Answers

1 Number

1.1 Number problems and reasoning

Purposeful practice 1

1 2 ways **2** 6 ways **3** 24 ways **4** 120 ways
5 60 ways **6** 20 ways **7** 5 ways **8** 1 way

Purposeful practice 2

1 120 **2** 720 **3** 5040

Purposeful practice 3

1 a 900 **b** 450
2 720

Problem-solving practice

1 15 **2** 81 **3** 5 **4** 5 735 160
5 a 36 **b** 9
6 a 216 **b** 2 more dice

Exam practice

1 54 516 060 **2** 132

1.2 Place value and estimating

Purposeful practice 1

1 a 126 000 **b** 12 600 **c** 1260 **d** 12.6
e 1.26 **f** 0.126 **g** 0.054 83 **h** 0.5483
i 5.483 **j** 548.3 **k** 5483 **l** 54 830
2 a 463.68 **b** 463.68 **c** 463.68
3 a 20.48 **b** 20.48 **c** 20.48

Purposeful practice 2

1 6 **2** 7 **3** 6 or 7 **4** 7 **5** 6

Purposeful practice 3

1 a 100 **b** 100 000 **c** 40
d 140 **e** 100 or 110 **f** 11
g 11 **h** 0.25
2 a i underestimate **ii** overestimate **iii** difficult to tell
b i underestimate **ii** overestimate **iii** difficult to tell

Problem-solving practice

1 0.3 to 0.7 **2** £300 to £315 **3** £75
4 2400 cm^2 **5** Car B
6 a 19 250 cm^2 **b** 0.1925 cm^2 **c** 1.925 cm^2
7 a Any answers where one number has been multiplied by a power of 10 and the other number has been divided by the same power of 10.
For example: $1790 \times 0.245 = 438.55$ or $17.9 \times 24.5 = 438.55$
b Any answers where both numbers have been multiplied or divided by the same power of 10. For example $4970 \div 284 = 17.5$ or $49.7 \div 2.84 = 17.5$
8 a 20 kg to 21 kg
b An underestimate. Both numbers in the estimate have been rounded down so the accurate answer is likely to be higher.

Exam practice

1 6

1.3 HCF and LCM

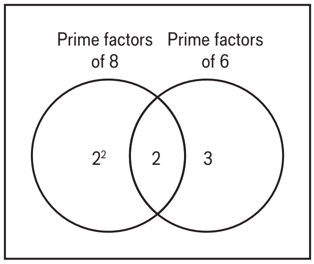
Purposeful practice 1

1 3×5 **2** $2 \times 3 \times 5$ **3** $2^2 \times 3 \times 5$
4 $2^2 \times 3 \times 5 \times 7$ **5** $2^2 \times 3^2 \times 5 \times 7$

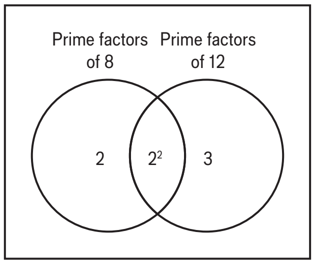
Purposeful practice 2

1 Any two of 105, 126, 140, 180
2 Factors, multiple

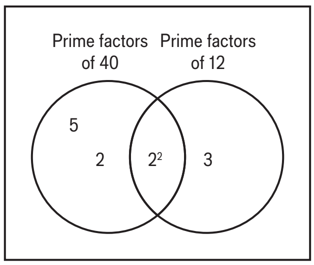
Purposeful practice 3

1


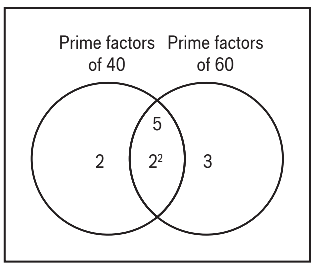
HCF = 2, LCM = 24

2


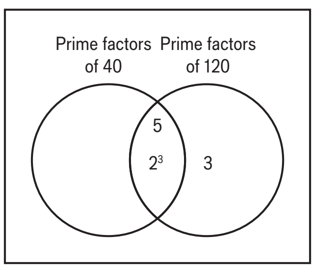
HCF = 4, LCM = 24

3


HCF = 4, LCM = 120

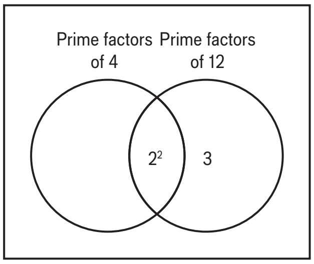
4


HCF = 20, LCM = 120

5


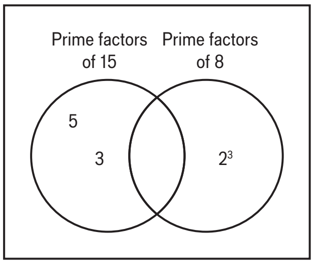
HCF = 40, LCM = 120

6



HCF = 4, LCM = 12

7



HCF = 1, LCM = 120

Purposeful practice 4

1 HCF = 1 LCM = 210
2 HCF = 6 LCM = 12
3 HCF = 1 LCM = 330
4 HCF = 50 LCM = 300
5 HCF = 24 LCM = 72
6 HCF = 1 LCM = 600
7 HCF = 3 LCM = 45
8 HCF = 2 LCM = 56

Problem-solving practice

1 a 6 is a factor because within the prime factors you can have 2×3.
b 21 is not a factor of 1320 because 21 cannot be made by multiplying any of the prime factors of 1320.
2 24 and 30
3 Any pair of 2-digit numbers with no common factors (apart from 1), for example, 15 and 16
4 600 cm
5 $a = 540$ $b = 1$

Exam practice

1 9.42 am
2 a 390 b 45

1.4 Calculating with powers (indices)

Purposeful practice 1

1 3^4 2 3^6 3 3^8 4 3^{10}
5 3^8 6 3^6 7 3^4 8 $3^0 = 1$

Purposeful practice 2

1 a 3^4 b 3^6 c 3^8 d 3^8 e 3^4
2 a 7^4 b 7^6 c $3^4 \times 7^1$ or $3^4 \times 7$
d 7^7 e 7^7 f 7^{17}
g 7^{12} h 7^{15} i 7^{12}
j 7^8 k $6^2 \times 7^2$ l $6^{20} \times 7^7$

Problem-solving practice

1 3^6
2 a 5 b 9
3 3, 3
4 a 2^7 b 2^9 c 5^6
5 a $n = 10$ b $n = 10$
6 a 3^8 b 3^4
7 2^6

Exam practice

1 3^{12}
2 a $x = 5$ b $y = 5$

1.5 Zero, negative and fractional indices

Purposeful practice 1

1 5^4 2 5^3 3 5^2 4 5^1
5 5^0 6 $5^{-1} = \frac{1}{5}$ 7 $5^{-2} = \frac{1}{5^2}$

Purposeful practice 2

1 a $\sqrt{7}$ b $\sqrt[3]{7}$ c $\sqrt[4]{7}$ d $\sqrt[5]{7}$ e $\frac{1}{\sqrt{7}}$ f $\frac{1}{\sqrt[3]{7}}$
2 a $6^{\frac{1}{2}}$ b $6^{\frac{1}{3}}$ c $6^{\frac{1}{4}}$ d $6^{-\frac{1}{2}}$ e $6^{-\frac{1}{3}}$ f $6^{-\frac{1}{4}}$

Purposeful practice 3

1 $6^{\frac{9}{2}} = (\sqrt{6})^9$ 2 $6^{-\frac{3}{2}} = \frac{1}{(\sqrt{6})^3}$ 3 $6^{\frac{3}{2}} = (\sqrt{6})^3$
4 $6^{-\frac{9}{2}} = \frac{1}{(\sqrt{6})^9}$ 5 $6^{\frac{3}{2}} = (\sqrt{6})^3$ 6 $6^{-\frac{9}{2}} = \frac{1}{(\sqrt{6})^9}$
7 $6^{\frac{9}{2}} = (\sqrt{6})^9$ 8 $6^{-\frac{3}{2}} = \frac{1}{(\sqrt{6})^3}$ 9 $6^{\frac{9}{2}} = (\sqrt{6})^9$
10 $6^{-\frac{3}{2}} = \frac{1}{(\sqrt{6})^3}$ 11 $6^{\frac{3}{2}} = (\sqrt{6})^3$ 12 $6^{\frac{9}{2}} = (\sqrt{6})^9$

Problem-solving practice

1 a $\frac{1}{2}$ b 125 c $\frac{1}{27}$ d $\frac{9}{4}$
2 a $\frac{5}{2}$ b $\frac{7}{2}$
3 $6^{\frac{1}{6}}$
4 1
5 a -3 b 0 c -2 d $-\frac{1}{2}$ e 0 f 0
6 a $x = -1.5$ b $x = -2$
7 $\frac{100}{9}$
8 a $16^{-\frac{3}{4}} \times 27^{\frac{2}{3}} = \frac{1}{(\sqrt[4]{16})^3} \times (\sqrt[3]{27})^2 = \frac{1}{2^3} \times 3^2 = \frac{1}{8} \times 9 = \frac{9}{8}$
b She thought that $16^{-\frac{3}{4}} \times -16^{\frac{3}{4}}$

Exam practice

1 $x = \frac{1}{4}$

1.6 Powers of 10 and standard form

Purposeful practice 1

1 10^4 2 10^3 3 10^2
4 10 or 10^1 5 10^0 6 10^{-1}
7 10^{-2} 8 10^{-3} 9 10^{-4}
10 10^{-5} 11 10^{-6}

Purposeful practice 2

1 a 6 300 000 b 630 000 c 63 000 d 63
e 630 f 63 g 6.3 h 0.63
i 0.063 j 0.006 3 k 0.000 63 l 0.000 063
2 a 3.425×10^6 b 3.425×10^5
c 3.425×10^4 d 3.425×10^3
e 3.425×10^2 f 3.425×10^1
g 3.425 or 3.425×10^0 h 3.425×10^{-1}
i 3.425×10^{-2} j 3.425×10^{-3}
k 3.425×10^4 l 3.425×10^5
m 3.425×10^6 n 3.425×10^{-2}
o 3.425×10^{-3} p 3.425×10^{-4}
q 3.425×10^2 r 3.425×10
s 3.425 t 3.425×10^3
u 3.425×10^{-5}

Purposeful practice 3

1 6×10^8 2 6×10^{10} 3 6×10^5
4 6×10^2 5 6 6 6×10^{-12}
7 1.2×10^9 8 1.2×10^{-3} 9 1.2×10^{-11}
10 3×10^2 11 3×10^{-2} 12 3×10^{-12}

Problem-solving practice

1 Students' own answers, for example, two of 0.01, $\frac{1}{10^2}$ and 10^{-2}

2 -2×10^3, -6.9×10^{-5}, 8×10^3, 0.0016×10^8, 2.6×10^6, 28×10^5

3 1.8×10^{-2}

4 **a** 300 **b** 200

5 4.29×10^2 hours

6 $(2 \times 10^3)\boxed{\div}(5 \times 10^5) = \boxed{4} \times 10^{-3}$

7 6×10^{26}

8 8.0808×10^6

9 The indices are 3, 4 and 6 in any order

Exam practice

1 7.08×10^{-4} 2 2.5×10^{25}

1.7 Surds

Purposeful practice 1

1 $2\sqrt{2}$ 2 $3\sqrt{2}$ 3 $4\sqrt{2}$ 4 $5\sqrt{2}$

5 $6\sqrt{2}$ 6 $7\sqrt{2}$ 7 $2\sqrt{3}$ 8 $3\sqrt{3}$

9 $4\sqrt{3}$ 10 $5\sqrt{3}$ 11 $6\sqrt{3}$ 12 $7\sqrt{3}$

13 $\sqrt{15}$ 14 $\sqrt{21}$ 15 $\sqrt{35}$

Purposeful practice 2

1 $\frac{1}{2}$ 2 $\frac{\sqrt{3}}{2}$ 3 $\frac{\sqrt{3}}{4}$ 4 2 5 $\frac{2\sqrt{3}}{3}$ 6 $2\sqrt{2}$

Purposeful practice 3

1 $\frac{\sqrt{2}}{2}$ 2 $\frac{\sqrt{3}}{3}$ 3 $\frac{\sqrt{5}}{5}$

4 $\sqrt{2}$ 5 $\frac{3\sqrt{2}}{2}$ 6 $\frac{\sqrt{3}}{3}$

Problem-solving practice

1 $\sqrt{2} + \sqrt{15} + 7 + \frac{6\sqrt{5}}{5}$

2 **a** $k = 63$

3 **a** 9 **b** 4 and 3

4 **a** 4 **b** 108

5 8 square units

6 $6\sqrt{2}$

7 $a = 2\sqrt{2}$ $b = 20\sqrt{2}$ m

Exam practice

1 **a** $\frac{3\sqrt{5}}{5}$ **b** $\frac{2\sqrt{5}}{5}$ 2 $\frac{13\sqrt{3}}{6}$

2 Algebra

2.1 Algebraic indices

Purposeful practice 1

1 **a** $\frac{1}{x}$ **b** $\frac{1}{x^2}$ **c** $\frac{1}{x^3}$ **d** $\frac{2}{x^3}$

e $\frac{7}{x^3}$ **f** $\frac{7}{2x^3}$ **g** $\frac{7}{2x^2}$

2 **a** $\sqrt{x}$ **b** $\frac{1}{\sqrt{x}}$ **c** $\sqrt[3]{x}$ **d** $\frac{1}{\sqrt[3]{x}}$

e $\frac{1}{\sqrt[4]{x}}$ **f** $\sqrt{x}$ **g** $\frac{1}{\sqrt{x}}$ **h** $4\sqrt{x}$

i $\frac{4}{\sqrt{x}}$

Purposeful practice 2

1 x^4 2 x^6 3 $21x^6$ 4 $\frac{x^6}{21}$

5 x^4 6 x^{-6} 7 x^{-2} 8 $18x^{-2}$

9 x or x^1 10 $x^{\frac{5}{2}}$ 11 x^2 12 x or x^1

13 1 14 3 15 $3x^3$

Purposeful practice 3

1 x^{12} 2 x^{21} 3 $\frac{x^{-12}}{81}$ 4 $\frac{x^{-12}}{16}$

5 $81x^2$ 6 $3x^2$ 7 $3x^2y^3$

Problem-solving practice

1 **A** and **D**

2 **a** $\left(\frac{2a^2\boxed{s^4}}{\boxed{3}}\right)^3 = \frac{\boxed{8}a^6s^{12}}{27}$ **b** $\sqrt{9\boxed{p^2}t^4} = \boxed{3}p\boxed{t^2}$

c $(\boxed{4}c^3k^2)^{\boxed{2}} = 16c^6k^{\boxed{4}}$

3 $9x^4\boxed{\div}3xy\boxed{\times}2y^3 = 6x^3y^2$

4 **a** $4x^2$ **b** $64x^6$ **c** $64x^4$ **d** $8x^2$

5 **a** pq **b** $\frac{a}{p}$ **c** p^2 **d** $\frac{q}{9}$

6 $2^4x^{\frac{1}{2}}$

7 $6x^{\frac{11}{2}}$

Exam practice

1 **a** a^6 **b** $4b^4c^6$ **c** $\frac{5de}{2}$

2.2 Expanding and factorising

Purposeful practice 1

1 $x^2 + 2x$ 2 $5x^2 + 10x$ 3 $5x^2 - 10x$

4 $5x^2 - 2x$ 5 $2x - 5x^2$ 6 $5x^2 - 2x$

7 $2y - 5xy$ 8 $5xy + 2y$ 9 $5xy - 2y$

10 $5xy - 2y^2$ 11 $2y^2 - 5xy$ 12 $2xy - 5x^2$

13 $5x^2 - 2xy$ 14 $2xy + 5x^2$ 15 $2y^2 + 5xy$

Purposeful practice 2

1 **a** $5x - 12$ **b** $5x$ **c** $x - 12$

d $x - 2$ **e** $-x - 2y$ **f** $-x^2 - 2xy$

2 **a** $3(x + 2)$ **b** $3(2x + 5)$ **c** $6(2a + c)$

d $d(3b + 1)$ **e** $6k(gh - 2)$ **f** $x^2(y + 2)$

g $2y^2(x + a)$ **h** $3xy^2(2x - 3y)$ **i** $5b(1 + 10a)$

3 **a** $(f + 2)(f + 5)$ **b** $(f + 2)(f - 1)$ **c** $(p + 2)[2(p + 2) + 3]$

d $2(p + 2)(p + 5)$ **e** $2(r + 2)(r - 1)$ **f** $2(r + 2s)(r + 2s - 3)$

g $(t + 2)(t + 3)$ **h** $(t + 2)(t + 1)$

Problem-solving practice

1 $2(y + 6)(y + 8)$

2 $20x + 42$

3 5, 3

4 $(x + 3)^2$

5 **a** $c, d, 2c, 2d, d^2, 2d^2, cd, 2cd, cd^2, 2cd^2$

b $2cd^2$

c $4c^2d^2(2c + 3d^2 - 4cd^3)$

Exam practice

1 $3m(4 - 3m)$ 2 $5ab - 18ag - 8bg$

2.3 Equations

Purposeful practice 1

1 $x = 2$ 2 $x = 3$ 3 $x = \frac{3}{2}$ 4 $x = -3$

5 $x = -8$ 6 $x = 8$ 7 $x = 1$ 8 $x = \frac{1}{2}$

Purposeful practice 2

1 $x = 3$ 2 $x = 0$ 3 $x = -6$ 4 $x = -6$

5 $x = 6$ 6 $x = 30$ 7 $x = 7\frac{1}{2}$

Purposeful practice 3

1 $x = -18$ 2 $x = -7\frac{1}{2}$ 3 $x = 7$ 4 $x = -5$

5 $x = -7\frac{1}{4}$ 6 $x = -2\frac{3}{4}$ 7 $x = 4$ 8 $x = \frac{7}{18}$

Problem-solving practice

1 26° 2 270° 3 $\frac{9}{13}$

4 **a** $5(x - 23) = 2(x + 7)$ **b** The man is 43.

