Answers

1 Number

1.1 Calculations

Purposeful practice 1
1 16 **2** 16 **3** 40 **4** 24
5 0 **6** 0 **7** 16 **8** 16
9 16 **10** 8 **11** 4 **12** 2

Purposeful practice 2
1 5 **2** 8 **3** 5 **4** 12.5 **5** 5 **6** 5

Purposeful practice 3
1 $33 \div 3 = 11$ **2** $\sqrt[3]{1331} = 11$ **3** $\sqrt[3]{(1334 - 3)} = 11$

4 $\sqrt[3]{2744} - 3 = 11$ **5** $\sqrt[3]{\dfrac{3993}{3}} = 11$ **6** $\sqrt[3]{\dfrac{363 \times 11}{3}} = 11$

Problem-solving practice
1 Students' own answers, for example, $(1 + 2 + 3) \times 4 + 5$.

2 $7 + 5 \times (3 + 8) = 62$
3 Students' own answers, for example,

−3	2	1
4	0	−4
−1	−2	3

4 Pole C is 6 m long.

5 $\dfrac{2 \times (11 - \mathbf{7})}{\mathbf{8}} = \mathbf{1}$ or $\dfrac{2 \times (11 - \mathbf{7})}{\mathbf{1}} = \mathbf{8}$

6 Sarah is incorrect. To find the cost of 80 tins of paint, she needs to calculate $80 \times £4 = £320$.
7 Students' own answers, for example,
$(6 - 5) \times (4 - 3) = 1$ $(6 - 5) \div (4 - 3) = 1$
$(6 - 4) \div (5 - 3) = 1$

Exam practice
1 £316 **2** 38

1.2 Decimal numbers

Purposeful practice 1
1 300 **2** 30 **3** 3 **4** 0.3 **5** 0.03 **6** 0.03
7 3 **8** 30 **9** 300 **10** 30 **11** 3 **12** 0.3

Purposeful practice 2
1 2 **2** 5 **3** 2 **4** 5 **5** 4 **6** 10 **7** 4
8 10 **9** 1 **10** 1.6 **11** 1 **12** 10 **13** 100 **14** 100
15 25 **16** 25 **17** 2.5 **18** 2

Purposeful practice 3
1 3.5 **2** 3.5 **3** 3.5 **4** 11.5 **5** 0.5
6 3.8 **7** 3.9 **8** 4.0 **9** 11.0 **10** 1.0
11 0.1 **12** 0.0 **13** 9.8 **14** 9.1 **15** 20.0

Problem-solving practice
1 a £4.67 **b** £0.47 (rounded) **2 a** $3.84\dot{9}$ **b** 3.75
3 Calculations C and D
4 Students' own answers, for example, $10 \times 0.5 = 5$
5 £303.75 **6** No, he only has 15 kg of flour.
7 20 packs **8** £6.40

Exam practice
1 4.2 **2** 212.5

1.3 Place value

Purposeful practice 1
1 100 **2** 200 **3** 1000 **4** 1000 **5** 1 **6** 0.1
7 0.001 **8** 0.002 **9** 0.0017 **10** 0.0011 **11** 0.0010 **12** 3.0

Purposeful practice 2
1 20 000 **2** 20 **3** 1 **4** 500 **5** 1200
6 500 **7** 4000 **8** 1200 **9** 300

Purposeful practice 3
1 a 8640 **b** 270 **c** 86.4 **d** 2.7
e 270 **f** 27 **g** 27 **h** 2.7
2 a 295 **b** 354 **c** 236 **d** 300

Problem-solving practice
1 a £500 a month
b Yes, he will save £6000 which is greater than £5775.
2 a No, an estimate of five times their yearly earnings is £180 000.
b It is an underestimate, because Carrie and Arjun's earnings were rounded down.
3 a 12 ounces
b It is an underestimate, because both values were rounded down.
c The estimated weight will increase to 15 because 4.7 rounds up to 5.
4 Students' own answers, for example, 0.54×8.7
5 Sam should have found 400×60 and then divided by 0.5 or multiplied by 2.
6 Approximately 20 minutes

Exam practice
1 $\dfrac{200}{4^2 + 4} = \dfrac{200}{16 + 4} = \dfrac{200}{20} = 10$
Billy's answer is correct.
2 90

1.4 Factors and multiples

Purposeful practice 1
1 a 20, 22, 24, 26, 28, 30 **b** 21, 24, 27, 30
c 20, 25, 30 **d** 21, 28 **e** 22 **f** 23, 29
2 31, 37 **3** 41, 43, 47

Purposeful practice 2
1 a 1, 2, 3, 6 **b** 1, 5 **c** 1, 2, 3, 5, 6, 10, 15, 30
d 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
e 1, 3, 5, 9, 15, 45
2 a 6, 12, 18, 24, 30 **b** 5, 10, 15, 20, 25
c 30, 60, 90, 120, 150 **d** 60, 120, 180, 240, 300
e 45, 90, 135, 180, 225

Purposeful practice 3
1 a 15 **b** 6 **c** 30 **d** 1 **e** 15
2 a 90 **b** 30 **c** 60 **d** 30 **e** 180

Problem-solving practice
1 Students' own answers, for example, 6 and 12.
There is more than one possible answer.
2 30 and 40, or 10 and 120.
3 Students' own answers, for example,
$45 \div 6 = 7.5$. This is not an integer, therefore 6 is not a factor of 45.
4 Tom is wrong because 2 is a prime number and $2^2 = 4$, which is not odd.
5 Students' own answers, for example,
$254 \div 8 = 31.75$. This is not an integer, therefore 8 is not a factor of 254.
6 $678 \div 3 = 226$. This is an integer, therefore 678 is a multiple of 3.
7 9.00 am **8** £13 **9** Paul is 30; Luca is 45. **10** 10 boxes

Exam practice
1 a 24 **b** 2, 17
2 Students' own answers, for example, 5 and 7.

1.5 Squares, cubes and roots

Purposeful practice 1
1 4 **2** 9 **3** 16 **4** 4 **5** 9 **6** −9 **7** 16
8 8 **9** 27 **10** 64 **11** −8 **12** −27 **13** −64 **14** −64

Purposeful practice 2

1 a 2 **b** 2, −2 **c** 3, −3 **d** 4, −4 **e** 3 **f** −3
2 a 1.26 **b** 0.4 **c** 1.39 **d** 0.646 **e** 0.3 **f** 0.03
g 1.58 **h** 0.5 **i** 0.158 **j** 2 **k** 0.431 **l** 0.2

Purposeful practice 3

1 7 **2** 5 **3** 9 **4** 3 **5** $2\sqrt{3}$
6 $3\sqrt{3}$ **7** $\sqrt{3}$ **8** 6 **9** 2 **10** $2\sqrt{3}$

Problem-solving practice

1 1 or 0 **2** 21.16 cm² **3** 24 cm
4 4 cm **5** 3 and 4 or −3 and −4 **6** 4 and 5 or −4 and −5
7 10 or −10 **8** 10 times longer
9 Students' own answers, for example, $(-3)^2 = 9$ but $(-3)^3 = -27$, so the square is larger.
10 Students' own answers, for example, $\sqrt{0.16} = 0.4$ and $0.4 > 0.16$.

Exam practice

1 27
2 $15 + 11 + 23 = 49$. $\sqrt{49} = 7$, so yes, Jordan is correct.
3 −125

1.6 Index notation

Purposeful practice 1

1 3^4 **2** $3^4 \times 5^2$ **3** $3^4 \times 5^2$
4 $3^4 \times 5^4$ (or 15^4) **5** $3^4 \times 5^4$ (or 15^4)

Purposeful practice 2

1 6^5 **2** 6^5 **3** 5^6 **4** 5^6 **5** 5^{12} **6** 5^7
7 5^3 **8** 5^4 **9** 5^7 **10** 5^{14} **11** 5^{17} **12** 5^4

Purposeful practice 3

1 a 10^2 **b** 10^3 **c** 10^4 **d** 10^1
e 10^0 **f** 10^{-1} **g** 10^5 **h** 10^{-2}
2 a 3×10^2 **b** 5×10^2 **c** 5.3×10^3 **d** 3×10^{-2} **e** 3.8×10^{-3}

Problem-solving practice

1 2^2 **2** 3^2 **3** 18
4 Yes − 15^2
Students' own reasoning, for example, simplifying the expression gives $3 \times 5 \times 15$ which can be written as 15×15 or 15 squared.
5 Change the 2 to a 3 or the 3 to a 2. The expression can then be written as 10^5.
6 125 **7** $g = 6$
8 Amal is wrong because $4^3 + 5^3 = 189$ and $9^3 = 729$.
9 Allison is correct because $\square \times 3 \times 3$ simplifies to $\square \times 3^2$.

Exam practice

1 a $x = 10$ **b** $y = 2$ **c** $a = 2$

1.7 Prime factors

Purposeful practice 1

1 $20 = 2^2 \times 5$ **2** $40 = 2^3 \times 5$
3 $120 = 2^3 \times 3 \times 5$ **4** $60 = 2^2 \times 3 \times 5$
5 $15 = 3 \times 5$ **6** $45 = 3^2 \times 5$
7 $180 = 2^2 \times 3^2 \times 5$ **8** $360 = 2^3 \times 3^2 \times 5$
9 $300 = 2^2 \times 3 \times 5^2$ **10** $200 = 2^3 \times 5^2$

Purposeful practice 2

1 $16 = 2^4$ **2** $36 = 2^2 \times 3^2$ **3** $81 = 3^4$
4 $100 = 2^2 \times 5^2$ **5** $64 = 2^6$ **6** $216 = 2^3 \times 3^3$
7 $1728 = 2^6 \times 3^3$ **8** $7056 = 2^4 \times 3^2 \times 7^2$

Purposeful practice 3

1 5 **2** 5 **3** 10 **4** 60
5 20 **6** 100 **7** 10 **8** 1

Purposeful practice 4

1 360 **2** 600 **3** 600 **4** 600
5 720 **6** 720 **7** 3420 **8** 3420

Problem-solving practice

1 No, it will be 540 days before they are taken to the vet together again and there are 365 days in a year.

2 Yes, 48 cans of orange juice and 3 tubs of ice cream (to serve 48 people) costs £30, so it is possible.
3 LCM = 1050
4 No, because 96 is not a factor of 144.
5 No, Thomas is not correct. The LCM of 1 and 81 is 81 but 1 is not in the 3 times table.
6 It will cost Ms Case £38. There will be no spare erasers but 20 spare pencils.
7 a 24 cakes **b** 4:45 pm

Exam practice

1 $2^5 \times 5$ **2** 736

2 Algebra

2.1 Algebraic expressions

Purposeful practice 1

1 $9x$ **2** $3x$ **3** x **4** $4x$ **5** x **6** x
7 $2b$ **8** $-3b$ **9** $-b$ **10** $+6b$ **11** $+6b$ **12** $-3b$

Purposeful practice 2

1 $10y + 3$ **2** $4y + 9$ **3** $10 + 5y$ **4** $7t - 7$ **5** $-2t + 2$
6 $8 - 5t$ **7** rs **8** $5rs$ **9** $10rs$ **10** $10rs$
11 $10rt$ **12** $6rt$ **13** $12rt$ **14** $3rst$ **15** $12rst$
16 $\frac{a}{b}$ **17** $\frac{b}{a}$ **18** $\frac{a}{2}$ **19** $\frac{b}{2}$ **20** $\frac{2}{a}$
21 $\frac{2}{b}$

Problem-solving practice

1 $2x + 5$ cm
2 $2x - 1 - 4x + 1$ (all the rest simplify to $6x$)
3 $7x + 8 - 2x - 5 + 3x = 8x + 3$
4 $10q$ cm² **5** $4b$
6 Students' own answers, for example, $\frac{36x}{3}, 4x \times 3, 5x + 7x$
7 Every row and column adds to $2x + 3y$

x	$3y$	x
$-y$	$2x - y$	$5y$
$4y + x$	y	$x - 2y$

Exam practice

1 a $15p$ **b** $2b$

2.2 Simplifying expressions

Purposeful practice 1

1 x^2 **2** x^3 **3** x^4 **4** x^5 **5** x^7 **6** x^8
7 x^9 **8** x^6 **9** n^3 **10** n^4 **11** n^6 **12** n^8
13 n^3 **14** n **15** n^4 **16** n^3

Purposeful practice 2

1 $6u$ **2** $-6u$ **3** $6u$
4 $-6uv$ **5** $-6uv$ **6** $-6uv$
7 $-6v^2$ **8** $-5xy$ **9** $5xy$
10 $10xy$ **11** $-10xy$ **12** $10xy$
13 $\frac{1}{6}p$ or $\frac{p}{6}$ **14** $6p$ **15** $3p$
16 $3p^2$ **17** $\frac{1}{3}p^2$ or $\frac{p^2}{3}$ **18** $\frac{1}{3}p$ or $\frac{p}{3}$
19 $-\frac{1}{3}t^3$ or $-\frac{t^3}{3}$ **20** $-3t^3$ **21** $3t^3$
22 $3t^2$ **23** $-3t^2$ **24** $3t$
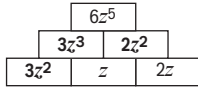
Problem-solving practice

1
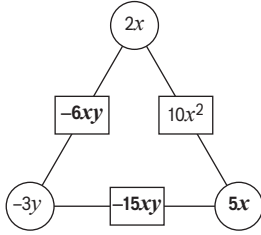
2 Students' own answers, for example, $15 \times -c \times d$ or $3c \times -5d$

3 a $t^2 \times t^3 = t^5$ **b** $\dfrac{x^5}{x^3} = x^2$ **c** $\dfrac{n^6}{n^2} = n^4$ **d** $\dfrac{r^3 \times r^3}{r^4} = r^2$

4 a $4x^3$ **b** x^6

5 Students' own answers, for example, $\dfrac{8x^3}{2}$ or $\dfrac{4x^4}{x}$

6

7 $4n$

Exam practice

1 **a** n^6 **b** $2x^2$ **c** $14ac$

2.3 Substitution
Purposeful practice 1

1 9	**2** 1	**3** -1	**4** 3
5 2	**6** 6	**7** -6	**8** 7

Purposeful practice 2

1 18	**2** -3	**3** -12	**4** 2
5 1	**6** -1	**7** 2	**8** $\dfrac{3}{2}$
9 -6	**10** 3	**11** $-\dfrac{1}{2}$	**12** $\dfrac{1}{2}$
13 -6	**14** 18	**15** -3	**16** 6

Purposeful practice 3

1 22	**2** -6	**3** 10	**4** 18	**5** 19
6 0	**7** 24	**8** 1	**9** -2	**10** 5
11 -6	**12** 6	**13** 4	**14** 16	**15** -16
16 20	**17** 14	**18** 36	**19** -10	**20** 4
21 -4	**22** 25	**23** $\dfrac{4}{10}$ or 0.4	**24** 1.6	

Problem-solving practice

1 a Students' own answers, for example, $m = 1$ and $n = 6$
b Students' own answers, for example, $m = 10$ and $n = -3$

2 Alex has forgotten that the expression st means $s \times t$.

The answer should be $8 \times \dfrac{1}{2} = 4$

3 Students' own answers, for example,
$2 \times 3 \times 4 = 3 \times 4 \times 2 = 4 \times 3 \times 2$.
Three numbers multiplied together in any order give the same answer.

4 1, 3, 5, 7, 9. The odd numbers.

5 a 21 **b** 22

6 Yes, Dilip is correct. Any negative number squared is positive, zero squared is zero and any positive number squared is positive.

7 $x = 2$

8 Many possible answers, for example $p = 1$, $q = 10$

Exam practice

1 14 **2** $c = 27$

2.4 Formulae
Purposeful practice 1

1 20 pence, 30 pence $n \times 10$ or $10n$ pence. $C = 10n$ pence
2 $2b$ pence, $3b$ pence $m \times b$ or mb or bm pence. $C = bm$ pence

Purposeful practice 2

1 a $C = 5p$	**b** $C = 5n$	**c** $C = 4t$
d $C = mt$	**e** $C = mt + 4$	**f** $C = mt + r$
2 a $N = y$	**b** $N = y - 20$	**c** $N = xy$
d $N = xy + 30$	**e** $N = xy - 5x$	

Problem-solving practice

1 $C = 3n$ the cost of n items at £3 each
$C = 10n$ the cost of n items at £10 each
$C = 3n + 10$ the cost of n items at £3 each plus £10 delivery charge
$C = 10n + 3$ the cost of n items at £10 each plus £3 delivery charge

2 a Rope A **b** Rope B

3 a x = price of a cake **b** y = price of a box **c** 9 cakes

4 $C = 1.5n + 4$

Exam practice

1 $L = 4x + 1$

2.5 Expanding brackets
Purposeful practice 1

1 $3x + 3$	**2** $3x + 6$	**3** $3x + 30$
4 $3x - 3$	**5** $3x - 6$	**6** $3x - 15$

Purposeful practice 2

1 $2m + 2$	**2** $2m - 2$	**3** $-2m - 2$
4 $-2m + 2$	**5** $-m - 7$	**6** $-m + 7$
7 $-m - 7$	**8** $-m + 7$	**9** $m - 7$

Purposeful practice 3

1 $4n + 12$	**2** $12 + 4n$	**3** $-15 - 5n$
4 $-5n - 15$	**5** $-5n + 15$	**6** $3r - 18$
7 $3r + 18$	**8** $6r + 18$	**9** $12r + 18$
10 $-6r - 18$	**11** $-6r + 18$	**12** $-18 - 6r$
13 $t^2 + 3t$	**14** $2t^2 + 3t$	**15** $2t^2 - 3t$
16 $k^2 + k$	**17** $2k^2 + 2k$	**18** $2k^2 - 2k$
19 $2k^2 + 8k$	**20** $2k^2 - 8k$	**21** $-2k^2 + 8k$

Problem-solving practice

1 a $2(a + \mathbf{4}) = 2a + 8$ **b** $2(y - 7) = \mathbf{2y} - 14$
c $\mathbf{n}(n + \mathbf{5}) = n^2 + 5n$ **d** $-3(\mathbf{3} + c) = -9 - \mathbf{3c}$
e $\mathbf{t}(6t - \mathbf{2}) = 6t^2 - 2t$ **f** $5p(\mathbf{p} - 3) = 5p^2 - \mathbf{15p}$

2 The expansion is correct, but $12d - 8$ cannot be simplified as the two terms are not alike.

3 Students' own examples, for example, $3(5x + 2) = 15x + 6$ and $2x(x + 1) = 2x^2 + x$

4
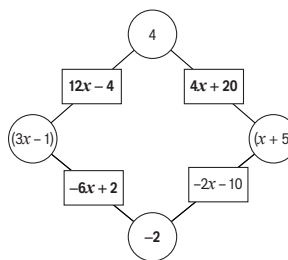
5 a $5x + 17$ **b** $20 + 3n$

Exam practice

1 a $2c^2 + 10c$ **b** $8d - 4d^2$ **c** $14x - 3$

2.6 Factorising
Purposeful practice 1

1 $2(x + 3)$	**2** $2(x + 2)$	**3** $2(x + 1)$
4 $2(x - 5)$	**5** $2(x - 4)$	**6** $2(x - 1)$
7 $3(y + 1)$	**8** $3(y + 2)$	**9** $3(y - 2)$

Purposeful practice 2

1 $4(a + 2)$	**2** $4(a - 3)$	**3** $6(a + 2)$	**4** $6(a - 4)$
5 $6(2t - 3)$	**6** $4(3t + 5)$	**7** $6(2t + 1)$	**8** $3(4t - 3)$
9 $3(7t - 3)$	**10** $7(3t + 5)$	**11** $5(4m + 7)$	**12** $20(m - 2)$
13 $m(m + 1)$	**14** $m(m + 2)$	**15** $m(m - 3)$	**16** $m(5 + m)$

17 $m(4 - m)$ **18** $m(2m - 1)$ **19** $m(m - 2)$ **20** $m(3m + 1)$
21 $m(m + 3)$ **22** $2b(b + 1)$ **23** $2b(b + 2)$ **24** $2b(b - 3)$
25 $b(b + g)$ **26** $b(b - g)$ **27** $b(3b - g)$ **28** $3b(b - 2g)$
29 $3b(b + 3g)$ **30** $3b(b - g)$ **31** $3b(2b + 3g)$ **32** $6b(b + 2g)$

Problem-solving practice

1 $n^2 - n \longrightarrow n(n - 1)$
  $2n - n^2 \longrightarrow n(2 - n)$
  $n - 2n^2 \longrightarrow n(1 - 2n)$
  $n^2 - 2n \longrightarrow n(n - 2)$
  $-n^2 + 2n \longrightarrow n(-n + 2)$
  $2n^2 - n \longrightarrow n(2n - 1)$
  $2n^2 - 2n \longrightarrow 2n(n - 1)$

2 a $3x - 24 = 3(x - 8)$ **b** $20x + 15 = 5(4x + 3)$
  c $x^2 - 2x = x(x - 2)$ **d** $4x^2 + 6x = 2x(2x + 3)$
  e $3x^2 - ax = x(3x - a)$

3 a Mo has tried to add the two terms (incorrectly, as they are not like terms) rather than factorise.
  b $3(y + 5)$

4 Students' own answers, for example, $15x + 6 = 3(5x + 2)$ and $2x^2 + x = 2x(x + 1)$

5 a $6x - 18$ **b** $3(2x - 1)$ **c** $7x - 1$

6 a $4(x + 9)$ **b** $3n(4n + 1)$

Exam practice

1 a $4(n - 3)$ **b** $x(x+1)$

2.7 Using expressions and formulae

Purposeful practice 1

1 $T = 8$ **2** $R = -4$ **3** $S = 4$
4 $V = -8$ **5** $L = 10$ **6** $B = 24$
7 $C = 3$ **8** $M = 12$ **9** $K = -12$
10 $P = \frac{2}{6}$ or $\frac{1}{3}$ **11** $N = 3$ **12** $Z = -3$
13 $D = 3$ **14** $F = \frac{2}{6}$ or $\frac{1}{3}$ **15** $H = -\frac{2}{6}$ or $-\frac{1}{3}$

Purposeful practice 2

1 $A = 11$ **2** $B = 5$ **3** $C = 0$
4 $D = 13$ **5** $D = 61$ **6** $E = 25$
7 $F = 31$ **8** $G = 29$ **9** $H = 26$
10 $J = 8$ **11** $J = 16$ **12** $K = 11$
13 $L = 24$ **14** $M = 32$ **15** $P = 13$
16 $Q = -8$ **17** $R = 9$ **18** $S = -1$
19 $W = -2\frac{1}{2}$ or $-2\frac{4}{8}$ **20** $X = 3\frac{1}{2}$ or $3\frac{4}{8}$

Problem-solving practice

1 Charlie's answer is wrong, because she has forgotten that st means $s \times t$.
2 Students' own answers, for example, $k = na - 10$
3 C. All the others give the value $T = -12$, whereas C gives the value $T = 6$.
4 16 m/s
5 a $v = 58.8$ **b** $v = 3$

Exam practice

1 a $X = 16$ **b** $m = 5$ **2** $p = -12$

3 Graphs, tables and charts

3.1 Frequency tables

Purposeful practice 1

1 Students' own answers, for example,
  a number of matches in a box, shoe size
  b heights or weights of people

2 a

Dice result	Tally	Frequency
1		
2		
3		
4		
5		
6		

b

Total dice score	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

c Example (students' own answers will vary)

Total dice score	Tally	Frequency
10–20		
21–30		
31–40		
41–50		
51–60		

Purposeful practice 2

1 a

Mark	Tally	Frequency
1	||||	4
2	ⅬⅠⅠⅠⅠ	5
3	|||	3
4	ⅬⅠⅠⅠⅠ ||||	9
5	ⅬⅠⅠⅠⅠ |||	8
6	|	1
Total		30

b 4

2 The number of books on shelves is discrete, so it can only take whole number values. Therefore, each row has a simple range. The length of books is continuous, so it can take any number within the range. Therefore, each row has an inequality with no gaps.

Problem-solving practice

1 The ranges 53–54 and 54–56 overlap.
2 Items are discrete, not continuous, so the groups should not be defined using inequalities.
3 Items are continuous, not discrete, so groups should be defined using inequalities.
4 There is nowhere to record a road width of 4.0 m or 5.0 m.

Exam practice

1 Frequency and tally for Friday do not match.

3.2 Two-way tables

Purposeful practice 1

1 a 213 km **b** 24 km **c** 105 km **d** 3532 km **e** 3745 km

Purposeful practice 2

1

	Year 7	Year 8	Year 9	Total
French	20	15	35	**70**
German	15	25	**10**	50
Spanish	10	**10**	**30**	**50**
Total	**45**	50	**75**	170

Problem-solving practice

1

Phil		
105	Mike	
53	207	Tony

2 a Car, train and history, geography

b

	Car	Train	Total
History	14	4	18
Geography	24	8	32
Total	38	12	50

c 18   **d** 4

3 Male music and total number of males.

Exam practice

1

	Children	Adults	Total
Theatre	12	10	22
Cinema	10	8	18
Total	22	18	40

3.3 Representing data
Purposeful practice 1

1 Example answers (students' answers may vary).

a

Number of driving test passes at centre A and centre B over 4 days
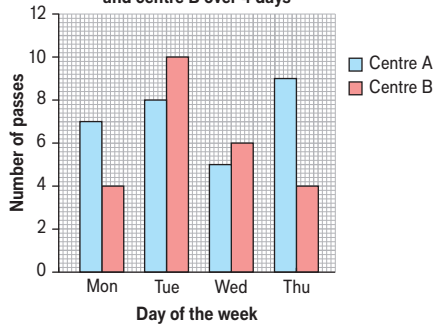
b

Number of driving test passes at centre A and centre B over 4 days
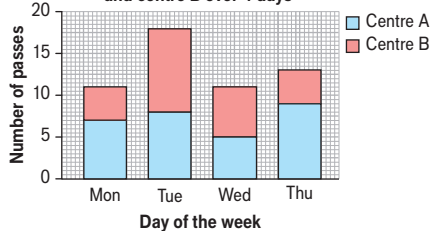
c Number of driving test passes at centre A and centre B over 4 days

Purposeful practice 2

1
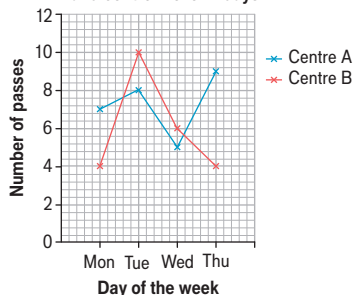
Time taken, t (minutes)	Frequency
$0 < t \leqslant 15$	5
$15 < t \leqslant 30$	14
$30 < t \leqslant 45$	9
$45 < t \leqslant 60$	4

Time taken to travel to school for 32 students

2
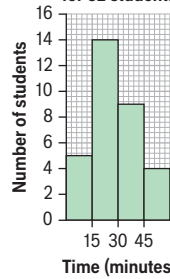
Car speed, s (mph)	Frequency
$5 < s \leqslant 10$	3
$10 < s \leqslant 15$	6
$15 < s \leqslant 20$	9
$20 < s \leqslant 25$	15
$25 < s \leqslant 30$	20

Problem-solving practice

1 a The data is not grouped.
The data is not continuous.

b Shreeya's homework hours on Monday.

c

	Fri	Sat	Sun	Mon
Aum	1	4	3	2
Shreeya	2	5	4	3

d Number of hours spent on homework
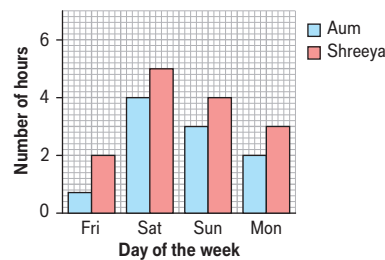
e $3 + 4 = 7$ hours

Exam practice

1 a Tuesday

b
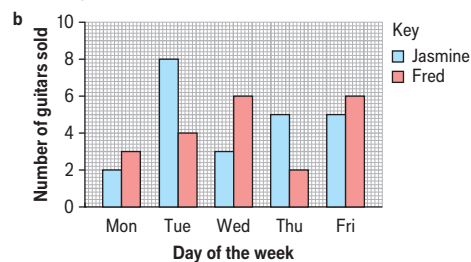
c 2   **d** 66

3.4 Time series
Purposeful practice 1

1 a Temperature of a town during one day
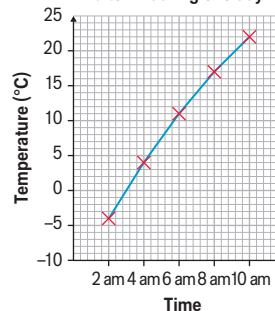
b Higher

c Accept answers between −2 °C and 2 °C.

d Accept answers between 13 °C and 16 °C.

Purposeful practice 2

1 a

Temperature of two towns during one day
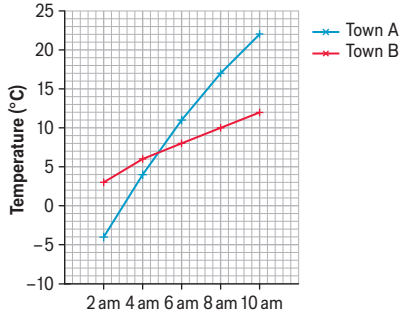
b 5 am

c Accept answers between 13°C and 15°C.

