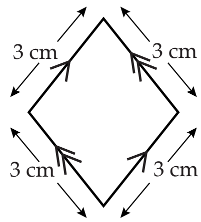
ISOSCELES TRAPEZIUM

b)



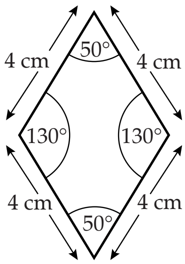
PARALLELOGRAM

c)



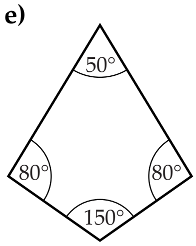
RHOMBUS

d)



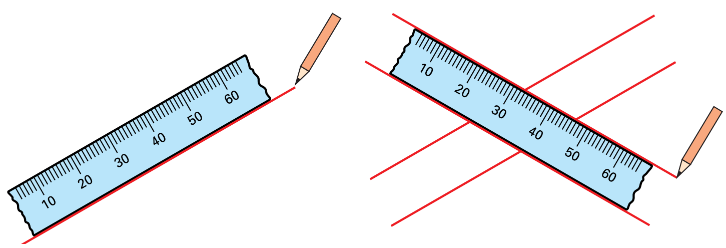
KITE

e)



DELTA

3 a) Draw along both edges of a ruler.
Take the ruler away and then place it across your lines at an angle. Draw along both edges of the ruler again.
Shade the shape enclosed by the lines. What is the name of the shape? Explain how you know.



b) It is possible to make another quadrilateral using the same method. What is the name of the quadrilateral? Explain how you know.

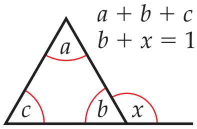
Investigation

4 You need a 3 by 3 pin board, or Resource sheet 61, and a rubber band. Make as many different quadrilaterals as you can. Draw and label each shape. Cut out your shapes and arrange them into groups that have similar properties. Stick the groups into your workbook and explain the properties that link the shapes in each group.

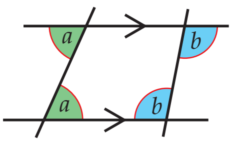
2.4 Solving geometrical problems

Solve geometrical problems using side and angle properties of special triangles and quadrilaterals

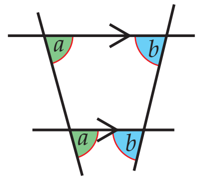
Key words
alternate
corresponding
vertically opposite



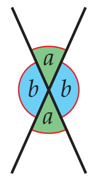
Angles in a triangle add up to 180°. Angles on a straight line add up to 180°.



Alternate angles are equal.



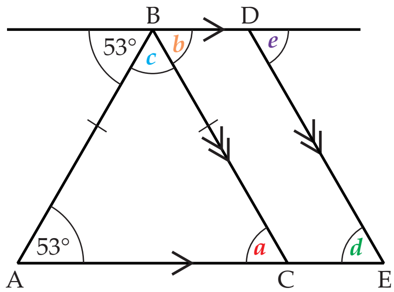
Corresponding angles are equal.



Vertically opposite angles are equal.

We can use this knowledge to find the size of missing angles. Since we are calculating the sizes of the angles by following a chain of reasoning and not measuring them, our diagrams do not have to be accurate.

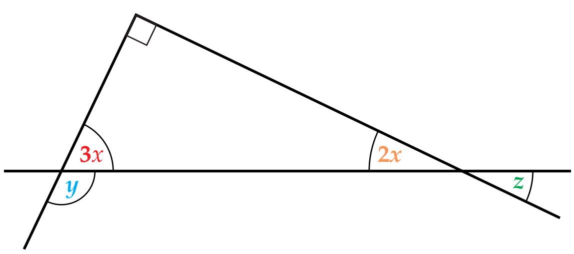
Example 1 Calculate the size of angles a, b, c, d and e, giving reasons for your answers.



We can see from the diagram that ABC is an isosceles triangle and that BD and CE are parallel. BC and DE are also parallel.

Angle a is 53° because triangle ABC is isosceles and the base angles of an isosceles triangle are equal.
Angle b is 53° because it is an alternate angle to angle a.
Angle c is 74° because angles on a straight line add up to 180° and $180° - 53° - 53° = 74°$.
Angle d is 53° because it is a corresponding angle to angle a.
Angle e is 53° because it is an alternate angle to angle d.

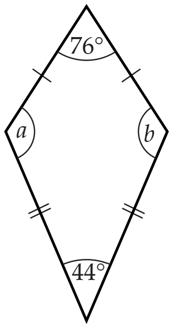
Example 2 Calculate the size of the missing angles giving reasons for your answers.