5 30

Exam practice

1 $3\frac{1}{7}$

2.4 Formulae

Purposeful practice 1

1 $b = \frac{a}{2}$
2 $b = \frac{a}{3d}$
3 $b = 3a$
4 $b = \frac{3a}{2}$
5 $b = a + 3$
6 $b = \frac{a + 3}{2}$
7 $b = 2(a + 3)$
8 $b = \frac{2(a + 3)}{3}$
9 $b = \frac{2a + 3}{2}$ or $b = a + \frac{3}{2}$
10 $b = a + 3$
11 $b = \frac{2a}{3cd} + 3$
12 $b = \sqrt{a}$
13 $b = \sqrt{a - 6}$
14 $b = \sqrt{\frac{a - 6}{3}}$
15 $b = \sqrt{a^2 - 9}$

Purposeful practice 2

1 a $u = 3$ b $u = 6$ c $u = 12$
2 a $a = 0$ b $a = -10$ c $a = 380$

Problem-solving practice

1 a 4 b Tetrahedron or triangular pyramid
2 $0.155\,\text{m}^3$
3 a area = $31.5\,\text{m}^2$ b $h = 8\,\text{m}$
c i $b = -6\,\text{cm}$
ii You cannot have a trapezium with a negative dimension.
4 Students' own answers, for example, $u - v = 10t$, $2(u - v) = 20t$, $2t = \frac{u - v}{5}$
5 $3.16\,\text{ms}^{-1}$
6 a $V = \frac{1}{3}\pi r^2 h$ b $h = \frac{3V}{\pi r^2}$ c 9.55 cm

Exam practice

1 $b = 3$

2.5 Linear sequences

Purposeful practice 1

1 a 18, 20 b $4\frac{3}{7}, 4\frac{6}{7}$ c 3.26, 3.54
d 0, −4 e $\frac{3}{4}, \frac{3}{8}$ f −3, 1
g −5, −4.5 h 8.5, 10 i 7, 15, 23
2 a 3, 5, 7, 9, 11 b 5, 7, 9, 11, 13 c 1, 3, 5, 7, 9
d 4, 7, 10, 13, 16 e 6, 9, 12, 15, 18 f 2, 5, 8, 11, 14
g 6, 11, 16, 21, 26 h 8, 13, 18, 23, 28 i 0, 1, 2, 3, 4
j 8, 15, 22, 29, 36 k 10, 17, 24, 31, 38 l 6, 13, 20, 27, 34

Purposeful practice 2

1 n
2 $2n$
3 $2n + 3$
4 $2n + 4$
5 $2n + 5$
6 $2n - 2$
7 $2n - 0.5$
8 $4n$
9 $4n + 3$
10 $4n + 4$
11 $4n + 5$
12 $4n - 2$
13 $4n - 0.5$
14 $7n$
15 $7n + 3$
16 $7n + 4$
17 $7n + 5$
18 $7n - 2$
19 $7n - 0.5$
20 $-3n$
21 $-3n + 3$
22 $-3n + 4$
23 $-3n + 5$
24 $-3n - 2$
25 $-3n - 0.5$

Problem-solving practice

1 a $6\frac{1}{2}, 7, 7\frac{1}{2}, 11$
b Jamie is correct. For even number positions $\frac{n}{2}$ gives an integer so $\frac{n}{2} + 6$ also gives an integer; for odd number positions $\frac{n}{2}$ is not an integer and so $\frac{n}{2} + 6$ is not an integer.
2 Students' answers may vary, for example, the common difference in the sequence is 4. The sequence starts with an even number and so all the terms will be even. 81 is odd and can't be in the sequence.
3 The nth term rule is $3n + 7$
81 would be the $\left(\frac{81 - 7}{3}\right) = \left(24\frac{2}{3}\right)$th term. n must be an integer, so 81 is not in the sequence.
OR The two closest terms are the 24th = 79 and 25th = 82. Therefore 81 cannot be a term in the sequence.
4 1004 (168th term)
5 −501 (73rd term)
6 a $4n + 1$ b $4n - 4$ c $8n - 3$

Exam practice

1 a $3n$
b The sequence is the 3 times table. 299 ÷ 3 is not an integer so 299 is not in the 3 times table and is not a term in this sequence.
c $3(n + 1)$ or $3n + 3$

2.6 Non-linear sequences

Purposeful practice 1

1 a $a = 1$ b $a = 1$ c $a = 1$ d $a = 2$
e $a = 2$ f $a = 2$ g $a = 3$ h $a = 3$
i $a = 3$ j $a = 4$ k $a = 4$ l $a = 10$
2 a $n^2 + n$ b $n^2 + 2n + 1$ c $n^2 + 5n - 2$
d $2n^2 + 3n + 4$ e $2n^2 + n$ f $2n^2 + 2n + 2$
g $3n^2 + 2n + 10$ h $3n^2 + 2n + 5$ i $3n^2 + 2n - 2$
j $4n^2 + 7n + 2$ k $4n^2 + n - 5$ l $4n^2 + 3$

Purposeful practice 2

1 $n^2 + 3n + 2$
2 $3n^2 + n - 4$
3 $2n^2 + 3n + 1$
4 $5n^2 + 5n + 6$
5 $n^2 + 6n + 10$
6 $2n^2 + 2n + 4$

Problem-solving practice

1 $n^2 + 1$
2 a $2n^2$ b $2n - 2$ c $2n^2 + 2n - 2$
3 Ben has subtracted each term in the original sequence from the corresponding term in the n^2 sequence, instead of the other way around. Subtracting corresponding terms in the correct order would have given the sequence 2, 7, 12, 17, 22, so the nth term rule of the original sequence is $n^2 + 5n - 3$.
4 a $0.5n^2 + 0.5n$
b The initial difference between terms is 2. The difference increases by 1 each time.
c The triangular number sequence

Exam practice

1 $2n^2 + 4n - 1$

2.7 More expanding and factorising

Purposeful practice 1

1 $x^2 + 2x + 1$
2 $x^2 + 3x + 2$
3 $x^2 + 3x + 2$
4 $x^2 - 2x - 3$
5 $x^2 + 2x - 3$
6 $x^2 + 3x - 10$
7 $x^2 - 4x + 4$
8 $x^2 - 9x + 18$
9 $x^2 - 8x + 16$
10 $x^2 + 6x + 9$
11 $x^2 - 2x + 1$
12 $x^2 - 4x + 4$

Purposeful practice 2

1 a $(x + 1)(x + 1)$ b $(x + 2)(x + 3)$ c $(x + 1)(x - 2)$
d $(x + 3)(x + 4)$ e $(x - 3)(x - 4)$ f $(x + 3)(x - 4)$
g $(x - 5)(x + 4)$ h $(x + 7)(x + 10)$ i $(x - 1)(x - 1)$
j $(x + 2)(x + 7)$ k $(x - 3)(x - 3)$ l $(x - 5)(x - 5)$
2 a $(x + 1)(x - 1)$ b $(p + 2)(p - 2)$ c $(c + 3)(c - 3)$
d $(x + 10)(x - 10)$ e $(a + 6)(a - 6)$ f $(k + 13)(k - 13)$
g $(10 + x)(10 - x)$ h $(5 + y)(5 - y)$ i $(2 + k)(2 - k)$
j $(2x + 2)(2x - 2)$ or $4(x + 1)(x - 1)$
k $(3x + 4)(3x - 4)$ l $(4 + 3x)(4 - 3x)$

Problem-solving practice

1 $x + 4$
2 $a = 6$ $b = 2$
3 a $x(x + 3) = x^2 + 6$ b $x = 2$
4 Aidan has worked out −6 × −6 incorrectly. The answer should be +36 not −36.
5 $x^2 + 10x$
6 $x = 2$

Exam practice

1 $(x - 4)(x + 2)$

3 Interpreting and representing data

3.1 Statistical diagrams 1

Purposeful practice

1 **a** School A 350, School B 240 **b** School A

2 **a** School A 350, School B 320 **b** School A

3 School B

Problem-solving practice

1 It is not possible to tell how many matches either team won from the pie charts as the total number of matches played by each team is not known. Therefore, Jo's statement might not be correct.

2 Maisie won 8 more matches than Luke (50 compared to 42).

Exam practice

1 Becky is incorrect: she cannot tell as there is no information about the population size for this week or last week.

3.2 Time series

Purposeful practice

1 **a** 3.4 (million), 3.8, 4.2 so the numbers are increasing.

b Second quarter: 5 (million), 5.2, 5.4 so the numbers are increasing.
Third quarter: 5.2 (million), 5.4, 5.6 so the numbers are increasing.
Fourth quarter: 4.4 (million), 4.8, 5.2 so the numbers are increasing.

c The overall number of visitors is increasing each year.

d Each year the least number of visitors is in the first quarter, then it increases for the second and third quarters, and then decreases for the fourth quarter.

2 **a** 370 (thousand), 340, 320 so the numbers are decreasing.

b Second quarter: 350 (thousand), 320, 300 so the numbers are decreasing.
Third quarter: 380 (thousand), 350, 340 so the numbers are decreasing.
Fourth quarter: 420 (thousand), 410, 380 so the numbers are decreasing.

c The overall number of visitors is decreasing each year.

d Each year the number of visitors decreases from the first to the second quarter, with the lowest numbers for the year in the second quarter; the numbers then increase for the third quarter and then increase again for the fourth quarter.

Problem-solving practice

1 Mila has described the variation, not the trend. The overall trend is that the amount spent is increasing.

2 The number of hours students spend watching TV varies from term to term. The overall trend is that the number of hours watched remains similar.

Exam practice

1 Average visitors per day

$$= \frac{700 + 600 + 300 + 400 + 800 + 1300 + 1000}{7} = 728.57$$

This is less than 750, so the attraction did not meet the predicted number.

3.3 Scatter graphs

Purposeful practice

1 **a** 53 kg

b Positive correlation

c The greater the height of a student, the greater their weight.

2 **a** £9900 (approximately)

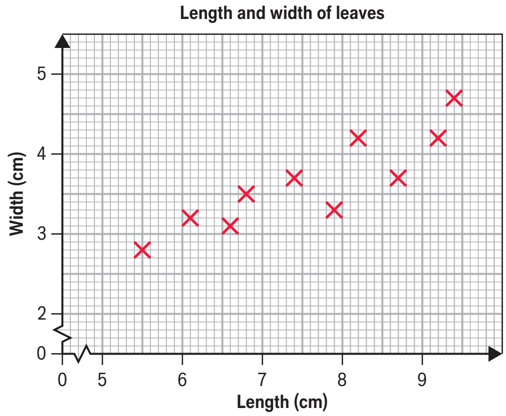
b Negative correlation

c The older the car, the lower its value.

Problem-solving practice

1 No, because there is no correlation, so there is no relationship between the height and test score of the students.

2



Yes, because there is positive correlation, so the longer the leaves, the wider they are.

Exam practice

1 **a** 25.8 cm

b Positive correlation

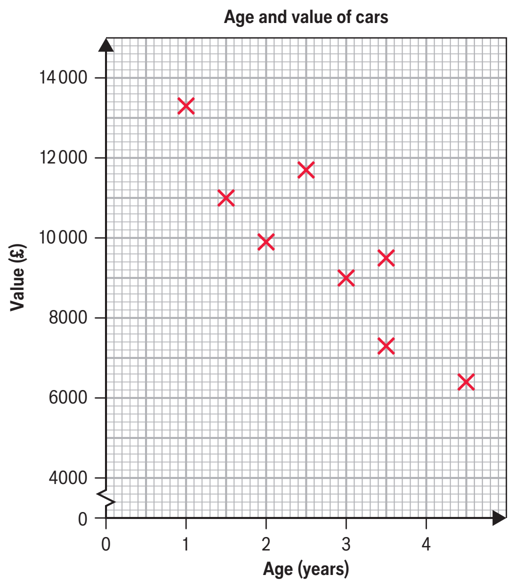
c Yes, the positive correlation shows that the longer the length of a student's hand, the greater the length of their foot.

3.4 Line of best fit

Purposeful practice

1 158.5 cm (approximately)

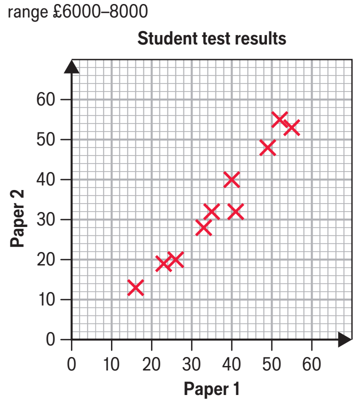
2 **a**



b Answers may vary. Check students' lines of best fit are reasonable.

c Students' own answers, depending on line of best fit drawn. Should fall in range £6000–8000

3 **a**



b 24–28

Problem-solving practice

1 Using a line of best fit suggests a more accurate estimate will be higher (8 cm–8.5 cm)

Exam practice

1 25.2–25.6 cm

3.5 Averages and range

Purposeful practice

1 a 0.5 hours
  b 1.5 hours, 2.5 hours, 3.5 hours, 4.5 hours
  c 205 hours
  d 80 students
  e 2.5625 hours
2 a 840 cm
  b 60 plants
  c 14 cm
3 a 125 staff
  b 34.8 years

Problem-solving practice

1 a The range of the data is from 80 to 180 cm so the mean should be somewhere in this range, but 1856 cm is outside this range.
  b Error 1 – Paul has not used the midpoint of each class interval; he has used the lowest value.
  Error 2 – Paul has divided by 5 instead of the total of the frequency column.
  c $(90 \times 12) + (110 \times 17) + (130 \times 25) + (150 \times 19) + (170 \times 6) = 10\,070$, $10\,070\text{ cm} \div 79 = 127.5\text{ cm}$

Exam practice

a £3000
b Yes, because the outliers in the range $5000 < x \leq 6000$ affect the mean.

3.6 Statistical diagrams 2

Purposeful practice

1 a

	Museum	Art Gallery	Theatre	Total
Male	29	7	19	55
Female	18	25	28	71
Total	47	32	47	126

  b 25
2 a

	Apple	Banana	Orange	Total
Girls	27	13	6	46
Boys	18	10	11	39
Total	45	23	17	85

  b 45
3 a 7
  b

	Home	UK	Abroad	Total
Girls	5	2	7	14
Boys	6	5	5	16
Total	11	7	12	30

  7

Problem-solving practice

1

	Walk	Car	Cycle	Total
Full-time	124	**48**	7	179
Part-time	**106**	26	**35**	**167**
Total	**230**	**74**	42	346

$\frac{106}{167}$

2

	$\frac{1}{2}$ litre bottles	1 litre bottles	2 litre bottles	Total
Saturday	9	16	19	44
Sunday	5	4	7	16
Total	14	20	26	60

$26 - 7 = 19$, so 19 2-litre bottles were sold on Saturday.
$60 - 16 = 44$, so 44 bottles in total were sold on Saturday.
So $44 - 9 - 19 = 16$, so 16 1-litre bottles were sold on Saturday.
$20 - 16 = 4$, so 4 1-litre bottles were sold on Sunday.
They sold the greatest number of 1 litre bottles on Saturday.