Purposeful practice 3

1 a 24 m   **b** 4 seconds

Problem-solving practice

1 a

	Oct	**Nov**	**Dec**
Earned (£)	**2200**	2550	**2680**

b

Amount earned by Sam over three months
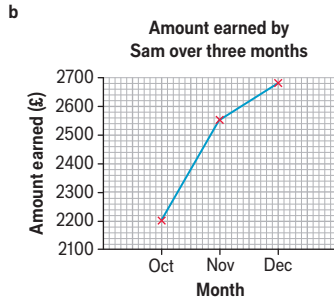
c Example (students' answers will vary): Sam's earnings increase between October and December. The amount by which they increase, decreases each month. More data would be required to make a confident prediction of Sam's earnings in January. An estimate might be earnings of £2750 in January.

Exam practice

1 No vertical axis label, no title, July appears twice/no August

3.5 Stem and leaf diagrams

Purposeful practice 1

1

1	9
2	2 5 5
3	1 7
4	0 4 7 8

Key 2 | 5 = 25 points

2 a

2	5 6 7
3	0 1 2
4	2 3 3 5 6 7
5	0 1 5 5 6
6	2 2
7	3

Key 2 | 5 = 25 marks

b 73   **c** 25   **d** 48

Purposeful practice 2

1 a

Boys' marks				Girls' marks				
	0 0	2	5 6 7			
8 5 4	3	0 1 2			
9 8 5 3	4	2 3 3 5 6 7
9 6 6 5 0	5	0 1 5 5 6
7 7 4 4	6	2 2
4 4	7	3

Key: boys' marks 0 | 2 = 20 marks

girls' marks 2 | 5 = 25 marks

b 54   **c** 2 − 1 = 1

Problem-solving practice

1 a

Vikram's plants		Penny's plants			
9 6 5 4	1			
1 1 0 0	2	7 8		
	3	4 7		
	4	2 4 4 5

Key: Vikram's plants 4 | 1 = 14 cm

Penny's plants 2 | 7 = 27 cm

b Vikram

c Penny

d Vikram = 7 cm, Penny = 18 cm

e Penny: all of Penny's plants are taller.

2

Percentage mark, m	Frequency
$40 < m \leq 50$	**5**
$50 < m \leq 60$	**7**
$60 < m \leq 70$	6
$70 < m \leq 80$	3

3

28	3 5 6 8
29	5
30	4 7 9 9
31	1 2
32	1

Key: 28 | 3 = 283 missed appointments

Exam practice

1

1	5 8 8 8 9
2	3 5 6 7 7 9
3	0 1 6 6 8 9
4	2 4 5

Key: 1 | 5 = 15 years old

3.6 Pie charts

Purposeful practice 1

1 a–b

c Final angle should measure 155°

Purposeful practice 2

1

	2000	**2005**	**2010**	**2015**
Tennis	200	500	1400	1750
Snooker	100	250	700	875
Cricket	100	250	700	875
Total	400	1000	2800	3500

2 Favourite film titles of students in Year 8

Favourite film types of students in Year 9
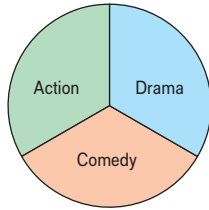
Favourite film types of students in Year 10

Problem-solving practice

1 a 150 kg peaches   **b** 120 kg apples
  c 765 kg pears

2

Club	Frequency
Football	42
Tennis	6
Photography	12
Coding	20
Netball	32
Origami	8

Exam practice

1 Transport used by teachers to get to work
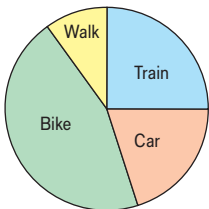
3.7 Scatter graphs

Purposeful practice

1
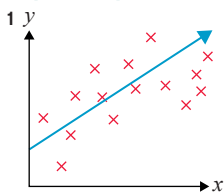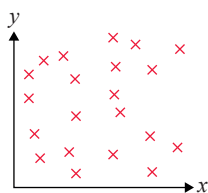
a Positive correlation   **b** No correlation
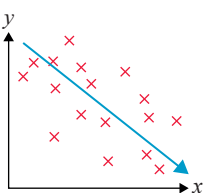
c Negative correlation

2 As **temperature** increases, sales of suntan lotion **increase.**
As **daily rainfall** increases, sales of suntan lotion **decrease.**

3 a The point shown in the top-right of the diagram.
  b (Moderate or strong) negative correlation.

4 a

Homework and test scores
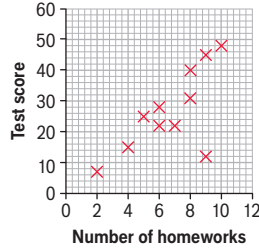
b (9, 12)   **c** Positive correlation

Problem-solving practice

1 a Positive correlation. More cars on the road will tend to lead to more accidents.
  b Positive correlation. People will tend to drink more when it is hotter.
  c Negative correlation. Car value will tend to decrease with age as people prefer to buy newer cars.
  d Negative correlation. As temperature increases, people will tend to spend less on heating.
  e Positive correlation. People will tend to buy more of an item that is advertised.
  f Negative correlation. Most people want a property that is close to public transport, so they will tend to pay more for it.
  g No correlation. Unlikely to be related.
  h No correlation. Unlikely to be related.

2 a

Marks of 7 students in two Chemistry exams
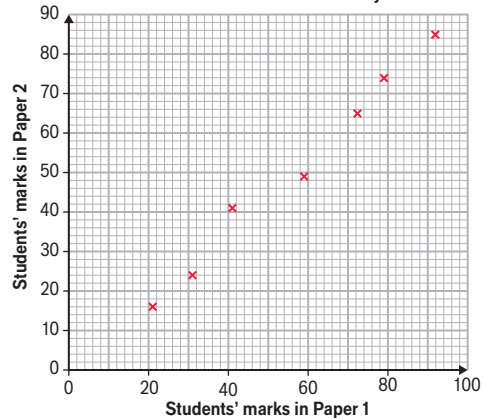
b Positive correlation

Exam practice

1 a (68, 45)   **b** Negative correlation

3.8 Line of best fit

Purposeful practice 1

1 Example answer (students' lines of best fit and estimations will vary).

a, b

Heights and widths of boxes
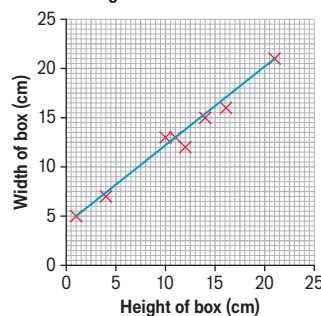
c Example answer (students' lines of best fit and estimations will vary): Between 10 and 12 cm.

d Example answer (students' lines of best fit and estimations will vary): Between 5 and 7 cm.

2 Example answer (students' lines of best fit and estimations will vary):

a, b

Heights and widths of boxes
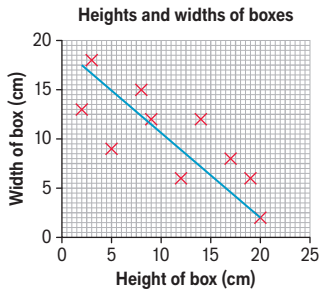
c Example answer (students' lines of best fit and estimations will vary): Between 4 and 6 cm.

3 a, b Example answer (students' lines of best fit and estimations will vary):

Distance travelled on 1 litre of diesel
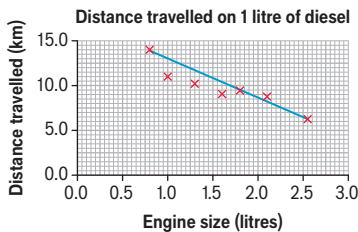
c Example answer (students' lines of best fit and estimations will vary): Between 6 and 8 km.

Problem-solving practice

1

Latitude and temperature of cities

21 °C

2 a

Birds
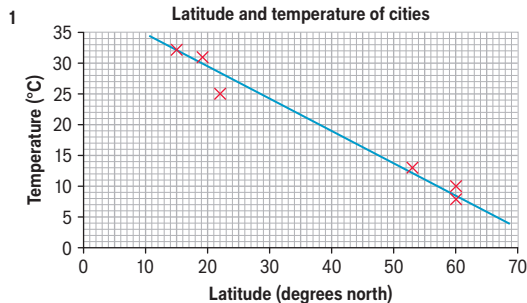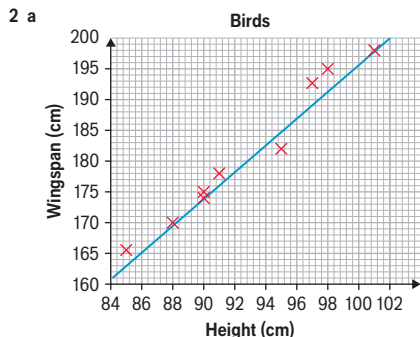
b Positive correlation

c No, the data point for the bird would lie a long way from the line of best fit so it is unlikely to be from the same family.

Exam practice

1 a (8, 25) **b** Negative correlation

c Between 80 and 90

d Yes, as the temperature increases, fewer cartons of soup are sold.

4 Fractions and percentages

4.1 Working with fractions

Purposeful practice 1

1 $\frac{2}{4}$ **2** $\frac{1}{2}$ **3** $\frac{1}{2}$ **4** $\frac{1}{2}$

5 $\frac{5}{8}$ **6** $\frac{2}{3}$ **7** $\frac{2}{3}$ **8** $\frac{7}{8}$

Purposeful practice 2

1 $\frac{3}{4}$ **2** $\frac{3}{4}$ **3** $\frac{5}{8}$ **4** $\frac{7}{8}$

5 $\frac{9}{8}$ or $1\frac{1}{8}$ **6** $\frac{31}{24}$ or $1\frac{7}{24}$ **7** $\frac{19}{15}$ or $1\frac{4}{15}$ **8** $\frac{67}{40}$ or $1\frac{27}{40}$

Purposeful practice 3

1 $\frac{1}{8}$ **2** $\frac{7}{24}$ **3** $\frac{17}{40}$ **4** $\frac{5}{8}$ **5** $\frac{5}{8}$

6 $\frac{5}{8}$ **7** $\frac{2}{5}$ **8** $\frac{7}{5}$ or $1\frac{2}{5}$ **9** $-\frac{1}{5}$

Problem-solving practice

1 $\frac{1}{4}$ and $\frac{2}{3}$ **2** $\frac{3}{10} = \frac{1}{10} + \frac{1}{5}$

3 Yes, $\frac{3}{8} < \frac{1}{2}$ because $\frac{4}{8} = \frac{1}{2}$ and $\frac{3}{8} < \frac{4}{8}$

4 Students' answers will vary, for example

a $\frac{1}{15} + \frac{2}{15} + \frac{8}{15}$ **b** $\frac{1}{15} + \frac{4}{30} + \frac{24}{45}$

c $\frac{8}{120} + \frac{8}{60} + \frac{8}{15}$

5 Students' answers will vary, for example

a $\frac{4}{5} - \frac{3}{5}$ **b** $\frac{4}{5} - \frac{6}{10}$ **c** $\frac{12}{15} - \frac{12}{20}$

6 Students' answers will vary, for example

a $\frac{1}{3}$ and $\frac{2}{9}$

b $\frac{2}{6}$ and $\frac{4}{18}$, $\frac{4}{12}$ and $\frac{6}{27}$, $\frac{5}{15}$ and $\frac{8}{36}$

7 $\frac{7}{24}$

Exam practice

1 Michael did not multiply the 1 by 3 or the 2 by 4.

2 $\frac{15}{48}$

4.2 Operations with fractions

Purposeful practice 1

1 4 **2** 5 **3** 10 **4** 10
5 15 **6** 7.5 **7** 6 **8** 12
9 24 **10** 48 **11** 50 **12** 25
13 15 **14** 5 **15** 27 **16** 12

Purposeful practice 2

1 $1\frac{3}{7}$ **2** $2\frac{3}{7}$ **3** $2\frac{5}{7}$ **4** $2\frac{5}{7}$

5 $2\frac{5}{7}$ **6** $2\frac{5}{7}$ **7** $2\frac{31}{42}$ **8** $6\frac{19}{42}$

9 3 **10** $3\frac{1}{14}$ **11** $4\frac{1}{14}$ **12** $4\frac{3}{14}$

Purposeful practice 3

1 $\frac{6}{7}$ **2** $1\frac{6}{7}$ **3** $2\frac{1}{7}$ **4** $1\frac{6}{7}$

5 $1\frac{6}{7}$ **6** $1\frac{5}{21}$ **7** $\frac{11}{12}$ **8** $\frac{11}{12}$

Problem-solving practice

1 $\frac{1}{15}$ of his yearly pay is the better option.

Students' reasoning may vary, for example, $\frac{3}{4}$ of £1300 is £975

whereas $\frac{1}{15}$ of $(12 \times £1300)$ is £1040.

2 Tessa has switched the $\frac{1}{5}$ and $\frac{3}{5}$ so worked out $\frac{3}{5} - \frac{1}{5}$ and not the

other way around.

3 120   **4** 47 cm   **5** $1\frac{3}{4}$ hours

6 $2\frac{4}{7}$ **7** 195   **8** $5\frac{5}{24}$

9 Students' own answers, for example,

$2\frac{1}{2} + 7\frac{1}{2} = 10,\ 5\frac{1}{4} + 4\frac{3}{4} = 10,\ 3\frac{4}{11} + 6\frac{6}{11} = 10$

Exam practice

1 $4\frac{8}{15}$

4.3 Multiplying fractions
Purposeful practice 1
1 18   **2** 18   **3** 18   **4** 18

Purposeful practice 2

1 Yes, simplify to $\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$ **2** Yes, simplify to $\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

3 No   **4** No   **5** No   **6** Yes, simplify to $2 \times \frac{1}{3} = \frac{2}{3}$

Purposeful practice 3

1 $\frac{1}{10}$ **2** $\frac{1}{10}$ **3** $\frac{1}{20}$ **4** $\frac{1}{20}$

5 $\frac{3}{10}$ **6** $\frac{3}{10}$ **7** $\frac{9}{5} = 1\frac{4}{5}$ **8** $\frac{9}{5} = 1\frac{4}{5}$

9 $\frac{3}{4}$ **10** $\frac{4}{5}$ **11** $\frac{4}{5}$ **12** $\frac{10}{3} = 3\frac{1}{3}$

13 $\frac{15}{2} = 7\frac{1}{2}$ **14** $\frac{15}{2} = 7\frac{1}{2}$ **15** $\frac{9}{2} = 4\frac{1}{2}$ **16** $\frac{8}{3} = 2\frac{2}{3}$

17 $\frac{12}{3} = 4$ **18** $\frac{14}{4} = 3\frac{1}{2}$ **19** $\frac{33}{4} = 8\frac{1}{4}$ **20** $\frac{38}{5} = 7\frac{3}{5}$

Problem-solving practice
1 48   **2** 0
3 Students' own answers, for example, 1 and 4 or 2 and 8
4 $\frac{12}{35}$ **5** $\frac{2}{15}$ **6** $\frac{1}{30}$ **7** 69

8 Jo spends longest. Tim spends $1\frac{1}{2}$ hours. Sam spends $\frac{1}{2} \times \frac{3}{2}$ hours =

$\frac{3}{4}$ hours. Jo spends $3 \times \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$ hours.

Exam practice

1 $\frac{21}{40}$

4.4 Dividing fractions
Purposeful practice 1

1 4   **2** 4   **3** 6   **4** 6

5 8   **6** 10   **7** 15   **8** $7\frac{1}{2}$

9 $7\frac{1}{2}$ **10** 5   **11** $6\frac{2}{3}$ **12** $2\frac{2}{3}$

13 $1\frac{1}{2}$ **14** $1\frac{1}{2}$ **15** $2\frac{1}{4}$ **16** $1\frac{1}{5}$

Purposeful practice 2

1 $\frac{1}{4}$ **2** $\frac{1}{4}$ **3** $\frac{1}{6}$ **4** $\frac{1}{9}$

---

5 $\frac{1}{6}$ **6** $\frac{5}{12}$ **7** $\frac{5}{12}$ **8** $\frac{5}{9}$

Purposeful practice 3
1 1   **2** 1   **3** $2\frac{1}{2}$ **4** $1\frac{2}{3}$

5 $1\frac{1}{3}$ **6** $2\frac{2}{3}$ **7** $1\frac{3}{5}$ **8** $1\frac{1}{5}$

Problem-solving practice
1 12

2 Several possible answers, for example, $\frac{4}{5}, \frac{8}{10}, \frac{12}{15}$

3 $\frac{2}{3}$ **4** $\frac{7}{8}$ **5** **a** $\frac{9}{10}$ **b** 54 minutes

6 **a** 96

   b Yes, as he can now answer $60 \div \frac{3}{8} = 60 \times \frac{8}{3} = 160$ questions.

Exam practice

1 $\frac{6}{7}$ **2** 63

4.5 Fractions and decimals
Purposeful practice 1

1 $\frac{1}{10}$ **2** $\frac{2}{5}$ **3** $\frac{1}{100}$ **4** $\frac{1}{25}$ **5** $\frac{7}{50}$

6 $\frac{1}{1000}$ **7** $\frac{7}{500}$ **8** $\frac{7}{50}$ **9** $\frac{18}{125}$ **10** $\frac{13}{125}$

Purposeful practice 2
1 0.5   **2** 0.25   **3** 0.125   **4** 0.1   **5** 0.3
6 0.9   **7** 0.01   **8** 0.27   **9** 0.61   **10** 0.061
11 0.2   **12** 0.6   **13** 0.05   **14** 0.85   **15** 0.04
16 0.36   **17** 1.5   **18** 2.5   **19** 1.75   **20** 2.25
21 $0.\dot{3}$ **22** $0.\dot{6}$ **23** $0.1\dot{6}$ **24** $0.8\dot{3}$ **25** $0.08\dot{3}$

Purposeful practice 3

1 $0.03, \frac{2}{25}, 0.3, \frac{4}{5}$ **2** $0.03, \frac{2}{25}, 0.3, \frac{2}{5}$

3 $\frac{2}{25}, 0.23, 0.3, \frac{4}{5}$ **4** $\frac{2}{25}, 0.23, 0.3, \frac{2}{3}$

5 $\frac{2}{25}, 0.23, 0.3, \frac{1}{3}$ **6** $\frac{2}{25}, 0.3, 0.33, \frac{1}{3}$

7 $\frac{2}{25}, 0.33, \frac{1}{3}, 0.34$ **8** $\frac{1}{4}, 0.33, \frac{1}{3}, 0.34$

9 $3.38, 3.4, \frac{7}{2}, \frac{17}{4}$ **10** $-\frac{7}{2}, 3.38, 3.4, \frac{17}{4}$

11 $-1, -\frac{4}{5}, -0.27, 0$ **12** $-1, -\frac{4}{5}, 0, 0.27$

Problem-solving practice
1 Any decimal that is greater than 0.4 but less than 0.5,
for example, 0.45

2 £8

3 Students' reasoning may vary, for example, $\frac{1}{6} = 0.1\dot{6}$. This is very

close to 0.16 but is not exactly equal to it. Therefore, multiplying by

0.16 is not the same as multiplying by $\frac{1}{6}$

4 Naz worked out $125 \div 8$ instead of $8 \div 125$

5 **a** Harry has not divided 3 by 2 to give 1.5, he's just written the
3 and 2 around a decimal point.
   b Harry has not divided 1 by 5 to give 0.2, he's just written the 5 after
a decimal point.
   c $\frac{1}{5}, 0.45, \frac{3}{2}, 2.3$

6 $\frac{9}{20}$ **7** Any decimal in between 0.65 and 0.72   **8** 43.2 g

Exam practice
1 0.03   **2** 0.4

4.6 Fractions and percentages

Purposeful practice 1

1 $\frac{1}{100}$ **2** $\frac{1}{10}$ **3** $\frac{1}{5}$ **4** $\frac{1}{4}$ **5** $\frac{1}{2}$

6 $\frac{1}{20}$ **7** $\frac{3}{20}$ **8** $\frac{7}{20}$ **9** $\frac{1}{25}$ **10** $\frac{8}{25}$

Purposeful practice 2

1 5% **2** 50% **3** 5% **4** 0.5%

Purposeful practice 3

1 50% **2** 25% **3** 75% **4** 60% **5** 60%
6 38% **7** 10% **8** 5% **9** 4% **10** 33.3% (1 d.p.)

Purposeful practice 4

1 5% **2** 50% **3** 0.5% **4** 25%
5 12.5% **6** 17.5% **7** 28% **8** 76%

Problem-solving practice

1 No, it is 75% **2** blue doors
3 Yes, Zainab's score is 64%
4 No, they are awake for 33.3% (1 d.p.).
5 Shop B as $\frac{1}{5} = 20\%$
6 Jazmin as $\frac{64}{80} = 80\%$, Laura got 75% and Kate got $\frac{5}{8}$ of the marks or 62.5%.

Exam practice

1 a $\frac{2}{5}$ **b** $\frac{65}{100} = \frac{13}{20}$ **2 a** 12% **b** 35%

4.7 Calculating percentages 1

Purposeful practice 1

1 3 **2** 30 **3** 60 **4** 15
5 75 **6** 45 **7** 33 **8** 1.5
9 1.5 **10** 15 **11** 30 **12** 7.5

Purposeful practice 2

1 1 **2** 0.1 **3** 0.01 **4** 0.001
5 0.05 **6** 0.15 **7** 0.2 **8** 0.005

Purposeful practice 3

1 150 **2** 30 **3** 150 **4** 30
5 150 **6** 1500 **7** 150 **8** 150

Purposeful practice 4

1 £190 **2** £190 **3** £380 **4** £95

Problem-solving practice

1 8% of £120
2 Katie has divided by 20 and multiplied by 100. She should have divided by 100 (to find 1%) and then multiplied by 20 (to find 20%). 20% of 180 g = 36 g
3 219 **4** 0.4%, 0.04, 40%, 0.44
5 £1638 is 10% of her annual salary. £1842.75 is 15% of her salary over $\frac{3}{4}$ of a year. She should take the second offer.
6 65% of 1200 = 780 students in school A voted.
45% of 960 = 720 students in school B voted.
So, more students voted in school A.
7 The bond gives £750 interest. The art would increase in value by £600. Isadora should put all of her money into the savings bond.
8 4 days **9** £14.60

Exam practice

1 36

4.8 Calculating percentages 2

Purposeful practice 1

1 120 **2** 96 **3** 88 **4** 80.8 **5** 160 **6** 176

Purposeful practice 2

1 80 **2** 64 **3** 72 **4** 79.2 **5** 0 **6** 75.2

Purposeful practice 3

1 a 110% **b** 200% **c** 90%
2 a 1.1 **b** 2 **c** 0.9
3 a £330 **b** £600 **c** £270
4 a 203.4 **b** 198 **c** 99 **d** 101.7
5 a 78.3 **b** 88.65

Problem-solving practice

1 £2575
2 20% off each item is a saving of £9.
60% off the total would be a saving of £27.
3 a 2550 copies **b** £13 260
4 346.5 days
5 a £263 680 **b** £24 840 **c** £27 896

Exam practice

1 £37.80

Mixed exercises A

Mixed problem-solving practice A

1 40p **2** $5x$
3 a $x = 8$ **b** $y = 2$
4 Tia has just replaced a with 4 and t with 5, but at means a multiplied by t. Tia has not multiplied 4 by 5 to give 20.
5 a Both the values Kamil has used are larger than the actual values, so the actual cost will be lower than his estimate.
b £167.20
6 The HCF of $8x^2 + 4x$ is $4x$, not $2x$

7 a
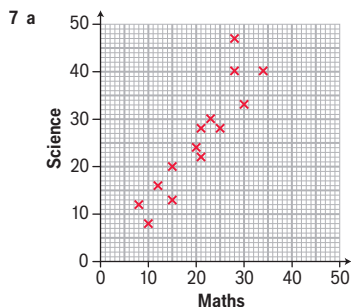
b Positive correlation **c** 34 (students' answers may vary – found by line of best fit)
d Yes, as the majority of the points for high science scores appear when there are high marks for maths tests (positive correlation).
8 Taylor is 9 and Callum is 8.

9 $A = 33$ **10** $\frac{1}{10}$

11 a
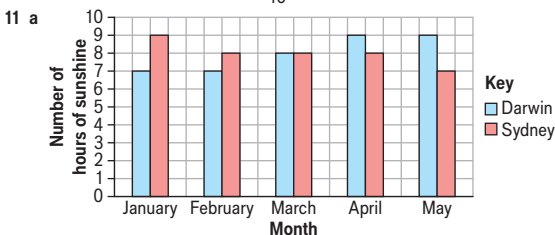
b Students' own answers, for example,
Overall, the number of hours of sunshine, increases over the five months for Darwin but decreases for Sydney. In March, both Darwin and Sydney have the same number of hours of sunshine.
12 $3(y - 5) = 3y - 15$
13 Emily because she scored 38 marks. Finn scored 36 marks. Toby scored 24 marks.
14 3 boxes of crisps and 10 boxes of juice
15 a $\frac{1}{3}$ **b** 1800

Exam practice

16 18
17 $L = 4x + 4$

18

	14 years old	15 years old	Total
Glasses	7	6	13
No glasses	43	34	77
Total	50	40	90

19 $\frac{5}{9}$, $\frac{5}{9}$ is $\frac{4}{9}$ smaller than 1, $\frac{9}{5}$ is $\frac{4}{5}$ greater than 1. $\frac{4}{9}$ is smaller than

$\frac{4}{5}$ so $\frac{5}{9}$ is closer to 1 than $\frac{9}{5}$.

20 a 1000   **b** 5600

   c Electric, because the bars get proportionally longer over time.

   d No, because we do not know the cost, prices or profit of the different cars.

21 £1836

22 No, Claire is wrong, a 25% decrease would give a force of 2.25 N/cm^2 but the force is 2.2 N/cm^2 which is lower, and therefore is a larger percentage decrease.

5 Equations, inequalities and sequences

5.1 Solving equations 1
Purposeful practice 1

1 $a + 3 = 12$, $a = 12 - 3$, $a = 9$
$a - 3 = 12$, $a = 12 + 3$, $a = 15$

$3a = 12$, $a = \frac{12}{3}$, $a = 4$

$\frac{a}{3} = 12$, $a = 3 \times 12$, $a = 36$

2 a $b = 24 - 8 = 16$ **b** $b = 24 \div 8 = 3$ **c** $b = 24 \times 8 = 192$
  d $b = 24 - 8 = 16$ **e** $b = 24 + 8 = 32$ **f** $b = 8 - 24 = -16$

Purposeful practice 2

1 $x = 11$ **2 a** $x + 14 = 36$ **b** $x = 22$

3 a $x - 9 = 15$ **b** $x = 24$ **4** $7x = 42$, $x = 6$

Problem-solving practice

1 35

2 Abi has 9 cards, Bashar has 27 and Chan has 18.

3 135°   **4** 81 cm^2

5 A rectangle with height 3 cm and width 10 cm.

6 128 cm^2

Exam practice

1 $x = 30°$

5.2 Solving equations 2
Purposeful practice 1

1 a $x = 3$ **b** $x = 6$ **c** $x = 2$ **d** $x = 4$
2 a $x = 3$ **b** $x = 6$ **c** $x = 2$ **d** $x = 4$
  e $x = 3$ **f** $x = 6$ **g** $x = 2$ **h** $x = 4$

Purposeful practice 2

1 $x = -12$ **2** $x = -6$ **3** $x = -2$ **4** $x = -4$

Purposeful practice 3

1 $y = \frac{4}{3}$ **2** $y = \frac{4}{3}$ **3** $y = \frac{8}{3}$ **4** $y = \frac{2}{3}$

5 $y = \frac{10}{3}$ **6** $y = \frac{5}{3}$ **7** $y = 12$ **8** $y = 48$

9 $y = 60$ **10** $y = 24$ **11** $y = 30$ **12** $y = 66$

13 $y = 1.2$ **14** $y = 5.25$ **15** $y = 1.625$

Problem-solving practice

1 12

2 a $x + 2x + 2x - 7 = 58$, $5x - 7 = 58$
  b Dec is 13, Emma is 26 and George is 19.

3 73°, 33.7°, 73.3°   **4** 20 cm   **5** $g = 11.25$

Exam practice

1 $f = 4$

5.3 Solving equations with brackets
Purposeful practice 1

1 a $x = 3$ **b** $x = 1.5$ **c** $x = 4$ **d** $x = 4.5$
2 a $x = 3$ **b** $x = 1.5$ **c** $x = 4$ **d** $x = 4.5$

Purposeful practice 2

1 $x = 10$ **2** $x = \frac{8}{3}$ **3** $x = \frac{8}{3}$ **4** $x = \frac{2}{3}$ **5** $x = -3$

6 $x = -1$ **7** $x = 0$ **8** $x = 2$ **9** $x = 4$

Purposeful practice 3

1 $x = 3$ **2** $x = 1.5$ **3** $x = 4$ **4** $x = 4.5$
5 $x = 2$ **6** $x = 3$ **7** $x = 2$ **8** $x = -2$
9 $x = 2$ **10** $x = -8$ **11** $x = -4$ **12** $x = 6$
13 $x = 5$ **14** $x = 2$ **15** $x = 9$ **16** $x = 7$
17 $x = -3$ **18** $x = -1$

Problem-solving practice

1 -5 **2** 5   **3** $h = 32°$

4 Rectangle B   **5** $y + 4 = 3y - 8$
                 $12 = 2y$
                  $y = 6$

Exam practice

1 $p = 7$

5.4 Introducing inequalities
Purposeful practice 1
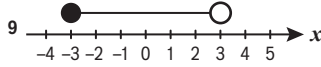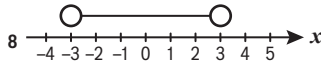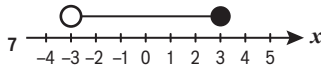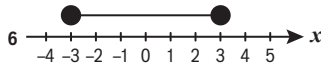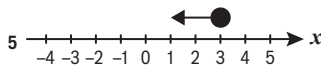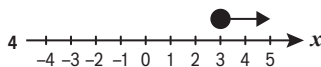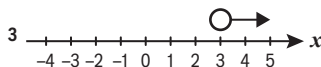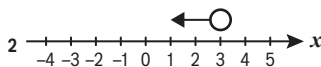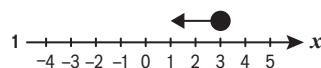
Purposeful practice 2

Students' own answers, for example,

1 3, 2, 1   **2** 2, 1, 0   **3** 4, 5, 6
4 3, 4, 5   **5** 3, 2, 1   **6** $-3, -2, -1$
7 $-2, -1, 0$ **8** $-2, -1, 0$ **9** $-3, -2, -1$

Purposeful practice 3

1 $x \leqslant 3$ **2** $x < 3$ **3** $x > 3$ **4** $x > 1$
5 $x > 2$ **6** $x > 0$ **7** $x > 12$ **8** $x \leqslant 12$
9 $x \leqslant 9$ **10** $x \leqslant 12$ **11** $x \geqslant 12$ **12** $x \geqslant 12$
13 $x \geqslant 3$ **14** $x > 3$ **15** $x > 9$ **16** $x < 9$
17 $x < 1$ **18** $x \geqslant 4$

Problem-solving practice

1 $x < 3$, so the smallest integer value x can take is 2.