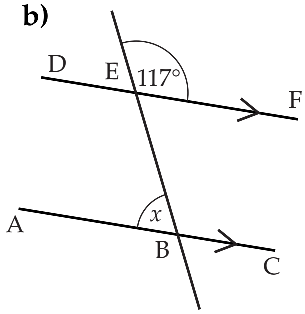


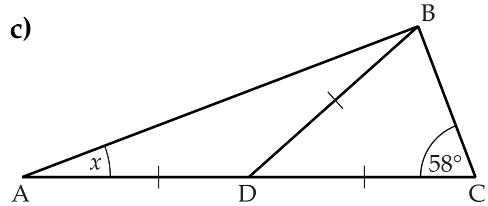
$90° + 3x + 2x = 180°$ because angles in a triangle add up to 180°
$90° + 5x = 180°$
$5x = 90°$
$x = 18°$ so $3x = 54°$ and $2x = 36°$
$y + 54° = 180°$ because angles in a straight line add up to 180° so $y = 126°$
$z = 36°$ because it is vertically opposite to the angle $2x$.

Exercise 2.4

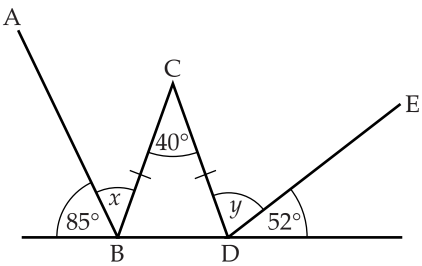
1 Calculate the size of the lettered angles, giving reasons for your answers.

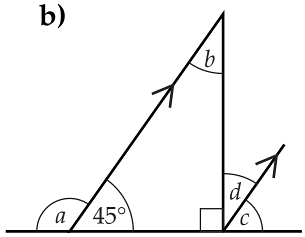
a)


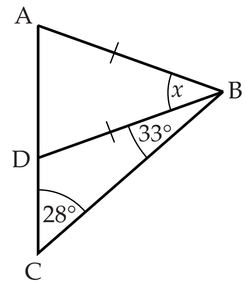
b)


c)


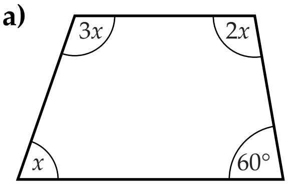
2 Calculate the size of the lettered angles, giving reasons for your answers.

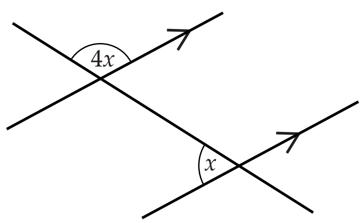
a)


b)


c)


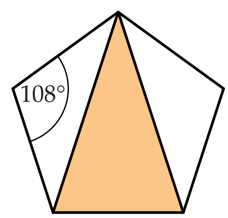
3 Calculate the size of each angle, giving reasons for your answers.

a)


b)


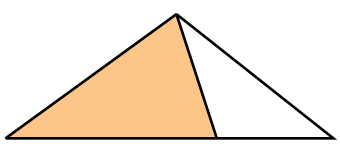
Investigation

4 The angle at the vertex of a regular pentagon is 108°.
Two diagonals are drawn to the same vertex to make three triangles.



a) Calculate the sizes of the angles in each triangle.

b) The middle triangle and one of the other triangles are placed together like this:



Explain why the triangles fit together to make a new triangle. What are its angles?

c) Investigate the other shapes you can make by putting the three triangles together.

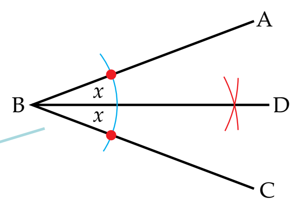
2.5 Construction

- Construct a bisector of an angle, using a ruler and compasses
- Construct the mid-point and perpendicular bisector of a line segment, using a ruler and compasses

Key words
bisector
compasses
equidistant
perpendicular bisector
mid-point
equidistant

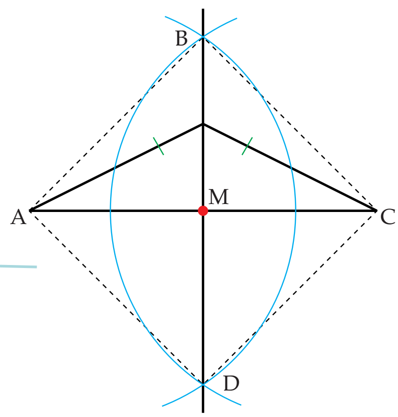
The **bisector** of an angle is a line that divides the angle into two equal parts. You can construct the bisector of an angle using **compasses**.

In this diagram, BD is the bisector of the angle ABC. Every point on the line BD is **equidistant** from the lines BA and BC.