3 13

Exam practice

1 $\frac{11}{37}$

4 Fractions, ratio and percentages

4.1 Fractions

Purposeful practice 1

1 a $\frac{1}{2}$ b $\frac{1}{3}$ c $\frac{1}{4}$ d $\frac{1}{5}$
2 a 2 b 3 c 4 d 5
3 a 5 b $3\frac{1}{3}$ c $\frac{10}{13}$ d $\frac{10}{17}$
  e $8\frac{1}{3}$ f $6\frac{1}{4}$ g $6\frac{2}{33}$ h $1\frac{87}{113}$
4 a $3\frac{1}{2}$ b $2\frac{1}{3}$ c $1\frac{3}{4}$ d $1\frac{2}{5}$
5 a $\frac{6}{7}$ b $\frac{7}{16}$ c $\frac{9}{31}$ d $\frac{11}{49}$

Purposeful practice 2

1 a $3\frac{31}{35}$ b $3\frac{46}{63}$ c $3\frac{55}{63}$ d $4\frac{7}{36}$
2 a $1\frac{6}{35}$ b $1\frac{5}{28}$ c $1\frac{22}{63}$ d $\frac{17}{36}$
3 a $1\frac{13}{35}$ b $4\frac{1}{2}$ c $4\frac{4}{5}$ d $5\frac{5}{8}$
4 a $1\frac{1}{15}$ b $1\frac{2}{7}$ c $2\frac{5}{36}$ d $3\frac{9}{10}$

Problem-solving practice

1 $2\frac{2}{9}$ 2 Perimeter $= 12\frac{1}{14}$ m, Area $= 9\frac{1}{28}\text{ m}^2$
3 7 strips 4 $1\frac{17}{20}$ m
5 3 tins. Total area for two coats is $14\frac{3}{10}\text{ m}^2$
6 $\frac{5}{7}$ 7 $2\frac{1}{3}$ 8 $2\frac{9}{14}$

Exam practice

1 0.625
2 a $\frac{83}{15}$ or $5\frac{8}{15}$ b $3\frac{1}{4}$

4.2 Ratios

Purposeful practice 1

1 a 1 : 1.5 b 1 : 2.5 c 1 : 3.5 d 1 : 0.75
  e 1 : 1.25 f 1 : 1.75
2 a $\frac{2}{3}$: 1 b $\frac{2}{5}$: 1 c $\frac{2}{7}$: 1 d $\frac{4}{3}$: 1
  e $\frac{4}{5}$: 1 f $\frac{4}{7}$: 1
3 a 1 : 300 b 1 : 200 c 1 : 100 d 1 : 40
  e 1 : 12.5 f 1 : $\frac{70}{3}$ g 1 : 16

Purposeful practice 2

1 a £80 : £160 b £60 : £180
  c £30 : £210 d £96 : £144
  e £100 : £140 f £64 : £176
2 a £60 : £120 : £180 b £45 : £135 : £180
  c £36 : £144 : £180 d £72 : £108 : £180
  e £60 : £90 : £210 f £80 : £60 : £220

3 a £16.67 : £33.33 : £50.00 **b** £12.50 : £37.50 : £50.00
c £10.00 : £40.00 : £50.00 **d** £16.67 : £25.00 : £58.33
e £5.56 : £16.67 : £77.78 **f** £33.33 : £22.22 : £44.44

Problem-solving practice

1 Ann 6, Bert 12, Callum 24
2 Doris 15, Ed 25, Frank 40
3 24 years old
4 No, Sandi needs 10 more grams of butter; she has enough of everything else.
5 22.5 cm
6 1 : 280 000
7 25 ml of white, 10 ml of green and 125 ml of blue

Exam practice

1 243 cm^2

4.3 Ratio and proportion

Purposeful practice 1

1 a 1 : 1.5, Q = 1.5P **b** 1 : 2.5, Q = 2.5P
c 1 : 0.75, Q = 0.75P **d** 1 : 1.25, Q = 1.25P
e 1 : 0.375, Q = 0.375P **f** 1 : 0.625, Q = 0.625P
2 a $\frac{2}{3} : 1, P = \frac{2}{3}Q$ **b** $\frac{2}{5} : 1, P = \frac{2}{5}Q$
c $\frac{4}{3} : 1, P = \frac{4}{3}Q$ **d** $\frac{4}{5} : 1, P = \frac{4}{5}Q$
e $\frac{8}{3} : 1, P = \frac{8}{3}Q$ **f** $\frac{8}{5} : 1, P = \frac{8}{5}Q$

Purposeful practice 2

1 a R = 2D **b** R = 3D **c** R = 0.5D
d R = 1.5D **e** R = 0.25D
2 a Y = 3X, 30 **b** N = 2.5M, 25
c T = 3.5S, 16 **d** V = 0.25W, 60.8

Problem-solving practice

1 5 : 6
2 a No **b** Yes, Y = 6X **c** No
3 688 km
4 £192.18
5 5 kg bag is better value, at £2.40 for 1 kg. 8 kg bag is £2.50 for 1 kg.
6 6.08 m
7 12 inches

Exam practice

1 Milk is better value for money in Australia. Students' own workings, for example, in England, 1 litre costs \$(0.49 ÷ 0.568 × 1.76) = \$1.52, compared to \$1.44 in Australia.

4.4 Percentages

Purposeful practice 1

1 a 20% **b** 50% **c** 100%
d 12.5% **e** 25% **f** 50%
g 150% **h** 200% **i** 320%
2 a 25% **b** 37.5% **c** 87.5%
d 5% **e** 20% **f** 90%
g 80% **h** 75% **i** 98%
3 a 30% increase **b** 6% decrease **c** 20% increase
d 7.5% decrease **e** 300% increase **f** 96% decrease

Purposeful practice 2

1 a £80 **b** £90 **c** £320 **d** £600
2 a £60 **b** £48 **c** £192 **d** £150
3 a £65 **b** £84 **c** £45 **d** £5500

Problem-solving practice

1 Bob by 0.8% **2** 61.4% (1 d.p.) **3** £250 000
4 £0 **5** 0.875 **6** £240

Exam practice

1 £400 **2** 8.8% **3** £166 154

4.5 Fractions, decimals and percentages

Purposeful practice 1

1 $0.\dot{3}$ **2** $0.\dot{6}$ **3** $0.1\dot{6}$ **4** $0.8\dot{3}$
5 $0.\dot{1}$ **6** $0.\dot{4}$ **7** $0.\dot{1}\dot{8}$ **8** $0.\dot{6}\dot{3}$
9 $0.\dot{1}4285\dot{7}$ **10** $0.\dot{4}2857\dot{1}$

Purposeful practice 2

1 a $\frac{7}{9}$ **b** $\frac{2}{9}$ **c** $\frac{8}{9}$ **d** $\frac{5}{9}$
e $\frac{13}{99}$ **f** $\frac{31}{99}$ **g** $\frac{6}{11}$ **h** $\frac{5}{11}$
i $\frac{41}{333}$ **j** $\frac{107}{333}$ **k** $\frac{44}{333}$ **l** $\frac{104}{333}$
2 a $\frac{11}{18}$ **b** $\frac{19}{30}$ **c** $\frac{23}{45}$ **d** $\frac{26}{45}$

Problem-solving practice

1 $0.\dot{2}\dot{6}$, 27%, $\frac{3}{11}$, $\frac{7}{25}$ **2** $\frac{3}{22}$ **3** $0.\dot{4}$
4 $2\frac{7}{22}$ **5** $1\frac{4}{33}$ **6** $1\frac{7}{11}$
7
$$\begin{aligned} x &= 0.878\,787\,878\,7\ldots \\ 100x &= 87.878\,787\,878\,7\ldots \\ 100x - x &= 87 \\ 99x &= 87 \\ x &= \frac{87}{99} = \frac{29}{33} \end{aligned}$$
8 Sarah is correct. Ryan has not subtracted 6 from 65.

Exam practice

1
$$\begin{aligned} x &= 0.218\,181\,818\ldots \\ 10x &= 2.181\,818\,181\,8\ldots \\ 1000x &= 218.181\,818\,181\ldots \\ 990x &= 216 \\ x &= \frac{216}{990} = \frac{12}{55} \end{aligned}$$
2
$$\begin{aligned} x &= 0.136\,363\,636\,3\ldots \\ 10x &= 1.363\,636\,363\,6\ldots \\ 1000x &= 136.363\,636\,36\ldots \\ 990x &= 135 \\ x &= \frac{135}{990} = \frac{3}{22} \\ y &= 0.444\,444\,44\ldots \\ 10y &= 4.444\,444\,4\ldots \\ 9y &= 4 \\ y &= \frac{4}{9} \\ x \times y &= \frac{3}{22} \times \frac{4}{9} = \frac{2}{33} \end{aligned}$$

Mixed exercises A

Mixed problem-solving practice A

1 No, you must multiply them, not add.
2 190
3 8.30 am
4 No, because the information about how many matches each team played is not given in the question. It is only possible to say what proportion of matches they won.
5 22 cm
6 a $7n - 5$
b No, solving $7n - 5 = 200$ doesn't give a whole number solution, so 200 is not in the sequence.
7 a 20 minutes and 15 seconds (or 20.25 minutes)
b Yes, as the mean is affected by the 8 higher values in the class interval $40 < t \leqslant 60$
8 No, as $5.8 \times 10^7 \times 100 = 5.8 \times 10^9$, $1.427 \times 10^9 < 5.8 \times 10^9$
9 a £113.04
b In Madrid the shirt costs £63.83, so it is £3.83 (or €4.40) cheaper in London.
10 Karen hasn't multiplied the first numerator by 5 and the second numerator by 3, so she has not replaced fractions with equivalent ones.

11 24

12 £234

13 $10x = 3.1515...$, $1000x = 315.1515...$, $990x = 312$, $x = \frac{312}{990}$, $x = \frac{52}{165}$

14 $2n^2 + n - 2$

Exam practice

15 Yes, as $\frac{80}{30} = 2\frac{1}{3}$, $\frac{90}{40} = 2\frac{1}{4}$

so the percentage decrease $= \dfrac{2\frac{1}{3} - 2\frac{1}{4}}{2\frac{1}{3}} \times 100 = 15.625\% < 20\%$

16 Yes, number of children $= 108 \div 3 \times 4 = 144$,
number of people $= 144 \div 2 \times 7 = 504$,
percentage of seats filled $= 504 \div 800 \times 100 = 63\%$, which is more than 60%.

17 **a** 13 years **b** Negative correlation

c Yes, as the points for dogs that are heavier appear where there are lower life expectancies.

d Answer in the range of 10.6–12.6 years.

18 **a** 0.2

b $\sqrt{\frac{1.05}{1.1}} \times 100 = 97.70...\%$, which is less than 100%, so it will decrease by 2.30%.

19 $x = \frac{34}{11}$

20 **a** $(p + q)(p - q)$ **b** $24(x^2 + 3)$

21 $-\frac{7}{10}$

22 Kate has written $\sqrt{20}$ as $5\sqrt{2}$ instead of $2\sqrt{5}$

5 Angles and trigonometry

5.1 Angle properties of triangles and quadrilaterals

Purposeful practice 1

1 $x = 140°, y = 40°$ **2** $x = 60°, y = 40°$ **3** $x = 20°, y = 100°$

4 $x = 80°, y = 20°$ **5** $x = 130°$ **6** $x = 120°$

Purposeful practice 2

1 $x = 120°$ **2** $x = 30°$ **3** $x = 30°$

Problem-solving practice

1 **a** Mia included the angle at the same vertex as the exterior angle. She needs to add the angles at the other two vertices.

b $x = 130°$

2 $x = 100°$ Angles within the equilateral triangle are 60°, so the larger angles within the isosceles triangle are $180° - 60° - 40° = 80°$ (angles on a straight line add to 180°). So $x = 180 - 80 = 100°$ (angles on a straight line add to 180°).

3 No.
Angle ACB = 43° (corresponding angles are equal).
Angle BAC = 43° (ABC is isosceles triangle).
Angle ABC = 94° (angles in a triangle add to 180°).
Therefore, quadrilateral is not a rectangle (at least one angle is not a right angle).

4 $x = 90°$. Students' reasoning may vary, for example,
Angle ACF = 135° (corresponding angles are equal).
Angle ACB = 45° (angles on a straight line add to 180°).
Angle ABC = 45° (ABC is isosceles triangle).
Therefore, $x = 90°$ (angles in a triangle add to 180°).

5 $y = 13°$. Students' reasoning may vary, for example
Angle DFE = 13° (angles on a straight line add to 180°).
Angle EDF = 13° (DEF is isosceles).
Therefore, angle $y = 13°$ (corresponding angles are equal).

Exam practice

1 Angle AFB = 40° (vertically opposite angles are equal)
Angle BAD = 65° (opposite angles of a parallelogram are equal)
Angle ABF = 180° − 40° − 65° = 75° (angles of a triangle add to 180°)

5.2 Interior angles of a polygon

Purposeful practice 1

1 156° **2** 157.5° **3** 158.8° **4** 160° **5** 161.1°

Purposeful practice 2

1 $x = 110°$ **2** $x = 120°$ **3** $x = 157°$

Purposeful practice 3

1 6 **2** 12 **3** 24

Problem-solving practice

1 Students' answers will vary, but should include a counter-example, for example, a regular heptagon has an interior angle of 128.57°.

2 No, a regular octagon has an interior angle of 135°.
Sum of interior angles $= (8 - 2) \times 180° = 1080°$.
In a regular octagon, angles are the same so each angle $= 1080° \div 8 = 135°$.

3 Students' own answers. Any combination of three angles that add to 360°.

4 Angle FEA = 60°. Students' reasons may vary, for example, the hexagon is regular so AGH is an equilateral triangle with interior angles of 60°.
Since FAE = 60° and AE = AF, angle AEF and angle AFE are both equal to $\frac{1}{2}(180° - 60°) = 60°$.

5 $x = 112°$

6 No, because the interior angles of a regular pentagon are all 108° and there is no combination of 108° that can sum to 360°, so there will always be a gap.

Exam practice

1 Angle BCD = 135°.
Students' own working, for example,
Let angle ABC be x. Therefore, angle BCD $= 3x$.
Sum of internal angles of a pentagon $= (5 - 2) \times 180° = 540°$.
So, $90° + 125° + 145° + 4x = 540°$.
Thus $4x = 180°$, so $x = 45°$.
Therefore, angle BCD $= 3 \times 45° = 135°$.

5.3 Exterior angles of a polygon

Purposeful practice 1

1 d, f, o, l, m

Purposeful practice 2

1 $w = 80°, x = 70°, y = 90°, z = 100°$; $70 + 90 + 100 + 100 = 360°$

2 $a = 50°, b = 70°, c = 135°, d = 120°, e = 85°, f = 130°$
$50 + 70 + 45 + 60 + 85 + 50 = 360°$

3 $g = 36°, h = 80°, i = 104°, j = 110°, k = 110°, l = 58°$
$36 + 80 + 76 + 110 + 58 = 360°$

Purposeful practice 3

1 36° **2** 32.7° **3** 30° **4** 27.7°

Problem-solving practice

1 Students' sketches of an equilateral triangle.
An equilateral triangle has exterior angles $= \frac{360°}{3} = 120°$. A 4-sided regular polygon (square) has exterior angles = 90°. As more sides get added, the angles get smaller, so the only regular polygon with obtuse exterior angles is an equilateral triangle.

2 $360° \div 72 = 5°$ so the shape would have 72 sides.

3 $y = 36°$ **4** 12 sides

5 16 sides **6** $x = 144°$

Exam practice

1 Angle of equilateral triangle = 60°
Sum of interior angles of 15-sided polygon $= 13 \times 180° = 2340°$
Interior angle of regular 15-sided polygon = 156°
Interior angle of polygon P $= 360° - 60° - 156° = 144°$
Exterior angle of polygon P $= 180° - 144° = 36°$
Number of sides of polygon P $= 360° \div 36° = 10$

5.4 Pythagoras' theorem 1

Purposeful practice 1

1 a, d

Purposeful practice 2

1 5 cm **2** 5.8 cm **3** 7.2 cm **4** 7.8 cm

Purposeful practice 3

1 Yes, $6^2 + 8^2 = 10^2$ **2** No, $6.1^2 + 8.1^2 \neq 10.1^2$

3 No, $5.9^2 + 7.9^2 \neq 9.9^2$

Problem-solving practice

1 0.2 m
2 £30 to buy 2 m² (or £17.11 to buy 1.14 m² if possible to buy the exact area required)
3 34.4 cm   4 217.6 cm   5 0.9 km (to 1 d.p.)

Exam practice

1 28.2 kg

5.5 Pythagoras' theorem 2

Purposeful practice 1

1 12 m   2 5 m   3 13 m
4 5.7 m   5 7.3 m   6 3.9 m

Purposeful practice 2

1 a $\sqrt{7}$ cm   b 2.6 cm
2 a $\sqrt{45}$ cm = $3\sqrt{5}$ cm   b 6.7 cm
3 a $\sqrt{5}$ cm   b 2.2 cm

Purposeful practice 3

1 2.4 cm   2 2.1 cm   3 3.9 cm

Problem-solving practice

1 12.6 m   2 43.3 cm²   3 5.4 m   4 110.9 cm²   5 0.6 m

Exam practice

1 $CD^2 = 45 - 9 = 36$, so CD = 6 cm
AE = 6 cm − 4 cm = 2 cm
AD = 3 cm + 1 cm = 4 cm
Area of triangle DEF =
$(6\text{ cm} \times 4\text{ cm}) - \frac{1}{2} \times 6\text{ cm} \times 3\text{ cm} - \frac{1}{2} \times 4\text{ cm} \times 1\text{ cm} - \frac{1}{2} \times 4\text{ cm} \times 2\text{ cm}$
$= 9\text{ cm}^2$

5.6 Trigonometry 1

Purposeful practice 1

1 8 cm   2 4.6 cm   3 8.7 cm   4 6.1 cm

Purposeful practice 2

1 2 cm   2 2.3 cm   3 2.9 cm   4 3.5 cm

Purposeful practice 3

1 3.5 cm   2 6.9 cm   3 3.5 cm   4 6.9 cm

Problem-solving practice

1 17.4 cm²   2 2.2 m   3 28.3 m   4 6.60 cm   5 8.7 km

Exam practice

1 19.2 cm

5.7 Trigonometry 2

Purposeful practice 1

1 48.6°   2 41.4°   3 53.1°   4 59.0°   5 36.9°   6 53.1°

Purposeful practice 2

1 a $x = 63.4°$ b $y = 63.4°$
2 a $x = 5.7°$ b $y = 5.7°$

Problem-solving practice

1 $x = 70.5°$
2 Angle ACB $= \cos^{-1}\left(\frac{5}{12}\right) = 65.4°$,
so angle ABC $= 180° - 40° - 65.4° = 74.6°$.
Triangle ABC has no equal angles and therefore is not isosceles.
3 60.9°   4 123.7°   5 41.8°, 66.4°, 71.8°

Exam practice

1 39.3°

6 Graphs

6.1 Linear graphs

Purposeful practice 1

1 a B, E, F, H, I   b A, C, D, G
c A, B, G, H   d A, C and D; B, E and F; H and I
2 B, A, E, D, C. (A and E are as steep as each other, so the order of those two can be swapped.)
3 A is $y = 2x + 4$, B is $y = 2x + 3$, C is $y = 2x$, D is $y = 2x - 1$

Purposeful practice 2

1 a 12   b 8   c 6   d 3
2 a (−6, 0)   b (4, 0)   c (−3, 0)   d (2, 0)
3 a 4   b 4

Problem-solving practice

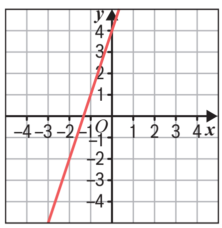
1 Peter is incorrect. B and C each have a gradient of 2, but A rearranges to $y = -2x + 8$ so it has gradient −2. D rearranges to $y = \frac{1}{2}x + 2$ so it has gradient $\frac{1}{2}$.
2 Only Rebecca is correct. It will cross the y-axis at (0, 4) so Sarah is wrong, and cross the x-axis at (2.5, 0) so Theresa is wrong.
3 Line A is $y = \frac{2}{3}x + 2$ and line B is $y = \frac{1}{2}x + \frac{3}{2}$
4 C $5x + 6y = 21$ (intercept 3.5), A $2y = 4x + 8$ (intercept 4), B $3y = 4x + 13$ (intercept $4\frac{1}{3}$), D $3y + 2x + 15 = 0$ (intercept −5)
5 a Students' answers will vary, so accept any equations where the coefficient of x in the first equation is half the coefficient of x in the second equation, for example, $y = 4x + 3$ and $2y = 8x + 4$
b Students' answers will vary, so accept any equations where the constant term in the first equation is half the constant term in the second equation, for example, $y = 2x + 3$ and $2y = 2x + 6$
c Any equations where the ratio of the coefficient of x to the constant term is the same for both equations, for example, $y = x + 3$ and $2y = 3x + 9$.