2 $2y + 3 < 20$

3 $3d < 19$, so the oldest David can be is 6.

4 $5t + 6 > 46$

$t > 8$

Therefore the smallest number Tumay could have started with is 9.

5 $m + 40 \geqslant 90$ **6** $6b \leqslant 34$ **7** c $\geqslant 7.5$

Exam practice

1 $-6, -5, -4, -3$ **2** $x > \dfrac{4}{3}$

5.5 More inequalities

Purposeful practice 1

1 $x = 2$ or 3 **2** $x = 2$ **3** $x = 2$ or 3 **4** $x = 2$

Purposeful practice 2

1 $2 < x < 3$ **2** $2 < x < 3$ **3** $2 < x < 3$

4 $2 < x < 3$ **5** $-1 < x < 3$ **6** $-2 < x < 2$

7 $-1 < x < 3$ **8** $-1 < x < 3$ **9** $1.5 < x < 3$

Purposeful practice 3

1 $-3 \leqslant x < -2$ and -3 **2** $-3 \leqslant x \leqslant -2$ and $-3, -2$

3 $-2.5 \leqslant x < -1.5$ and -2 **4** $-3.5 \leqslant x < -2.5$ and -3

5 $-3.5 \leqslant x < 1.5$ and $-3, -2, -1, 0, 1$

6 $-2.5 \leqslant x < 2.5$ and $-2, -1, 0, 1, 2$

Purposeful practice 4

1 4   **2** 2 (other answers possible)   **3** 2

4 4 (other answers possible)   **5** 3 or 4   **6** 2

Problem-solving practice

1 $220 < 20x + 80 < 340$ or $220 < y < 340$

2 $90 \leqslant m \leqslant 150$

3 a Yes. $x > 3$ (x must be greater than 3) for the rectangle to have an area greater than $44\,\text{cm}^2$. $x \leqslant 8$ for the triangle to have an area less than or equal to $38\,\text{cm}^2$.

b $6x + 12 \leqslant 12x + 8 \rightarrow x \geqslant \dfrac{4}{6}$

4 Toby has not reversed the inequality or the signs when dividing by a negative number. It should be $-5 \leqslant x < 4$

5 $2.5 \leqslant p \leqslant 3.5$ **6** $12.5 \leqslant d \leqslant 55$ **7** $2 \leqslant n \leqslant 6$

Exam practice

1 $4 \leqslant x < 7$

5.6 Using formulae

Purposeful practice 1

1 3   **2** 1   **3** -1 **4** -3 **5** -9

Purposeful practice 2

1 3   **2** 2.5   **3** 2   **4** 1.5   **5** 0

Purposeful practice 3

1 $x = \dfrac{y + 3}{2}$ **2** 3

Purposeful practice 4

1 $x = \dfrac{y}{4}$ **2** $I = \dfrac{V}{R}$ **3** $s = ut$

4 $x = y - 3$ **5** $c = f - d$ **6** $a = \dfrac{f}{m}$

Problem-solving practice

1 14 cm

2 a Student A has substituted the value of 48 for x instead of y.

b Student B has subtracted 4 instead of adding 4.

3 a $F = PA$ **b** 144 N

4 a 180 km   **b** 48 mph

5 $12\,\text{m/s}^2$

Exam practice

1 a $G = 17$ **b** $T = \dfrac{G - 2}{5}$

5.7 Generating sequences

Purposeful practice 1

1 Add 2

2 Add the two previous terms

3 Double the previous term   **4** Add 3

5 Add the two previous terms   **6** Multiply by 2

Purposeful practice 2

1 9, 11   **2** 7, 9   **3** $-1, 1$

4 $1, -1$ **5** $-10, -12$ **6** $\dfrac{9}{2}, \dfrac{11}{2}$

7 $-\dfrac{3}{2}, -\dfrac{4}{2}$ **8** $-\dfrac{3}{2}, -2$ **9** 3, 3.5

Purposeful practice 3

1 5, 10, 15, 20, 25   **2** 6, 11, 16, 21, 26

3 7, 12, 17, 22, 27   **4** 8, 13, 18, 23, 28

5 4, 9, 14, 19, 24   **6** 3, 8, 13, 18, 23

7 $-1, 1, 3, 5, 7$ **8** $-1, 0, 1, 2, 3$

9 $-1, -1, -1, -1, -1$ **10** $-1, -2, -3, -4, -5$

11 $-1, -2, -3, -4, -5$ **12** $-1, -3, -5, -7, -9$

Problem-solving practice

1 14 more chairs and 4 more tables

2 5 of each

3 a Yes, it will alternate between -2 and 1

b Start at 3

4 a No, 26 will not appear in the sequence. All of the terms, apart from the starting term, will be odd.

b No, because doubling any integer and subtracting 1 will give an odd number. Therefore, whatever integer is chosen to start the sequence, the rest of the terms will be odd.

5 2 more terms – using 9 and 11 matchsticks

Exam practice

1 a i 17

ii Add on three to the previous term

b 23

5.8 Using the nth term of a sequence

Purposeful practice 1

1 4, 8, 12, 16, 20   **2** 5, 9, 13, 17, 21

3 6, 10, 14, 18, 22   **4** 6, 8, 10, 12, 14

5 $-2, 0, 2, 4, 6$ **6** 2, 0, $-2, -4, -6$

Purposeful practice 2

1 $2n$ **2** $2n - 1$ **3** n **4** $n + 1$

5 $2n + 2$ **6** $4n$ **7** $-2n + 17$ **8** $-2n + 22$

9 $-n + 10$ **10** $-5n + 41$ **11** $-3n + 23$ **12** $-4n + 23$

Purposeful practice 3

1 a 50   **b** 100   **c** 101   **d** 150

e 149   **f** 24   **g** -151 **h** -101

2 a yes, 50th term   **b** yes, 25th term   **c** No   **d** No

e Yes, 17th term   **f** Yes, 102nd term   **g** No   **h** No

Problem-solving practice

1 The fourth terms of sequence A and B are the same (13). The numbers 23, 33, 43... appear in both sequences but in different positions.

2 a No, the pattern would need 60 tiles, which would cost £21.

b Multiple possible student answers that sum to < 50, for example, $3n$ gives 3, 6, 9, 12, which costs $3 \times £3.50$ or £10.50 for the first four terms

3 23, 27 or 31

4 a $4n + 8$
  b No, non-integer result for n.
5 $6n - 1$ **6** $3n - 1$
7 The first four terms of $8n - 3$ have a sum of 68. The first four terms of $5n + 3$ have a sum of 62. After the initial term, all terms of $8n - 3$ are greater than or equal to the equivalent term in $5n + 3$. Therefore any four consecutive terms of $8n - 3$ will have a sum greater than that of the equivalent four consecutive terms in $5n + 3$.
8 D, 10th term.
9 Students' own answers, for example, $-2n + 6$

Exam practice

1 141

6 Angles

6.1 Properties of shapes
Purposeful practice 1

Shape	Equal sides	Pairs of equal opposite sides	Pairs of parallel sides	Angles
	4	2	2	4 equal angles of 90°
	2 pairs, opposite	2	2	4 equal angles of 90°
	2 pairs, opposite	2	2	2 pairs equal, opposite
	4	2	2	2 pairs equal, opposite
	2 pairs, adjacent	0	0	1 pair equal, opposite
	0	0	1	2 equal angles of 90°, adjacent
	2 pairs, opposite	1	1	2 pairs equal, adjacent

Purposeful practice 2

1 a $x = 5\,$cm
  b $n = 8\,$cm, $t = 3\,$cm
  c $p = 80°$, $q = 100°$, $r = 7\,$mm
  d $j = 80°$, $k = 120°$, $l = 20\,$mm, $m = 15\,$mm
  e $u = 110°$, $v = 70°$, $w = 2\,$cm

Problem-solving practice

1 Students' own answers, for example,
  a   **b**
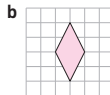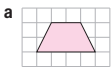
2 a Square, rhombus
  b Square – or four right angles
    Rhombus – no right angles
3 Yes, a rhombus has the same properties as a parallelogram, such as two pairs of equal sides, two pairs of opposite parallel sides, opposite angles equal. In addition, a rhombus has an extra property because all of its sides are equal. So, a rhombus is a special type of parallelogram.
4 D = (6, 3)

Exam practice

1 Parallelogram

6.2 Angles in parallel lines
Purposeful practice 1

1 a $a = 110°$ (corresponding angles are equal)
  b $b = 80°$ (alternate angles are equal)
    $c = 100°$ (angles on a straight line add to 180°)
  c $d = 120°$ (alternate angles are equal)
    $e = 60°$ (angles on a straight line add to 180°)
  d $f = 70°$ (corresponding angles are equal)
    $g = 110°$ (angles on a straight line add to 180°)
  e $h = 130°$ (alternate angles are equal)
    $i = 130°$ (corresponding angles are equal OR vertically opposite angles are equal)
  f $j = 130°$ (angles on a straight line add to 180°)
    $k = 130°$ (angles on a straight line add to 180° OR vertically opposite angles are equal)
    $l = 130°$ (alternate angles are equal OR corresponding angles are equal)

Purposeful practice 2

1 Students' reasoning may vary, for example,
  a $a = 60°$ (alternate angles are equal)
    $l = 100°$ (angles in a triangle add to 180°)
  b $n = 55°$ (alternate angles are equal)
    $m = 60°$ (angles on a straight line add to 180°)
    $p = 65°$ (angles in a triangle add to 180°)
  c $s = 50°$ (alternate angles are equal)
    $t = 45°$ (vertically opposite angles are equal)
    $u = 85°$ (angles in a triangle add to 180°)
  d $v = 100°$ (corresponding angles are equal)
    $w = 100°$ (alternate angles are equal)
  e $x = 70°$ (alternate angles are equal)
    $y = 70°$ (corresponding angles are equal)
    $z = 110°$ (angles on a straight line add to 180°)
  f $a + 33 = 125°$ (corresponding angles are equal), so $a = 92°$
    $b = 180 - 125 = 55°$ (corresponding angles are equal; angles on a straight line add to 180°)

Problem-solving practice

1 Students' reasoning may vary, for example,
    Angle $y = 115°$ (alternate angles are equal)
    Angle DBC = 65° (angles on a straight line add to 180°)
    Angle BCD = 40° (vertically opposite angles are equal)
    Angle $x = 75°$ (angles in a triangle add to 180°)
2 a Angles on a straight line add to 180°.
  b Corresponding angles are equal.
  c Vertically opposite angles are equal.
3 Students' own answers, for example,
    $p = 125°$ (corresponding angles are equal)
    $q = 55°$ (angles on a straight line add to 180°)
    So $p + q = 180°$
4 If ABC and DEF were parallel, angle DEB would be 90° (corresponding angles are equal). However, angle DEB is on a straight line with the 85° angle, so it must be 95° (angles on a straight line add to 180°). Therefore, ABC and DEF cannot be parallel.

Exam practice

1 Jamal should have written angle DEB = 119° because alternate angles are equal.

6.3 Angles in triangles
Purposeful practice 1

1 a b and c **b** e and f **c** g and h
  d j and k **e** m and n **f** q and r

Purposeful practice 2

1 a Scalene   **b** Scalene
  c Isosceles   **d** Equilateral
  e Right-angled, scalene   **f** Isosceles
2 a $d = 30°$, $e = 60°$ **b** $s = 55°$, $t = 55°$, isosceles
  c $f = 60°$, $g = 50°$ **d** $x = 30°$, $y = 75°$, $z = 75°$, isosceles

Problem-solving practice

1 Angles in a triangle add to 180°. Therefore, triangles can have a maximum of one obtuse angle, so must have at least two acute angles.

2 Angle ACB = 50° (vertically opposite angles are equal)
Angle CAB = 180° − 65° − 50° = 65° (angles in a triangle add to 180°)
Two angles in triangle ABC are equal, so it is isosceles.

3 Angle ZXY = 180° − 110° = 70° (angles on a straight line add to 180°)
Angle XZY + angle XYZ = 180° − 70° = 110° (angles in a triangle add to 180°)
Angle XZY = 110° ÷ 2 = 55° (base angles of an isosceles triangle are equal)

4 Angle PRQ = 38° (vertically opposite angles are equal)
Angle RPQ = 180° − 76° = 104° (angles on a straight line add to 180°)
Angle PQR = 180° − 104° − 38° = 38° (angles in a triangle add to 180°)
Two angles in triangle PQR are equal, so it is isosceles.

5 Students' own answers, for example,
Angle STR = 36° (base angles of an isosceles triangle are equal)
Angle RSU = angle RUS (base angles of an isosceles triangle are equal)
Angle RSU = angle RUS = $\frac{1}{2}$(180° − 36°) = 72° (angles in a triangle add to 180°)
Angle SUT = 180° − 72° = 108° (angles on a straight line add to 180°)
Angle UST = 180° − 36° − 108° = 36° (angles in a triangle add to 180°)
Triangle SUT has two equal angles, so it is isosceles.

Exam practice

1 Angle DBC = 180° − 96° = 84° (angles on a straight line add to 180°)
Angle BDC = 180° − 84° − 48° = 48° (angles in a triangle add to 180°)
Two angles in triangle BCD are equal, so it is isosceles.

6.4 Exterior and interior angles
Purposeful practice 1

1 Angles can be drawn in the 'opposite' direction from that shown, but all sides must be extended in the same way (i.e. clockwise or anticlockwise).
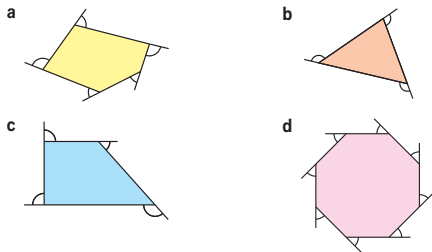
a
b
c
d

Purposeful practice 2

1 a $n = 65°$
b $w = x = 90°$, $y = 110°$
c $p = r = s = u = 50°$, $t = 80°$

Purposeful practice 3

1 a Exterior angle = 120°
Interior angle = 60°
b Exterior angle = 90°
Interior angle = 90°
c Exterior angle = 45°
Interior angle = 135°
d Exterior angle = 40°
Interior angle = 140°

Purposeful practice 4

1 36 **2** 24 **3** 18 **4** 12 **5** 5

Problem-solving practice

1 $x = \frac{360°}{5} = 72°$ (exterior angle of regular pentagon = $\frac{360°}{\text{number of sides}}$)
$y = 180° − 72° = 108°$ (interior angle and exterior angle add to 180°, all angles in a regular pentagon are equal)
Angle ABF = angle z (symmetry)
$z = \frac{1}{2}$(180° − 108°) = 36° (angles on a straight line add to 180°)

2 square

3 a $a = 45°$ **b** $b = 135°$

4 For a regular polygon, $\frac{360°}{\text{exterior angle}}$ = number of sides
$\frac{360°}{50°}$ is not a whole number, so Kelly must be wrong.

5 For a regular polygon, $\frac{360°}{\text{number of sides}}$ = exterior angle
Exterior angle of a regular 9-sided polygon = $\frac{360°}{9} = 40°$
Exterior angle of a regular 18-sided polygon = $\frac{360°}{18} = 20°$
40° is double 20°, so the exterior angle of a regular 9-sided polygon is double the exterior angle of a regular 18-sided polygon.
Exterior angle of a regular 9-sided polygon = $\frac{360°}{9} = 40°$
Exterior angle of an equilateral triangle = $\frac{360°}{3} = 120°$
$40 \times 3 = 120$
So the exterior angle of an equilateral triangle is 3 times the exterior angle of a regular 9-sided polygon.

6 $x = 120°$, with students' own reasoning, for example,
Exterior angle of regular hexagon is 360° ÷ 6 = 60°
(exterior angle of a regular polygon = 360° ÷ number of sides).
The interior angle of a regular hexagon is 180° − 60° = 120°
(angles on a straight line add to 180°).
Angle HEF = 60° (PEH is a line of symmetry that bisects angle DEF, which is an interior angle).
$x = 120°$ (angles on a straight line add to 180°)

Exam practice

1 Interior angle at C is 60° (angles on a straight line add to 180°)
Interior angle at A is 65° (angles in a triangle add to 180°)
$x = 115°$ (angles on a straight line add to 180°)

6.5 More exterior and interior angles
Purposeful practice 1

1

Polygon	Number of sides	Number of triangles	Angle sum
	3	1	180°
	4	2	2 × 180° = 360°
	5	3	**3 × 180° = 540°**
	6	4	**4 × 180° = 720°**
	7	5	**5 × 180° = 900°**
	8	6	**6 × 180° = 1080°**

Purposeful practice 2

1 a 1260° **b** 1440° **c** 1620°
2 a i 360° **ii** $x = 150°$ **b i** 540° **ii** $y = 80°$
c i 720° **ii** $z = 50°$
3 a 108° **b** 120° **c** 135°
d 140° **e** 144° **f** 150°

Problem-solving practice

1 11

2 $n = 60°$, with students' own working, for example,
Angle sum of polygon is (number of sides − 2) × 180°
So angle sum of hexagon is (6 − 2) × 180° = 720°
Hexagon is regular, so each interior angle is 720° ÷ 6 = 120°
Horizontal line is line of symmetry, so it bisects an interior 120° angle, therefore $n = 120° ÷ 2 = 60°$

3 $b = 162°$, with students' own working, for example,
Angle sum of polygon is (number of sides − 2) × 180°
So angle sum of pentagon is (5 − 2) × 180° = 540°
Pentagon is regular, so each interior angle is 540° ÷ 5 = 108°
Interior angle of square = 90°
Therefore $b = 360° − 108° − 90° = 162°$ (angles around a point add to 360°)

4 Angle sum of polygon is (number of sides − 2) × 180°
So angle sum of nonagon is (9 − 2) × 180° = 1260°
Nonagon is regular, so each interior
angle is = 1260° ÷ 9 = 140°. Thus, angle ABC = 140°
Triangle ABC is isosceles (sides AB = BC
(sides of regular nonagon are equal), so angle BAC = angle BCA
(base angles of an isosceles triangle are equal).

Therefore, angle BAC = $\frac{1}{2}(180° − 140°) = 20°$

5 135°

Exam practice

1 $w = 132°$, with students' own working, for example,
Angle sum of polygon is (number of sides − 2) × 180°
So angle sum of hexagon is (6 − 2) × 180° = 720° and angle sum of pentagon is (5 − 2) × 180° = 540°
Hexagon and pentagon are regular, so each interior angle of hexagon is 720° ÷ 6 = 120° and each interior angle of pentagon is 540° ÷ 5 = 108°
Therefore, $w = 360° − 120° − 108° = 132°$ (angles around a point add to 360°)

6.6 Geometrical problems
Purposeful practice 1

1 a $x = 35°$ **b** $y = 105°$
 c $z = 62°$ **d** $n = 130°$
2 a i $a = 40°$ **ii** 80°, 120°, 160°
 b i $b = 36°$ **ii** 36°, 36°, 108°
 c i $c = 36°$ **ii** 36°, 36°, 144°, 144°
 d i $d = 45°$ **ii** 45°, 45°, 90°, 90°, 90°
3 a $e = 60°$, $e + 10° = 70°$, $e − 10° = 50°$
 b $f + 60° = 80°$, $2f = 40°$, $3f = 60°$

Purposeful practice 2

1 angle ACB = 40°, angle CAB = 100°
2 angle DFE = 25°, angle FDE = 75°

Problem-solving practice

1 $m = 70°$, both angles are 110°
2 $x = 60°$
3 45°
4 Angles are 28°, 68°, 84° with students' own working, for example,
Smallest angle = x
Largest angle = $3x$
Other angle = $3x − 16°$
$7x − 16° = 180°$ (angles in a triangle add to 180°)
$7x = 196°$
$x = 28°$
So angles are 28°, 68° and 84°.
5 56°

Exam practice
 1 a i $x = 55°$ **ii** Angles in a triangle add to 180°
 b $y = 45°$

7 Averages and range

7.1 Mean and range
Purposeful practice 1
1 10   **2** 8   **3** 6   **4** 6
Purposeful practice 2
1 Route 1 range: 16 mins, Route 2 range: 7 mins
2 Route 1 mean: 9 mins, Route 2 mean: 13 mins

Purposeful practice 3
1 10.64   **2** 10.64

Value, v	Frequency, f	$v \times f$
7	3	21
8	2	16
9	3	27
10	6	60
11	2	22
12	1	12
13	4	52
14	4	56
Total	25	266

Problem-solving practice

1 a The way the calculation is written means that only the 9 is divided by 4. Emir should have found the total of the four numbers, by pressing the equals key, before dividing by 4. Alternatively, he could have used brackets.
 b The mean should lie within the range of the data.
2 18.95 s (2 d.p.)   **3** 3 hours 45 minutes

Exam practice

1 Raj read more consistently. His range of 4 books is smaller than Ellen's range of 12.

7.2 Mode, median and range
Purposeful practice 1

1 Median 12, range 20   **2** Median 12, range 20
3 Median 12, range 15   **4** Median 12, range 15
5 Median 13, range 23   **6** Median 13, range 23
7 Median 12, range 23   **8** Median 13, range 28
9 Median 13, range 23

Purposeful practice 2

1 a 0.6 kg is an outlier.
 b Range, excluding the outlier, is 1.8 kg.
 c Median, excluding the outlier, is 3.6 kg.
2 a Median 23.45, mode 24.6, range 5.5
 b Median 234.5, mode 246, range 55

Problem-solving practice

1 a Any number ⩾10   **b** Any number ⩽ 7
 c 22 is the only extra value that will give a range of 16. Negative 1 would give the required range but is not an acceptable value as the cards display positive numbers.
2 a i Median = 112   **ii** Mode = 112   **iii** Range = 46
 b The manager has only looked at the leaf section of the diagram so has mistakenly taken the highest value as 9 (instead of 139) and the lowest as 3 (instead of 93).
3 a Students' own answers, for example, change 62 to 70, giving a new median of 70.
 b Change 62 to any other integer from 55 to 67 inclusive.

Exam practice

1 137 cm

7.3 Types of average
Purposeful practice 1

Day 1 median: 3 kg, mode: 3 kg   **Day 2** median: 4 kg, mode: 4 kg
Day 3 median: 4 kg, mode: 5 kg   **Day 4** median: 4 kg, mode: 5 kg

Purposeful practice 2

1 10–14 cars (Dev's record); 0–4 (Daisy's record)
2 26 (Dev's record); 22 (Daisy's record)
3 10–14 (Dev's record); 5–9 (Daisy's record)

Problem-solving practice

1 Total frequency = 3 + 7 + 8 + 1 = 19
So median = 10th Value.
This lies in the $35 \leqslant x < 40$ height range and so the median height is between 35 and 40 cm.

2 a 1st week: mean = 5.5 minutes, median = 5 minutes
2nd week: mean = 10.5 minutes, median = 6 minutes
  b Students' own answers, for example, the mean is least useful because it gives a distorted average for week 2 because of the outlier.
3 a The mode
  b Range and mean cannot be calculated, because they can only be calculated from numerical data.

Exam practice

1 a The mean is 15.6 but there is no dress size of 15.6.
  b Students' reasoning may vary, for example, the mode will be the most useful average because it shows Sam that the most frequently bought size is 14.

7.4 Estimating the mean

Purposeful practice 1

1 25.0 **2** 30.0 **3** 25.0 **4** 25.0 **5** 31.7 **6** 131.7

Purposeful practice 2

1 11.875 **2 a** 13.125 **b** 12.1875

Problem-solving practice

1 a Estimated range is 19. **b** 10 to 14 complaints
  c Estimated mean is $10\frac{1}{2}$ complaints.
2 The missing frequency is 3.

Exam practice

1 a £305
  b Students' reasoning may vary, for example, Terry is correct. The mean may not be the best average to use as there are outliers.

7.5 Sampling

Purposeful practice 1

1 All Year 7 students who are 12 in Julie's school
2 All Year 7 students who are in that class
3 All Year 7 students who are in a Cardiff school
4 All Year 7 students who live in Cardiff
5 All Year 7 students who are in a Lancashire school
6 All Year 7 students who are in a Scottish school

Purposeful practice 2

1 Students' own answers, for example, make a numbered list of all the students in the year group and select numbers using a random number generator.
2 Students' own answers, for example, carrying out a national census is very costly and time-consuming.
3 a Students' own answers, for example, a sample taken at midday may largely involve people who are not working that day so may not be representative. The sample size is too small.
  b Students' own answers, for example, take the survey across a wider range of times. Increase the sample size.
4 Students' own answers, for example, the diet of people at the gym may not be representative of the diet of the rest of the population.

Problem-solving practice

1 a Students' own answers, for example, a sample of 5 is not big enough to be representative. It is not possible to see, from a sample of 5 customers, whether all the different sandwiches need to be ordered.
  b The owner should increase the number of customers in the sample to increase accuracy.
2 Students' own answers, for example, the last 10 beams may be very similar. They should randomly sample from the 100 beams.
3 a Students' own answers, for example, 10 am is early, so people won't have been on many rides. 20 is too small a sample.
  b Students' own answers, for example, take the survey at the park exit. Take a larger sample.
4 Diesel 20, Petrol 25

Exam practice

1 The survey suggests that about 400 people will like rock music. Assumptions made: The sample is representative of the group of people who will be attending the festival. People have one main type of music they like.

8 Perimeter, Area and Volume 1

8.1 Rectangles, parallelograms and triangles

Purposeful practice 1

1 60 cm^2 **2** 60 cm^2 **3** 60 cm^2

Purposeful practice 2

1 30 cm^2 **2** 30 cm^2 **3** 30 cm^2

Purposeful practice 3

1 18 cm^2 **2** 9 cm^2 **3** 36 cm^2
4 36 cm^2 **5** 24 cm^2 **6** 60 cm^2

Problem-solving practice

1 5 cm **2** 12 cm
3 Students' answers will vary. Length $\times$ perpendicular height should be 60 cm^2.
4 10 cm **5** 609 cm^2

Exam practice

1 8 cm
2 Sketch of a parallelogram with base = 6 cm and height = 3 cm

8.2 Trapezia and changing units

Purposeful practice

1 a 13 cm **b** 104 cm^2 **c** 52 cm^2
2 a 60 cm^2 **b** 30 cm^2
3 42 cm^2
4 a 42 cm^2 **b** 52.5 cm^2 **c** 54.6 cm^2

Problem-solving practice

1 7 mm **2** 3.375 m^2 **3** 14 cm **4** 23.4 cm^2

Exam practice

1 Students' answers will vary. Measurements should be clearly labelled with base length and perpendicular height such that $\frac{1}{2}$ base $\times$ height = 21 cm^2.
Example answer:
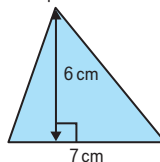
8.3 Area of compound shapes

Purposeful practice

1 a 6 cm **b** 72 cm^2 **c** 5 cm by 4 cm **d** 20 cm^2 **e** 92 cm^2
2 a Rectangle C is 7 cm by 5 cm and rectangle D is 10 cm by 3 cm.
  b Area of rectangle C = 35 cm^2 and area of rectangle D = 30 cm^2.
  c 65 cm^2
3 a Area of rectangle E = 36 cm^2 and area of rectangle F = 45 cm^2.
  b 81 cm^2
4 a 42 cm^2 **b** 330 cm^2 **c** 70 cm^2

Problem-solving practice

1 Shape B has the bigger area, by 4 cm^2. **2** 237 cm^2
3 Area = (7.5 m $\times$ 5.5 m) + (3 m $\times$ 4 m) = 53.25 m^2
53.25 m^2 $\div$ 1.75 m^2 = 30.4, so Caily needs 31 packs.