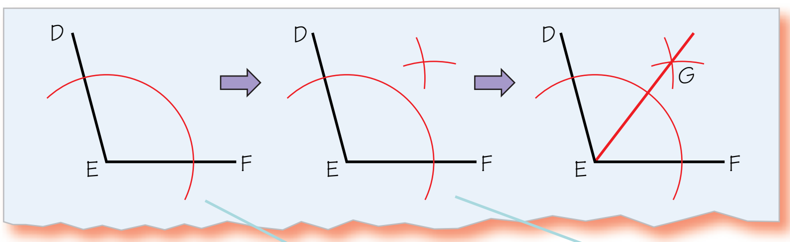


The **perpendicular bisector** of a line segment divides the line segment into two equal parts at right angles.

In this diagram, BD is the perpendicular bisector of AC. It crossses AC at the **mid-point** (M) of the line. Every point on the line BD is **equidistant** from both A and C. If you join ABCD, a rhombus is formed.



Example 1 ∠DEF is 100°. Construct the bisector of ∠DEF.

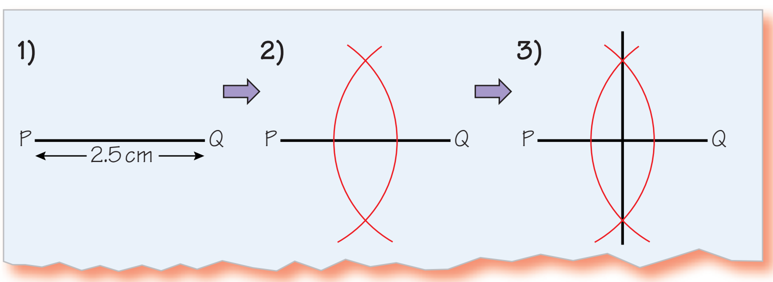


An arc is a part of a circle. You can draw arcs with compasses.

1) Open the compasses and put the point on E. Draw an arc that intersects with ED and EF.

2) Do not change the opening of the compasses. Put the point first on the intersection of the arc with ED and then with EF. Draw new arcs to intersect at G. Join EG.

Example 2 The line segment PQ is 2.5 cm long. Construct the perpendicular bisector of PQ.



1) Draw a line PQ of 2.5 cm. Open up the compasses to over half the length of PQ.

2) Place the point at P and draw an arc. Keep the opening of the compasses the same and repeat at Q. Join the points where the arcs intersect.

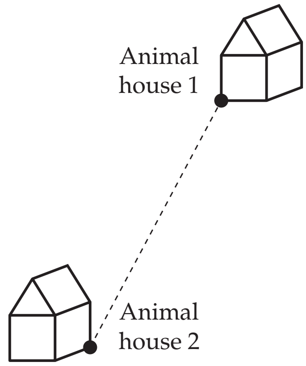
Exercise 2.5

1. Draw any acute angle.
Construct the angle bisector using only a ruler and compasses.

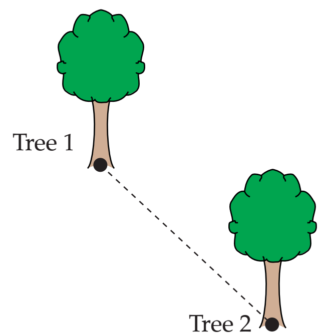
2. Draw a line 10 cm in length.
Construct the perpendicular bisector of the line using only a ruler and compasses.
Mark the mid-point (M) of the line.

3. Draw any obtuse angle.
Construct the angle bisector using only a ruler and compasses.
Check it is correct by measuring the two angles and making sure they are the same.

4. A new water channel is to be built at the zoo.
It is to be placed between two animal houses so that the houses are equidistant from the channel.
Copy this diagram, and construct a red line to show the position of the water channnel.



5. A new fence is to be built in the park.
The fence is to be placed between two large trees so that the trees are equidistant from the fence.
The head gardener decides to paint in a line to mark the position of the fence.
Copy this diagram and construct a blue line to show the position of the fence.



Investigation

6. **a)** Draw any triangle with sides longer than 4 cm but shorter than 8 cm. Construct the angle bisector for each of the three angles. Make sure the angle bisectors are long enough to cross each other. What do you notice?
b) Draw another triangle approximately the same size. Construct the perpendicular bisector for each of the three sides. Make sure the perpendicular bisectors are long enough to cross each other. What do you notice?