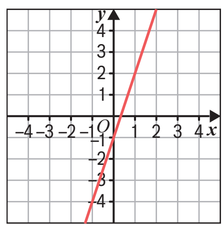
Exam practice

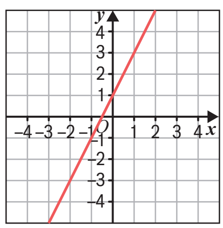
1 The equation for L2 can be rearranged to give $y = 2x + \frac{1}{2}$, so the gradient of both lines is 2. Same gradient shows the lines are parallel.
2 A and D

6.2 More linear graphs

Purposeful practice 1

1 a


b


c


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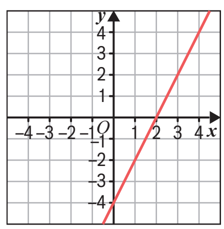
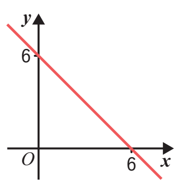


2 $y = 5x + 2$ = C; $y = 5x - 3$ = A; $y = 2 - 3x$ = D; $y = 4 - 3x$ = B

3 a


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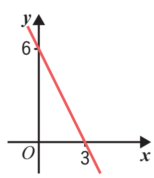
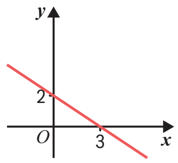


c



Purposeful practice 2

1 a (3, 14) b (3, 15) c (3, 9)

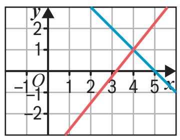
2 a (8, 8) b (8, 0) c (0, −4) and (8, −8)
d (8, 4), (0, 8) and (12, 2) e (8, 0), (0, −4), (−8, −8) and (12, 2)
f (0, −8) and (−8, −4)

Problem-solving practice

1 A has gradient of (6 − 2) ÷ (5 − 1) = 1
B has gradient of (4 − 2) ÷ (5 − 1) = $\frac{1}{2}$
C has gradient of (−6 − 2) ÷ (5 − 1) = −2
B is the only graph with a gradient of $\frac{1}{2}$

2 a $y = 2x - 1$ b $y = \frac{1}{2}x + 3$ c $y = -3x + 12$
d $y = 3x - 1$ e $y = -4x + 13$ f $y = -0.5x + 2.5$

3 a



b (4, 1)

4 $y = \frac{1}{2}x + \frac{5}{2}$

Exam practice

1 $y = 3x - 2$

6.3 Graphing rates of change

Purposeful practice 1

1 a 10 m/s b 1.5 seconds c At point D
d Between A and B (in the first second); between C and D (2.5 to 3.5 seconds)
e Slowing down and coming to rest

Purposeful practice 2

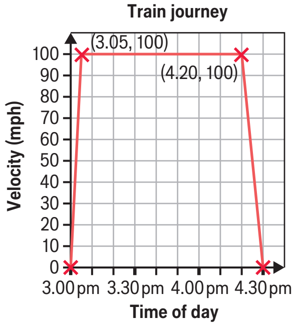
1 1600 km/h² 2 200 km 3 3600 km 4 200 km

Problem-solving practice

1 a 10 minutes b 12 km c 0.0083 m/s² d 7.2 km

2 a 60 m/s b 5 m/s² c 3
d Answers in the range 2750 m to 2800 m.

3 Train journey



Distance = $\frac{1}{2}$(1.5 + 1.25) × 100 = 137.5 miles

Exam practice

1 a $\frac{20}{40}$ = 0.5 m/s²
b $\frac{1}{2}$(50 + 30) × 40 + 80 × 50
= 1600 + 4000 = 5600 m
= 5.6 km

6.4 Real-life graphs

Purposeful practice

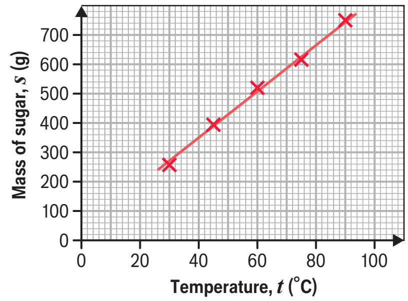
1 a $18 b £16 or £17
c i Allow between 1.8 and 1.9
ii You receive $1.80 – $1.90 for each £1

2 a £110 b 8 hours
c i £10 ii The cost to hire the hall, before time is taken into account. This could be called a 'standing charge'.
d i £25 ii The cost per hour

3 a i 24 °C ii The temperature of the room when the freezer was turned on
b 6 °C c 12 hours
d i −3 ii That the freezer gets 3 °C colder each hour

Problem-solving practice

1 a Sugar dissolving in tea



b 670 g (approx.) c 84 °C (approx.)
d The gradient is about 8. This means 8 g more sugar will dissolve for every 1 °C increase in temperature.

2 a Company A £1250; Company B £3500
b Company A £200 (approx); Company B £80 (approx). These are the costs per month.
c At 12 months, Company A is cheaper.
d At 2 years, Company B is cheaper.
e They cost the same after approximately 19 months.

Exam practice

1 a −2
b The rate at which the liquid flows from the container (2 litres per second)
c The volume of liquid in the container at the beginning

6.5 Line segments

Purposeful practice 1

1 (2, 4) 2 (3, 5) 3 $\left(2\frac{1}{2}, 4\frac{1}{2}\right)$
4 (0, 5) 5 (4, 0) 6 $\left(-\frac{1}{2}, 4\frac{1}{2}\right)$
7 (−3, −1) 8 (−4, 0) 9 $\left(-3\frac{1}{2}, 4\frac{1}{2}\right)$
10 $\left(3\frac{1}{4}, -\frac{3}{4}\right)$ 11 $\left(-\frac{3}{4}, 4\frac{1}{4}\right)$ 12 $\left(-\frac{1}{4}, -\frac{1}{4}\right)$

Purposeful practice 2

1 Just A 2 B and C 3 $y = -\frac{1}{2}x + 5$ 4 $y = 2x - 5$

Problem-solving practice

1 $y = -7x + 6$

2 Perpendicular gradient should be −7, not 7.
Tim needs to find midpoint of line segment and substitute its coordinates into equation $y = -7x + c$ (rather than coordinates of one end of segment, as he has done).
Tim has substituted values incorrectly (substituted x-value for y and y-value for x).

3 a (2, 3), (4, 1) and (4, 3)
b (4, 5), (6.5, 5), (5, 3) and (2.5, 3)
c (3.25, 4), (5.25, 5), (5.75, 4), (3.75, 3)

4 Eliza is correct. The gradient of the line segment from (−3, 2) to (−1, 5) is $\frac{3}{2}$. The gradient of the line segment from (−3, 2) to $(-2, 1\frac{1}{3})$ is $-\frac{2}{3}$. Therefore these line segments are perpendicular and $(-2, 1\frac{1}{3})$ could be the a vertex of the rectangle.

5 **a** $y = \frac{1}{2}x - \frac{1}{2}$ and $y = -2x + 12$

b (2, 3) and (4, 4) or (6, 0) and (4, −1)

c 3.2 (1 d.p.)

Exam practice

1 $y = -x + 11$

6.6 Quadratic graphs

Purposeful practice

1 A and D are quadratic graphs

2 B and C are quadratic equations

3 Equation **i** is graph C; equation **ii** is graph A; equation **iii** is graph D; equation **iv** is graph B

4 **a** 1 **b** 2 **c** 0

Problem-solving practice

1 **a** $x = -0.7$ and 3.7 **b** $x = -0.8$ and 1.3 **c** $x = 3$

2 **a** **i** $x = -3$ and 2 **ii** $x = 1.4$ and −1.4 (approx) **iii** $x = -4$ and 2

b The graphs of $y = x^2 + x - 6$ and $y = 3x - 8$ never meet, so the equation cannot have any solutions.

Exam questions

1 It should be joined with a smooth curve

6.7 Cubic and reciprocal graphs

Purposeful practice

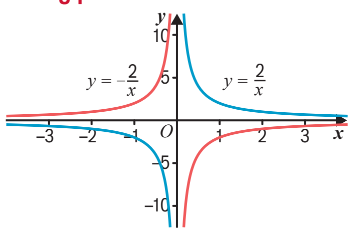
1 A and C are cubic graphs

2 **a** Cubic **b** Quadratic **c** Reciprocal **d** Cubic

3 **a** 2 **b** 3 **c** 1 **d** 1

Problem-solving practice

1 **a** and **b**



2 **a** $x = -2.9, -0.3, 1.2$ (approx) **b** $x = -3, 0, 1$

3 **a** $a = 1, b = 2$ **b** $a = 2, b = 1$ **c** $a = -1, b = -2$ **d** $a = -1, b = \frac{1}{2}$

Exam questions

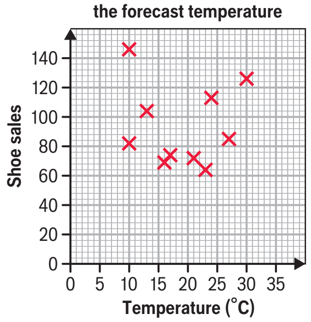
1 **i C** **ii D** **iii F** **iv E**

6.8 More graphs

Purposeful practice 1

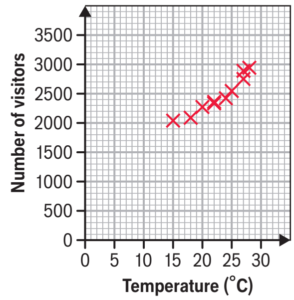
1 There is no relationship between the temperature and shoe sales.

Sales of shoes and the forecast temperature



2 There is a positive correlation between temperature and visitors, which suggests that the higher the temperature forecast, the more visitors come to the theme park.

Number of visitors and the forecast temperature



Purposeful practice 2

1 Only D will produce a circle.

2 **a** 2 **b** 4 **c** 8 **d** 10 **e** $\sqrt{7}$

3 Only A and B lie on the circle.

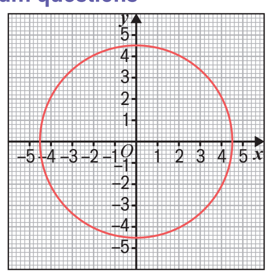
Problem-solving practice

1 **a** 2500 **b** 925 (approx) **c** 16 minutes **d** 7 minutes

2 **i** C **ii** A **iii** B

Exam questions

1



7 Area and volume

7.1 Perimeter and area

Purposeful practice 1

1 56 cm^2 2 56 cm^2 3 56 cm^2

Purposeful practice 2

1 28.7 cm^2 2 8.5 m^2 3 60 cm^2 4 60 cm^2

Problem-solving practice

1 **a** $h = 7.5$ cm **b** $a = 8$ cm **c** $b = 8$ cm

2 **a** James:

- forgot the $\frac{1}{2}$ from the formula
- used the side length of 8 cm, not the perpendicular height of 5 cm
- incorrectly worked out his calculation – he should have completed the calculation inside of the brackets first

b The correct answer is $\frac{1}{2}(6 + 10) \times 5 = 40$ cm^2

3 Area of A = 54 cm^2, b = 21 cm, c = 4.5 cm, d = 5 cm

Exam practice

1 Accept any triangle with area of 9 cm^2

7.2 Units and accuracy

Purposeful practice 1

1 **a** 48 cm^2 **b** 4800 mm^2

2 **a** 17.5 cm^2 **b** 1750 mm^2

3 **a** 11.2 cm^2 **b** 1120 mm^2

Purposeful practice 2

1 **a** **i** 13.5 mm $\leqslant l \leqslant$ 16.5 mm **ii** 135 cm $\leqslant l \leqslant$ 165 cm

iii 1.35 m $\leqslant l \leqslant$ 1.65 m **iv** 13.5 m $\leqslant l \leqslant$ 16.5 m

b **i** 14.25 mm $\leqslant l \leqslant$ 15.75 mm **ii** 142.5 cm $\leqslant l \leqslant$ 157.5 cm

iii 1.425 m $\leqslant l \leqslant$ 1.575 m **iv** 14.25 m $\leqslant l \leqslant$ 15.75 m

c i $14.625\text{ mm} \leqslant l \leqslant 15.375\text{ mm}$ ii $146.25\text{ cm} \leqslant l \leqslant 153.75\text{ cm}$
iii $1.4625\text{ m} \leqslant l \leqslant 1.5375\text{ m}$ iv $14.625\text{ m} \leqslant l \leqslant 15.375\text{ m}$

2 **a** $17.5\text{ cm} \leqslant l < 18.5\text{ cm}$ **b** $179.5\text{ mm} \leqslant l < 180.5\text{ mm}$
c $1.795\text{ m} \leqslant l < 1.805\text{ m}$ **d** $1.75\text{ m} \leqslant l < 1.85\text{ m}$
e $2.25\text{ m} \leqslant l < 2.35\text{ m}$ **f** $2.295\text{ m} \leqslant l < 2.305\text{ m}$
g $1.95\text{ km} \leqslant l < 2.05\text{ km}$ **h** $1.995\text{ km} \leqslant l < 2.005\text{ km}$

Problem-solving practice

1 **a** Holly has the wrong answer for 0.8×0.8. The answer is 0.64 cm^2.
b She might find it easier to find the area in mm^2 and then divide by 100 to give cm^2.

2 **a** $h = 6$ cm or 60 mm **b** $b = 40$ mm or 4 cm

3 Adam, Charlie and Daisy

4 **a** Lower bound 4932.25 m^2, upper bound 5078.25 m^2
b Lower bound 4974.0075 m^2, upper bound 4988.5975 m^2

5 No, because the actual measurements could be bigger than 4 m and 5 m, making the area more than 20 m^2. The upper bound is $4.5 \times 5.5 = 24.75\text{ m}^2$, which would require 25 carpet tiles.

Exam practice

1 $4.5\text{ cm} \leqslant L < 5\text{ cm}$

7.3 Prisms

Purposeful practice 1

1 222 cm^2 **2** 420 cm^2 **3** 247.2 cm^2 **4** 216 cm^2

Purposeful practice 2

1 **Q1:** 180 cm^3 **Q2:** 360 cm^3 **Q3:** 187.2 cm^3 **Q4:** 144 cm^3
2 **Q1:** 180 ml **Q2:** 360 ml **Q3:** 187.2 ml **Q4:** 144 ml

Problem-solving practice

1 6 units **2** 6.25 cm

3 Fifty 8 cm cubes have a surface area of $50 \times 6 \times 8^2 = 19\,200\text{ cm}^2$, so yes one container of paint is enough as $20\,000 - 19\,200 = 800\text{ cm}^2$ of paint remaining.

4 Square base has sides of 5 cm, cuboid is 20 cm tall. Volume $= 500\text{ cm}^3$

5 108 cm^2

Exam practice

1 Volume of cuboid $= 230\text{ cm} \times 120\text{ cm} \times 15\text{ cm} = 414\,000\text{ cm}^3 = 414$ litres
414 litres $\div$ 50 litres $= 8.28$, so the gardener needs 9 bags of compost.
Therefore, it will cost £63, which is £3 more than £60.