Exam practice

1 304 cm^2

8.4 Surface area of 3D solids
Purposeful practice
1 a and **b**
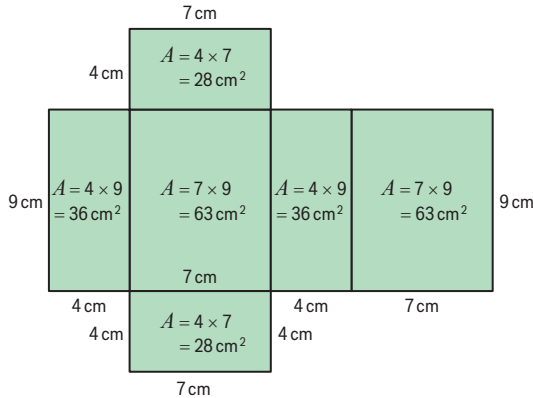
c 254 cm²

2 a 292 cm²   **b** 216 cm²
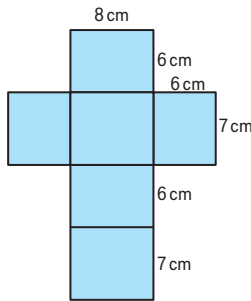
c 636 cm²
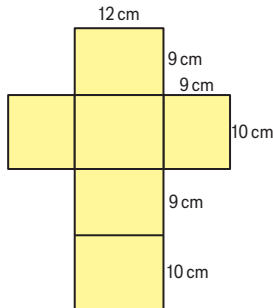
3 a 432 cm²
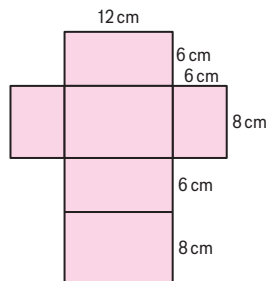
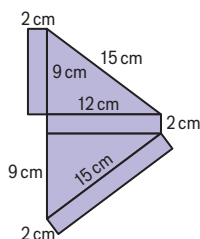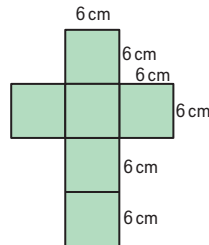**b** 180 cm²   **c** 216 cm²
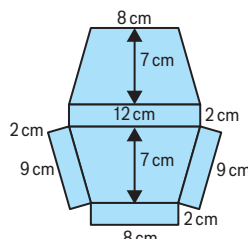
Problem-solving practice
1 7 cm   **2** £1800   **3** 7 cm   **4** 9 cm

Exam practice
1 7 cans

8.5 Volumes of prisms
Purposeful practice 1
1 a 12   **b** 12   **c** 8
2 a 12 cm³   **b** 12 cm³   **c** 8 cm³

Purposeful practice 2
1 a 96 cm³   **b** 90 cm³   **c** 315 cm³
2 a 160 cm³   **b** 54 cm³   **c** 252 cm³

Problem-solving practice
1 95 cm³   **2 a** 3 cm   **b** 27 cm³
3 a 69 m³   **b** £4968

Exam practice
1 512 cm³
2 a Yes, one cube has a volume of 125 cm³, $8 \times 125 = 1000$
   b i Students' answers will vary. Dimensions of cuboid should be 5 cm by 5 cm by 40 cm **or** 10 cm by 10 cm by 10 cm **or** 5 cm by 10 cm by 20 cm.
    ii Students' answers will vary. Surface area of cuboid should be 850 cm² **or** 600 cm² **or** 700 cm².

8.6 More volume and surface area
Purposeful practice 1
1 a 30 cm³   **b** 28 cm³   **c** 0.03 m³   **d** 0.028 m³

Purposeful practice 2
1 a 36 000 mm³   **b** 360 000 mm³
  c 105 000 000 cm³   **d** 378 000 000 cm³

Problem-solving practice
1 a 9 cm   **b** 7 cm   **c** 11 cm
2 225 mm
3 No.
  20 cm ÷ 2 cm = 10, so 10 dice will fit along the width of the box.
  12 cm ÷ 2 cm = 6, so 6 dice will fit along the depth of the box. This means that $10 \times 6 = 60$ dice will be in each layer.
  5 cm ÷ 2 cm = 2.5, so only 2 layers of dice will fit in the box, with a space left at the top.
  Therefore, only 120 dice will fit in the box.

Exam practice
1 32 cm

Mixed exercises B

Mixed problem-solving practice B
1 No, the nth term is $4n - 3$ and when $4n - 3 = 35$, $n = 9.5$
2 8 cm   **3** 12
4 a i 40   **ii** The term-to-term rule is 'add 9'
  b 112
5 $x = 30°$ **6** 12 cm
7 $x = 12$ cm **8** $a = 132°$
9 a Possible answers are 0, 1, 2 or 4   **b** 7, 7, 16
10 Angle BEF = 35°. Students' reasoning may vary, for example, AD and CE are parallel and angle ADF and angle BEF are alternate angles. So, angle BEF = angle ADF = 35° because alternate angles are equal.
11 260
12 a $x = 7$ m   **b** £159.29
13 Jakub, because the range for his scores, 3, is less than Kate's, 7.
14 $91° \leqslant x \leqslant 124°$

Exam practice
15 729 cm³   **16** $7n - 1$
17 Angle DEF = 145°.
  $x = 5 \times 42° = 210°$. The internal angles of a hexagon total 720°.
  Angle DEF = 720° − 138° − 42° − 210° − 95° − 90° = 145°.

18 a 30.6

   b Yes, because the mean is affected by outliers.

19 19°, 66° and 95°

   $x + 5x + 5x - 29 = 180°$, giving $11x - 29 = 180°$. So, $x = 19°$.

   Angles are 19°, $5 \times 19°$ and $(5 \times 19°) - 29°$ i.e. 19°, 95° and 66°.

20 Opposite sides of a rectangle are equal, so $2x + 5 = 4x - 9$.

   Solving these equations gives $x = 7\,\text{cm}$.

   The length of the rectangle is $19\,\text{cm}$ and its area is $95\,\text{cm}^2$,

   so $19y = 95$. $y = 5\,\text{cm}$.

9 Graphs

9.1 Coordinates

Purposeful practice 1

1 Student's own answers, for example, (1, 3), (2, 4), (3, 5)

2 a (−4, −3), (−3, −3), (−2, −3), (−1, −3), (0, −3), (1, −3), (2, −3), (3, −3), (4, −3)

   b (−4, 0), (−3, 0), (−2, 0), (−1, 0), (0, 0), (1, 0), (2, 0), (3, 0), (4, 0)

   c (−4, 3), (−3, 3), (−2, 3), (−1, 3), (0, 3), (1, 3), (2, 3), (3, 3), (4, 3)

   d (−4, −4), (−3, −3), (−2, −2), (−1, −1), (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)

   e (−4, −8), (−3, −6), (−2, −4), (−1, −2), (0, 0), (1, 2), (2, 4), (3, 6), (4, 8)

   f (−4, 8), (−3, 6), (−2, 4), (−1, 2), (0, 0), (1, −2), (2, −4), (3, −6), (4, −8)

3 a (−3, −4), (−3, −3), (−3, −2), (−3, −1), (−3, 0), (−3, 1), (−3, 2), (−3, 3), (−3, 4)

   b (0, −4), (0, −3), (0, −2), (0, −1), (0, 0), (0, 1), (0, 2), (0, 3), (0, 4)

   c (3, −4), (3, −3), (3, −2), (3, −1), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4)

   d (−4, −4), (−3, −3), (−2, −2), (−1, −1), (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)

   e (−2, −4), (−1.5, −3), (−1, −2), (−0.5, −1), (0, 0), (0.5, 1), (1, 2), (1.5, 3), (2, 4)

   f (2, −4), (1.5, −3), (1, −2), (0.5, −1), (0, 0), (−0.5, 1), (−1, 2), (−1.5, 3), (−2, 4)

Purposeful practice 2

1–6
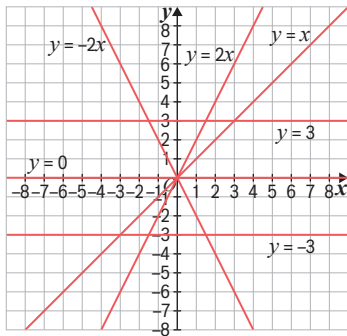
Problem-solving practice

1 (1, −3)

2 $x = 5$ (or $x = -3$)

3 Right-angled triangle

4 a

   b (0, 0)

5 a and b **i**

     ii (−2, 3)

c (1, 2)

d
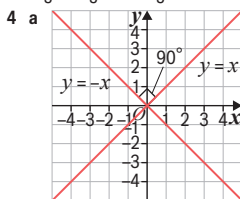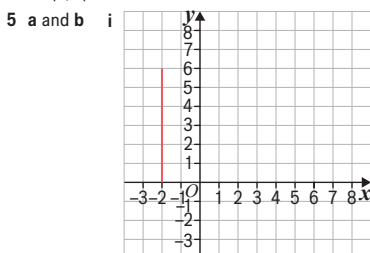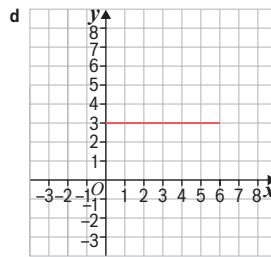
Exam practice

1 a (5, 4)

   b
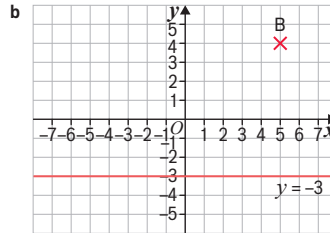
9.2 Linear graphs

Purposeful practice 1

1

x	−2	−1	0	1	2
$y = 2$	2	2	2	2	2
	(−2, 2)	(−1, 2)	(0, 2)	(1, 2)	(2, 2)

2

x	−2	−1	0	1	2
$y = x$	−2	−1	0	1	2
	(−2, −2)	(−1, −1)	(0, 0)	(1, 1)	(2, 2)

3

x	−2	−1	0	1	2
$y = 2x$	−4	−2	0	2	4
	(−2, −4)	(−1, −2)	(0, 0)	(1, 2)	(2, 4)

4

x	−2	−1	0	1	2
$y = 2x + 1$	−3	−1	1	3	5
	(−2, −3)	(−1, −1)	(0, 1)	(1, 3)	(2, 5)

5

x	−2	−1	0	1	2
$y = 2x − 1$	−5	−3	−1	1	3
	(−2, −5)	(−1, −3)	(0, −1)	(1, 1)	(2, 3)

Purposeful practice 2

1 a–c
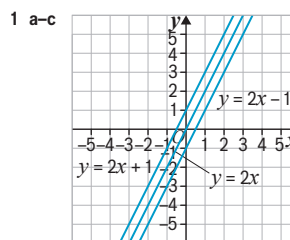
2 a i

x	−2	−1	0	1	2
$y = −2x$	4	2	0	−2	−4

   ii

x	−2	−1	0	1	2
$y = −2x + 1$	5	3	1	−1	−3

   iii

x	−2	−1	0	1	2
$y = −2x − 1$	3	1	−1	−3	−5

b

$y = -2x - 1$, $y = -2x$, $y = -2x + 1$
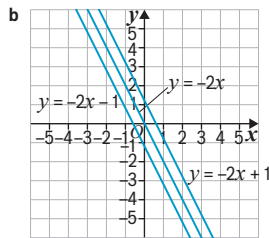
Problem-solving practice

1 a $(2, -4)$ **b** $(2, 0)$ **c** $(2, 8)$ **d** $(2, -8)$

2 a

x	-2	-1	0	1	2
$y = \frac{1}{2}x$	-1	-0.5	0	0.5	1

b

$y = \frac{1}{2}x$
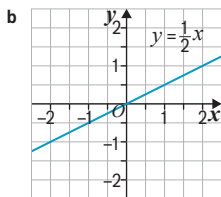
3 A $(2, -8)$; B $(2, 8)$; C $(2, 0)$; D $(2, -4)$

Exam practice

1 a C **b** A **c** D

9.3 Gradient

Purposeful practice 1

1 A, B, C, D, E, and F have positive gradients.

2 G, H and I have negative gradients.

3 Students' own answers, for example, I and H are parallel.

4 B: 2; C: 1; D: 3; E: 2; F: 1; G: -2; H: -4; I: -4

Purposeful practice 2

1 A: $\frac{1}{3}$; B: 2; C: $\frac{1}{2}$; D: 3

2 D: $y = 3x - 21$

A: $y = \frac{1}{3}x + 4$

B: $y = 2x$

C: $y = \frac{1}{2}x - 1$

Problem-solving practice

1 Line B is steeper because it has a greater gradient (gradient of 4, compared to gradient of 3).

2 Students' own answers, for example,

3
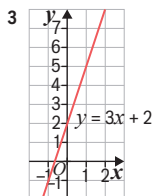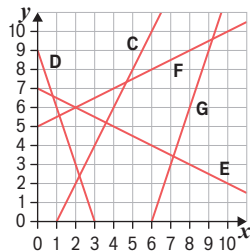
$y = 3x + 2$

4 a $-\frac{1}{3}$ **b** 2 **c** $\frac{1}{4}$

Exam practice

1 Lines B and E **2** $d = 27$

9.4 $y = mx + c$

Purposeful practice 1

1 The y-intercept of a line is the point where it crosses the y-axis.

2 a $(0, 5)$ **b** $(0, -2)$ **c** $(0, -2)$ **d** $(0, -2)$ **e** $(0, -3)$ **f** $(0, 4)$
  g $(0, -4)$ **h** $(0, 4)$ **i** $(0, 4)$ **j** $(0, -4)$ **k** $(0, -4)$ **l** $(0, 2)$

Purposeful practice 2

1 A **a** -3 **b** -2 **c** $y = -3x - 2$
  B **a** 3 **b** 2 **c** $y = 3x + 2$
  C **a** 2 **b** -4 **c** $y = 2x - 4$
  D **a** 0 **b** -4 **c** $y = -4$

Problem-solving practice

1 a $y = x$ **b** $y = x + 3$

c $y = -x$ **d** $y = -x - 1$

e $y = -2$ **f** $y = 2x + 1$

g $y = -3x$ **h** $y = -3x + 2$
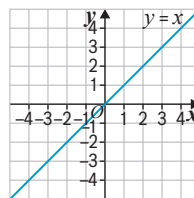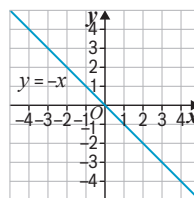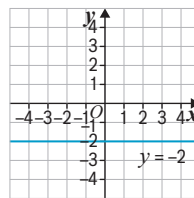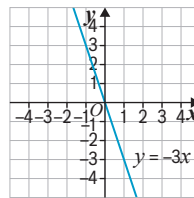
2 A: $y = 2x + 18$ B: $y = x + 9$ C: $y = 2x + 4$
  D: $y = x - 3$ E: $y = -4x + 20$

3 a $y = x - 3$ and $y = x - 4$, $y = -2x + 4$ and $y = -2x - 1$, $y = 3x + 1$ and $y = 3x$
  b $y = 2x + 5$ and $y = 5$, $y = 3x + 1$ and $y = -4x + 1$

Exam practice

1 $y = -2x + 4$

9.5 Real-life graphs

Purposeful practice 1

1 Brand A = Line 3, Brand B = Line 2, Brand C = Line 1

2 Brand A = £8/kg, Brand B = £4/kg, Brand C = £2/kg

3 £30

4 No, Brand A and Brand B will cost the same.

Purposeful practice 2

1 a

Strawberries bought (kg)	1	2	3	4	5	6
Cost (£)	1.88	3.76	5.64	7.52	9.40	11.28

b

Cost of strawberries
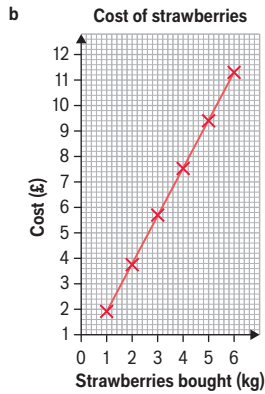
c £4.70

d 2.9 kg

Problem-solving practice

1 a

Electricity used (kWh)	0	200	400	600	800	1000
Brian pays (£)	0.00	**30.00**	**60.00**	**90.00**	**120.00**	150.00
Tom pays (£)	15.00	**39.00**	**63.00**	**87.00**	**111.00**	135.00

b

Cost of electricity used (£)
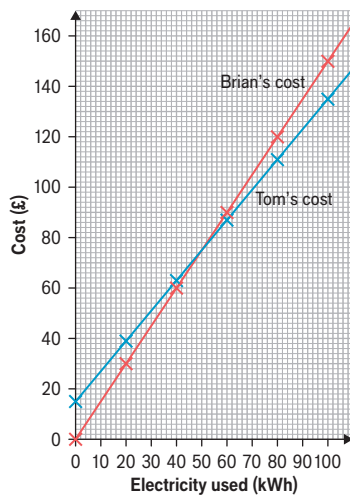
c Brian   **d** Tom

2 a

Fuel bought (litres)	2	4	6	8	10	12
Cost (£)	2.60	**5.20**	**7.80**	**10.40**	**13.00**	15.60

b

Cost of fuel (£)
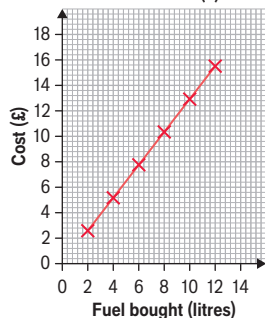
c £58.50   **d** Approximately 3.2 litres

Exam practice

1 £0.60

9.6 Distance–time graphs

Purposeful practice

1 Red line   **2** 40 km   **3** Kyle for 2 hours
4 2.30 pm and she stays for 1.5 hours   **5** 1.30 pm
6 Approximately 5.30 pm and they are 27.5 km from Bedford
7 Going back to Bedford   **8** 9.00 pm
9 a 16 km/hr   **b** 8 km/hr   **c** 16 km/hr (to 1d.p.)

Problem-solving practice

1 a

Bill's journey
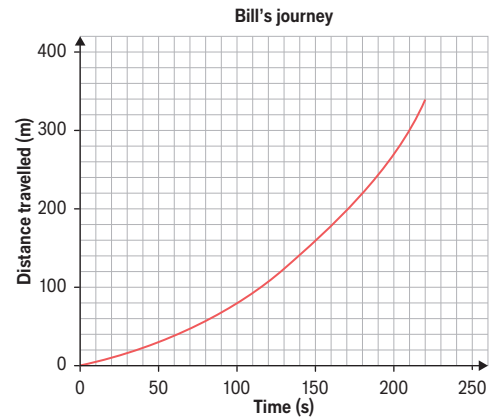
b Bill's speed is increasing, as the gradient of the distance–time graph becomes steeper.

2 a She has not taken into account the time that Sally was at the service station, stuck in roadworks and buying flowers.

b 48 mph

Exam practice

1 Dalva, with students' own working, for example,
from the graph, Dalva ran 80 metres in 11 seconds.

This is equivalent to $\frac{80}{11} \times 60 \times 60$ metres per hour = 26 200 metres per hour.
(to the nearest 100 m) 26 200 metres per hour = 26.2 km/h.

Dalva's speed is about 26.2 km/h whereas Seb's speed is 22.3 km/h, so Dalva ran faster.

9.7 More real-life graphs

Purposeful practice 1

1 B   **2** A   **3** A = 3; B = 1; C = 2

Purposeful practice 2

1 a 6.0 cm   **b** 3.8 minutes (or 3 minutes and 48 seconds)
  c 1.25 cm   **d** 1.25
2 a 3.0 cm   **b** 7.6 minutes (or 7 minutes and 36 seconds)
  c 0.625 cm   **d** 0.625

Problem-solving practice

1 a 2002–2012: either by working out the change in each 10-year period, or by noticing that this is the steepest section.
  b £1.10 (accept answers between £1.08 and £1.12)
  c Students' answers may vary, for example, the price has started to go down and we cannot be sure if it will continue going down, or if it will go up again.
2 a 29
  b This is not very reliable because few results lie on the line of best fit. However, most results lie within 10 marks to either side of the line of best fit, so David's score is likely to lie within the range 19–39.
  c No other students on the graph scored close to 80 in the history test, so it is not possible to directly predict Sakina's maths score from the graph. However, there is a fairly strong positive correlation between maths and history marks, so it is likely that Sakina scored very highly on the maths test.

Exam practice

1 £80 per hour

10 Transformations

10.1 Translation

Purposeful practice 1

1 a C　**b** H　**c** E　**d** G　**e** B　**f** F　**g** D

2 a i $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$　　**ii** $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$　　**iii** $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

b i $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$　　**ii** $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$　　**iii** $\begin{pmatrix} 10 \\ 2 \end{pmatrix}$

c i $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$　　**ii** $\begin{pmatrix} -4 \\ 8 \end{pmatrix}$　　**iii** $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$

Problem-solving practice

1 $y = -2$　　**2** C　　**3** (56, 26)

4 Q $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$　　R $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$　　S $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

Exam practice

1 Translation of 2 units to the left and 4 units down, so $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$.

2
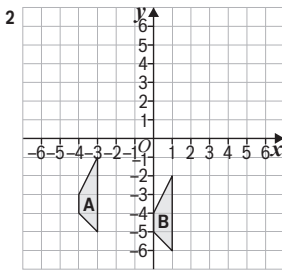
10.2 Reflection

Purposeful practice 1

1 a $x = 2$　　**b** $y = 0$　　**c** $x = -1$
d $x = 3$　　**e** $y = 4$　　**f** $y = -2$

Purposeful practice 2

1 a Reflections are labelled A′, B′, C′ and, D′ on the diagram below.
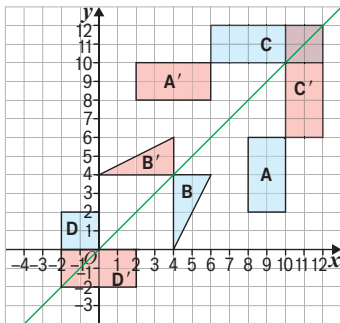
b A: (8, 2), (10, 2), (10, 6), (8, 6)
B: (4, 0), (6, 4), (4, 4)
C: (6, 10), (12, 10), (12, 12), (6, 12)
D: (0, −2), (−2, −2), (−2, 2), (0, 2)
c A: (2, 8), (2, 10), (6, 10), (6, 8)
B: (0, 4), (4, 6), (4, 4)
C: (10, 6), (10, 12), (12, 12), (12, 6)
D: (−2, 0), (−2, −2), (2, −2), (2, 0)

Problem-solving practice

1 a The mirror line is $y = 2$.
b The shape is reflected in the y-axis.
c The mirror line is $x = 4$.
2 a $x = 1$　　**b** $y = 4 - x$　　**c** $x = -1$
d $y = 4$　　**e** $y = 2$

Exam practice

1 Reflection in the x-axis (or the line $y = 0$)
2 Reflection in the y-axis (or the line $x = 0$)

10.3 Rotation

Purposeful practice 1
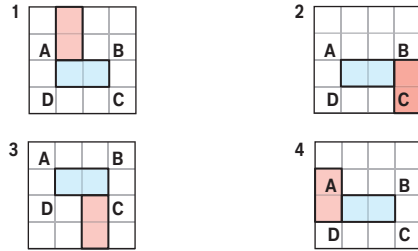
Purposeful practice 2

1 180°　　**2** 90° anticlockwise (or 270° clockwise)
3 180°　　**4** 90° clockwise (or 270° anticlockwise)

Purposeful practice 3

1 a H　**b** D　**c** G　**d** E　**e** F
2 a R　**b** S　**c** Q　**d** P

Problem-solving practice

1 Alex rotated anticlockwise instead of clockwise.
Alex has labelled the corners incorrectly.
2 a A to C is a rotation 180° around (4, 2).
b C to E is a rotation 90° anticlockwise around (5, 5).
c E to A is a rotation 90° anticlockwise around (7, 1).
3 This is a rotation of 180°. The centre of rotation is the point (1, 0).

Exam practice

1
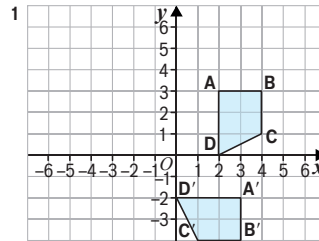
10.4 Enlargement

Purposeful practice 1

1 Shape C　　**2** Shape E　　**3** Shape B

Purposeful practice 2

1

2

V   W

X   Y
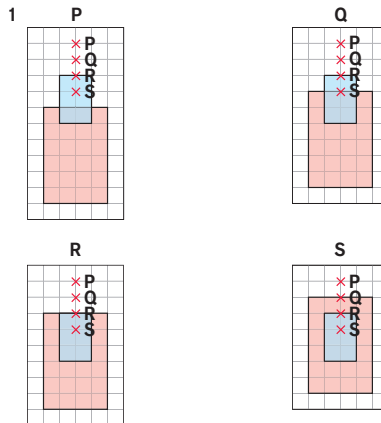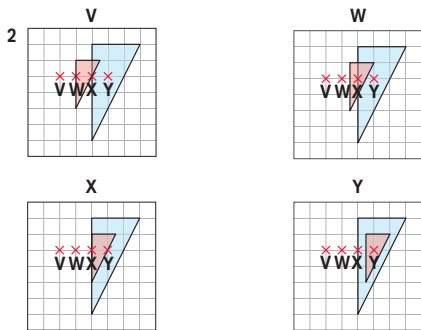
Problem-solving practice

1 a $2\frac{1}{2}$

  b width = 9 cm, height = 6 cm

2 a Scale factor needs to be larger than 1. Check centre of enlargement to ensure the enlarged shape will be in quadrant 1.

  b Scale factor needs to be between 0 and 1. Check centre of enlargement to ensure the enlarged shape will be in quadrant 1.

  c Scale factor needs to be larger than 1. Check centre of enlargement to ensure the enlarged shape will be in quadrant 2.

  d Scale factor needs to be between 0 and 1. Check centre of enlargement to ensure the enlarged shape will be in the same quadrant.

3 Anna: The perimeter should be 36 cm (she has doubled when she should have tripled).

Paul: The difference will triple if the side length triples (i.e. it will become 6 cm not 2 cm).

Charlie: The new area will be 3^2 times larger (54 cm), not 3 times larger.

Exam practice

1 Correct enlargement shown from any centre of enlargement. For example,
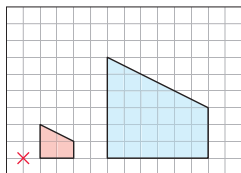
10.5 Describing enlargements

Purposeful practice 1

1 a Scale factor 2   **b** Scale factor 4

  c Scale factor $\frac{1}{2}$ **d** Scale factor $\frac{1}{2}$

2 a $\frac{1}{4}$ **b** 8   **c** $\frac{1}{8}$ **d** 2

Purposeful practice 2

1 Centre (1, −1), scale factor 2   **2** Centre (1, −1), scale factor $\frac{1}{2}$

3 Centre (0, 8), scale factor 2   **4** Centre (0, 8), scale factor $\frac{1}{2}$

Purposeful practice 3

1 Scale factor 2   **2** Scale factor 3

Problem-solving practice

1 Centre of enlargement (0, 3), scale factor 2

2 Centre of enlargement (9, 4), scale factor $\frac{1}{2}$

3 An enlargement by scale factor $\frac{1}{3}$, centre (3, 2)

Exam practice

1 Enlargement, scale factor 2, centre of enlargement P

10.6 Combining transformations

Purposeful practice 1

1 a P   **b** P   **c** T   **d** R   **e** S   **f** S

2 a Translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ **b** Translation of $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$

  c Rotation of 180° about (2, 3)   **d** Rotation of 180° about (4, −1)

Problem-solving practice

1
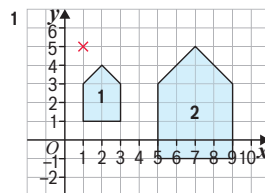
2 a The reflection in the line $y = x$ is labelled (a) on the following diagram; the second reflection in the x-axis is labelled 'End result'

  b The translation is labelled (b) on the following diagram; the final position is the same as (a).
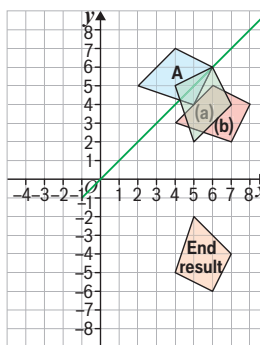
3 After rotation, shape T becomes shape B. This reflects to make shape Q.

Exam practice

1 a and **b**
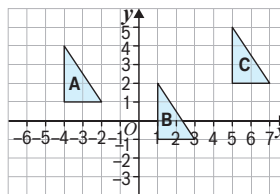
  c $\begin{pmatrix} 9 \\ 1 \end{pmatrix}$

11 Ratio and proportion

11.1 Writing ratios

Purposeful practice 1

1 □ △ △   **2** □ □ □ △ △

3 □ □ △ △ △   **4** □ □ □ □ △ △ △ △

5 □ □ □ □ △ △ △ △ △   **6** □ □ □ □ △ △ △

7 □ □ □ □ △ △   **8** □ □ △

Purposeful practice 2

1 Yes   **2** No   **3** No   **4** No

5 No   **6** Yes   **7** No   **8** Yes

Purposeful practice 3

1 2 : 3	**2** 2 : 3	**3** 3 : 2	**4** 3 : 2
5 4 : 3	**6** 8 : 3	**7** 4 : 3	**8** 1 : 1
9 2 : 3 : 4	**10** 2 : 4 : 3	**11** 2 : 4 : 3	**12** 2 : 4 : 3
13 1 : 4 : 3	**14** 1 : 8 : 3	**15** 5 : 3	**16** 1 : 7

Problem-solving practice

1 2 : 5 **2** 2 : 3 **3** 1 : 8

4 Harriet is wrong because the ratio 1 : 3 means 3 times as many green marbles as red marbles.

5 4 : 16 : 1

6 a James has not fully simplified, he has just halved the ratio.
8 : 10 simplifies to 4 : 5.

b Karis has written the ratio back-to-front.