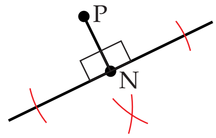
2.6 Perpendiculars

Use a straight edge and compasses to construct the perpendicular from a point **to** a line and from a point **on** a line

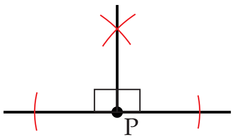
Key words
construct
perpendicular
arc

We can use a ruler and compasses to:

- **construct** the **perpendicular** from a point **to** a line

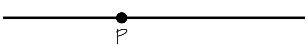


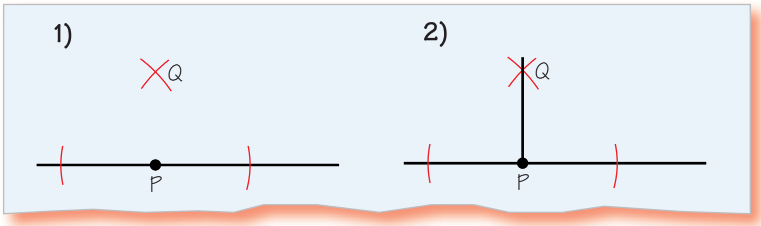
- construct the **perpendicular** from a point **on** a line



An **arc** is part of a circle. You can draw arcs with your compasses.

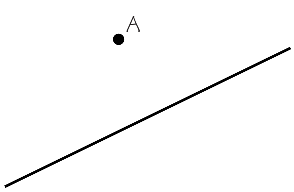
Example 1 Make a copy of the diagram.
Using only a ruler and compasses draw
a perpendicular from the point P on the line.

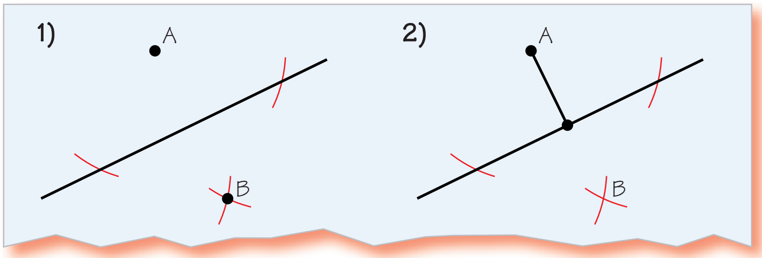




1) Using compasses, draw two **arcs** from P to make two intersections on the line. From the two intersections, draw two **arcs** that intersect and label the intersection Q.
2) Place a straight edge from P to Q and join them with a straight line.

Example 2 Make a copy of the diagram.
Using only a ruler and compasses draw
a perpendicular from the point A to the line.

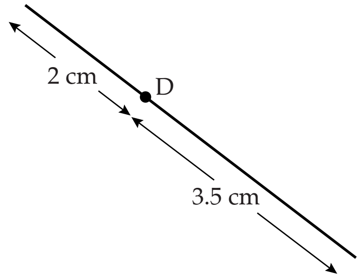




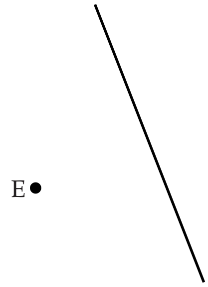
1) Using compasses, draw two **arcs** from the point A that intersect with the line. From the two **arcs** draw two more **arcs** that intersect and label the intersection B.
2) Place a straight edge from A to B and join A to the straight line.

Exercise 2.6

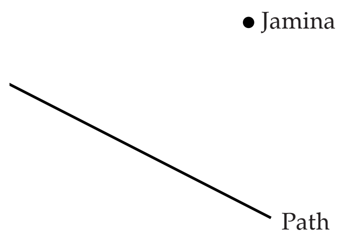
1. Make a copy of the diagram. Using a ruler and compasses, construct a perpendicular from the point D on the line.



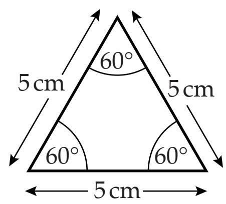
2. Make a copy of the diagram. Using a ruler and compasses, construct a perpendicular from the point E to the line.



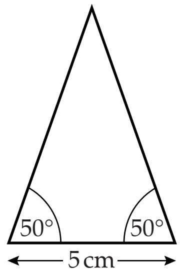
3. Jamina is walking her dog in the park when it starts to rain. She wants to take the shortest route back to the path. Copy the diagram and construct the shortest route to the path, using a ruler and compasses.



4. **a)** Draw an equilateral triangle, using a ruler and a protractor, with sides 5 cm in length, as shown here. Construct the perpendiculars from each vertex to the opposite side. What do you notice?



   b) Draw an isosceles triangle with base 5 cm and base angles of 50°, as shown here. Construct the perpendiculars from each vertex to the opposite side. What do you notice?



   c) Draw a scalene triangle with base 5 cm and base angles of 70° and 30°, as shown here. Construct the perpendiculars from each vertex to the opposite side. What do you notice?