7.4 Circles

Purposeful practice 1

1 **a** 56.5 cm **b** 113.1 cm **c** 22.0 mm
d 44.0 mm **e** 47.1 m **f** 94.2 m

2 **a** 18π cm **b** 36π cm **c** 7π mm
d 14π mm **e** 15π m **f** 30π m

Purposeful practice 2

1 **a** 254.5 cm^2 **b** 1017.9 cm^2 **c** 38.5 mm^2
d 153.9 mm^2 **e** 176.7 m^2 **f** 706.9 m^2

2 **a** $81\pi\text{ cm}^2$ **b** $324\pi\text{ cm}^2$ **c** $\frac{49}{4}\pi\text{ mm}^2$
d $49\pi\text{ mm}^2$ **e** $\frac{225}{4}\pi\text{ m}^2$ **f** $225\pi\text{ m}^2$

Problem-solving practice

1 50.3 cm **2** 4 units

3 The father's wheel turns 2273 times, the daughter's 5305 times. The daughter's wheel turns 3032 more times.

4 32.93 m^2 **5** Area $= 9.42\text{ cm}^2$

Exam practice

1 $x = 6.38$

7.5 Sectors of circles

Purposeful practice 1

1 **a** 6.3 cm **b** 8.0 cm **c** 14.6 cm

2 **a** 11.3 cm **b** 17.7 cm **c** 40.6 cm
d 54.6 cm **e** 56.5 cm **f** 43.9 cm

Purposeful practice 2

1 **a** 12.6 cm^2 **b** 15.9 cm^2 **c** 29.2 cm^2

2 **a** $4\pi\text{ cm}^2$ **b** $\frac{76\pi}{15}\text{ cm}^2$ **c** $\frac{418}{45}\pi\text{ cm}^2$

Problem-solving practice

1 **a** Graham:
- missed out a 2 from his calculation, so the arc length should be 50.3 cm
- used the wrong units
- forgot to add on the two radii to give a perimeter

b 102.7 mm

2 The 40° sector has an area of 50.3 m^2. The smaller sector has an area of 19.6 m^2. The shaded area is 30.6 m^2.

3 There are two ways to solve this. Students will need to work out the interior angles of the hexagon (120°). Then either:
- find the area of one sector $\frac{120}{360} \times \pi \times 5^2 = 26.18\text{ cm}^2$ and multiply this by 6 to give 157.1 cm^2

or
- realise they have six thirds of a circle – which is the same as two complete circles – and so calculate $2 \times \pi \times 5^2 = 157.1\text{ cm}^2$

Exam practice

1 29.3 cm

7.6 Cylinders and spheres

Purposeful practice 1

1 628.3 cm^3 **2** 785.4 cm^3 **3** 7068.6 cm^3

Purposeful practice 2

1 478 mm^2 **2** 729 mm^2 **3** 2790 mm^2

Purposeful practice 3

1 **a** 2144.66 mm^3 **b** 804.25 mm^2
2 **a** 57.91 cm^3 **b** 72.38 cm^2
3 **a** 24.43 m^3 **b** 40.72 cm^2

Problem-solving practice

1 Volume of cylinder A $= 3769.91\ (1200\pi)\text{ cm}^3$
Volume of cylinder B $= 30\,159.29\ (9600\pi)\text{ cm}^3$
The volume of cylinder B is $8 \times$ larger than the volume of cylinder A.
Maria might have realised this by looking at the formula, $\pi r^2 h$. When the r is doubled then the volume will be multiplied by 4, and then when the height is doubled this multiplies by a further 2, making the volume 8 times greater overall.

2 $37\,900\,000\text{ km}^2$ **3** 161 cm^3 **4** 3.23 cm

Exam practice

1 236 cm^2 (to 3 s.f.), students' working may vary. Volume of sphere is $\frac{4}{3}\pi r^3$, so volume of hemisphere is $\frac{2}{3}\pi r^3$.
For P, $\frac{2}{3}\pi r^3 = \frac{250}{3}\pi$. $r^3 = 125$, so radius of P $= 5$ cm.
Surface area of sphere $= 4\pi r^2$, so area of curved surface of hemisphere $= 2\pi r^2$.
Area of flat surface of hemisphere $= \pi r^2$, so total surface area of a hemisphere $= 3\pi r^2$.
Surface area of P is $3\pi \times 5^2\text{ cm}^2 = 236\text{ cm}^2$ (to 3 s.f.)

7.7 Pyramids and cones

Purposeful practice 1

1 **a** 405 cm^3 **b** 300 cm^3 **c** 500 cm^3
2 **a** $225\pi\text{ cm}^3$ **b** $150\pi\text{ cm}^3$ **c** $100\pi\text{ cm}^3$

Purposeful practice 2

1 **a** $96\pi\text{ cm}^2$ **b** $160\pi\text{ cm}^2$ **c** $300\pi\text{ cm}^2$
2 **a** 169.6 cm^2 **b** 301.6 cm^2 **c** 16.5 cm^2

Problem-solving practice

1 The volume is $2\,592\,100\text{ m}^3$, so 2 592 100 stones were needed to fill this volume. Even with some smoothing needed on the sloping edges this answer would be fairly accurate.

2 2.7 cm

3 1 single ice cream has a volume of 208.69 cm^3 so she can make 23 complete ice creams from a 5 litre container.

4 579.41 cm^2

Exam practice

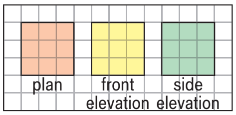
1 230 m (to nearest m)

8 Transformations and constructions

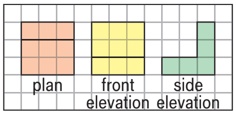
8.1 3D solids

Purposeful practice 1

1



2 a



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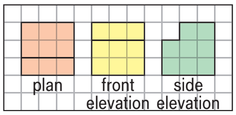

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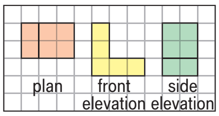
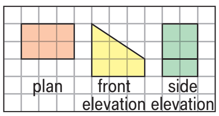


Purposeful practice 2

1 a



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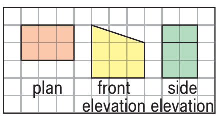
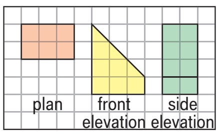


2 a



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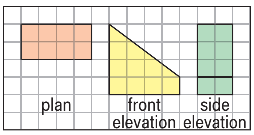

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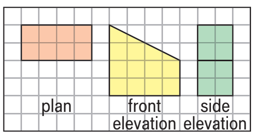
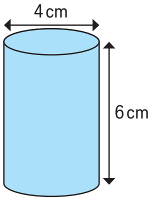


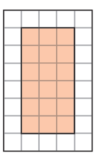
Problem-solving practice

1 a



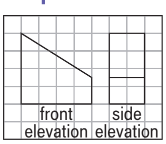
b Cylinder

2



Exam practice

1



8.2 Reflection and rotation

Purposeful practice 1

1 a Reflection in the y-axis or line $x = 0$

b Reflection in the line $y = 5$

c Reflection in the line $y = -2$

d Reflection in the line $x = -3$

Purposeful practice 2

1 a Rotation 90° clockwise about (0, 0)

b Rotation 90° clockwise about (0, –1)

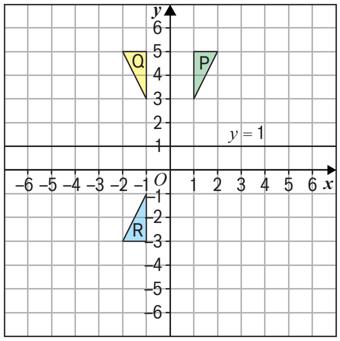
c Rotation 90° clockwise about (–1, –1)

d Rotation 90° clockwise about (–2, –1)

Problem-solving practice

1 Sophie has stated the angle and direction correctly (90° anticlockwise) to score 1 mark. The centre of rotation is (–1, 0) not (0, 1). She needed to state that the transformation is a rotation for a third mark.

2 a–c



d Rotation 180° about (0, 1)

Exam practice

1 Rotation 180° about (–1, –1)

2 Rotation 90° clockwise about (0, 1)

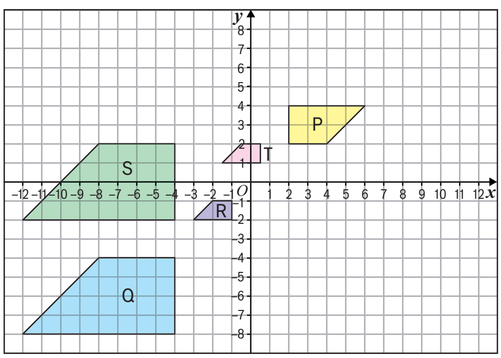
8.3 Enlargement

Purposeful practice 1

1 B: $\frac{1}{2}$, C: 2, D: $-\frac{1}{2}$, E: –1, F: –2

Purposeful practice 2

1–4

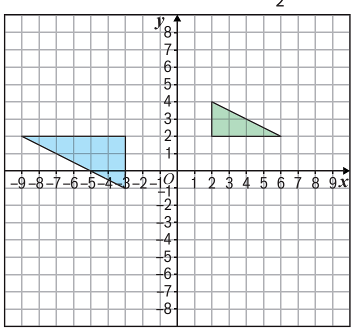


Problem-solving practice

1 a Olivia has given the scale factor from triangle Q to triangle P rather than from triangle P to triangle Q. The centre of enlargement is not (0, 0).

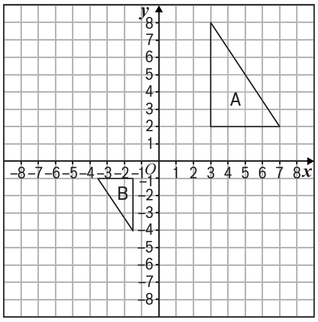
b An enlargement with scale factor $-\frac{1}{2}$, centre of enlargement (4, 2)

2



Exam practice

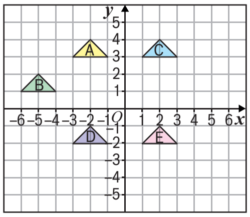
1



8.4 Translations and combinations of transformations

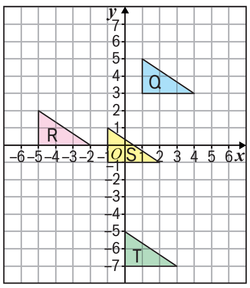
Purposeful practice 1

1–3



Purposeful practice 2

1, 2, 4, 5



3 $\begin{pmatrix}-7\\-2\end{pmatrix}$ 6 $\begin{pmatrix}-2\\-9\end{pmatrix}$

Problem-solving practice

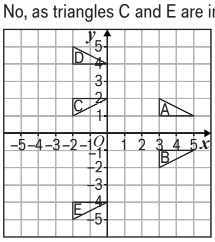
1 $\begin{pmatrix}-4\\5\end{pmatrix}$

2 Students' vectors that total $\begin{pmatrix}9\\-4\end{pmatrix}$, for example $\begin{pmatrix}5\\-2\end{pmatrix}$ and $\begin{pmatrix}4\\-2\end{pmatrix}$; $\begin{pmatrix}-1\\1\end{pmatrix}$ and $\begin{pmatrix}10\\-5\end{pmatrix}$

3 Translation by the vector $\begin{pmatrix}-7\\2\end{pmatrix}$

Exam practice

1 No, as triangles C and E are in different positions.



8.5 Bearings and scale drawings

Purposeful practice 1

1 a i 050° ii 230°
b i 070° ii 250°
c i 140° ii 320°

2 a i 130° ii 310°
b i 100° ii 280°
c i 060° ii 240°

3 a 080° b 335°

Purposeful practice 2

1 a 210° b 245° c 315°

2 a 010° b 045° c 125°

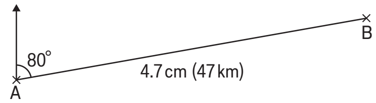
Problem-solving practice

1 220°

2 054°

3 a Sam is incorrect as 110° is the bearing of B from A, not A from B.
b Paul is incorrect as he has worked out the acute angle at B (anticlockwise angle from north), not the reflex angle (clockwise angle from north).

4



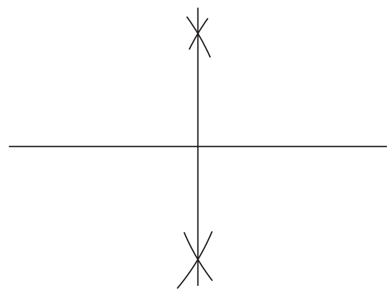
Exam practice

1 132°

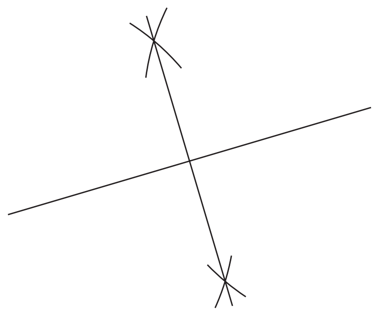
8.6 Constructions 1

Purposeful practice

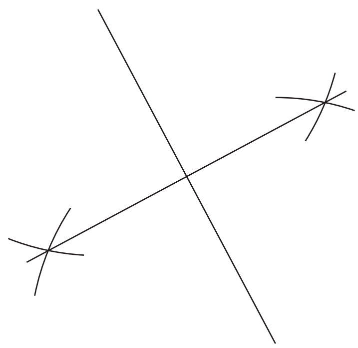
1 a



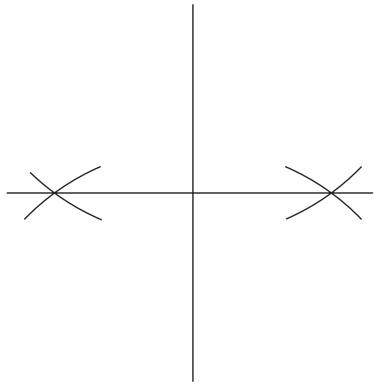
b



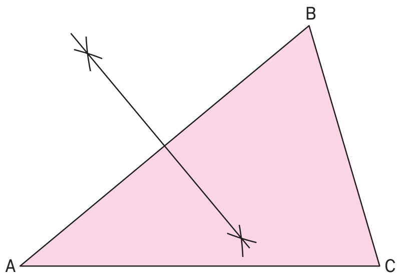
c



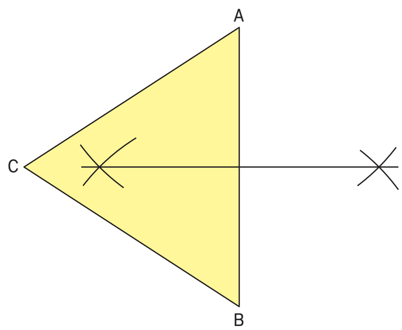
d



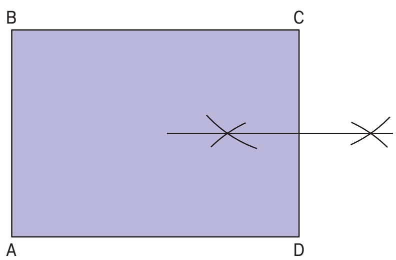
2 a



b



3

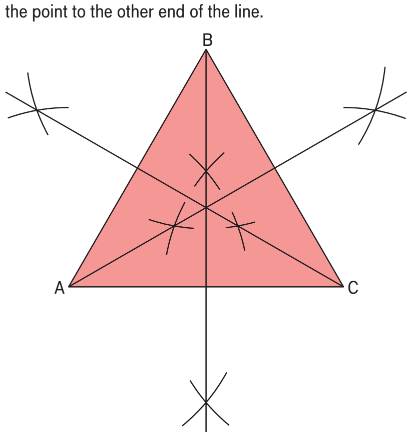


Problem-solving practice

1 a Jake has opened his compasses to less than half the length of the line, not more than half, so the arcs do not intersect.

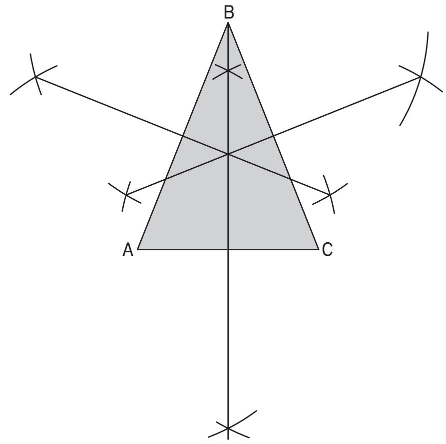
b Emily did not keep her compasses at the same distance when she moved the point to the other end of the line.

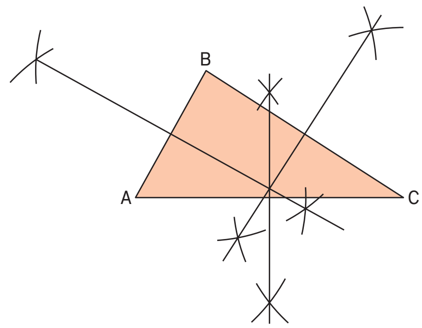
2 a



b All of the perpendicular bisectors intersect at the centre of the equilateral triangle.

c Students draw their own isosceles and scalene triangles and bisect each side. For example:

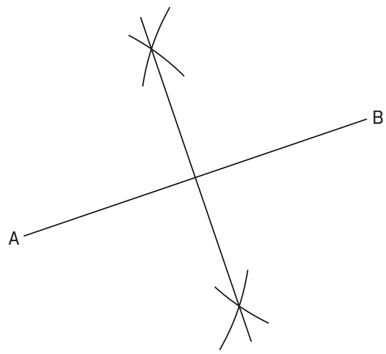




All three perpendicular bisectors intersect in any triangle but not necessarily in the centre of the triangle.

Exam practice

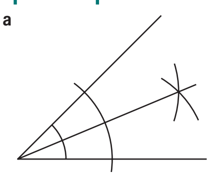
1



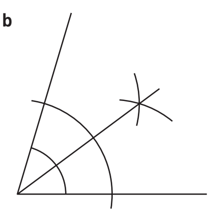
8.7 Constructions 2

Purposeful practice

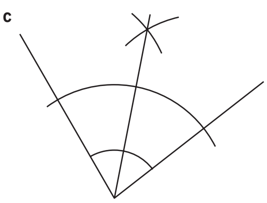
1 a



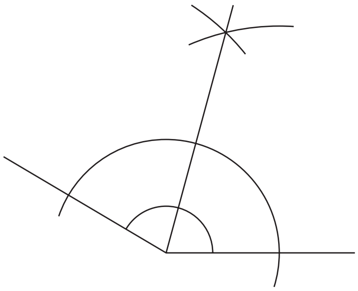
b



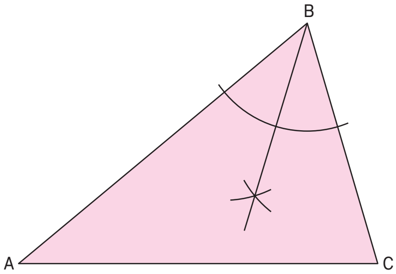
c



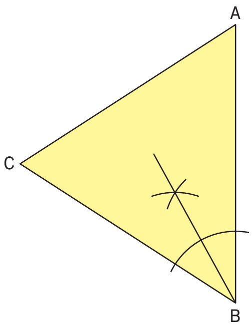
d



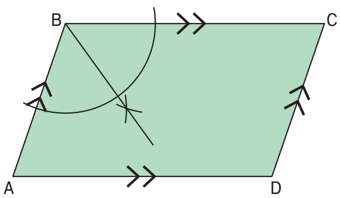
2 a



b



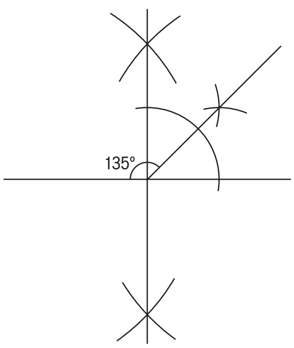
3



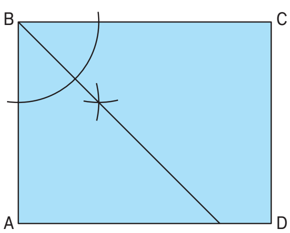
Problem-solving practice

1 George has drawn his arcs from the end of each arm of the angle. He should have first drawn an arc that crosses each arm of the angle from the vertex. Then he should have drawn arcs from where the first arc intersects each arm of the angle.