7 The ratio of flour to butter to caster sugar is 9 : 2 : 1

8 No, the ratio 5 : 2 is equivalent to 30 : 12 so the ratio of 30 g : 16 g is not suitable for growing mushrooms.

9 No, 5 adults can only take 60 children, 6 adults are needed for 62 children.

Exam practice

1 1 : 4

11.2 Using ratios 1

Purposeful practice 1

1 16	**2** 4	**3** 8	**4** 2
5 0.5	**6** 1	**7** 10	**8** 40

Purposeful practice 2

1 3 : 4	**2** 3 : 4	**3** 15 : 2	**4** 75 : 1
5 15 : 2	**6** 3 : 4	**7** 3 : 40	**8** 3 : 400

Purposeful practice 3

1 24   **2** 24   **3** 6   **4** 24   **5** 36   **6** 18

Problem-solving practice

1 300 ml   **2 a** 1056   **b** 1200

3 a 24 km   **b** 6 hours

4 a 0.6 m

b No, the ratio 9 : 2 is equivalent to 1.8 : 0.4 so it should be 0.4 m

5 Daniel is wrong because there are still decimal points in his ratio. It should be 1 : 4.

6 3   **7** 3

8 This year is 7 : 3, so 2 : 1, 9 : 5 and 5 : 3

Exam practice

1 54

11.3 Ratio and measures

Purposeful practice 1

1 a 120 minutes **b** 30 minutes **c** 150 minutes

2 a 4.8 km **b** 8 km **c** 1.875 miles **d** 3.125 miles

3 a 6.6 pounds **b** 11 pounds **c** 1.36 kg (to 2 d.p.) **d** 2.27 kg (to 2 d.p.)

Purposeful practice 2

1 €224   **2** $256   **3** £178.57   **4** £156.25

Purposeful practice 3

1 a 1 : 2	**b** 1 : 4	**c** 1 : 8
2 a 1 : 3	**b** 1 : 9	**c** 1 : 27
3 a 1 : 4	**b** 1 : 16	**c** 1 : 64

Problem-solving practice

1 7.92 pounds   **2** 48 km/h   **3** 6.6 pounds or 3 kg

4 $2.5 \times 1.6 = 4$, therefore Nana walks 4 km. Charlie walks 3.5 km, so Nana walks further.

5 America, as $425 = £332.03

6 Callum is incorrect because he should divide by 1.12

7 Ollie gains money, because £300 = ¥42 300, ¥42 300 = £306.52

8 40 cm^2

Exam practice

1 204 m

11.4 Using ratios 2

Purposeful practice 1

1 £20 : £80	**2** £20 : £80	**3** £30 : £70
4 £60 : £140	**5** £6 : £14	**6** £6 : £14
7 £9 : £21	**8** £10 : £20	**9** £15 : £15
10 £10 : £9 : £1	**11** £10 : £8 : £2	**12** £15 : £12 : £3
13 £20 : £16 : £4	**14** £25 : £20 : £5	**15** £30 : £24 : £6

Purposeful practice 2

1 60 ml : 100 ml : 40 ml

2 600 ml : 1000 ml : 400 ml

3 0.6 litres : 1 litre : 0.4 litres

4 0.3 litres : 0.5 litres : 0.2 litres

5 150 ml : 250 ml : 100 ml

6 0.15 litres : 0.25 litres : 0.1 litres

7 1.5 litres : 2.5 litres : 1 litre

8 1.8 litres : 3 litres : 1.2 litres

Purposeful practice 3

1 0.8 kg : 1.2 kg	**2** 8 g : 12 g	**3** 80 g : 120 g
4 800 g : 1200 g	**5** 1.6 kg : 2.4 kg	**6** 0.4 kg : 0.6 kg
7 2 kg : 3 kg	**8** 2.4 kg : 3.6 kg	

Problem-solving practice

1 100 ml of red paint and 200 ml of yellow paint

2 a Pavlo £12 000, Erik £18 000

b £9000   **c** Masha £4000, Julia £5000

3 a 252 g silver and 108 g gold   **b** £3327.84

4 Franci should have divided by the total of the ratios, so £60 ÷ 12 = 5. Then she should have multiplied this by her part of the ratio, 5, to give £25.

5 48 jelly beans   **6** 18 cows and 30 sheep

7 400 books

Exam practice

1 72 chocolates

11.5 Comparing using ratios

Purposeful practice 1

1 3 : 1   **2** 1 : 3   **3** 1 : 2   **4** 2 : 4 or 1 : 2

Purposeful practice 2

1 $\frac{2}{3}$ **2** $\frac{2}{3}$ **3** $\frac{1}{3}$ **4** $\frac{3}{10}$ **5** $\frac{3}{10}$

Purposeful practice 3

1 1 : 4	**2** 1 : 2	**3** 1 : 1	**4** 1 : 0.5	**5** 1 : 5
6 1 : 0.2	**7** 1 : 0.3	**8** 1 : 3	**9** 1 : 6	**10** 1 : 0.6

Purposeful practice 4

1 Paint A is 1 : 1.33 blue to yellow.
Paint B is 1 : 0.75 blue to yellow.
So Paint A has more yellow.

2 Paint A is 1 : 1.33 blue to yellow.
Paint B is 1 : 1.25 blue to yellow.
So Paint A has more yellow.

Problem-solving practice

1 2 : 1   **2** $\frac{5}{7}$

3 Louise is not correct. The ratio of sugar to other ingredients is $\frac{3}{10}$ to $\frac{7}{10}$, which is equivalent to 3 : 7.

4 1 : 1.5   **5** 1 : 2.5   **6** $\frac{14}{25}$

Exam practice

1 a $\frac{6}{11}$ **b** 2 : 1

11.6 Using proportion

Purposeful practice 1

1 30 g   **2** 60 g   **3** 180 g   **4** 270 g

Purposeful practice 2

1 7.5 kg   **2** 15 kg   **3** 45 kg   **4** 67.5 kg

Purposeful practice 3

1 £1 : 175 ml   **2** £1 : 180 ml   **3** £1 : 191.49 ml
4 £1 : 200 ml   **5** £1 : 200 ml   **6** £1 : 190.22 ml

Purposeful practice 4

1 £0.0057 : 1 ml   **2** £0.0055 : 1 ml   **3** £0.0052 : 1 ml
4 £0.005 : 1 ml   **5** £0.005 : 1 ml   **6** £0.0053 : 1 ml

Problem-solving practice

1 $330 \div 75 = 4.4$, so 4.4 ml per pence.
$500 \div 125 = 4$, so 4 ml per pence.
Therefore, the can is better value for money.

2 The 25 kg bag is better value. 11 kg bag is 32p per kg.
25 kg bag is 23p per kg

3 $300 \div 80 = 3.75$, so 3.75p per tea bag.
$450 \div 160 = 2.8125$, so 2.81p per tea bag.
$575 \div 240 = 2.3958333$, so 2.4p per tea bag.
Therefore, the large box at £5.75 is the best value for money.

4 Shop A: $30 \div 3 \times 1 = £10$.
Shop B: $30 \div 5 \times £1.50 = £9$, so shop B is cheaper.

Exam practice

1 For one biscuit: 17.5 g of flour, 5 g butter, 8.75 g brown sugar, 3 g syrup, 0.05 eggs. Matt has only 100 g of syrup so he can make a maximum of 33 biscuits.

11.7 Proportion and graphs

Purposeful practice 1

1 **A** and **D** show direct proportion

2 a
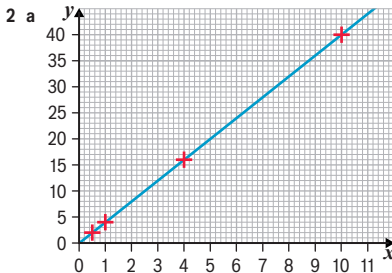
b Yes, the values are in direct proportion as the graph is a straight line passing through the origin.

Purposeful practice 2

1

Amount of sugar in teaspoons
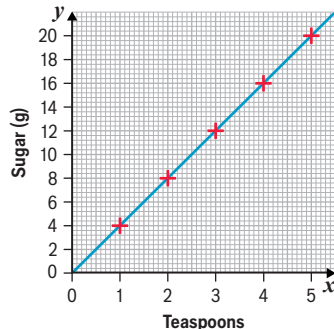
2 2.5 teaspoons   **3** 25 teaspoons   **4** 0 g
5 24 g   **6** 240 g

Problem-solving practice

1 a

Plumber's charges per hour
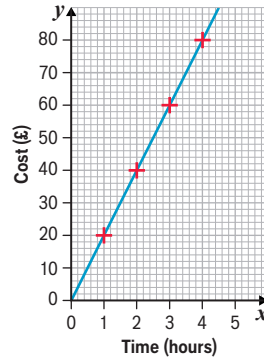
b The plumber's charges are in direct proportion to the hours she works because the graph is a straight line which goes through the origin.

2 a

Salesperson's earnings per sale
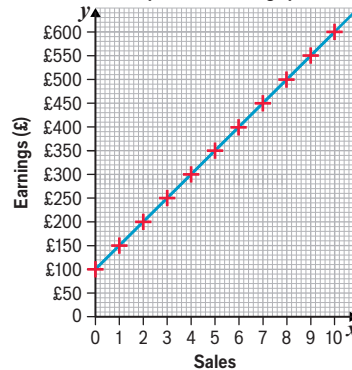
b This graph does not show direct proportion because the line does not go through the origin.

c They make £560 instead of £500, so they are £60 better off.

3 No, because the graph of the fare compared to the distance travelled would not go through the origin.

Exam practice

1 Approximately 10.5 miles

11.8 Proportion problems

Purposeful practice 1

1 £23.60   **2** 16 chairs   **3** £1

Purposeful practice 2

1 £25   **2** 4 hours   **3** 300 ml

Purposeful practice 3

1 28 hours
2 a 1.5 days   **b** 40 bananas   **c** 28 monkeys

Problem-solving practice

1 a No, 4 people would take half as long as 2 people to make the table so the cost would be the same (£115.50).

  b $1\frac{3}{4}$ hours

2 It has doubled.
3 a 20 hours   **b** 10 hours
4 No, because the temperature has halved, but the number of ice creams sold has not doubled.
5 a 3 surgeons   **b** 3 hours   **c** 30 patients
6 a £111   **b** £111   **c** 3 people

Exam practice

1 a 7.5 days
  b i The rate at which each decorator paints is the same, all the time.
    ii If the rate is slower, it will take longer. If the rate is faster, it will take less time.

12 Right-angled triangles

12.1 Pythagoras' theorem 1

Purposeful practice 1
1 a c **b** b, a **2 a** z **b** x, y
3 a i **b** g, h **4 a** d **b** e, f

Purposeful practice 2
1 5 cm   **2** 10 cm   **3** 20 cm   **4** 13 cm   **5** 26 cm   **6** 25 cm

Purposeful practice 3
1 8.5 cm   **2** 14.4 cm   **3** 24.5 cm

Problem-solving practice
1 1.4 m   **2** 12.8 m
3 It is quicker to travel directly from A to C, by 14.6 minutes (or 14 minutes and 36 seconds).
4 a 60.2 cm   **b** Wall is vertical, shelf is horizontal.

Exam practice
1 21.6 cm^2

12.2 Pythagoras' theorem 2

Purposeful practice 1
1

Coordinates	x length	y length	Length of a line between the points
(0, 0), (3, 4)	3	4	5
(0, 0), (4, 3)	4	3	5
(1, 1), (5, 4)	4	3	5
(1, 2), (5, 5)	4	3	5
(−1, −2), (−5, −5)	4	3	5

Purposeful practice 2
1 a $x^2 + 4^2 = 5^2$ **b** 3.0 cm   **2 a** $x^2 + 4^2 = 6^2$ **b** 4.5 cm
3 a $x^2 + 4^2 = 6^2$ **b** 4.5 cm   **4 a** $6^2 + 4^2 = x^2$ **b** 7.2 cm

Problem-solving practice
1 a Ladder B
  b Using Pythagoras' theorem
    Length of ladder A = 11.4 m (1 d.p.)
    Length of ladder B = 13.6 m (1 d.p.)
    Height ladder A reaches = 12.4 m (1 d.p.)
    Height ladder B reaches = 14.5 m (1 d.p.).
    So ladder B reaches 2.04 m further up the wall.
2 6.6 m (1 d.p.)   **3** 7.6 cm (1 d.p.)
4 $9^2 + 40^2 = 1681 = 41^2$
5 The hypotenuse of the right-angled triangle is $\sqrt{(7^2 + 3^2)} = 7.6$ cm, so its perimeter is 7 cm + 3 cm + 7.6 cm = 17.6 cm.
Perimeter of the equilateral triangle = 3×6 cm = 18 cm. So the statement is incorrect.
6 120.7 cm

Exam practice
1 No, it is not, because Pythagoras' theorem does not hold true.
AC = 9 cm, but $\sqrt{7^2 + 4^2} = 8.06$ cm.
2 44 cm^2

12.3 Trigonometry: the sine ratio 1

Purposeful practice 1
1 $\sin \theta = \dfrac{a}{c}$ **2** $\sin \theta = \dfrac{b}{c}$ **3** $\sin \theta = \dfrac{r}{t}$ **4** $\sin \theta = \dfrac{s}{t}$

Purposeful practice 2
1 5.0 cm   **2** 0.5 cm   **3** 0.9 cm   **4** 8.7 cm   **5** 7.1 cm
6 0.7 cm   **7** 20.0 cm   **8** 11.5 cm   **9** 14.1 cm

Problem-solving practice
1 3.83 m   **2** 3.19 m   **3** 2.74 cm   **4** 5.7 m   **5** 3.95 m

Exam practice
1 9 cm

12.4 Trigonometry: the sine ratio 2

Purposeful practice 1
1 30°   **2** 30°   **3** 19.5°

Purposeful practice 2
1 19.5°   **2** 41.8°   **3** 75.2°   **4** 14.5°   **5** 11.5°   **6** 9.6°

Problem-solving practice
1 28.1°   **2** 14.5°   **3 a** 9.59°   **b** 10.5°   **c** 11.5°   **4** 51.1°

Exam practice
1 43.9°

12.5 Trigonometry: the cosine ratio

Purposeful practice 1
1 $\cos \theta = \dfrac{b}{c}$ **2** $\cos \theta = \dfrac{a}{c}$ **3** $\cos \theta = \dfrac{s}{t}$ **4** $\cos \theta = \dfrac{r}{t}$

Purposeful practice 2
1 8.7 cm   **2** 5.0 cm   **3** 7.1 cm   **4** 11.5 cm
5 20.0 cm   **6** 14.1 cm   **7** 15.6 cm   **8** 6.4 cm

Purposeful practice 3
1 41.4°   **2** 60.0°   **3** 75.5°   **4** 70.5°   **5** 60.0°   **6** 48.2°

Problem-solving practice
1 3.46 m   **2** 0.67 m   **3** 36.9°   **4** 41 m

Exam practice
1 33.9°

12.6 Trigonometry: the tangent ratio

Purposeful practice 1
1 $\tan \theta = \dfrac{a}{b}$ **2** $\tan \theta = \dfrac{b}{a}$ **3** $\tan \theta = \dfrac{r}{s}$ **4** $\tan \theta = \dfrac{s}{r}$

Purposeful practice 2
1 5.8 cm   **2** 17.3 cm   **3** 10 cm   **4** 17.3 cm
5 5.8 cm   **6** 10 cm   **7** 11.9 cm   **8** 8.4 cm

Purposeful practice 3
1 63.4°   **2** 53.1°   **3** 45°   **4** 38.7°   **5** 33.7°   **6** 56.3°

Problem-solving practice
1 26.6°   **2** 6.7 m   **3 a** 8.4 m   **b** 14.6 m
4 Option B shows the correct calculation.

Exam practice
1 30.7°

12.7 Finding lengths and angles using trigonometry

Purposeful practice 1
1 a $\tan 48° = \dfrac{x}{6}$ **b** 6.7 cm
2 a $\cos \theta = \dfrac{6}{10}$ **b** 53.1°
3 a $\sin \theta = \dfrac{6}{10}$ **b** 36.9°
4 a $\tan \theta = \dfrac{6}{10}$ **b** 31.0°
5 a $\sin 48° = \dfrac{6}{y}$, $\cos 48° = \dfrac{x}{y}$, $\tan 48° = \dfrac{6}{x}$
  b $x = 5.4$ cm, $y = 8.1$ cm
6 a $\sin 48° = \dfrac{x}{y}$, $\cos 48° = \dfrac{6}{y}$, $\tan 48° = \dfrac{x}{6}$
  b $x = 6.7$ cm, $y = 9.0$ cm

Purposeful practice 2
1 45°   **2** 2 cm   **3** 2 cm
4 2 cm   **5** 3 cm   **6** 2 cm

Problem-solving practice
1 2
2 $\sin 30°$, $\tan 30°$, $\sin 45°$, $\cos 30°$, $\tan 45°$

3 17.0 cm   **4** 11.7 cm

5 No, it makes an angle of 50.3°.

Exam practice

1 29.4°

Mixed exercises C

## Mixed problem-solving practice C

1 a and **b**
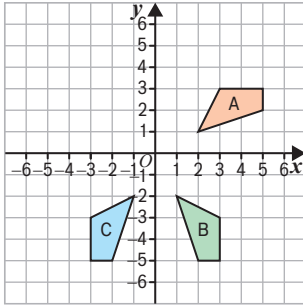
  c A reflection in the line $y = -x$

2 Charlie, because Akram saves 60%, Beth 58% and Charlie 65%

3 £80

4 $325 \div 100 = 3.25$ so Caitlin has enough flour to make $3.25 \times 8 = 26$ pancakes.
$9 \div 2 = 4.5$ so she has enough eggs to make $4.5 \times 8 = 36$ pancakes.
$825 \div 300 = 2.75$ so she has enough milk to make $2.75 \times 8 = 22$ pancakes. The greatest number of pancakes Caitlin can make is 22.

5 Frozen burgers (12 burgers will cost £6.25 compared to £6.39)

6 $y = 2x - 4$

7 a 08 20   **b** 8 km

  c
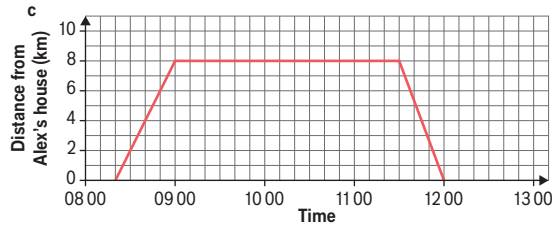
8 24.2 kg   **9** 34.6 cm

10 a
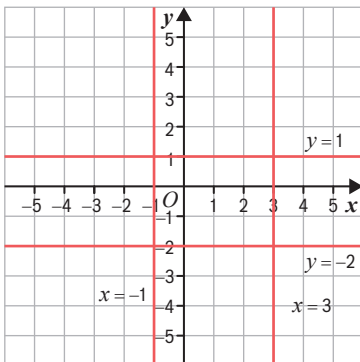
  b 12 squares

Exam practice

11 £42

12
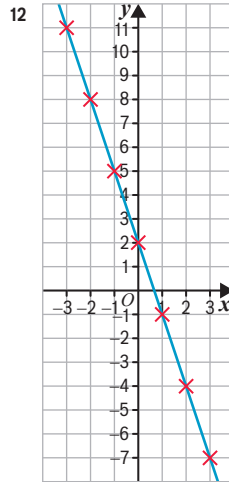
13 A translation by $\begin{pmatrix} -5 \\ 5 \end{pmatrix}$

14 a 15.4 pounds (allow answers between 15.2 and 15.6)   **b** 52.3 kg

15 70   **16** 5 : 3

17 Height = 6.42 cm, volume = 360 cm³

18 50.3°

13 Probability

13.1 Calculating probability

Purposeful practice 1

1 $P(R) = \dfrac{2}{3}$ $P(W) = \dfrac{1}{3}$

2 $P(R) = \dfrac{2}{4}$ or $\dfrac{1}{2}$ $P(W) = \dfrac{2}{4}$ or $\dfrac{1}{2}$

3 $P(R) = \dfrac{2}{5}$ $P(W) = \dfrac{3}{5}$

4 $P(R) = \dfrac{2}{6}$ or $\dfrac{1}{3}$ $P(W) = \dfrac{4}{6}$ or $\dfrac{2}{3}$

5 $P(R) = \dfrac{2}{7}$ $P(W) = \dfrac{4}{7}$ $P(B) = \dfrac{1}{7}$

6 $P(R) = \dfrac{2}{8}$ or $\dfrac{1}{4}$ $P(W) = \dfrac{4}{8}$ or $\dfrac{1}{2}$ $P(B) = \dfrac{2}{8}$ or $\dfrac{1}{4}$

Purposeful practice 2

1 $P(B) = \dfrac{1}{6}$ $P(Y) = \dfrac{5}{6}$

2 $P(B) = \dfrac{2}{6}$ or $\dfrac{1}{3}$ $P(Y) = \dfrac{4}{6}$ or $\dfrac{2}{3}$

3 $P(B) = \dfrac{3}{6}$ or $\dfrac{1}{2}$ $P(Y) = \dfrac{3}{6}$ or $\dfrac{1}{2}$

4 $P(B) = \dfrac{4}{6}$ or $\dfrac{2}{3}$ $P(Y) = \dfrac{2}{6}$ or $\dfrac{1}{3}$

5 $P(B) = \dfrac{5}{6}$ $P(Y) = \dfrac{1}{6}$

6 $P(B) = \dfrac{6}{6}$ or 1 $P(Y) = 0$

Purposeful practice 3

1 $\dfrac{1}{10}$ **2** $\dfrac{3}{10}$ **3** $\dfrac{2}{10}$ or $\dfrac{1}{5}$

4 $\dfrac{4}{10}$ or $\dfrac{2}{5}$ **5** $\dfrac{4}{10}$ or $\dfrac{2}{5}$ **6** $\dfrac{3}{10}$

7 $\dfrac{5}{10}$ or $\dfrac{1}{2}$ **8** $\dfrac{6}{10}$ or $\dfrac{3}{5}$ **9** $\dfrac{8}{10}$ or $\dfrac{4}{5}$

10 $\dfrac{9}{10}$ **11** $\dfrac{7}{10}$ **12** $\dfrac{7}{10}$

13 $\frac{8}{10}$ or $\frac{4}{5}$ **14** $\frac{6}{10}$ or $\frac{3}{5}$ **15** $\frac{5}{10}$ or $\frac{1}{2}$

16 $\frac{6}{10}$ or $\frac{3}{5}$ **17** $\frac{3}{10}$ **18** $\frac{4}{10}$ or $\frac{2}{5}$

Problem-solving practice

1 Students' own answers, for example, 4 black and 1 white, or 8 black and 2 white

2 $\frac{3}{8}$ **3** 5%   **4** $\frac{2}{3}$ **5** $\frac{3}{10}$

6 P(1) = P(4) = 0.25

Exam practice

1 $\frac{3}{14}$

13.2 Two events
Purposeful practice 1

1 a 2   **b** 3   **c** HR, HB, HY, TR, TB, TY   **d** 6
2 a 2   **b** 4   **c** HR, HB, HY, HG, TR, TB, TY, TG   **d** 8
3 a 2   **b** 4   **c** H1, H2, H3, H4, T1, T2, T3, T4   **d** 8
4 a 2   **b** 3   **c** HR, HB, HY, TR, TB, TY   **d** 6

Purposeful practice 2

1

		4-sided spinner			
		1	**2**	**4**	**5**
3-sided spinner	**1**	2	3	5	6
	2	3	4	6	7
	3	4	5	7	8

2 a $\frac{2}{12}$ or $\frac{1}{6}$ **b** $\frac{3}{12}$ or $\frac{1}{4}$ **c** $\frac{7}{12}$ **d** $\frac{9}{12}$ or $\frac{3}{4}$ **e** $\frac{5}{12}$

Problem-solving practice

1 a 6   **b** $\frac{1}{18}$

2 $\frac{4}{12}$ or $\frac{1}{3}$ **3** 12   **4** $\frac{9}{18}$ or $\frac{1}{2}$

5 No, it is not fair.
P(more than 6) = $\frac{21}{36}$ and P(6 or less) = $\frac{15}{36}$

Exam practice

1 $\frac{4}{9}$

13.3 Experimental probability
Purposeful practice 1

1 Dice A $\frac{18}{100}$, Dice B $\frac{17}{100}$

No, if the dice were fair the probability of rolling a 6 would be $\frac{1}{6}$

2 Dice A   **a** 2   **b** 9   **c** 36
     Dice B   **a** 2   **b** 9   **c** 34

3

	Number of rolls	Money paid to arcade	Estimated prize money paid	Estimated profit
Dice A	200	£200	36 × £3 = £108	£92
Dice B	200	£200	34 × £3 = £102	£98

Purposeful practice 2

1 37   **2** 19   **3** $\frac{19}{37}$ **4** 7   **5** $\frac{7}{37}$ **6** $\frac{7}{19}$ **7** $\frac{5}{17}$

Problem-solving practice

1 a 5   **b** £60 − £30 = £30
   c Students' own answers, for example probability is not certainty.
2 a 1 person   **b** 280

3 a

	Milk	Plain	Total
Mint	18	7	25
Orange	14	13	27
Total	32	20	52

   b $\frac{18}{52}$ or $\frac{9}{26}$

Exam practice

1 68

13.4 Venn diagrams
Purposeful practice 1

1 a 1, 8, 10

   b
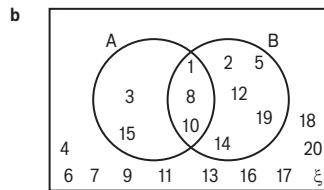
   c   **i** 1, 2, 3, 5, 8, 10, 12, 14, 15, 19
     ii 1, 8, 10
     iii 3, 4, 6, 7, 9, 11, 13, 15, 16, 17, 18, 20
     iv 4, 6, 7, 9, 11, 13, 16, 17, 18, 20

2 a 12, 24

   b
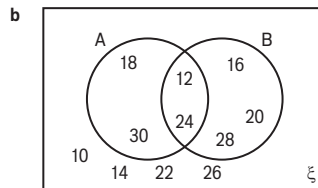
   c   **i** 12, 16, 18, 20, 24, 28, 30   **ii** 12, 24
     iii 10, 14, 18, 22, 26, 30   **iv** 10, 14, 22, 26

3 a 5

   b
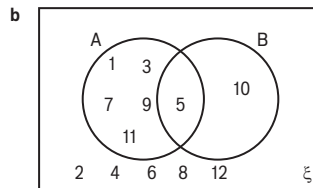
   c   **i** 1, 3, 5, 7, 9, 10, 11   **ii** 5
     iii 1, 2, 3, 4, 6, 7, 8, 9, 11, 12   **iv** 2, 4, 6, 8, 12

Purposeful practice 2

1 11   **2** 2

3 a $\frac{2}{11}$ **b** $\frac{5}{11}$ **c** $\frac{7}{11}$ **d** $\frac{4}{11}$

Problem-solving practice

1 $\frac{2}{9}$

2 a

   b $\frac{4}{8}$ or $\frac{1}{2}$

3

3

The probability that Nic rolls exactly one 3 is 0.375

Exam practice

1 a

b $\frac{6}{9}$ or $\frac{2}{3}$

Exam practice

1
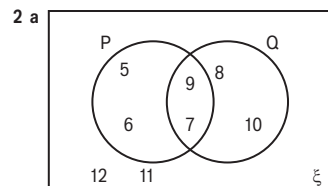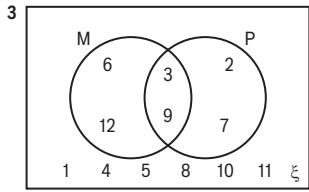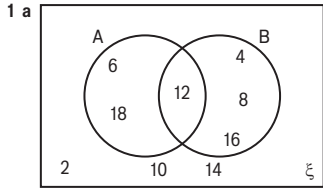
13.5 Tree diagrams

Purposeful practice 1

1 40   **2** 19   **3** $\frac{19}{40}$ **4** 23   **5** 11   **6** $\frac{11}{23}$

Purposeful practice 2

1

2
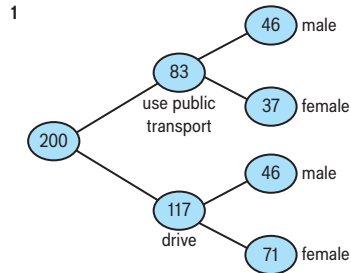
13.6 More tree diagrams

Purposeful practice 1

1 a
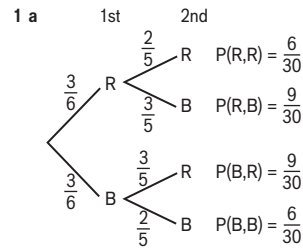
b i $\frac{6}{30}$ or $\frac{1}{5}$ **ii** $\frac{18}{30}$ or $\frac{3}{5}$ **iii** $\frac{24}{30}$ or $\frac{4}{5}$

Purposeful practice 2

1 a

b $\frac{68}{110}$ or $\frac{34}{55}$

2 a

b $\frac{26}{56}$ or $\frac{13}{28}$

Problem-solving practice

1 a

b $\frac{4}{53}$

2 $\frac{15}{39}$
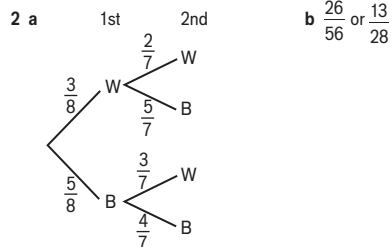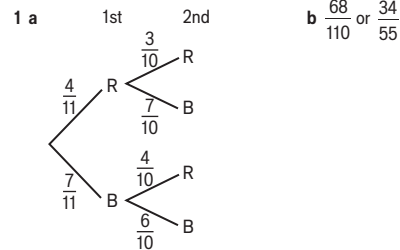
Problem-solving practice

1 $\frac{70}{132}$ **or** $\frac{35}{66}$ **2** $\frac{12}{90}$ **or** $\frac{4}{30}$

3 a

b 0.06

4 0.54

312
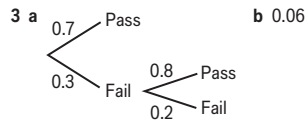
Exam practice

1 The probabilities for the first game do not sum to 1.
On the second game, the first branch is incorrect as the probabilites are on the wrong branches.