2

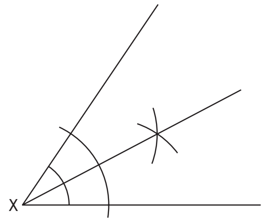


3



Exam practice

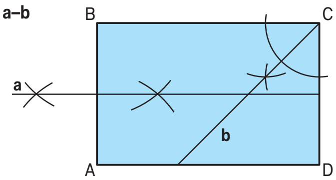
1



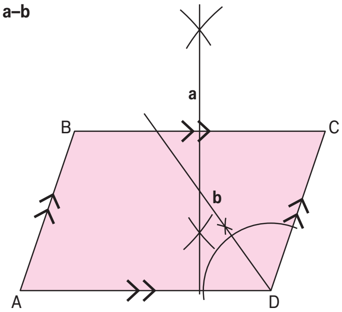
8.8 Loci

Purposeful practice

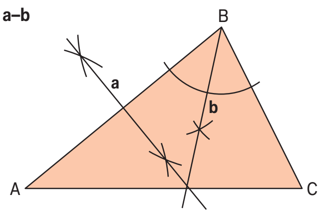
1 a–b



2 a–b

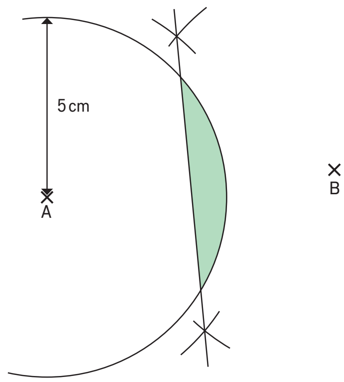


3 a–b

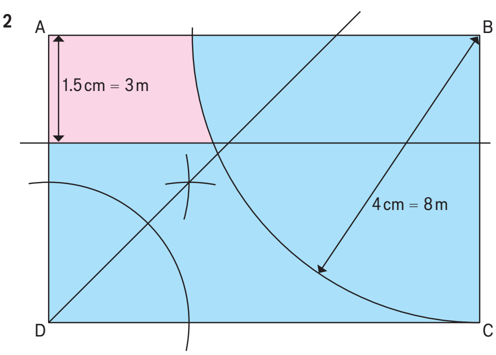


Problem-solving practice

1

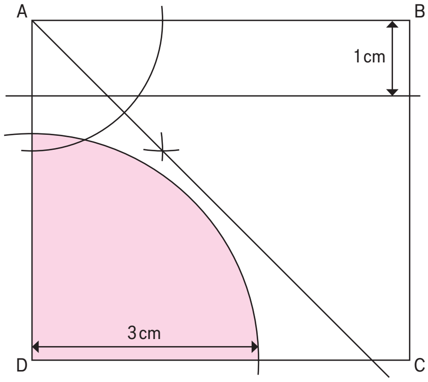


2



Exam practice

1



Mixed exercises B

Mixed problem-solving practice B

1 Sean used straight lines to join the points but should have used a smooth curve.

2 a I b C c D d G

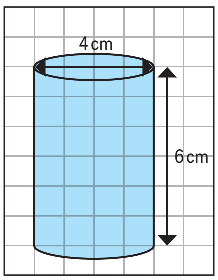
3 a $-\frac{1}{2}$

b The rate at which the water in the barrel is changing, in litres per second.

The negative sign tells us that the barrel is emptying at the rate of $\frac{1}{2}$ litre per second.

c $L = 20$ represents the volume of water, in litres, in the barrel at the start.

4



5 Area $= 5 \times 1.8 + \frac{1}{2} \times 0.8 \times (3 + 5) = 12.2\,\text{m}^2$

Cost = 20% off 13 × 28 = 20% off 364 = £291.20 < £300

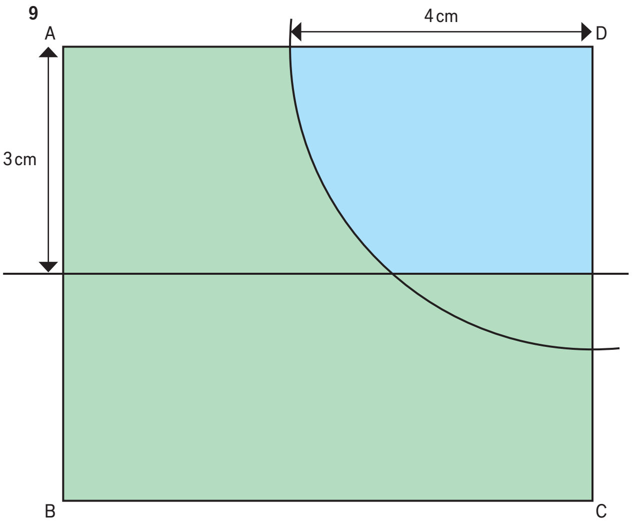
Yes, Charlotte has enough money to buy all of the tiles she needs.

6 A rotation 90° clockwise about (2, −2)

7 Rearranging L_2 gives $3y = 12x - 7$, $y = 4x - \frac{7}{3}$, therefore the gradient of L_2 is 4. The gradient of L_1 is 4. As the gradients are equal the two lines are parallel.

8 $829\,\text{cm}^3$

9

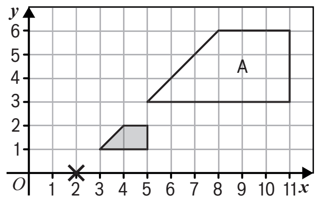


10 20.7 cm

Exam practice

11 $y = \frac{1}{2}x - 2$

12



13 Angle sum = (10 − 2) × 180 = 1440, angle ABC = 1440 ÷ 10 = 144°

14 6.38 cm

15 Volume of sphere $= 4 \times 243\pi = 972\pi = \frac{4}{3}\pi r^3$

$729 = r^3$

$r = 9\,\text{cm}$

Surface area of S $= \frac{1}{4} \times 4\pi r^2 + \pi r^2 = \pi \times 9^2 + \pi \times 9^2 = 509\,\text{cm}^2$

9 Equations and inequalities

9.1 Solving quadratic equations 1

Purposeful practice 1

1 a $3^2 + 2 \times 3 = 9 + 6 = 15$ b $3^2 + 3 = 9 + 3 = 12$

c $3^2 - 3 \times 3 = 9 - 9 = 0$

2 a $x = -3$ and $x = 2$ b $x = -3$ and $x = -2$

c $x = 2$ and $x = 2$ (repeated root)

Purposeful practice 2

1 a $x = -3$ and $x = 2$ b $x = -2$ and $x = 3$

c $x = -3$ and $x = -2$ d $x = 2$ and $x = 3$

2 a $x = -4$ and $x = -3$ b $x = -4$ and $x = 3$

c $x = -3$ and $x = 4$ d $x = 3$ and $x = 4$

Problem-solving practice

1 a Salma has only found one solution, but a quadratic equation has two solutions. She needs to rearrange the equation to equal zero before solving it.

b $x = -3$ or $x = 4$

2 $x = -3$ or $x = 5$ 3 $x = -1$ and $x = 1$

4 It factorises to $(x - 4)^2$, so its roots are $x = 4$ and $x = 4$ (repeated root).

5 a $y = x^2 - 6x + 9$ b $y = x^2 - 9$ c $y = x^2 + 6x + 9$

6 6 or −6

7 $x = -6$

8 $y = x^2 + 18x + 72$

9 There is a repeated root of $x = -5$. So the sketch cannot be correct because it shows two different roots.

Exam practice

1 $x = -5$, $x = 4$

9.2 Solving quadratic equations 2

Purposeful practice 1

1 a $x = -\frac{3}{2}$ and $x = 2$ b $x = -\frac{2}{3}$ and $x = 3$

c $x = -2$ and $x = -\frac{3}{2}$ d $x = \frac{2}{3}$ and $x = 3$

2 a $x = -\frac{1}{2}$ and $x = 3$ b $x = \frac{1}{2}$ and $x = 3$

c $x = 2$ and $x = -\frac{1}{2}$ d $x = -2$ and $x = -\frac{1}{2}$

Purposeful practice 2

1 a i $x = -3.58$ or $x = -0.42$ ii $x = -2.22$ or $x = -0.45$

iii $x = -1.59$ or $x = -0.16$

b i $x = -2.62$ or $x = -0.38$ ii $x = -4.79$ or $x = -0.21$

iii $x = -6.85$ or $x = -0.15$

c i $x = -4.30$ or $x = -0.70$ ii $x = -4$ or $x = -1$

iii $x = -3.62$ or $x = -1.38$

d i $x = -0.25$ or $x = 2.45$ ii $x = -0.55$ or $x = 0.82$

iii $x = -1.25$ or $x = 2.92$

e **i** $x = -0.25$ (repeated) **ii** $x = 0.6$ (repeated)
iii $x = -0.625$ (repeated)

Purposeful practice 3

1 **a** $x = -3 + \sqrt{2}$ or $x = -3 - \sqrt{2}$ **b** $x = 3 + \sqrt{2}$ or $x = 3 - \sqrt{2}$
c $x = 3 + \sqrt{10}$ or $x = 3 - \sqrt{10}$

2 **a** $x = 2 + \frac{\sqrt{6}}{2}$ or $x = 2 - \frac{\sqrt{6}}{2}$ **b** $x = -\frac{1}{2} + \frac{\sqrt{21}}{6}$ or $x = -\frac{1}{2} - \frac{\sqrt{21}}{6}$
c $x = \frac{9}{4} + \frac{\sqrt{113}}{4}$ or $x = \frac{9}{4} - \frac{\sqrt{113}}{4}$

Problem-solving practice

1 **a** Mark has forgotten the negative sign in front of the 3 at the beginning of the formula, and dropped the negative sign from the 4 in the equation.
His initial equation should be $x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-4)}}{2 \times 2}$
b $x = -2.35$ or $x = 0.85$

2 **a** **i** 73 **ii** −55 **iii** 0
b **i** A **ii** C **iii** B

3 All rearrange to the same quadratic, all with $x = -5$ or $x = -7$

4 **a** $1 + \frac{1}{x} = x$
$x + 1 = x^2$
$x^2 - x - 1 = 0$
b $x = -0.618$ or $x = 1.618$

Exam practice

1 $x = 2 \pm \sqrt{3}$

9.3 Completing the square

Purposeful practice

1 **a** $x^2 + 6x + 9$ **b** $x^2 - 6x + 9$ **c** $2x^2 + 12x + 18$
d $2x^2 - 12x + 18$ **e** $5x^2 + 30x + 45$ **f** $5x^2 - 30x + 45$

2 **a** $x^2 + 6x + 12$ **b** $x^2 + 6x + 5$ **c** $x^2 + 6x - 6$
d $x^2 - 6x + 12$ **e** $x^2 - 6x + 5$ **f** $x^2 - 6x - 6$

3 **a** $(x + 3)^2 + 3$ **b** $(x + 3)^2 + 1$ **c** $(x + 3)^2 + 6$
d $(x + 3)^2 - 10$ **e** $(x + 3)^2 - 20$ **f** $(x + 3)^2 - 100$

4 **a** $(x - 3)^2 + 3$ **b** $(x - 3)^2 - 21$ **c** $(x - 3)^2 - 9$
d $(x + 3)^2 + 3$ **e** $(x + 3)^2 - 21$ **f** $(x + 3)^2 - 9$

5 **a** $2(x + 3)^2 + 2$ **b** $2(x + 3)^2 - 10$ **c** $2(x + 3)^2 - 36$
d $3(x + 2)^2 + 9$ **e** $3(x + 2)^2 + 6$ **f** $3(x + 2)^2 - 30$

6 **a** $9(x + 1)^2 - 12$ **b** $9(x + 1)^2 - 9$ **c** $9(x + 1)^2 + 10$
d $16(x + 1)^2 - 12$ **e** $16(x + 1)^2 - 16$ **f** $16(x + 1)^2 + 48$

7 **a** $(x + 4)^2 + 1$ **b** $4\left(x - \frac{5}{2}\right)^2 - 10$ **c** $2\left(x + \frac{3}{2}\right)^2 - \frac{11}{2}$
d $(x + 2.5)^2 - 5.25$ **e** $(x + 0.5)^2 + 0.75$ **f** $10(x = 0.1)^2 + 0.9$

Problem-solving practice

1 **a** Jenny used the wrong number inside the bracket. This should be 4. The $-8x$ has been incorrectly included.
b $(x + 4)^2 + 34$

2 $x^2 + 4x + 10 = (x + 2)^2 + 6$, so square A has side $x + 2$ and rectangle B has area 6.

3 **a**
$$\begin{aligned} 3x^2 + 12x + 7 &= 3\left(\frac{x^2 + 4x + 7}{3}\right) \\ &= 3\left[\frac{(x + 2)^2 - 5}{3}\right] \\ &= 3(x + 2)^2 - 5 \end{aligned}$$
b
$$\begin{aligned} 3x^2 + 12x + 7 &= 3(x^2 + 4x) + 7 \\ &= 3[(x + 2)^2 - 4] + 7 \\ &= 3(x + 2)^2 - 12 + 7 \\ &= 3(x + 2)^2 - 5 \end{aligned}$$

4 **a** $3(x + 3)^2 - 6$
b $3(x + 3)^2 - 6 = 0$
$3(x + 3)^2 = 6$
$(x + 3)^2 = 2$
$x + 3 = \pm\sqrt{2}$
$x = -3 + \sqrt{2}$ or $x = -3 - \sqrt{2}$

5 **a** $(x + 3)^2 - 8$
b Substitution: $2y = x$, so $(2y + 3)^2 - 8 = 0$, giving $y = -0.086$ or -2.9

6 **a** $(x + 2)^2 + 6$
b Setting this to be zero would give $(x + 2)^2 = -6$, which is not possible because square numbers are never negative.

7 **a** $n^2 + n + 0.25 = (n + 0.5)^2$
(This is a perfect square.)
b $n^2 + n + 0.25 = n(n + 1) + 0.25$
c Part **a** gives $(2 + 0.5)^2 = 2.5^2 = 6.25$, and part **b** gives $2 \times 3 + 0.25 = 6.25$. As the answers are the same, Anna is correct.
d $(n + 0.5)^2 = n(n + 1) + 0.25$ so substituting $n = 5$ gives $5.5^2 = 5 \times 6 + 0.25 = 30.25$

Exam practice

1 $(x + 4)^2 - 21$

9.4 Solving simple simultaneous equations

Purposeful practice 1

1 **a** $x = 2, y = 8$ **b** $x = 4, y = 8$ **c** $x = 4, y = 12$
2 **a** $x = 1.5, y = 7$ **b** $x = 1, y = 8$ **c** $x = 3, y = 4$

Purposeful practice 2

1 **a** $x = 4, y = 2$ **b** $x = 12, y = -14$
c $x = 3, y = 15$ **d** $x = 2, y = 18$

2 **a** $x = 3, y = 4$ **b** $x = 3, y = 4$ **c** $x = 3, y = 4$
d $x = 2, y = 3$ **e** $x = 2, y = 3$ **f** $x = 2, y = 3$

3 **a** $x = 4, y = 2$ **b** $x = 2, y = 6$ **c** $x = \frac{1}{2}, y = -\frac{1}{4}$

4 **a** $x = 3, y = 1$ **b** $x = -2, y = 3$ **c** $x = \frac{1}{2}, y = -4$

Problem-solving practice

1 The numbers are 3.5 and 2.5
2 $x = 8$ and $y = 7$, giving an area of $144\,\text{cm}^2$.
3 A cup of coffee costs £2.70 and a cake costs £1.89.
4 **a** 8p or £0.08 **b** 12p or £0.12
5 Cost per day is 5p; cost per unit is 2p.
6 Simultaneous equations are $2x + y = 12$ and $x - y = 3$, or $2x + y = 12$ and $y - x = 3$. The combinations are 3, 3, 6 ($x = 3, y = 6$) or 5, 5, 2 ($x = 5, y = 2$)

Exam practice

1 $x = 3, y = 4$

9.5 More simultaneous equations

Purposeful practice 1

1 **a** $x = 2, y = -1$ **b** $x = 2, y = -1$
2 **a** **i** ① × 7 and ② × 3 **b** $x = -3, y = 1$
ii ① × 3 and ② × 4

Purposeful practice 2

1 **a** $x = 4, y = 4$ **b** $x = 1, y = -3$ **c** $x = -7, y = 2$
2 **a** $x = 3, y = 1$ **b** $x = 5, y = -2$ **c** $x = \frac{1}{2}, y = 2$

Purposeful practice 3

1 $x = \frac{2}{3}, y = \frac{19}{8}$ **2** $x = \frac{3}{8}, y = \frac{1}{3}$ **3** $x = \frac{3}{2}, y = -\frac{1}{5}$