14 Multiplicative reasoning

14.1 Percentages
Purposeful practice 1

1 a £125　　　**b** £110　　　**c** £105　　　**d** £102.50
2 a £80　　　**b** £90.91　　**c** £95.24　　**d** £97.56
3 a £75　　　**b** £90　　　**c** £95　　　**d** £97.50
4 a £133.33　**b** £111.11　**c** £105.26　**d** £102.56

Purposeful practice 2

1 a 10%　**b** 10%　**c** −9.1%　**d** −10%　**e** 11.1%
　f 11.1%　**g** 11.1%　**h** −10%　**i** −99.1%
2 a 11.1%　**b** −10%　**c** 11.1%　**d** −10%

Problem-solving practice

1 £11 111.11
2 It is better to buy the small bottle because you get 750 ml for £1.75 instead of £2.50.
3 £266.67
4 a 6923 words　**b** 16.7% (1 d.p.)
5 With the dessert because it costs £27.12.
6 100%　　　**7 a** −4%　**b** −5%

Exam practice

1 £219.60　　　　**2** 11.1%

14.2 Growth and decay
Purposeful practice 1

1 £1050　**2** £1102.50　**3** £1157.63　**4** £1340.10

Purposeful practice 2

1 £950　**2** £902.50　**3** £857.38　**4** £735.09

Purposeful practice 3

1 £55.75　**2** £1677.14　**3** £55.17　**4** £1712.06

Purposeful practice 4

1 21%　**2** 4.5%　**3** 15.5%　**4** −1%

Problem-solving practice

1 Increase of 2% every year for 23 years would give expected price of $12p \times 1.02^{23}$, which is 18.9p (to 1 d.p.). The price has increased to 20p, so the increase in price is more than expected.
2 The first loan (0.2% per day) will involve Chelsea paying back £1586.31 in total. The second loan (2.5% per annum) will involve paying back £1575.94 in total. The 2-year loan is cheaper.
3 2019　　　**4** 2008
5 No, Adib will lose £148.40. After the first increase, Adib's shares are worth £968.50. After the second increase, they are worth £1491.49. After the decrease, they are worth £596.60.
6 12.5%

Exam practice

1 £753.91
2 The total amount with Friendly Bank will be $£52\,000 \times 1.021^4$, which is £56 507.53. The total amount with Aspiration Bank after 1 year will be $£52\,000 \times 1.05$, which is £54 600 and then after 3 further years will be $£54\,600 \times 1.008^3$, which is £55 920.91. Monica should use Friendly Bank.

14.3 Compound measures
Purposeful practice 1

1 3 kg/m³　**2** 6 kg/m³　**3** 3 kg/m³
4 1.5 kg/m³　**5** 0.67 kg/m³　**6** 0.33 kg/m³

Purposeful practice 2

1 a 3 N/m²　**b** 6 N/m²　**c** 4 N/m²　**d** 3 N/m²
2 a 3 litres per minute　　**b** 1.5 litres per minute
　c 0.75 litres per minute　**d** 0.2 litres per minute
　e 0.1 litres per minute　　**f** 720 litres per minute

Purposeful practice 3

1 20 kg　**2** 18 N　**3** 5 m³　**4** 2 m²

Problem-solving practice

1 Yes, she is right. The pressure exerted will be 2.45 times greater on Earth.
2 a The second bowl　**b** 2.9167 litres
3 a £7.74　**b** £7.29　**c** Sandra (she is paid £1695).
4 a 246　**b** 123 sentences and 41 quotes
5 336 kg
6 a 20.98 g　**b** 3.86 g　**c** 11.3 g/cm³ (1 d.p.)

Exam practice

1 1.01 g/cm³

14.4 Distance, speed and time
Purposeful practice 1

1 a 2 m/s　**b** 0.5 m/s　**2 a** 18 m　**b** 18 m
3 a 0.5 hrs (or 30 minutes)　**b** 2 hrs

Purposeful practice 2

1 a 23 m/s　**b** 17 m/s　**2 a** 65 m　**b** 44 m
3 a 7 m/s　**b** 7 m/s

Problem-solving practice

1 54.7 mph (1 d.p.)
2 a 2.04 seconds (2 d.p.)　**b** 20.41 m (2 d.p.)
3 a 1 hour　**b** 85 km/hr
4 690 m　　　**5** 6.56 m/s
6 No, the car is going faster at 5.6 m/s.

Exam practice

1 70.2 km/h

14.5 Direct and inverse proportion
Purposeful practice 1

1 $4x = y$　　**2** $x = 4y$　　**3** $2x = 3y$
4 $3x = 2y$　　**5** $3x = y$　　**6** $3x = y$

Purposeful practice 2

1 a i $g = 2.5h$ (or equivalent equation)
　　ii 4.8　　　　**iii** 22.5
　b i $g = 0.4h$ (or equivalent equation)
　　ii 30　　　　**iii** 3.6
2 a i $f = \dfrac{10}{w}$ (or equivalent equation)
　　ii 0.83 (2 d.p.)　**iii** 1.11 (2 d.p.)
　b i $w = \dfrac{10}{f}$ (or equivalent equation)
　　ii 0.83 (2 d.p.)　**iii** 1.11 (2 d.p.)

Problem-solving practice

1 a $c = \dfrac{k}{h}$　　**b** $c = \dfrac{24}{h}$ (or equivalent equation)
　c 6 hours
2 a $w = 6t$　**b** 60 litres
　c Yes, there will be 90 litres of water.
3 14 books
4 a Inversely proportional　**b** Neither
　c Directly propotional
5 a 6.7 hours (or 6 hours and 40 minutes)
　b Yes, because the total number of hours of work is fixed; if there are more workers, they will each work fewer hours so the total cost will be the same.

Exam practice

1 a 12 days
  b i It is assumed they paint at the same rate.
  ii If they paint slower, they will take longer.
  If they paint faster, they will take a
  shorter amount of time.

15 Constructions, loci and bearings

15.1 3D solids

Purposeful practice 1

1 a 6 **b** 12 **c** 8   **2 a** 6 **b** 12 **c** 8
3 a 6 **b** 12 **c** 8   **4 a** 6 **b** 12 **c** 8

Purposeful practice 2

1 a cone   **b** cylinder   **c** sphere
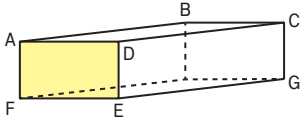
Purposeful practice 3

1 a Triangle and square   **b** 5
2 a i 8   **ii** 12   **iii** 18   **b i** 5   **ii** 7   **iii** 10

Problem-solving practice

1 An octagon
2 Each edge of a cube joins two squares together, so two edges from the squares produce only one edge of the cube. The answer is half of 24.
3 Evan is not correct. He has confused edges and vertices. There are 14 faces, 36 edges and 24 vertices.

Exam practice

1 a

  b 6
2 a 7 faces   **b** 12 edges

15.2 Plans and elevations

Purposeful practice 1

1
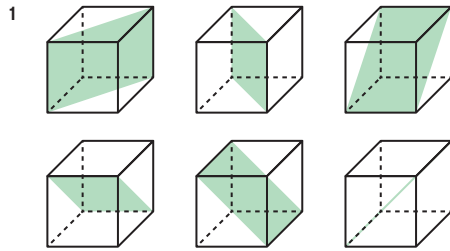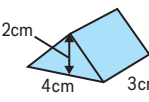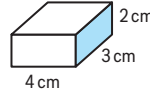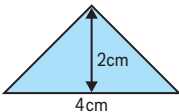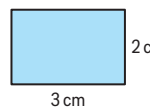
2 a 6 planes of symmetry.
  b 5 planes of symmetry.
  c 7 planes of symmetry.
  d 6 planes of symmetry.

Purposeful practice 2

1 Students' answers may vary, for example
  a A cuboid, a rectangular pyramid
  b A triangular prism, a pentagonal prism
2 a

  b

  c
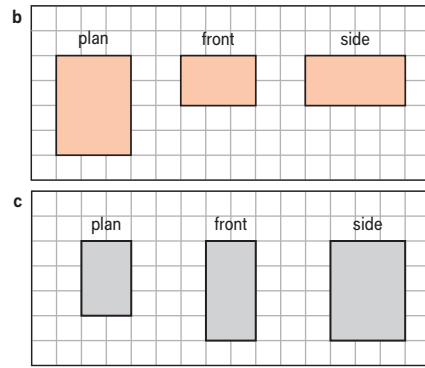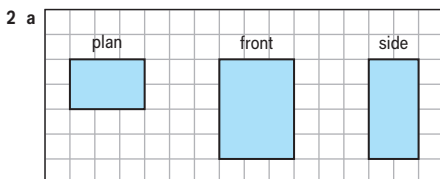
Problem-solving practice

1 a Triangular prism   Cuboid

  b Side elevation of triangular prism   Side elevation of cuboid

2

3 a

  b

  c

Exam practice

1

front view   side elevation

2
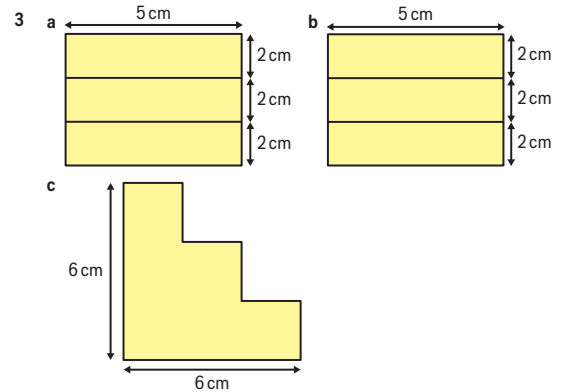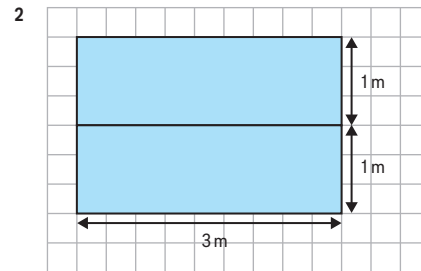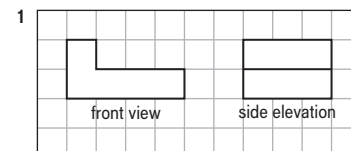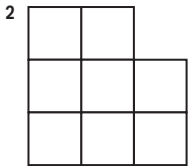
15.3 Accurate drawings 1
Purposeful practice 1

(These diagrams are not to scale.)

a

60°
7.7 cm   8.5 cm
65°   55°
8.1 cm

b

7.7 cm
65°   60°
8.1 cm   8.5 cm
55°

c

55°
8.1 cm   8.5 cm
65°   60°
7.7 cm

Purposeful practice 2

(These diagrams are not drawn to scale.)

1

4 cm
60°
5 cm

60°
4 cm
5 cm

2

6 cm
36°
8 cm
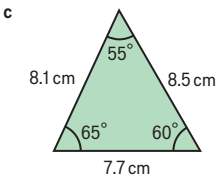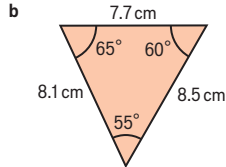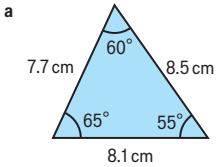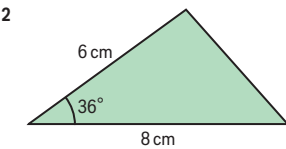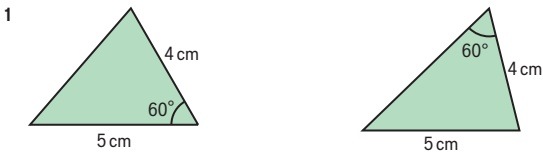
Purposeful practice 3

1 **A** and **C** are congruent (ASA); **B** and **E** are congruent (SAS).

Problem-solving practice

1 a Jamie is incorrect. **A** and **B** are congruent because of the ASA of 50°, 8 cm, 70°. C is not congruent to **A** and **B** because its sides are of different lengths.
2 Abbie has measured angle B instead of angle A. The angle at A is 106°.
3 a RHS   **b** 12 cm   **c** 5 cm   **d** 67.4°

Exam practice

(This diagram is not drawn to scale.)

1

5.5 cm
85°   3 cm
30°   65°
6 cm
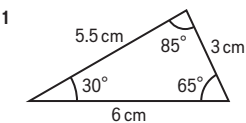
15.4 Scale drawings and maps
Purposeful practice 1

1 a Accurate drawing of a rectangle measuring 18 cm by 12 cm
 b Accurate drawing of a rectangle measuring 9 cm by 6 cm
 c Accurate drawing of a rectangle measuring 6 cm by 4 cm
 d Accurate drawing of a rectangle measuring 4.5 cm by 3 cm
 e Accurate drawing of a rectangle measuring 3 cm by 2 cm
 f Accurate drawing of a rectangle measuring 3.6 cm by 2.4 cm

Purposeful practice 2

1 a i 25 000   **ii** 250   **iii** 0.25   **iv** 4
 b i 2500   **ii** 1250   **iii** 500   **iv** 2000

c i 2.5   **ii** 1.25   **iii** 0.5   **iv** 2
d i 40 cm   **ii** 24 cm   **iii** 7 cm

Purposeful practice 3

1 a 1 cm × 2 cm   **b** 2 cm × 6 cm
 c 1.5 cm × 3 cm   **d** 2 cm × 2.5 cm
 e 1.5 cm × 1 cm

Problem-solving practice

1 a Town A to town B is 20 km; town C to town B is 25 km. So the total distance you travel is 45 km.
 b James is correct. 40 km would be represented by 12 cm on the map. You can quickly try the points that look furthest from town C and discover they are less than 12 cm away.
2 a 150 km = 600 cm; far too large for a printed map
 b 150 km = 150 cm; still too large for a printed map
 c 150 km = 30 cm; a good size for a printed map
 d 150 km = 1.5 cm; too small to be a useful map
3 a

Swimming pool

10 cm

School   4 cm   Railway station

8 cm

Car park
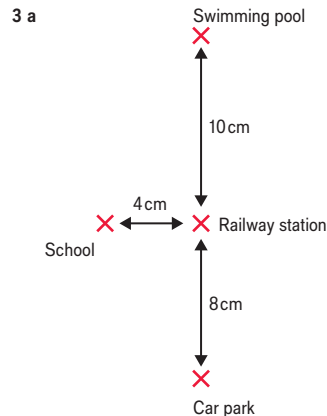
 b i 5.4 km   **ii** 4.5 km

Exam practice

1 7 cm
2 a 8 km

 b

Rainham
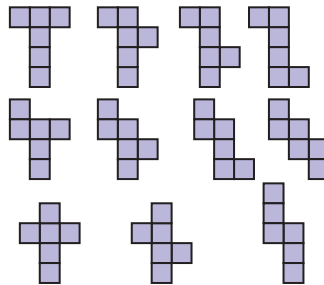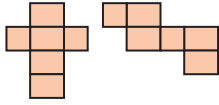N
Park   Purfleet
5 cm
Scale: 1 cm represents 2 km
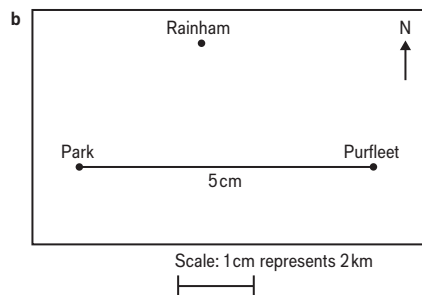
## 15.5 Accurate drawings 2
Purposeful practice 1

1 a Diagram C shows the net of a cube.
 b Students' answers may vary. There are 11 possible nets of a cube.

2 Students' own answers, with examples shown below. Measurements should be accurate with sides of length 3.5 cm and 5 cm.
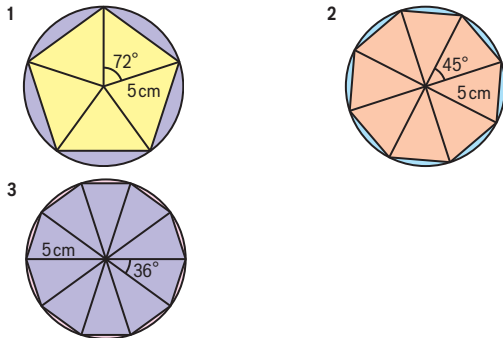

3 Students' own answers, with examples shown below. Measurements should be accurate with sides of length 3.5 cm and 5 cm.


Purposeful practice 2

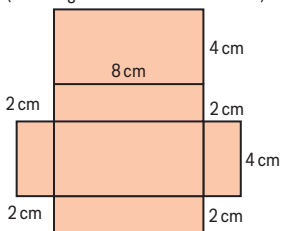(These diagrams are not drawn to scale.)

1

72°
5 cm

2

45°
5 cm

3

5 cm
36°
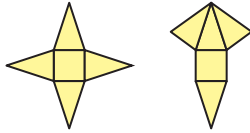
Problem-solving practice

1 It is not possible to form a triangle with these side lengths. The third side of a triangle must be smaller than the total of the other two sides. If AB = 8.2 cm and BC = 7.2 cm then AC would have to be smaller than 15.4 cm.

2 30°

3 A scale of 1 cm to 1 m would be appropriate. (This diagram is not drawn to scale.)

12 cm
2.4 cm
2.35 cm
2.4 cm
2.35 cm
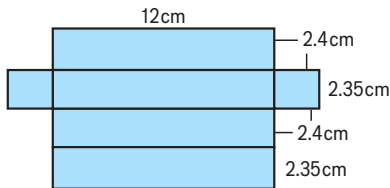
4 a The 8 cm rectangles should alternate between 2 cm and 4 cm tall. The squares on the sides should be rectangles 2 cm wide by 4 cm tall.

b (This diagram is not drawn to scale.)

4 cm
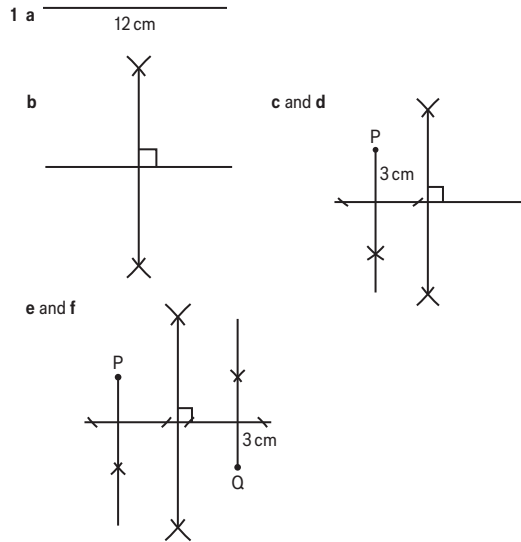8 cm
2 cm
2 cm
4 cm
2 cm
2 cm

Exam practice

1 Students' diagrams will vary but should show accurate net, for example (diagram not drawn to scale):

3 cm
3 cm 4 cm
2 cm
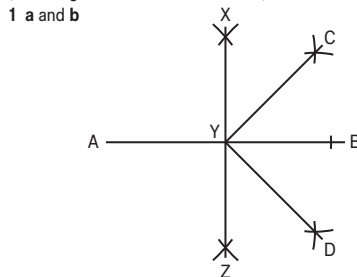3 cm 4 cm
3 cm
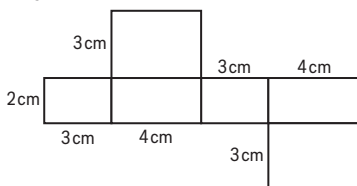
15.6 Constructions

Purposeful practice 1

(These diagrams are not drawn to scale.)

1 a
12 cm

b

c and **d**

P
3 cm

e and **f**

P
3 cm
Q

Purposeful practice 2

(This diagram is not drawn to scale.)

1 a and **b**

X
C
A   Y   B
D
Z

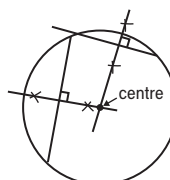## Purposeful practice 3

1 a and **b** Accurate scalene triangle with three acute angles and angle bisectors accurately constructed

2 a and **b** Accurate scalene triangle with one obtuse angle and angle bisectors accurately constructed

3 a and **b** Accurate scalene triangle with three acute angles and perpendicular bisectors accurately constructed

4 a and **b** Accurate scalene triangle with one obtuse angle and perpendicular bisectors accurately constructed

Problem-solving practice

1 Angle constructed correctly: extend the original line; draw a circle centred on the end of the line; use the diameter of the circle as a new line and construct the perpendicular bisector of this line.

2 Jenna is nearest to the bottom edge of the field. The distance is 75 m.

3 (This diagram is not drawn to scale.)


6 cm   6 cm
6 cm   6 cm
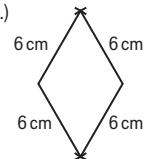
4 Angle constructed correctly (by constructing equilateral triangle, then bisecting one of the angles).

5 Circle with two chords marked. Accurate construction of perpendicular bisector for each chord. Centre of circle is point where perpendicular bisectors intersect.

centre

Exam practice

1 Perpendicular bisector accurately constructed

2 (This diagram is not drawn to scale.)
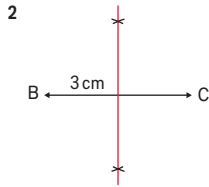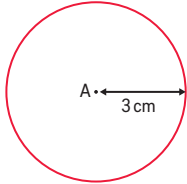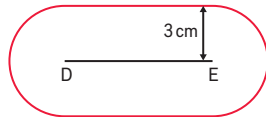

6 cm 6 cm
6 cm
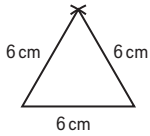
15.7 Loci and regions
Purposeful practice 1

(These diagrams are not drawn to scale.)

1

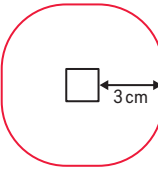A • → 3 cm

2

B ← 3 cm → C

3

3 cm
D   E

4

3 cm

5

3 cm

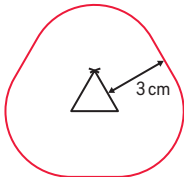### Purposeful practice 2

(These diagrams are not drawn to scale.)

1

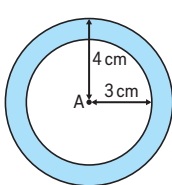4 cm
A 3 cm

2

B •   • C

3

3 cm
D ←→ E

4

3 cm 5 cm

5

3 cm
2 cm
4 cm

Problem-solving practice

(These diagrams are not drawn to scale.)

1

4 m
2 m
1 m

2 a Garden shown by rectangle ACIF.
b Patio shown by rectangle ABGF
c The tree may be planted in the shaded region between line EH and the curve.

A   B D E   C
Back of house
6 m   4 m
F   G   H   I
4 m   3 m   3 m
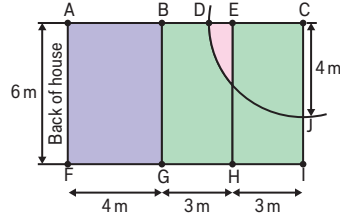
3 a The corners of the locus should be rounded so the outer line is always 2 cm from the rectangle.
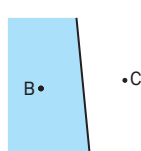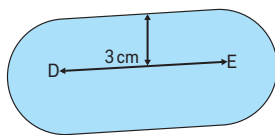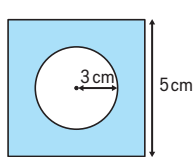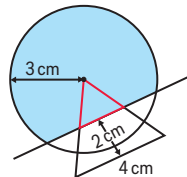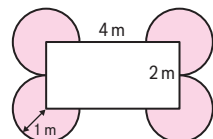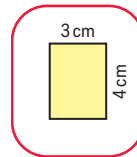**b**

3 cm
4 cm

Exam practice

(These diagrams are not drawn to scale.)

1

9 cm
A   B
4 cm   5 cm 1.5 cm
45°
D   C

2

B
A
2 cm
C
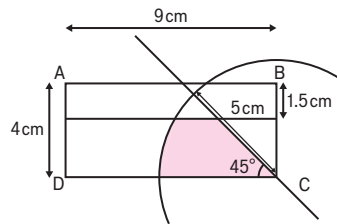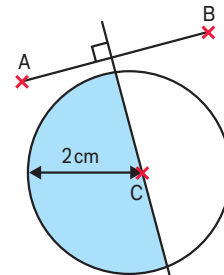
15.8 Bearings
Purposeful practice 1

1

B 025°	**C** 070°	**D** 115°	**E** 160°
F 205°	**G** 250°	**H** 295°	**I** 340°

2 a 220°   **b** 285°   **c** 330°   **d** 195°

Problem-solving practice

1 a and **b** (diagram not drawn to scale)

X   10 km   M
51°   39°
shelter
N 5 km
43°
walkers
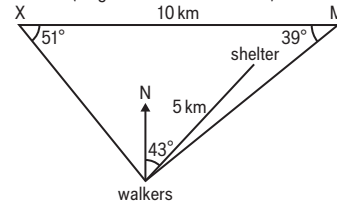
2 a Accurate scale diagram
b 12.8 miles
c 333°
3 a Accurate scale diagram
b Any answer between 252 and 253 km, at a bearing of 206°

Exam practice

1 a 0.6 km   **b** 110°   **c** 235°
2 1 The angle is 50° not 60°.
2 Bearings are always given using three figures, so even if the angle was 60°, the bearing would be written 060°.

16 Quadratic equations and graphs

16.1 Expanding double brackets

Purposeful practice 1

1 $x^2 + 8x + 7$ **2** $x^2 + 8x + 7$ **3** $z^2 - 5z + 6$
4 $z^2 - 5z + 6$ **5** $y^2 + 3y - 4$ **6** $y^2 + 3y - 4$

Purposeful practice 2

2 a 18 **b** $x^2 + 9x + 18$ **3 a** 12 **b** $x^2 + 8x + 12$
4 a 2 **b** $x^2 + 3x + 2$ **5 a** 20 **b** $x^2 + 12x + 20$
6 a −20 **b** $x^2 + 8x - 20$ **7 a** −6 **b** $x^2 + x - 6$
8 a −6 **b** $x^2 - x - 6$ **9 a** −10 **b** $x^2 - 3x - 10$
10 a 4 **b** $x^2 + 4x + 4$ **11 a** 9 **b** $x^2 + 6x + 9$
12 a 9 **b** $x^2 - 6x + 9$ **13 a** 16 **b** $x^2 - 8x + 16$

Purposeful practice 3

1 $x^2 - 1$ **2** $x^2 - 4$ **3** $x^2 - 9$ **4** $x^2 - 16$

Problem-solving practice

1 a A width must be positive, so for $x - 5$ to be more than 0, x must be greater than 5.
  b $x^2 - 25$
2 $x^2 + 8x + 15$
3 a 2 **b** 2 **c** 6 **d** 1 **e** 4
4 $(y - 1)(y + 4) = y^2 + 3y - 4$ and $(y + 1)(y - 4) = y^2 - 3y - 4$;
  Rowan is wrong because the two expansions have different y terms.
5 Area = length × width
   $= (x + 2)(x - 3)$
   $= x^2 + 2x - 3x - 6$
   $= x^2 - x - 6$
  So the width must be $x - 3$
6 $6x + 8$ or $2(3x + 4)$
7 $a = 17$

Exam practice

1 $n^2 + 9n + 14$

16.2 Plotting quadratic graphs

Purposeful practice 1

1 a

x	−3	−2	−1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = x^2 + 1$	10	5	2	1	2	5	10
$y = x^2 - 2$	7	2	−1	−2	−1	2	7
$y = 2 + x^2$	11	6	3	2	3	6	11

b
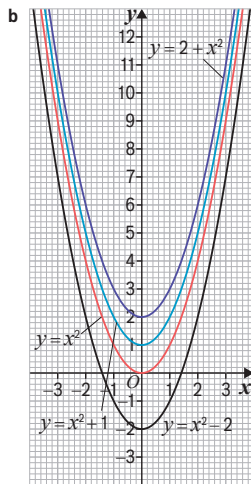
2 a

x	−3	−2	−1	0	1	2	3
$y = -x^2$	−9	−4	−1	0	−1	−4	−9
$y = -x^2 + 5$	−4	1	4	5	4	1	−4
$y = 4 - x^2$	−5	0	3	4	3	0	−5
$y = -x^2 - 1$	−10	−5	−2	−1	−2	−5	−10

b
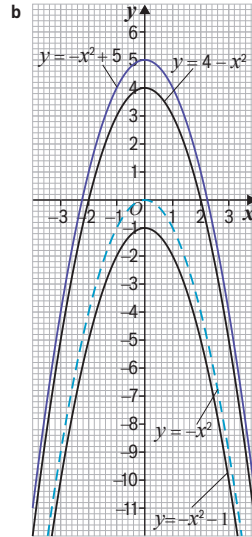
Purposeful practice 2

1 a

x	−3	−2	−1	0	1	2
x^2	9	4	1	0	1	4
$x^2 + x$	6	2	0	0	2	6
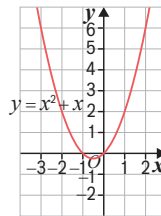
b

x	−4	−3	−2	−1	0	1	2
x^2	16	9	4	1	0	1	4
$2x$	−8	−6	−4	−2	0	2	4
$x^2 + 2x + 1$	9	4	1	0	1	4	9

318

c
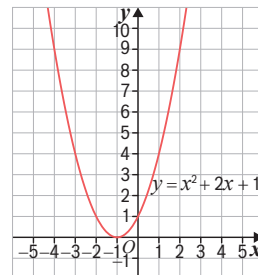
x	−2	−1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-2x$	4	2	0	−2	−4	−6	−8
$x^2 - 2x - 1$	7	2	−1	−2	−1	2	7

$y = x^2 - 2x - 1$
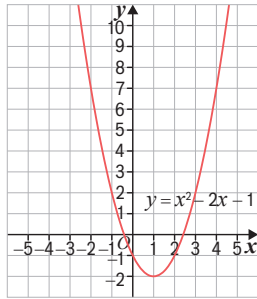
Problem-solving practice

1 B and C

2 a
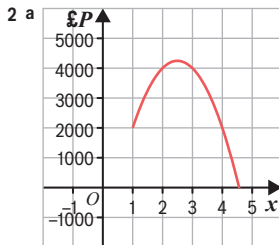
b Between £4100 and £4400 (exact answer is £4250).