Problem-solving practice

1 One bag of sand is 20 kg, therefore 20 bags of sand can be carried.
2 Adult tickets cost £8.50, child tickets cost £5.50. Offer is £4 less.
3 **a** When $x = 4, y = 19$, so $19 = 4m + c$
When $x = 8, y = 31$, so $31 = 8m + c$
b Solving the equations gives $m = 3$ and $c = 7$, so equation of line is $y = 3x + 7$
c Yes, if $x = 6$ then $y = 3 \times 6 + 7 = 25$
4 40 sheep, 75 chickens
5 $y = 5x^2 + 11$
6 $x = 3, y = -2$
7 Shorts = £4.99 and t-shirts = £5.99
8 (2, 1)

Exam practice

1 $x = -2, y = 3$

9.6 Solving linear and quadratic simultaneous equations

Purposeful practice 1

1 a $x = -2, y = -2$ or $x = 3, y = 3$
  b $x = -3, y = 3$ or $x = 2, y = -2$
  c $x = -2, y = -4$ or $x = 3, y = 6$
  d $x = -3, y = 6$ or $x = 2, y = -4$
2 a $x = -3, y = -1$ or $x = -2, y = 0$
  b $x = -4, y = -6$ or $x = -3, y = -5$
  c $x = 1, y = 5$ or $x = 7, y = 23$
  d $x = -6, y = -20$ or $x = 5, y = 13$
3 a $x = -1.58, y = 1.42$ or $x = 1.58, y = 4.58$
  b $x = -2.16, y = 0.84$ or $x = 1.16, y = 4.16$
  c $x = -2.44, y = 2.56$ or $x = 1.44, y = 6.44$

Purposeful practice 2

1 a $x = -3, y = -10$ or $x = 0, y = -1$
  b $x = -3.56, y = -9.68$ or $x = 0.56, y = 2.68$
  c $x = -8, y = -13$ or $x = 0, y = 3$
  d $x = -10.12, y = -23.25$ or $x = -1.88, y = 6.75$
2 a $x = -2, y = -1$ or $x = 1, y = 2$
  b $x = -1, y = -2$ or $x = 2, y = 1$
  c $x = -2.2, y = -0.4$ or $x = -1, y = 2$
  d $x = 2.2, y = 0.4$ or $x = 1, y = -2$
3 a $x = 3, y = 18$ (repeated)
  b $x = 2, y = 15$ (repeated)
  c $x = -4, y = 23$ (repeated)

Problem-solving practice

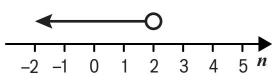
1 $a = 3$
2 $x = -2.45, y = 7.55$ and $x = 2.45, y = 12.45$
3 480 m
4 a $(-5, 9)$ and $(1, 3)$ b distance = 8.49 c $(0, -1)$
5 $x = 3, y = 4$ and $x = -4, y = -3$
6 $a = 25$
7 $c = 0.75$

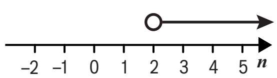
Exam practice

1 $x = 3, y = 5$ or $x = -5.4, y = 2.2$

9.7 Solving linear inequalities

Purposeful practice 1

1 a


b


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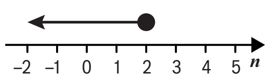
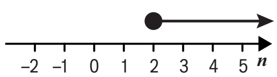
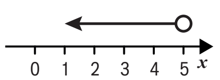
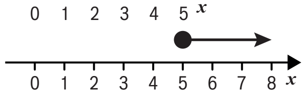


d


2 a


b


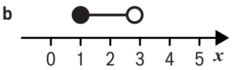
c


3 a $3x < 15$ b $4x < 20$ c $4x + 3 < 23$ d $4x - 1 < 19$
4 a $3x \geqslant 15$ b $4x \geqslant 20$ c $4x + 3 \geqslant 23$ d $4x - 1 \geqslant 19$
5 a $\{x : x < 4\}$ b $\{x : x \leqslant -2\}$ c $\{x : x \geqslant 4\}$ d $\{x : x < 8\}$

Purposeful practice 2

1 a $-2 > -5$ b $-4 < -1$ c $3 > -7$ d $2 < 6$
2 a $x > -5$ b $x < -1$ c $x > -7$ d $x < 6$
3 a $x < 3$ b $x \geqslant -7$ c $x < -4$ d $x \geqslant -4$
  e $x \leqslant -5$ f $x \geqslant 4$ g $x \geqslant -6$ h $x < -6$
  i $x < -3$ j $x > -7$ k $x \leqslant 7$ l $x \leqslant 4$

Problem-solving practice

1 A4, B1, C3, D2
2 a i $x < \frac{8}{3}$ ii $x \geqslant 1$
  b


3 $x = 1, 2, 3, 4$
4 150 cm^2
5 105 cm^2
6 Students' inequality with solution set $\{x : x > -2\}$
7 Between the first and second line Esther should have subtracted 2 so the line ends in -24 not -20. Between the second and third lines she should have reversed the inequality sign. The final answer should be $x \leqslant 8$
8 The values of x^2 would be 16, 9, 4, 1, 0, 1, 4, 9, 16, so if $x^2 > 9$ then $x < -3$ or $x > 3$.

Exam practice

1 $x > 2$

10 Probability

10.1 Combined events

Purposeful practice 1

1 a H, T b 1, 2, 3 c H1, H2, H3, T1, T2, T3
2 a H, T b 1, 2, 3, 4, 5, 6
  c H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6
3 a 7, 8, 9 b 1, 2, 3, 4, 5, 6
  c 7, 1; 7, 2; 7, 3; 7, 4; 7, 5; 7, 6; 8, 1; 8, 2; 8, 3; 8, 4; 8, 5; 8, 6; 9, 1; 9, 2; 9, 3; 9, 4; 9, 5; 9, 6

Purposeful practice 2

1

		Four-sided spinner			
		1	2	3	4
Three-sided spinner	1	2	3	4	5
	4	5	6	7	8
	9	10	11	12	13

2 a $\frac{1}{12}$ b $\frac{7}{12}$ c $\frac{4}{12}$ or $\frac{1}{3}$
  d $\frac{5}{12}$ e $\frac{8}{12}$ or $\frac{2}{3}$

Problem-solving practice

1 $\frac{15}{36}$ or $\frac{5}{12}$ 2 $\frac{6}{36}$ or $\frac{1}{6}$ 3 $\frac{3}{6}$ or $\frac{1}{2}$
4 a 10 b $\frac{5}{60}$ or $\frac{1}{12}$
5 No, it is not fair. P(12 or more) = $\frac{17}{36}$ and P(less than 12) = $\frac{19}{36}$ therefore Kim is more likely to win.
6 Students' own answers, for example, cards should be organised into a set of 3 cards and a set of 5 cards to give 15 possible outcomes.

Exam practice

1 a $\frac{4}{40} = \frac{1}{10}$ b $\frac{11}{40}$

10.2 Mutually exclusive events

Purposeful practice 1

1 $\frac{1}{10}$ 2 $\frac{2}{10}$ or $\frac{1}{5}$ 3 $\frac{3}{10}$ 4 $\frac{4}{10}$ or $\frac{2}{5}$
5 $\frac{3}{10}$ 6 $\frac{4}{10}$ or $\frac{2}{5}$ 7 $\frac{5}{10}$ or $\frac{1}{2}$ 8 $\frac{6}{10}$ or $\frac{3}{5}$
9 $\frac{7}{10}$ 10 $\frac{9}{10}$ 11 $\frac{8}{10}$ or $\frac{4}{5}$ 12 $\frac{8}{10}$ or $\frac{4}{5}$
13 $\frac{7}{10}$ 14 $\frac{6}{10}$ or $\frac{3}{5}$ 15 $\frac{5}{10}$ or $\frac{1}{2}$ 16 $\frac{7}{10}$
17 $\frac{4}{10}$ or $\frac{2}{5}$ 18 $\frac{3}{10}$

Purposeful practice 2

1 0.4 2 0.05 3 $\frac{1}{8}$ 4 45%

Problem-solving practice

1 0.97 2 > 90% 3 0.3 4 0.2

5 Students' own reasoning, for example, the probability of picking a black counter is $\frac{1}{12}$. This means that $\frac{1}{12}$ of the counters must be black. $\frac{1}{12}$ of 6 is $\frac{1}{2}$ and it is not possible to have $\frac{1}{2}$ a counter, so there cannot only be 6 counters in the bag.

6 a Black

b There are half as many black as pink, so 6 black. There are $1\frac{1}{2}$ times as many white as pink, so 18 white. There are three times as many pink as green, so 4 green.

Exam practice

1 $1 - 0.4 - 0.45 = 0.15$.
P(blue) $= 2 \times$ P(green), so P(blue) $= 0.1$, P(green) $= 0.05$.
A probability of 0.4 represents 8 cubes, so 0.1 represents 2 cubes and 0.05 represents 1 cube. Therefore, there is 1 green cube.

10.3 Experimental probability

Purposeful practice 1

1 a 21 b 15 c 32 d 52

2 a

Score	1	2	3	4	5	6
Experimental probability	0.15	0.175	0.23	0.2	0.16	0.085

b i 69 ii 48
iii 70.5 so estimate is 70 or 71 iv 138

Purposeful practice 2

1 a $\frac{15}{33} = 0.45$ b $\frac{49}{110} = 0.45$
c $\frac{12}{20} = 0.60$ d $\frac{76}{163} = 0.47$

Problem-solving practice

1 28 2 6 red, 8 blue, 2 green, 4 white

3 a i, iii, v, vi b ii, iv

4 P(6) $= 0.16$, $0.16 \times 120 = 19.2$. Estimate 19.

5 a Students' answers will vary, for example, it is likely to be fair because for a fair five-sided spinner the expected number of each score in 80 spins is 16, and all the frequencies are close to this.
OR Students may calculate all experimental probabilities and compare them to the theoretical probability of 0.2 for each score.

b Increase the number of trials

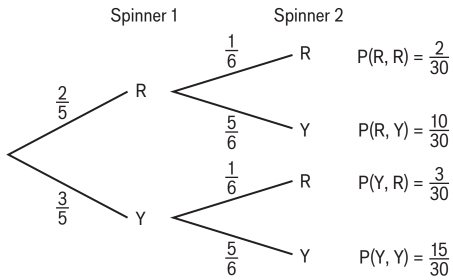
Exam practice

1 Min's results give the best estimate because she carried out the largest number of trials.

10.4 Independent events and tree diagrams

Purposeful practice

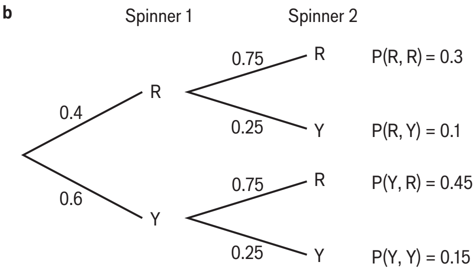
1 a



b Yellow, yellow c 5

2 a Spinner 1 P(R) $= 0.4$, Spinner 2 P(R) $= 0.75$

b



c 0.55 d 45

Problem-solving practice

1 a i $\frac{3}{4}$ ii $\frac{3}{8}$

b Students' answers will vary, for example, spinner 1 with P(B) $= \frac{1}{4}$ e.g. 4 sections, 1 blue and 3 green. Spinner 2 with P(B) $=$ P(G) $= \frac{1}{2}$, so with equal number of green and blue sections.

2 14 3 $\frac{1}{72}$

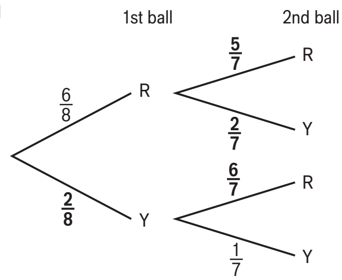
Exam practice

1 The probability for not white on the first spin should be 0.55 (not 0.65)
On the second spin, P(white) should be 0.45 and P(not white) should be 0.55. Jake has written them the wrong way round.

10.5 Conditional probability

Purposeful practice 1

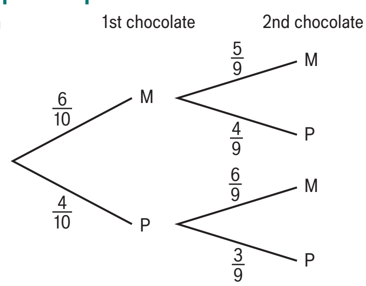
1



2 $\frac{2}{56} = \frac{1}{28}$

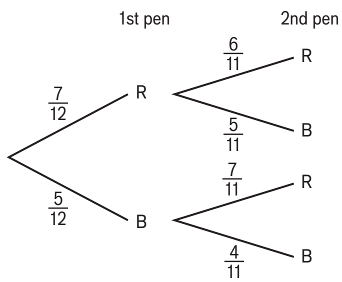
Purposeful practice 2

1 a



b $\frac{48}{90} = \frac{8}{15}$

2 a



b $\frac{112}{132} = \frac{28}{33}$

Problem-solving practice

1 One of:

the probabilities on the branches for the second sweet do not add to 1

she has changed the numerators and not the denominators for the second sweet

on the top pair, the probabilities should be $\frac{14}{27}$ and $\frac{13}{27}$

on the bottom pair, the probabilities should be $\frac{15}{27}$ and $\frac{12}{27}$

the probabilities for mints and toffees are the wrong way around

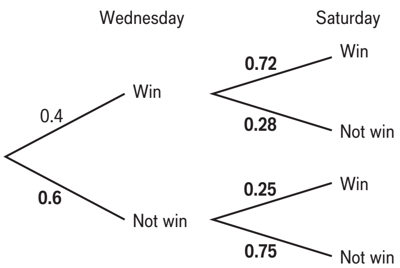
2 $\frac{210}{1320} = \frac{7}{44}$

3 a $\frac{40}{72} = \frac{5}{9}$

b If he has to take a third sock, it means he already has one black and one white. If he takes another sock it will be either black or white, so will make a pair with one of the ones he has already.

Exam practice

1 a



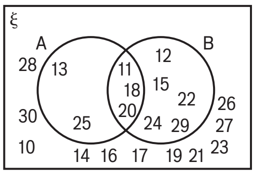
b 0.262

10.6 Venn diagrams and set notation

Purposeful practice

1 a 11, 18, 20

b



c i 11, 12, 13, 15, 18, 20, 22, 24, 25, 29

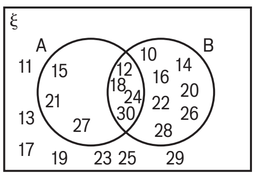
ii 10, 12, 14, 15, 16, 17, 19, 21, 22, 23, 24, 26, 27, 28, 29, 30

iii 12, 15, 22, 24, 29

iv 10, 14, 16, 17, 19, 21, 23, 26, 27, 28, 30

2 a 12, 18, 24, 30

b



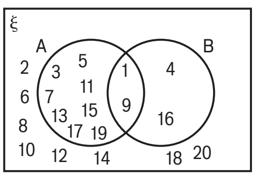
c i 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 26, 27, 28, 30

ii 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 28, 29

iii 10, 14, 16, 20, 22, 26, 28 iv 11, 13, 17, 19, 23, 25, 29

3 a 1, 9

b



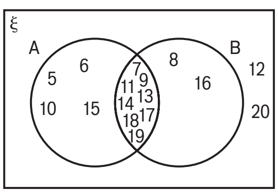
c i 1, 3, 4, 5, 7, 9, 11, 13, 15, 16, 17, 19

ii 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

iii 4, 16 iv 2, 6, 8, 10, 12, 14, 18, 20

4 a 7, 9, 11, 13, 14, 17, 18, 19

b

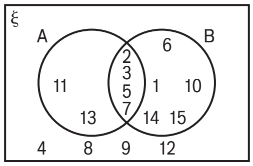


c i 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19

ii 8, 12, 16, 20 iii 8, 16 iv 12, 20

5 a 2, 3, 5, 7

b



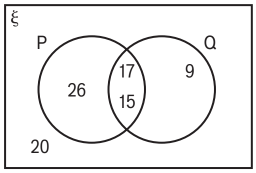
c i 1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15

ii 1, 4, 6, 8, 9, 10, 12, 14, 15

iii 1, 6, 10, 14, 15 iv 4, 8, 9, 12

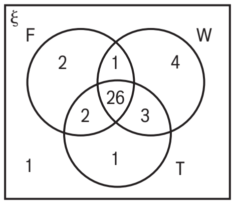
Problem-solving practice

1 a



b $\frac{2}{5}$

2 a

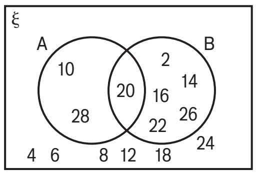


b $\frac{32}{40} = \frac{4}{5}$

3 a $\frac{27}{29}$ b $\frac{14}{29}$ c $\frac{25}{29}$ d $\frac{2}{29}$

Exam practice

1 a



b $\frac{8}{14} = \frac{4}{7}$

11 Multiplicative reasoning

11.1 Growth and decay

Purposeful practice 1

1 a £52.50 b £55.13 c £57.88

d £81.44 e £85.52 f £132.66

2 a £55 b £60.50 c £66.55

d £129.69 e £142.66 f £336.37

Purposeful practice 2

1 a £47.50 b £45.13 c £42.87

d £29.94 e £28.44 f £17.92

2 a £59.87 b £56.88 c £35.85

Purposeful practice 3

1 15.5% increase 2 4.5% increase 3 1% decrease

Problem-solving practice

1 34.6%

2 £21 125.63 (or £21 133.68 using exact value of multiplier)

3 Put it in savings because that yields 10.25% interest overall not a 10% increase.

4 3.125%

5 a The first option is better, as on the fourth day you will get £84.38 from the first option or £0.08 from the second.

b The second option is better. On the 28th day you will get £0.08 from the first option but £1 342 177.28 from the second option.

c On the 12th day.