3 a Graph is not symmetrical, and is not a parabola or a ∪-shaped curve. Pat has wrongly calculated $(-3)^2$ as −9, and so on.

b

$y = x^2 + 3$
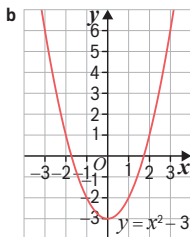
c (0, −3) is a minimum.

Exam practice

1 Olivia has used straight lines between the points instead of drawing a smooth curve.

16.3 Using quadratic graphs

Purposeful practice 1

1 a $x = -2, x = -3$ **b** $x = 1$

Purposeful practice 2

1 a $x = 0.37, x = -5.37$ (accept answers between $x = 0.35$ and $x = 0.4$, $x = -5.35$ and $x = -5.4$)

b $x = -6, x = 1$

c $x = -4.56, x = -0.44$ (accept answers between $x = -4.5$ and $x = -4.6$, and between $x = -0.3$ and $x = -0.5$)

Problem-solving practice

1 a

$y = -x^2 + 6x + 5$

b $x = -0.7, x = 6.7$ (approximately)

c Line at $y = 14$ only touches the graph at one point. Solution is $x = 3$

d $x = 0, x = 6$

2 a $x = 3.7$ (accept between 3.6 and 3.8) and $x = 0.3$ (accept between 0.2 and 0.4)

b $x = 2$ (repeated root)

c $x = 0$ and $x = 4$

Exam practice

1 a

x	−3	−2	−1	0	1	2	3
y	5	1	−1	−1	1	5	11

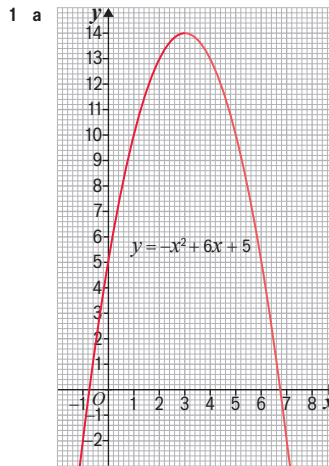**b**

$y = x^2 + x - 1$
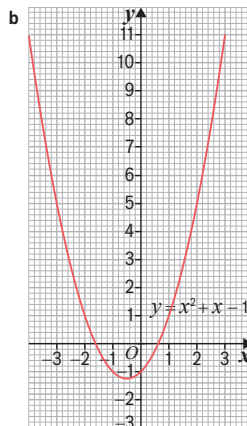
c $x = -2.5, x = 1.5$ (approximately)

16.4 Factorising quadratic expressions

Purposeful practice 1

1 $(x+1)(x+7)$ **2** $(x-1)(x+7)$ **3** $(x+1)(x-7)$
4 $(x-1)(x-7)$ **5** $(x+1)(x-5)$ **6** $(x+1)(x+5)$
7 $(x-1)(x+5)$ **8** $(x-1)(x-5)$

Purposeful practice 2

1 a $(x+1)(x+4)$ **b** $(x-1)(x-4)$ **c** $(x+2)(x+2)$
d $(x-2)(x-2)$ **e** $(x-1)(x+4)$ **f** $(x+1)(x-4)$
2 a $(x+1)(x-6)$ **b** $(x+1)(x+6)$ **c** $(x+2)(x+3)$
d $(x-2)(x+3)$ **e** $(x-1)(x+6)$ **f** $(x+2)(x-3)$
g $(x-1)(x-6)$ **h** $(x-2)(x-3)$
3 a $(x+3)(x-6)$ **b** $(x-2)(x+9)$ **c** $(x-3)(x-6)$
d $(x+1)(x+18)$ **e** $(x+2)(x+9)$ **f** $(x+2)(x-9)$
g $(x-1)(x+18)$ **h** $(x+3)(x+6)$ **i** $(x-2)(x-9)$
j $(x+1)(x-18)$ **k** $(x-3)(x+6)$ **l** $(x-1)(x-18)$

Problem-solving practice

1 The two answers are equivalent, as the order of two brackets multiplied together is not important. Alternatively, students may give working to show both Ben and Jill's answers expand to the original quadratic expression.

2 a $(x-3)(x+5)$
  b Desi has taken the x out of the first two terms as a common factor, but has not factorised all three terms in the expression.
3 a $(x-3)$ and $(x-5)$
  b $(x-7)$ and $(x-5)$
  c $(x-7)$ and $(x-4)$
4 Students' answers may vary,
  a of form $x^2 \pm a$ **b** of form $x^2 - a^2$
5 a $(x+2)(x-6) = x^2 - 4x - 12$
  b $(x-3)(x+7) = x^2 + 4x - 21$
  c $(x-5)(x-1) = x^2 - 6x + 5$

Exam practice
1 $(x-3)(x-3)$
2 $(x+3)(x+2)$

16.5 Solving quadratic equations algebraically
Purposeful practice 1
1 $x=7, x=-7$ **2** $x=7, x=-7$ **3** $x=7, x=-7$
4 $x=8, x=-8$ **5** $x=8, x=-8$ **6** $x=8, x=-8$

Purposeful practice 2
1 a 0   **b** 0   **c** 0   **d** 0
  e 0   **f** 0   **g** 0   **h** 0
2 a One of a or b or both are zero.
  b One of a or $(x-3)$ or both are zero.
  c One of $(x+4)$ or $(x-3)$ is zero.
3 a $x=0$ **b** $x=5$ **c** $x=-3$
  d $x=5$ **e** $x=0, x=5$ **f** $x=0, x=-3$
  g $x=-3, x=5$ **h** $x=-3, x=3$

Purposeful practice 3
1 a $x=-4, x=-3$ **b** $x=4, x=-3$ **c** $x=4, x=3$
  d $x=4, x=3$ **e** $x=-4, x=6$ **f** $x=4, x=-6$
  g $x=4, x=6$ **h** $x=-4, x=-6$ **i** $x=3, x=5$
  j $x=3, x=-5$ **k** $x=-3, x=-5$ **l** $x=-3, x=5$
2 a $x=-2, x=-2$ **b** $x=2, x=2$ **c** $x=3, x=3$
  d $x=-3, x=-3$ **e** $x=-1, x=-1$ **f** $x=5, x=5$
  g $x=-7, x=-7$ **h** $x=10, x=10$

Problem-solving practice
1 Every quadratic equation has two solutions. The square root of any number has a positive and negative value, as the product of two negative numbers is positive.
2 a $x(x+3)$ **b** $x=0, x=-3$ **c** $x=-1, x=-2$
3 a $x^2 - 16 = 0$ or $x^2 = 16$
  b $x^2 + 17x + 72 = 0$
  c $x^2 + 10x + 25 = 0$
  d $x(x+7) = 0$ or $x^2 + 7x = 0$
4 a Either substitute $x=-2$ into $x^2 - 3x - 10$ to show it gives zero or factorise to find both solutions and show that one is $x=-2$
  b $x=5$
5 a $x=-13, x=13$
  b $x=-19, x=19$
  c $x=-25, x=25$
6 $x=-15, x=-15$
7 Solutions are $x=-2$ and $x=4$. Positive solution is $x=4$
8 $x=10$
9
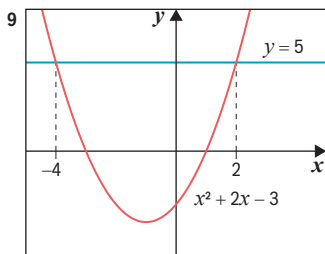
The x-coordinates are 2 and -4.

Exam practice
1 a $x=4, x=-9$ **b** $x=4, x=8$

Mixed exercises D
Mixed problem-solving practice D
1 a

  b 60 cm³

2 $\dfrac{1}{16}$

3 a 0.29   **b** 116   **c** 156

4 a
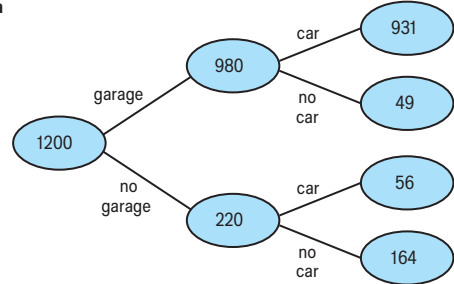
  b $\dfrac{56}{220}$ or $\dfrac{14}{55}$

5 PQ on the map is 3.7 cm. 3.7×100 km = 370 km, so the distance between the ports in real life is 370 km. $370 \div 40 = 9.25$, so the journey will take $9\frac{1}{4}$ hours or 9 hours and 15 minutes.

6 Accurate drawing of net. See example below (not drawn to scale).

4.5 cm

6 cm
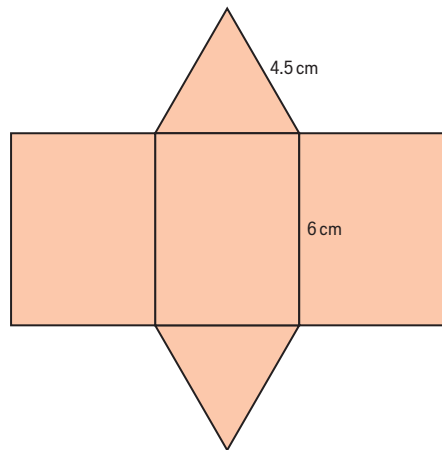
7 (This diagram is not drawn to scale.)

8 $x^2 + 6x + 9$
9 a $-3, -4, 0, 5$

b
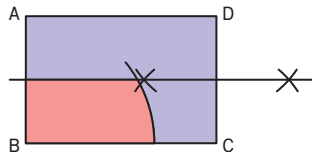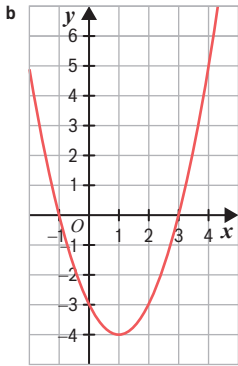
c −0.7 and 2.7 (approximately)

10 The branches for the first spin total 0.9, not 1, P(not red) should be 0.45
The 0.45 and 0.55 on the first two branches of the second spin are the wrong way around.

11 $p = 17.5$

Exam practice

12 Jenny has joined the points with straight lines instead of a smooth curve.
13 360 m   **14** 48   **15** £9
16 Bank A, as Bank A gives £2048.43 interest and Bank B gives £2040.28 interest
17 $x = 3$

17 Perimeter, area and volume 2

17.1 Circumference of a circle 1
Purposeful practice 1

1 a 37.68 cm   **b** 37.68 m   **c** 75.36 cm
d 75.36 cm   **e** 15.70 m   **f** 15.70 m
g 1.57 m   **h** 1.57 m   **i** 157.00 cm
2 a 37.70 cm   **b** 37.70 m   **c** 75.40 cm
d 75.40 cm   **e** 15.71 m   **f** 15.71 m
g 1.57 m   **h** 1.57 m   **i** 157.08 cm

Purposeful practice 2

1 6.28 cm   **2** 12.57 cm   **3** 25.13 cm
4 125.66 cm   **5** 29.53 m   **6** 20.14 km

Problem-solving practice

1 7.96 cm
2 Kasia needs to type $2\pi \times 6$; she has missed out the multiplication sign.
3 9.42 m   **4** 433 complete revolutions

Exam practice

1 31.42 (to 2 d.p.)

17.2 Circumference of a circle 2
Purposeful practice 1

1 a $3495 \leqslant 3500 < 3505$ **b** $3499.5 \leqslant 3500 < 3500.5$
c $349\,500 \leqslant 350\,000 < 350\,500$ **d** $349\,950 \leqslant 350\,000 < 350\,050$
e $349\,995 \leqslant 350\,000 < 350\,005$ **f** $349\,999.5 \leqslant 350\,000 < 350\,000.5$
2 Answers should be given to 2 decimal places. You should use the same level of accuracy in your answers as in the information in the question.

Purposeful practice 2

1 a 2000   **b** 2400   **c** 2400
d 2396   **e** 2396.3   **f** 2396.35
g 2396.3   **h** 2396.35   **i** 2396.346
2 a $2500 \leqslant 3000 < 3500$ **b** $3495 \leqslant 3500 < 3505$
c $34.5 \leqslant 35 < 35.5$ **d** $3.45 \leqslant 3.5 < 3.55$
e $0.345 \leqslant 0.35 < 0.355$
3 a Rounded to the nearest 1000
b Rounded to the nearest 10
c Rounded to the nearest unit
d Rounded to the nearest tenth
e Rounded to the nearest hundredth

Problem-solving practice

1 a 810   **b** 14 000   **c** 8565
d 182 125   **e** 24 097 500   **f** 11.8125
2 79.3 m
3 a $6650\,\text{km} \leqslant \text{radius} < 6750\,\text{km}$ **b** 42 000 km
4 6500 km   **5** 9.95 m or 995 cm
6 a $0.75\,\text{m} \leqslant \text{side length} < 0.85\,\text{m}$ **b** 0.421875 m³   **c** 3.375 m²

Exam practice

1 a £17 500   **b** £18 499.99

17.3 Area of a circle
Purposeful practice

1 a 28.27 cm²   **b** 113.10 cm²   **c** 113.10 cm²
d 0.09 km²   **e** 2.54 m²   **f** 0.64 m²
2 a 256π cm²   **b** 0.64π m²   **c** $\frac{1}{36}\pi$ m²   **d** $\frac{1}{144}\pi$ m²
3 a 3.99 cm   **b** 5.64 cm   **c** 7.98 cm   **d** 0.89 m
4 a 3 cm   **b** 9 cm   **c** 2.5 m   **d** $\frac{1}{4}$ km

Problem-solving practice

1 25 cm   **2** 3.39 m   **3** 18.75π cm²
4 Hoorain should have worked out the area of each circle separately, then subtracted the smaller area from the larger area. This would give 36π cm².

Exam practice

1 Area $= 36\pi$ m² ≈ 113 m². $113 \div 23 \approx 4.9$, so Jo must buy 5 boxes.

17.4 Semicircles and sectors
Purposeful practice 1

1 $\frac{1}{8}$ **2** $\frac{3}{8}$ **3** $\frac{5}{8}$ **4** $\frac{8}{9}$ **5** $\frac{7}{9}$ **6** $\frac{5}{9}$

Purposeful practice 2

1 a 4.36 cm²   **b** 1.75 cm
2 a 8.73 cm²   **b** 3.49 cm
3 a 34.9 cm²   **b** 6.98 cm
4 a 56.5 m²   **b** 18.8 m
5 a 339 mm²   **b** 56.5 mm
6 a 0.885 m²   **b** 1.36 m
7 a 3.54 m²   **b** 2.72 m
8 a 1.77 m²   **b** 2.72 m

Problem-solving practice

1 36.54 cm   **2** 8.02 m
3 a 3180 cm²   **b** 411 cm
c Work out the area of the whole circle and divide by 2.
4 a 110 cm   **b** 6.54 cm
5 0.691 m²

Exam practice

1 8.02 cm²

17.5 Composite 2D shapes and cylinders
Purposeful practice 1

1 2260 cm³   **2** 9050 cm³   **3** 18 100 cm³
4 0.339 m³   **5** 1.36 m³   **6** 2.71 m³

Purposeful practice 2

1 4.08 m²   **2** 0.589 m²   **3** 4.52 m²

Purposeful practice 3

1 942 cm²   **2** 4360 cm²   **3** 12 700 cm² or 1.27 m²   **4** 1.75 m²

Problem-solving practice

1 11 300 m²   **2** 6706 cm²   **3** 392 cm²   **4** 7540 m²   **5** 5 tins

Exam practice

1 339 cm³ (to nearest whole cm³)

17.6 Pyramids and cones
Purposeful practice 1
1 $18\,000\,\text{cm}^3$ or $0.018\,\text{m}^3$ **2** $128\,000\,\text{cm}^3$ or $0.128\,\text{m}^3$
3 $1.5\,\text{m}^3$ **4** $120\,000\,\text{cm}^3$ or $0.12\,\text{m}^3$

Purposeful practice 2
1 $20\,900\,\text{cm}^3$ **2** $83\,800\,\text{cm}^3$ **3** $41\,900\,\text{cm}^3$
4 $168\,000\,\text{cm}^3$

Purposeful practice 3
1 $0.146\pi\,\text{m}^2$ **2** $0.15\pi\,\text{m}^2$ **3** $65\pi\,\text{m}^2$

Problem-solving practice
1 $260\,\text{cm}^2$ **2 a** $1395\,\text{cm}^2$ **b** £20.92   **3** $6\frac{2}{3}\,\text{cm}^3$

Exam practice
1 A has the greater volume.
Volume of A $= 30.8\,\text{cm}^3$.
Volume of B $= 29.3\,\text{cm}^3$.

17.7 Spheres and composite solids
Purposeful practice 1
1 a $36\pi\,\text{cm}^3$ **b** $36\pi\,\text{cm}^2$
2 a $288\pi\,\text{cm}^3$ **b** $144\pi\,\text{cm}^2$
3 a $972\pi\,\text{cm}^3$ **b** $324\pi\,\text{cm}^2$
4 a $2304\pi\,\text{cm}^3$ **b** $576\pi\,\text{cm}^2$

Purposeful practice 2
1 a $1436.76\,\text{cm}^3$ **b** $615.75\,\text{cm}^2$
2 a $7696.90\,\text{cm}^3$ **b** $2507.00\,\text{cm}^2$
3 a $9133.66\,\text{cm}^3$ **b** $2814.87\,\text{cm}^2$
4 a $2565.63\,\text{cm}^3$ **b** $1264.22\,\text{cm}^2$

Problem-solving practice
1 a $24\,200\,\text{cm}^3$ **b** $5660\,\text{cm}^2$
2 $3811\,\text{cm}^3$ **3** $1767\,\text{cm}^3$
4 a $244\,\text{cm}^3$ **b** $214\,\text{cm}^2$

Exam practice
1 $603.2\,\text{cm}^2$

18 Fractions, indices and standard form

18.1 Multiplying and dividing fractions
Purposeful practice 1
1 a $\dfrac{21}{4}$ **b** $\dfrac{16}{3}$ **c** $\dfrac{10}{3}$ **d** $\dfrac{11}{3}$

2

$\times$	$3\frac{2}{3}$	$5\frac{1}{4}$
$3\frac{2}{3}$	$\frac{121}{9}=13\frac{4}{9}$	$\frac{231}{12}=19\frac{1}{4}$
$5\frac{1}{4}$	$\frac{231}{12}=19\frac{1}{4}$	$\frac{441}{16}=27\frac{9}{16}$

Purposeful practice 2
1 a $\dfrac{1}{4}$ **b** $\dfrac{1}{4}$ **c** 4 **d** 4
2 a 9 **b** 6 **c** 16 **d** 10
3 a $3\frac{1}{3}$ **b** $3\frac{7}{9}$ **c** $9\frac{9}{35}$ **d** $1\frac{11}{24}$

4

$\div$	$3\frac{2}{3}$	$5\frac{1}{4}$
$3\frac{2}{3}$	1	$\frac{44}{63}$
$5\frac{1}{4}$	$1\frac{19}{44}$	1

Problem-solving practice
1 $18\frac{2}{15}$ **2 a** 3 **b** 5 **c** 7 **d** 11
3 $4\frac{3}{5}\,\text{cm}$ **4 a** $\dfrac{2}{5}$ **b** $\dfrac{2}{7}$ **c** $\dfrac{2}{13}$
5 a 40 Egyptian pounds   **b** $\dfrac{27}{4}=6\frac{3}{4}$ pounds

Exam practice
1 Students' reasoning may vary, for example, Tim has calculated
2×6 and $\dfrac{3}{7}\times\dfrac{1}{2}$ but has forgotten to calculate $\dfrac{3}{7}\times 6$ and $2\times\dfrac{1}{2}$.

18.2 The laws of indices
Purposeful practice 1
1 a 2^6 **b** 2^6 **c** 2^0 **d** 2^{15} **e** 2^{15} **f** 2^0
  g 2^0 **h** 2^{12} **i** 2^{4a} **j** 2^0 **k** 2^{10a} **l** 2^{10a}
2 a 2^{10} **b** 2^4 **c** 2^6 **d** 2^8 **e** 2^0 **f** 2^3
  g 2^6 **h** 2^9

Purposeful practice 2
1 $\dfrac{1}{3}$ **2** $\dfrac{1}{5}$ **3** 3 **4** 5 **5** $\dfrac{5}{2}$ **6** $\dfrac{1}{9}$
7 $\dfrac{1}{16}$ **8** $\dfrac{1}{25}$ **9** 9 **10** 25 **11** $\dfrac{25}{4}$ **12** $\dfrac{125}{8}$

Purposeful practice 3
1 a 1 **b** 1 **2 a** $\dfrac{1}{8}$ **b** $\dfrac{1}{8}$ **3 a** $\dfrac{1}{9}$ **b** $\dfrac{1}{9}$

Purposeful practice 4
1 m^3 **2** m^6 **3** m^0 **4** m^4 **5** m^4
6 m^4 **7** m^4 **8** m^{10} **9** m^{-4}

Problem-solving practice
1 a Tina has calculated the reciprocals of the whole number and the fraction parts of the mixed number separately and then added these reciprocals together.
  b $\dfrac{2}{9}$
2 a 2^3 **b** 2^{-3} **c** 2^5 **d** 2^{-5} **e** 2^{15} **f** 2^{-15}
3 3^8
4 a -3 **b** 2 **c** 9 **d** 0
5 a $y=6$ **b** $y=-1$ **c** $y=10$ **d** $y=4$

Exam practice
1 a $\dfrac{1}{9}$ **b** 1

18.3 Writing large numbers in standard form
Purposeful practice 1
1 a 5.2×10^1 **b** 5.2×10^2 **c** 5.2×10^3
  d 5.2×10^4 **e** 5.2×10^7
2 a 1.01×10^2 **b** 1.1×10^2 **c** 1.01×10^4
  d 1.1×10^3 **e** 1.001×10^5 **f** 1.001×10^3

Purposeful practice 2
1 a 3010 **b** $3\,010\,000$ **c** 301 **d** $30\,100$ **e** 30.1
2 a 2050 **b** 2500 **c** 2005
  d $5\,210\,000$ **e** $5\,020\,100$ **f** $5\,021\,000$

Purposeful practice 3
1 $507, 570, 5007, 5070, 5700$
2 $5.07\times 10^2, 5.7\times 10^2, 5.007\times 10^3, 5.07\times 10^3, 5.7\times 10^3$

Problem-solving practice
1

Country	Poupulation
Canada	$3.628\,64\times 10^7$
India	1.339×10^9
Thailand	6.7959×10^7
Monaco	3.84×10^4

2 a 2900 kg   **b** 3400 kg   **c** 4300 kg
   d 82 000 kg   **e** 25 000 kg

3 a 5×10^6 **b** 4.8×10^8 **c** 2.5×10^4
   d 2.5×10^5 **e** 3.2×10^9

4 a 3×10^{14} **b** 3×10^{16} **c** 4×10^{10} **d** 6.5×10^7

5 a 1.64×10^4 kg   **b** 5.1×10^5 kg   **c** 7.48×10^4 kg
   d 8.36×10^1 kg   **e** 4.725×10^3 kg

6 2.5×10^7, 250 000, 2.5×10^4, 2.5×10^3, 250

Exam practice

1 a 25 630   **b** 8093

2 a 4.3×10^7 **b** 4.0506×10^4

18.4 Writing small numbers in standard form

Purposeful practice 1

1 a 5.2×10^{-1} **b** 5.2×10^{-2} **c** 5.2×10^{-4} **d** 5.2×10^{-7}

2 a 1.025×10^{-3} **b** 1.025×10^{-4} **c** 9.52×10^{-2} **d** 9.52×10^{-1}

3 a 4.04×10^{-3} **b** 4.4×10^{-4} **c** 4.04×10^{-1} **d** 4.4×10^{-1}

Purposeful practice 2

1 a 0.0035   **b** 0.35   **c** 0.000 000 35   **d** 0.035
   e 0.0109   **f** 0.109   **g** 0.000 000 2508   **h** 0.02508
   i 0.0103   **j** 0.13   **k** 0.000 000 1003   **l** 0.01003

2 As an ordinary number: 0.00031, 0.0031, 0.00301, 0.0032
In order (smallest first): 0.00031, 0.00301, 0.0031, 0.0032

3 As an ordinary number: 0.651, 0.00615, 0.0651, 0.00614
In order (largest first): 0.651, 0.0651, 0.00615, 0.00614

4 In standard form: 1.02×10^{-4}, 1.2×10^{-5}, 1.002×10^{-4}, 1.2×10^{-4}
In order (smallest first): 1.2×10^{-5}, 1.002×10^{-4}, 1.02×10^{-4}, 1.2×10^{-4}

5 In standard form: 7.008×10^{-4}, 7.8×10^{-5}, 7.08×10^{-4}, 7.8×10^{-4}
In order (largest first): 7.8×10^{-4}, 7.08×10^{-4}, 7.008×10^{-4}, 7.8×10^{-5}

Problem-solving practice

1 a 2.5×10^0 litres   **b** 5.0×10^{-1} litres   **c** 1.2×10^1 litres
   d 3.5×10^{-1} litres   **e** 1.5×10^{-2} litres

2 Students could correct either side of the equals sign.
   a $500 = 5 \times 10^2$ or $0.05 = 5 \times 10^{-2}$
   b $0.0021 = 2.1 \times 10^{-3}$ or $0.002\,01 = 2.01 \times 10^{-3}$
   c $0.000\,02 = 2 \times 10^{-5}$ or $0.05 = 5 \times 10^{-2}$
   d Robyn is already correct.
   e $0.000\,012 = 1.2 \times 10^{-5}$ or $0.001\,024 = 1.024 \times 10^{-3}$

3 a 4×10^{-6} **b** 1.5×10^{-4} **c** 6.5×10^{-5}
   d 4.1×10^{-4} **e** 7.6×10^{-4} **f** 2.3×10^{-9}

4 a James: The power should be -4 not 4.
   Hannah: The initial number should be 4.06 not 4.6.
   Katy: The initial number 40.6 in standard form should be between 1 and 10 so should be 4.06 and so the power needs to be -4.
   b 4.06×10^{-4}

Exam practice

1 a 0.017   **b** 0.007 34

2 a 2.5×10^{-2} **b** 5×10^{-1}

18.5 Calculating with standard form

Purposeful practice 1

1 a and **b** are both 7×10^5 **c** and **d** are both 7×10^3

2 All answers are 9.8×10^5

Purposeful practice 2

Answers to **Q1–3** are the same:
1 1.6×10^{12} **2** 1.6×10^{12} **3** 1.6×10^{12}

Purposeful practice 3

1 a 3×10^{-2} **b** 3×10^{-2} **c** 3×10^2
   d 3×10^2 **e** 1.2×10^3

2 a 4×10^1 **b** 4×10^2 **c** 4×10^{-2} **d** 4×10^{-2}
   e 4×10^{-2} **f** 4×10^{-2}

3 a 1600   **b** 16 000   **c** 160

Problem-solving practice

1 a Alpha Centauri is 3.78×10^{13} km away
   b Procyon is 1.026×10^{14} km away

2 a 2.3×10^{-10} kg   **b** 1.15×10^{-13} kg

3 3.01×10^{25}

---

4 a 3×10^{-2} m³   **b** $400 = 4 \times 10^2$

5 Students' working may vary, for example,
   a time for light to reach Earth from the Sun = $(1.496 \times 10^{11}) \div (3 \times 10^8)$
   $\approx 0.5 \times 10^3 = 500$ seconds
   500 seconds $= (500 \div 60)$ minutes ≈ 8.3 minutes.
   So, light takes about 8 minutes to reach Earth from the Sun.
   b The distance from the Moon to Earth is about $\dfrac{1}{400}$ of the distance from the Sun to Earth, so the time light takes to travel from the Moon to Earth $\approx 500 \div 400$ seconds ≈ 1.25 seconds

Exam practice

1 8×10^{-2} **2** 7.452×10^{-3} **3** 3.7×10^4

19 Congruence, similarity and vectors

19.1 Similarity and enlargement

Purposeful practice 1

1 Set 1 a $1:2, 2$ **b** $1:4, 4$ **c** $2:1, \dfrac{1}{2}$ **d** $4:1, \dfrac{1}{4}$

   Set 2 a $1:2, 2$ **b** $1:4, 4$ **c** $2:1, \dfrac{1}{2}$ **d** $4:1, \dfrac{1}{4}$

Purposeful practice 2

1 Corresponding angles: angle BAC and angle EFD, angle ABC and angle FDE, angle ACB and angle FED
Corresponding sides: AB and FD, AC and FE, BC and DE

2 Corresponding angles: angle PMN and angle QTR, angle MNP and angle TRQ, angle MPN and angle TQR
Corresponding sides: MP and TQ, NP and RQ, MN and TR

3 Corresponding angles: angle VUW and angle XZY, angle UWV and angle ZYX, angle UVW and angle ZXY
Corresponding sides: UV and ZX, UW and ZY, VW and XY

Problem-solving practice

1 A and C are similar rectangles because they have corresponding sides in a ratio of $2:3 \left(\dfrac{2}{3}\right)$

2 B and D are similar triangles because they have corresponding angles and sides in a ratio of $1:3 \left(\dfrac{1}{3}\right)$
A and D are also similar triangles because they have corresponding angles and sides in a ratio of $2:3 \left(\dfrac{2}{3}\right)$

3 Q1 ratio of corresponding sides is $2:3 \left(\dfrac{2}{3}\right)$
Q2 ratio of corresponding sides is $1:3 \left(\dfrac{1}{3}\right)$

Exam practice

1 All sides correspond in a ratio of $9:2 \left(\dfrac{9}{2}\right)$

19.2 More similarity

Purposeful practice 1

1 Scale factor = 2, $a = 6$ cm   **2** Scale factor = 2, $b = 4$ cm

3 Scale factor = 1.5, $c = 7.5$ cm   **4** Scale factor = 0.5, $d = 4.5$ cm

5 Scale factor = 0.4, $e = 1.6$ cm

Purposeful practice 2

1 angle CED   **2** angle CDE   **3** angle ECD

4 CE   **5** CD   **6** DE

7 a 2   **b** 2   **c** 2   **8** 2

Problem-solving practice

1 6 cm   **2** 1 cm

Exam practice

1 a 2.88 cm   **b** 9.5 cm

19.3 Using similarity
Purposeful practice 1
1 Yes   **2** No   **3** No   **4** Yes
5 No   **6** Yes   **7** Yes

Purposeful practice 2
1 $1:3\left(\dfrac{1}{3}\right)$ **2** 48 mm   **3** 144 mm   **4** $1:3\left(\dfrac{1}{3}\right)$

Problem-solving practice
1 **a** 24 cm   **b** 24 cm   **c** 40.5 cm
2 33 cm   **3** 48 cm   **4** 80 cm

Exam practice
1 **a** 15.5 cm   **b** 4.6 cm

19.4 Congruence 1
Purposeful practice
1 **a** SAS   **b** RHS   **c** ASA
2 Yes. BC = DE, AC = FE, AB = FD
3 A and C

Problem-solving practice
1 Yes, by RHS. XY = XZ. Angle XMY = angle XMZ = 90°, XM shared.
2 $x = 30°, y = 55°$
3 $x = 40°$. Students' working may vary, for example, the two triangles are congruent (SSS), so the two unmarked angles in the bottom triangle are 120° and 20°. Angles in a triangle add up to 180°, so $x = 180° - 120° - 20° = 40°$

Exam practice
1 **a** 3 cm   **b** **i** 30°   **ii** 80°

19.5 Congruence 2
Purposeful practice
1 **a**
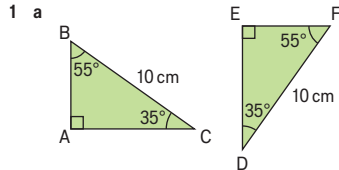
   i Angles are 90°, 35° and 55°.
   ii AB = EF, AC = ED, BC = FD

b
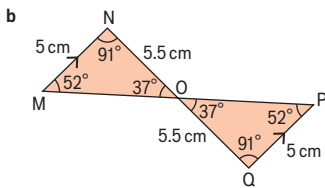
   i Angle MNO = 91°, angle QPO = 52°
   ii MN = PQ, NO = QO, MO = PO

2 **a** AB = ZY, BC = YX, angle ABC = angle ZYX, congruent by SAS
   b PR = MN, PQ = MO, angle PQR = angle MON = 90°, congruent by RHS
   c GI = LK, angle HGI = angle JLK, angle GIH = angle LKJ, congruent by ASA

Problem-solving practice
1 Students' own answers, for example, BD is a common side, AD = CB (opposite sides in a rectangle are equal), angle BAD = angle BCD = 90° (angles in a rectangle are 90° degrees), so triangles are congruent by RHS.
2 Students' own answers, for example, FH is a common side, EF = GH (opposite sides in a parallelogram are equal), FG = HE (opposite sides in a parallelogram are equal), so triangles are congruent by SSS.
3 **a** Students' own answers, for example, XW = ZY (given), angle WXV = angle YZV (alternate angles are equal), angle XWV = angle ZYV (alternate angles are equal), so triangles are congruent by ASA.
   b 6 cm   **c** 5 cm

Exam practice
1 Angle BAC = 50° because it is an isosceles triangle.
Angle ACB = 80° because angles in a triangle sum to 180°.
Side PQ is 3 cm because it is an isosceles triangle.
SAS: triangles must be congruent.

19.6 Vectors 1
Purposeful practice 1
1 $\begin{pmatrix}3\\2\end{pmatrix}$ **2** $\begin{pmatrix}-2\\3\end{pmatrix}$ **3** $\begin{pmatrix}-2\\-3\end{pmatrix}$ **4** $\begin{pmatrix}2\\-3\end{pmatrix}$ **5** $\begin{pmatrix}-3\\2\end{pmatrix}$

Purposeful practice 2
1 **a** $\begin{pmatrix}3\\8\end{pmatrix}$ **b** $\begin{pmatrix}-1\\-2\end{pmatrix}$ **c** $\begin{pmatrix}1\\2\end{pmatrix}$ **d** $\begin{pmatrix}3\\-8\end{pmatrix}$ **e** $\begin{pmatrix}-3\\8\end{pmatrix}$ **f** $\begin{pmatrix}-3\\-8\end{pmatrix}$

2 **a** $\overrightarrow{AC}$ **b** $\overrightarrow{AB}$ **c** $\overrightarrow{BD}$ **d** $\overrightarrow{OE}$ **e** $\overrightarrow{XZ}$ **f** $\overrightarrow{OY}$

Problem-solving practice
1 **a** $\begin{pmatrix}4\\1\end{pmatrix}$ **b** $\begin{pmatrix}-1\\6\end{pmatrix}$ **c** $\begin{pmatrix}3\\2\end{pmatrix}$ **d** $\begin{pmatrix}4\\4\end{pmatrix}$

2 **a** $x = 5$ **b** $x = 2, y = 7$
   c $x = 4, y = -9$ **d** $x = 5, y = 14$
3 $\begin{pmatrix}2m+1\\3n+8\end{pmatrix}$ **4** $\overrightarrow{AB} = \begin{pmatrix}-7\\-9\end{pmatrix}$ **5** $x = 4, y = -1$

Exam practice
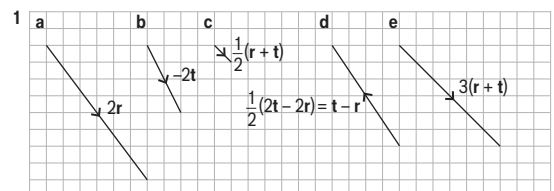**1** $\begin{pmatrix}9\\5\end{pmatrix}$

19.7 Vectors 2
Purposeful practice 1
1 $\begin{pmatrix}1\\5\end{pmatrix}$ **2** $\begin{pmatrix}2\\4\end{pmatrix}$ **3** $\begin{pmatrix}-1\\-5\end{pmatrix}$ **4** $\begin{pmatrix}-2\\-4\end{pmatrix}$ **5** $\begin{pmatrix}10\\2\end{pmatrix}$

6 $\begin{pmatrix}-4\\-2\end{pmatrix}$ **7** $\begin{pmatrix}1\\-4\end{pmatrix}$ **8** $\begin{pmatrix}-1\\4\end{pmatrix}$ **9** $\begin{pmatrix}-1\\4\end{pmatrix}$

Purposeful practice 2
1 **a** $\begin{pmatrix}-6\\10\end{pmatrix}$ **b** $\begin{pmatrix}-9\\15\end{pmatrix}$ **c** $\begin{pmatrix}3\\-5\end{pmatrix}$ **d** $\begin{pmatrix}-4\\1\end{pmatrix}$

   e $\begin{pmatrix}-8\\2\end{pmatrix}$ **f** $\begin{pmatrix}-16\\4\end{pmatrix}$

2 **a** $\begin{pmatrix}-3\\2\end{pmatrix}$ **b** $\begin{pmatrix}1\\-4\end{pmatrix}$ **c** $\begin{pmatrix}6\\-4\end{pmatrix}$ **d** $\begin{pmatrix}-2\\8\end{pmatrix}$

   e $\begin{pmatrix}5\\0\end{pmatrix}$ **f** $\begin{pmatrix}11\\-14\end{pmatrix}$

3 **a** $\begin{pmatrix}8\\12\end{pmatrix}$ **b** $\begin{pmatrix}16\\24\end{pmatrix}$ **c** $\begin{pmatrix}-12\\-18\end{pmatrix}$ **d** $\begin{pmatrix}2\\-3\end{pmatrix}$

Problem-solving practice
1

2 **a** $x = 3, y = 2$ **b** $y = 4$
3 $x = 8, y = 5$
4 $x = -2, y = 2$
5 $p = 4, q = 1$

Exam practice
1 $\begin{pmatrix}9\\-2\end{pmatrix}$

20 More algebra

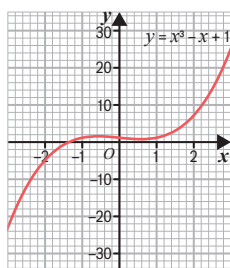## 20.1 Graphs of cubic and reciprocal functions

Purposeful practice 1

1 a $y = x^3 + 1$

x	–3	–2	–1	0	1	2	3
$x^3 + 1$	–26	–7	0	1	2	9	28
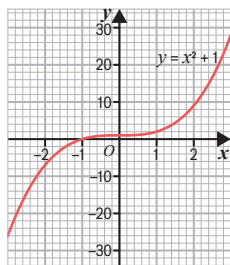
b $y = x^3 - x + 1$

x	–3	–2	–1	0	1	2	3
$x^3 - x + 1$	–23	–5	1	1	1	7	25

c $y = x^3 + x^2 - x + 1$

x	–3	–2	–1	0	1	2	3
$x^3 + x^2 - x + 1$	–14	–1	2	1	2	11	34
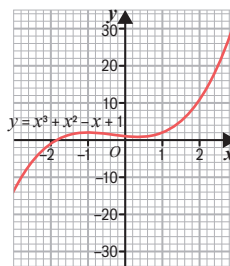
d $y = x^3 + x^2 - x - 1$

x	–3	–2	–1	0	1	2	3
$x^3 + x^2 - x - 1$	–16	–3	0	–1	0	9	32
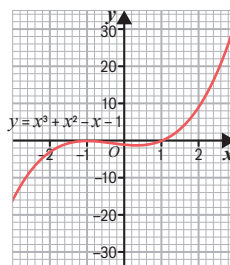
e $y = -x^3 + 1$

x	–3	–2	–1	0	1	2	3
$-x^3 + 1$	28	9	2	1	0	–7	–26
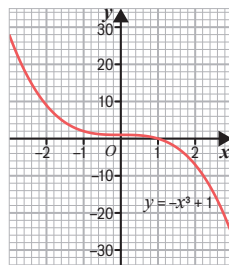
f $y = -x^3 + 2x + 1$

x	–3	–2	–1	0	1	2	3
$-x^3 + 2x + 1$	22	5	0	1	2	–3	–20
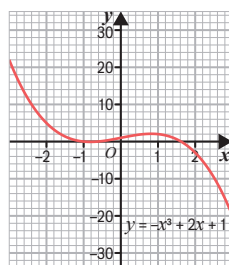
g $y = -x^3 - x^2 + 2x + 1$

x	–3	–2	–1	0	1	2	3
$-x^3 - x^2 + 2x + 1$	13	1	–1	1	1	–7	–29
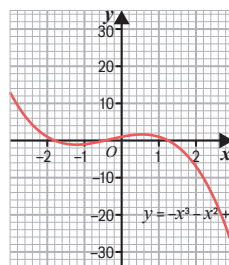
h $y = -x^3 - x^2 + 2x - 3$

x	–3	–2	–1	0	1	2	3
$-x^3 - x^2 + 2x - 3$	9	–3	–5	–3	–3	–11	–33
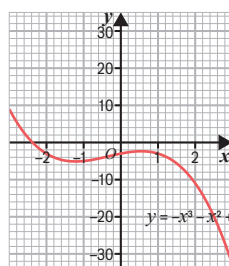
Purposeful practice 2

1 a $y = \dfrac{2}{x}$

x	0.25	0.5	1	2	4	5	10
$y = \dfrac{2}{x}$	8	4	2	1	0.5	0.4	0.2
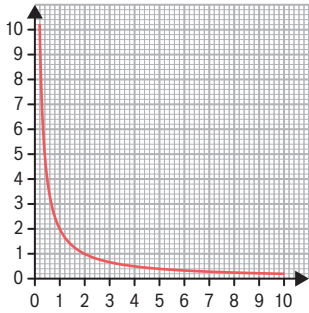
b $y = \dfrac{3}{x}$

x	0.5	1	2	3	4	5	10
$y = \dfrac{3}{x}$	6	3	1.5	1	0.75	0.6	0.3
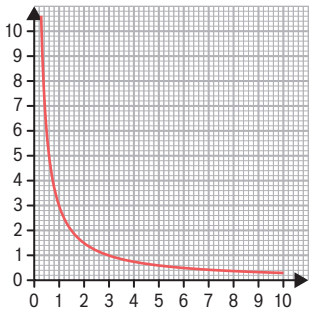
c $y = \dfrac{4}{x}$

x	0.5	1	2	4	5	8	10
$y = \dfrac{4}{x}$	8	4	2	1	0.8	0.5	0.4
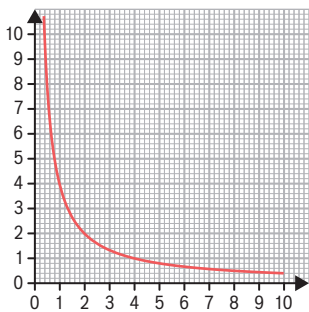
Problem-solving practice

1 a Graph ii shows $y = \dfrac{5}{x}$ **b** Graph i shows $y = \dfrac{8}{x}$

2 a Graph iii   **b** Graph iv   **c** Graph i   **d** Graph ii

Exam practice

1 a

x	0.5	1	1.5	2	3	4	5	6
y	6	3	**2**	1.5	**1**	0.75	**0.6**	**0.5**

b
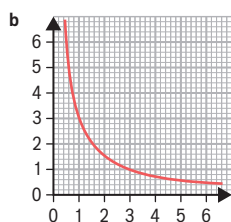
20.2 Non-linear graphs
Purposeful practice 1

1 a 100 ants   **b** 200 ants   **c** Month 5

2 a 100°C   **b** 22°C   **c** 3 minutes and 48 seconds   **d** 56°C

3 a Initial/starting height is 40 cm.   **b** Initial distance is 40 km.
   c Initial balance/amount saved is £40.   **d** Initial temperature is 40°C.
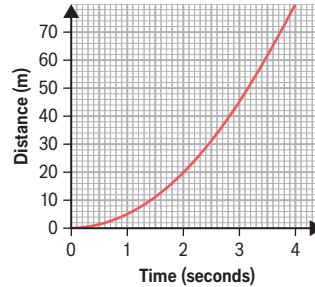
Problem-solving practice

1 a Approximately $3\frac{1}{4}$ hours   **b** 60

2 a 3.6 kg   **b** Week 6 and week 7

3 31 m

Exam practice

1 a 13 m/s   **b** 20 m/s   **c** 7 seconds and 15 seconds

20.3 Solving simultaneous equations graphically
Purposeful practice 1

1 a

x	−2	−1	0	2	3.5	4
y	11	9	7	3	0	−1

b Students' own answers, for example, $x = 1, y = 5$ and $x = 5, y = -3$

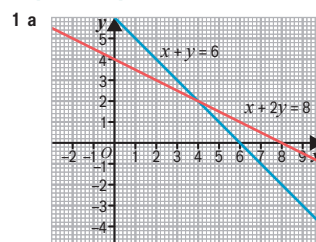**c** $x = 2$ and $y = 3$

d
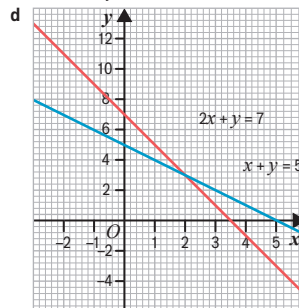
e $x = 2$ and $y = 3$. This is the same as the answer to **Q1c**. The point at which the two lines intersect is the only point where both equations are satisfied.
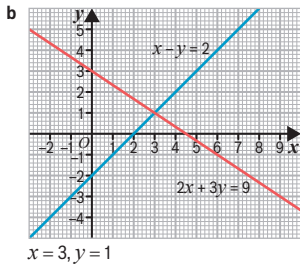
f Students' own answers, for example, I found eight pairs of values but the line extends beyond the graph I have drawn so there are infinite possible pairs.

g There are infinite possible solutions to $2x + y = 7$ but only one that also satisfies $x + y = 5$.
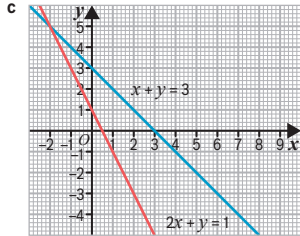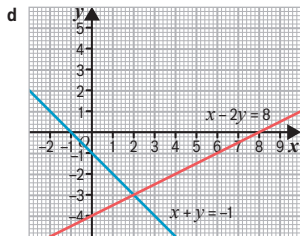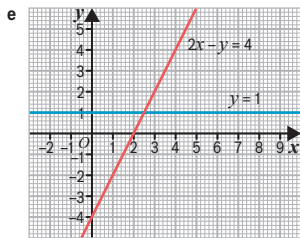
Purposeful practice 2

1 a

$x = 4, y = 2$

b

$x - y = 2$

$2x + 3y = 9$

$x = 3, y = 1$

c

$x + y = 3$

$2x + y = 1$

$x = -2, y = 5$

d

$x - 2y = 8$

$x + y = -1$

$x = 2, y = -3$

e

$2x - y = 4$

$y = 1$

$x = 2.5, y = 1$

f

$4x - y = 3$

$2x + y = 6$

$x = 1.5, y = 3$

g

$2x + 4y = 5$

$x + y = 2$

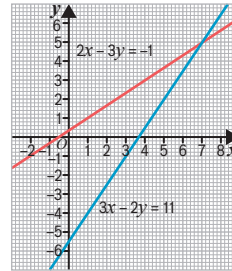$x = 1.5, y = 0.5$

h

$x - y = 2$

$3x + y = -4$
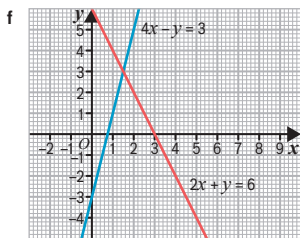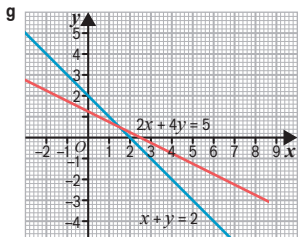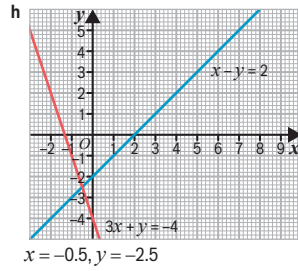
$x = -0.5, y = -2.5$

Problem-solving practice

1 Students' own answers – any two graphs that intersect at (4, 1), for example, graphs to show equations $x + y = 5$ and $x - y = 3$

2 $x = -3, y = 4$. Students may or may not draw a graph.

3 –0.5 and 9.5. Students may or may not draw graphs of $x + y = 9$ and $x - y = 10$

4 $x = 7, y = 5$. Students may or may not draw a graph showing $2x - 3y = -1$ **and** $3x - 2y = 11$

$2x - 3y = -1$

$3x - 2y = 11$

5 Students may or may not draw a graph showing $2x + y = 4.30$ **and** $3x + 5y = 9.25$
   a £1.75   **b** 80p

Exam practice

1 a   **b** $x = 2, y = -1$

$y = 3 - 2x$

$y = x - 3$

20.4 Solving simultaneous equations algebraically

Purposeful practice 1

1 a $3x = 6, x = 2$ **b** $5x + 2y = 16$ **c** $-2y = 4, y = -2$
2 a $3x = 6, x = 2$ **b** $-4y = -24, y = 6$ **c** $x - 2y = 0$

Purposeful practice 2

1 a $4x + 8y = 16$ **b** $3x + y = 8$ **c** $2x - 3y = 7$
     $4x + y = 9$ $3x - 6y = -6$ $2x + 2y = 12$
2 a $x + 2y = 4$ **b** $6x + 2y = 16$ **c** $2x - 3y = 7$
     $8x + 2y = 18$ $x - 2y = -2$ $3x + 3y = 18$
3 a $6x - 2y = 2$ **b** $10x + 5y = 25$ **c** $21x + 7y = 21$
     $6x + 3y = 27$ $10x + 4y = 28$ $21x + 6y = 15$
4 a $2x + 6y = 24$ **b** $5x + 10y = 55$ **c** $5x - 20y = 55$
     $3x - 6y = 6$ $8x + 10y = 70$ $16x + 20y = 40$

Problem-solving practice

1 $x = 4, y = 3$
2 a Danny has not multiplied the total of (2) by 3
   b Correct working to give $x = 7$ and $y = 2$
3 Students' own answers, for example, $2x - y = 8$ and $x + 3y = 11$.

4 Solution from graph is $x = -1, y = 5$

Students' own working to show solution algebraically is $x = -1, y = 5$. The answers should be the same.

5 317 and 586

6 Using simultaneous equations, adult ticket is £12 and child ticket is £6. So, adult ticket costs twice as much as child ticket. Students may use another method.

Exam practice

1 $x = -0.5, y = -3$

20.5 Rearranging formulae
Purposeful practice

1 a $t = \dfrac{m}{3}$ **b** $t = 2p$ or $t = \dfrac{p}{0.5}$ **c** $t = \dfrac{q}{n}$ **d** $t = \dfrac{r}{4n}$ **e** $t = \dfrac{v}{x}$

f $t = \dfrac{w}{2x}$ **g** $t = \dfrac{z}{nx}$ **h** $t = \dfrac{2y}{x}$ **i** $t = \dfrac{2s}{n}$

2 a $h = 3c$ **b** $h = 10d$ **c** $h = as$ **d** $h = abf$

e $h = \dfrac{1}{g}$ **f** $h = \dfrac{2}{m}$ **g** $h = \dfrac{r}{n}$ **h** $h = \dfrac{4s}{p}$

i $h = \dfrac{t}{4r}$ **j** $h = \dfrac{sv}{a}$ **k** $h = \dfrac{uw}{a}$ **l** $h = \dfrac{vwx}{a}$

3 a $x = y - 2$ **b** $x = a - b$ **c** $x = v + b$ **d** $x = \dfrac{w - a}{2}$

e $x = \dfrac{c + a}{2}$ **f** $x = \dfrac{g - a}{3}$ **g** $x = \dfrac{h + a}{3}$ **h** $x = \dfrac{k - 3}{b}$

i $x = 2(z - 3)$ **j** $x = 2(f - c)$ **k** $x = a(m - c)$ **l** $x = a(m + d)$

4 a $y = \dfrac{t}{2x}$ **b** $y = \dfrac{n}{az}$ **c** $y = \dfrac{10p}{a}$ **d** $y = \dfrac{10q}{z}$

e $y = \dfrac{2r}{z}$ **f** $y = \dfrac{2s}{az}$ **g** $y = \sqrt{t}$ **h** $y = \sqrt{\dfrac{u}{3}}$

i $y = \sqrt{\dfrac{v}{5}}$ **j** $y = \sqrt{\dfrac{x}{b}}$ **k** $y = \dfrac{\sqrt{z}}{2}$ **l** $y = \dfrac{b}{3}$

Problem-solving practice

1 a distance = speed × time **b** time = $\dfrac{\text{distance}}{\text{speed}}$

2 9 seconds

3 a She has divided by 2 first, instead of adding 3 to both sides first. She has not divided all terms by 2.

b $y = \dfrac{x + 3}{2}$

4 $y = 5x + 3$ **and** $5x - y = -3$

5 a $r = \dfrac{C}{2\pi}$ **b** $r = \sqrt{\dfrac{A}{\pi}}$

Exam practice

1 $n = \dfrac{D - 5}{3}$ **2** $x = 2(y + 3z)$

20.6 Proof
Purposeful practice 1

1 Ensure students have expanded and simplified correctly.

2 a $x^2 + 3x + 2 - x^2 = 3x + 2$

b $x^2 + 3x + 2 - x^2 - x = 2x + 2 = 2(x + 1)$

Purposeful practice 2

1 Even number: $2n$
Odd number: $2n + 1$
Multiple of 3: $3n$
3 consecutive integers: $n + n + 1 + n + 2$

2 a $n + n + 1 = 2n + 1$. $2n$ is even, so $2n + 1$ is odd.

b $2m - 2n = 2(m - n)$ which is a multiple of 2, so even.

c $2n + 1 - (2m + 1) = 2n + 1 - 2m - 1 = 2n - 2m = 2(n - m)$ which is a multiple of 2, so even.

d $2n + 1 - 2m = 2n - 2m + 1 = 2(n - m) + 1$ which is a multiple of 2, plus 1, so odd.

Problem-solving practice

1 a Students' answers will vary, for example, $1 + 2 + 3 = 6$, which is a multiple of 3.

b Three consecutive integers are a, $a + 1$, $a + 2$
$a + a + 1 + a + 2 = 3a + 3 = 3(a + 1)$, which is a multiple of 3.

2 a Perimeter $P = 3a + b + 3a + b + 3a + b + 3a + b = 12a + 4b$

b Yes, Bella is correct. $12a + 4b = 4(3a + b)$, which is a multiple of 4.

3 a Students' answers will vary, for example, $1 + 2 = 3$, which is not even.

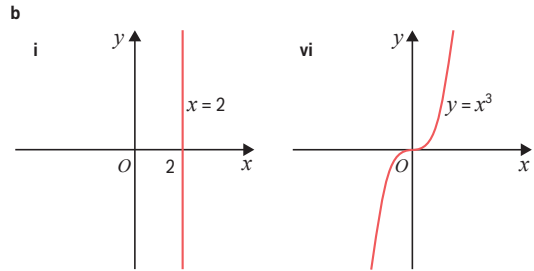**b** $2m + 2n + 1 = 2(m + n) + 1$, which is 1 more than an even number, so is odd.

Exam practice

1 Area $(x + 2)^2 = x^2 + 4x + 4 = 9$. Therefore, $x^2 + 4x = 5$

Mixed exercises E

Mixed problem-solving practice E

1 a A and iii, B and v, C and viii, D and vii, E and ii, F and iv

b

2 Jane has ordered the digits at the start of each distance and has not looked at the place value of the digits. The order should be Mercury, Earth, Jupiter, Saturn.

3 79.6 cm

4 Total area = $\pi \times 12^2 = 144\pi$;
Shaded area = $144\pi - \pi \times 6^2 = 144\pi - 36\pi = 108\pi$;
108π is not half of 144π, so Farrah is not correct.

5 $4\dfrac{15}{32}$

6 Students' estimates may vary. For example, $5 \times 10^7 \div 5 \times 10^4 = 1 \times 10^3$ hours

7 For the second triangle, use Pythagoras' theorem to find the missing side: $\sqrt{60^2 + 11^2} = 61$ cm. Both triangles have a right angle, a hypotenuse of 61 cm and a side of 60 cm and so are congruent (RHS).

8 $x = -\dfrac{1}{2}$ and $y = -5$

9 a Students' own answers, for example, $3 \times (1 + 3) = 3 \times 4 = 12$

b An odd number plus an odd number is always an even number, so $m + n$ is always even.
3 multiplied by an even number is always an even multiple of 3, which is also a multiple of 6. Therefore, the value of $3(m + n)$ will always be a multiple of 6.

10 Angle AMB = angle CMD because opposite angles are equal, AM = CM because M is the midpoint of AC and angle MAB = angle MCD because alternate angles are equal. Therefore, triangles ABM and CDM are congruent (ASA).

11 Squaring both sides gives $A^2 = x + y$ and then subtracting x from both sides gives $A^2 - x = y$ or $y = A^2 - x$

12 $\left(\dfrac{p}{q}\right)^{-2} = \dfrac{1}{\left(\dfrac{p}{q}\right)^2} = 1 \times \left(\dfrac{q}{p}\right)^2 = \dfrac{q^2}{p^2}$

13 36.1 cm

Exam practice

14 $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$

15 1.4×10^{-4} **16** £2588.84

17 a 9.5 cm **b** 10.4 cm

18 The equation of the line L_2 rearranges to give $y = 4x + \dfrac{7}{3}$.
The gradient of both L_1 and L_2 is 4 so the lines are parallel.

19 $(x + 4)(x + 5) = 30$, $x^2 + 9x + 20 = 30$, $x^2 + 9x = 10$