6 a 3249 b 77.2% c 16 minutes

7 £802.82

Exam practice

1 £21 640.32

11.2 Compound measures

Purposeful practice 1

1 a 15 words per minute b 8.3 words per minute
c 120 words per minute

Purposeful practice 2

1 a 216 000 m/h b 108 000 m/h c 21 600 m/h d 2160 m/h
2 a 216 km/h b 108 km/h c 21.6 km/h d 2.16 km/h

Purposeful practice 3

1 a $-1/3\,\text{m/s}^2$ b $-1/3\,\text{m/s}^2$
2 a $-1.29\,\text{m/s}^2$ b $-1.66\,\text{m/s}^2$
3 a $-2.5\,\text{m/s}^2$ b $-3.5\,\text{m/s}^2$

Problem-solving practice

1 Jahidul – he writes at a rate of $\frac{60}{5} \times 140 = 1680$ words per hour compared to Angela's rate of 1570 words per hour.
2 4.17 seconds (3 s.f.)
3 No, using formula $s = ut + \frac{1}{2}at^2$:
Distance cheetah travels $= 0 + \frac{1}{2} \times 8.93 \times 11^2 = 540.3$ m (1 d.p.)
Distance gazelle travels $= 0 + \frac{1}{2} \times 4.2 \times 11^2 = 254.1$ m (1 d.p.)
Cheetah starts 300 m behind gazelle.
$(254.1 + 300) - 540.3 = 13.8$, so the gazelle will be 13.8 m ahead of the cheetah after 11 seconds.
4 Rowan: $a = \frac{v-u}{t} = \frac{3.5}{2.5} = 1.4\,\text{m/s}^2$
Nurhad: $a = \frac{v-u}{t} = \frac{3.8}{3} = 1.2\dot{6}\,\text{m/s}^2$
So Rowan has the greater acceleration.
5 The cyclist will win in 4.19 s compared to the car's 4.63 s
6 £63
7 a Grant would finish first. Archie plants $5y$ flowers in 2 minutes, which is $150y$ in an hour. This is a slower rate than Grant.
b 5 minutes and 36 seconds

Exam practice

1 For first 10 seconds: $u = p$, $v = 3p$, $t = 10$
Using $v = u + at$
$3p = p + 10a$
$\frac{2p}{10} = a$, or $a = \frac{p}{5}$
$s = 10p + \frac{1}{2} \times \frac{p}{5} \times 10^2$
$s = 20p$
For final 20 seconds: $u = 3p$, $a = 0$, $t = 20$
$s = ut + \frac{1}{2}at^2$
$s = 3p \times 20 = 60p$
Total distance $= 20p + 60p = 80p$

11.3 More compound measures

Purposeful practice 1

1 a $3\,\text{N/cm}^2$ b $6\,\text{N/cm}^2$ c $3\,\text{N/cm}^2$
2 a $3\,\text{g/cm}^3$ b $1.5\,\text{g/cm}^3$ c $3\,\text{g/cm}^3$

Purposeful practice 2

1 a 12 g b 6 g c 3 g
2 a 12 N b 24 N c 48 N
3 a $3\,\text{cm}^3$ b $1.5\,\text{cm}^3$ c $3\,\text{cm}^3$
4 a $3\,\text{cm}^2$ b $6\,\text{cm}^2$ c $3\,\text{cm}^2$

Problem-solving practice

1 $16\,100\,\text{cm}^3$ of feathers.
2 1.006 g/ml
3 The second person exerts greater pressure.
First person:
Force $= (67 \times 9.8)$ N $= 656.6$ N
Area $= 2 \times (40 + 0.25)\,\text{cm}^2 = 80.5\,\text{cm}^2$
Pressure = Force ÷ Area $= 8.16\,\text{N/cm}^2$ (2 d.p.)
Second person:
Force $= (75 \times 9.8)$ N $= 735$ N
Area $= 2 \times (40 + 0.7)\,\text{cm}^2 = 81.4\,\text{cm}^2$
Pressure = Force ÷ Area $= 9.03\,\text{N/cm}^2$ (2 d.p.)
4 No, it has a density of $0.32\,\text{g/cm}^3$.
5 Yes. The swimming pool only exerts $1.25\,\text{N/cm}^2$
6 Yes, its density is $650\,\text{kg/m}^3$, which is less than that of water.

Exam practice

1 0.71 grams per cm^3

11.4 Ratio and proportion

Purposeful practice 1

1 a 16 patients b 9 nurses c 10 patients
d 9 nurses e 3 hours f 12 nurses

Purposeful practice 2

1 a $y = 4x$ b $y = 12$ c $x = 0.75$
2 a $y = \frac{100}{x}$ b $y = 33\frac{1}{3}$ c $x = 33\frac{1}{3}$
3 a $y = 4.2x$ b $y = 12.6$ c $x = 0.71$ (to 2 d.p.)
4 a $y = \frac{420}{x}$ b $y = 140$ c $x = 140$

Problem-solving practice

1 10.9 hours
2 a 110 b 2 m
3 £4212
4 5.625 hours
5 It will cost the same amount to hire 10 or 12 workers.
6 a 12 hours 10 minutes b 6 hours 51 minutes (to the nearest minute)
7 4 hours

Exam practice

1 £37.20

12 Similarity and congruence

12.1 Congruence

Purposeful practice 1

1 SAS 2 AAS 3 RHS 4 AAS 5 SSS 6 SAS

Purposeful practice 2

1 Triangles B and C are congruent to triangle A. B by SAS (as the missing angle in triangle A is 90°). C by RHS (as the missing angle in triangle A is 90°).

Problem-solving practice

1 a True, SSS
b False, it can only be RHS if both hypotenuses are the same and one of the other sides is the same, but we are not told which side is which. It can only be SAS if the right angle is the included angle between the 6 cm and 8 cm sides for both triangles.
c False, it can only be SAS if the 55° angle is the included angle between the 7 cm and 10 cm sides for both triangles.
d False, the corresponding angles may all be equal but the sides may not be equal.
2 No, Tiff is incorrect because for triangle Y, the 100° angle is not the included angle between 4 cm and 7.5 cm.
3 Yes, using Pythagoras' theorem AC = 8 cm, so the triangles are congruent, RHS
4 Two angles and a corresponding side are equal, AAS, so triangle PQM and triangle RSM are congruent.

Exam practice

Angle AEB = Angle DEC (vertically opposite angles)
Angle ABE = Angle EDC (alternate angles are equal)
Angle BAE = Angle ECD (alternate angles are equal)
As DC = AB and the angles in each triangle are the same, triangle ABE is congruent to triangle DEC using the AAS condition.

12.2 Geometric proof and congruence

Purposeful practice

1 a Angle AEB = angle CED because vertically opposite angles are equal.
Angle BAE = angle CDE because alternate angles are equal.
AB = CD

b AAS

2 a, b Pairs of corresponding sides from: AB with BC or CD, AD with BC or CD (accept BD with BD)

c Yes, either because alternate angles are equal or opposite angles in a rhombus are equal (depending on answer to **Q2a** and **b**).

d SAS

3 a AB = CD because opposite sides in a rectangle are equal.
AE = CE (or DE), BE = DE (or CE) because the diagonals of a rectangle are equal and intersect at their midpoints.

b SSS

Problem-solving practice

1 Students' own proofs, for example, AB = BC as ABC is an equilateral triangle.
Both triangles have the common side BD and angle ADB = angle CDB = 90°.
AB is the hypotenuse of triangle ABD and BC is the hypotenuse of triangle BCD, therefore triangle ABD is congruent to triangle BCD by RHS.

2 Students' own proofs, for example, QR = PS as opposite sides of a parallelogram are equal.
Angle MQR = angle MSP because alternate angles are equal.
Angle QMR = angle PMS as vertically opposite angles are equal.
Therefore triangle PSM is congruent to triangle QRM by AAS.

3 AB = BC, AN = CM, AM = CN and both triangles have a common side AC, therefore SSS, so triangle AMC is congruent to triangle CNA.
Students' own proof, for example,
AB = BC and AM = $\frac{1}{2}$AB, CN = $\frac{1}{2}$BC, so AM = CN.
Angle MAC = angle NCA (base angles of isosceles triangle ABC).
AC is a common side in triangles AMC and CNA.
Therefore triangle AMC is congruent to triangle CNA by SAS.

4 Students' own proofs, for example,
AD = GD as ADG is an isosceles triangle.
AD is a side of the square ABCD, GD is a side of the square DEFG therefore these squares are congruent.
So, DE = DC as these are sides of congruent squares.
Angle ADC and angle GDE are angles in squares so they both equal 90°.
Therefore, angle ADE = 90° + angle CDE and angle GDC = 90° + angle CDE, so angle ADE = angle GDC.
Thus, triangle ADE is congruent to triangle GDC by SAS.

Exam practice

AX = YC, AD = CD because adjacent sides of a kite are equal and angle XAD = angle DCY because the base angles of an isosceles triangle are equal, therefore SAS, so triangle ADX is congruent to triangle CDY. Students' own proof, for example, AD = CD as a kite has two pairs of adjacent equal sides. Angle DAX = angle DCY as these are base angles in the isosceles triangle ACD. AX = CY, therefore triangle ADX is congruent to triangle CDY by SAS.

12.3 Similarity

Purposeful practice 1

1 a i $\frac{\text{length of B}}{\text{length of A}} = 2$ **ii** $\frac{\text{width of B}}{\text{width of A}} = 2$

b Yes, the rectangles are similar as the ratios of corresponding sides are the same.

2 a i $\frac{\text{base of B}}{\text{base of A}} = 3$ **ii** $\frac{\text{height of B}}{\text{height of A}} = 3$

b Yes, the triangles are similar as the ratios of corresponding sides are the same, and the included angles are equal.

3 a 3 ÷ 1.2 = 2.5 and 4.25 ÷ 1.7 = 2.5 so the parallelograms are similar.

b 5 ÷ 3 = $1\frac{2}{3}$ and 4 ÷ 2 = 2 so the triangles are not similar.

Purposeful practice 2

1 B x = 6, C x = 5, D x = 11.25

2 A x = 0.8, C x = 3.6, D x = 17.1

Problem-solving practice

1 Ben is not right because the corresponding angles in similar shapes are equal.

2 22.5 cm **3** $3.3\dot{3}$ cm

Exam practice

1 58.5 ÷ 13 = 4.5, 54 ÷ 12 = 4.5 and 22.5 ÷ 5 = 4.5. All ratios for corresponding sides are the same so the two triangles are mathematically similar.

12.4 More similarity

Purposeful practice 1

1 a Angle ECD = 30° (alternate angles are equal).
Angle CDE = 58° (alternate angles are equal).
Angle CED = 92° (vertically opposite angles are equal).

b From **Q1a** all corresponding angles are equal, so triangles ABE and CDE are similar.

2 a Angle ABC = 80° (corresponding angles are equal)
Angle ACB = 60° (corresponding angles are equal)

b Both triangles have a common angle of 40° and from **Q2a**, all corresponding angles are equal, so triangles ADE and ABC are similar.

Purposeful practice 2

1 a CED = 92° (vertically opposite angles are equal); EDC = 58° and DCE = 30° (alternate angles are equal). Therefore triangles ABE and DCE are similar (AAA).

b 2.8 cm

2 a Angle DAE = angle BAC (common); angle ADE = angle ABC and angle AED = angle ACB (corresponding angles are equal). Therefore triangles ABC and ADE are similar (AAA).

b 42 cm

Problem-solving practice

1 No. WX = 20 cm and XY = 10 cm, 30 ÷ 20 = 1.5 and 20 ÷ 10 = 2.
Corresponding sides are not in the same ratio, so rectangle ABCD and rectangle WXYZ are not mathematically similar.

2 4.5 cm

3 a AE = 19 cm

b CD = 7 cm

Exam practice

1 Assuming that AE is parallel to BD, 9 ÷ 3 × 2 = 6, x = 6 cm
Assuming that the corresponding sides are EC and BC, 12 ÷ 2 × 3 − 2 = 16, x = 16 cm

12.5 Similarity in 3D solids

Purposeful practice 1

1

Question	Linear scale factor	Surface area of A	Surface area of B	Area scale factor	Volume of A	Volume of B	Volume scale factor
1	2	6 cm^2	24 cm^2	4	1 cm^3	8 cm^3	8
2	2	**10 cm^2**	**40 cm^2**	**4**	**2 cm^3**	**16 cm^3**	**8**
3	3	**10 cm^2**	**90 cm^2**	**9**	**2 cm^3**	**54 cm^3**	**27**
4	4	**22 cm^2**	**352 cm^2**	**16**	**6 cm^3**	**384 cm^3**	**64**

Purposeful practice 2

1

Linear scale factor	Area scale factor	Surface area of A	Surface area of B	Volume scale factor	Volume of A	Volume of B
2	**4**	52 cm^2	**208 cm^2**	**8**	24 cm^3	**192 cm^3**
3	**9**	52 cm^2	**468 cm^2**	**27**	24 cm^3	**648 cm^3**
5	**25**	52 cm^2	**1300 cm^2**	**125**	24 cm^3	**3000 cm^3**
7	**49**	52 cm^2	**2548 cm^2**	**343**	24 cm^3	**8232 cm^3**

Problem-solving practice

1 10 125 cm^2 **2** 10 cm

3 a 320 cm^3 **b** 72.5 cm^2

4 55.6 cm^2 **5** 1 : 9 **6** 160 cm^3

Exam practice

1 9.16 cm

Mixed exercises C

Mixed problem-solving practice C

1 **a** 0.15 **b** 60

2 Students' own answer, for example, $(x + 5)(x - 3) = 0$

3 $2\frac{3}{4}$ hours

4 Bank A: $8000 \times 1.028^3 = 8690.99$, so £690.99 interest
Bank B: $8000 \times 1.04 \times 1.022^2 = 8690.11$, so £690.11 interest
Sasha should choose bank A.

5 49.25 cm

6 No, Lauren is not correct. The triangles are mathematically similar but just because the angles are the same, it does not mean that the sides are the same length.

7 **a**

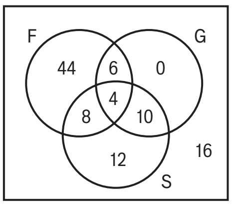
		Tom					
		1	1	1	4	5	6
Sasha	1	(1, 1)	(1, 1)	(1, 1)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 1)	(2, 1)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 1)	(3, 1)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 1)	(4, 1)	(4, 4)	(4, 5)	(4, 6)

b No, as the probability that Tom will win is $\frac{11}{24}$, which is higher than the probability that Sasha will win, which is $\frac{9}{24}$

c Tom 66 and Sasha 54

8 40 clips in a tub and 72 clips in a box.

9 **a**



b $\frac{2}{17}$

10 **a** $5 < 2n - 7 < 12$ **b** $6 < n < 9.5$ **c** 7, 8 or 9

11 $3x^2 - 9x + 4 = 0$

12 **a** Kit has substituted $y = 4x + 3$ for x, instead of y. Kit should have written $x^2 + 3x - 9 = 4x + 3$

b $x = 4$ and $y = 19$ or $x = -3$ and $y = -9$

13 0.3688

14 **a** 12.5 cm **b** 168 cm²

15 3.64

Exam practice

16 £14 550.73

17 A tea costs £1.60 and a coffee costs £2.80.

18 Ratio of the length of cone A to the length of cone B is $\sqrt[3]{64} : \sqrt[3]{27} = 4:3$
Ratio of the area of cone A to the area of cone B is $4^2 : 3^2 = 16 : 9$
$592 \div 16 \times 9 = 333$, therefore, the surface area of cone B = 333 cm²

19 P(GB or BG) $= \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{5}{8} = \frac{40}{72} = \frac{5}{9}$, students may draw a probability tree diagram to help.

20 Length upper bound = 14.25, length lower bound = 14.15
Width upper bound = 17.05, width lower bound = 16.95
Height upper bound = 22.75, height lower bound = 22.65
Mass upper bound = 1982.5, mass lower bound = 1977.5
Density upper bound $= \frac{1982.5}{14.15 \times 16.95 \times 22.65} = 0.364\,937\,798$
Density lower bound $= \frac{1977.5}{14.25 \times 17.05 \times 22.75} = 0.357\,763\,345$
Density = 0.36 g/cm³ as the upper and lower bounds both round to 0.36 to 2 decimal places (or 2 significant figures)

21 **a** PS = QR as opposite sides of a parallelogram are equal,
angle TQR = angle PSU as opposite angles of a parallelogram are equal and angle TRQ = angle SPU is given,
so using ASA, triangle TRQ is congruent to triangle SPU.

b TQ = SU as triangle TRQ is congruent to triangle SPU and TQ is parallel to SU, so TQUS is a parallelogram. Opposite sides of a parallelogram are parallel, so TS is parallel to QU.

13 More trigonometry

13.1 Accuracy

Purposeful practice 1

1 **a** 3.55 and 3.45, 3.45 and 3.35 **b** 3.55 and 3.45, 3.35 and 3.25
c 3.45 and 3.35, 3.35 and 3.25

2 **a** 45.8°, 46.7°, 45.8°, 45° **b** 45.8°, 46.7°, 46.7°, 47.5°
c 46.7°, 45.8°, 45°, 45.9°

Purposeful practice 2

1 **a** 4.3 and 5.7 **b** 5.1 and 6.5 **c** 5.7 and 7.1

2 **a** 45° and 60.9° **b** 52.1° and 65.6° **c** 45° and 57.5°

Problem-solving practice

1 Students' own answer ⩾ 16.5 cm

2 The upper bound of the angle is 44.4° so it cannot be too steep.

3 0.35 m and 0.30 m

4 Simon is incorrect. The bounds when the side lengths are rounded to 1 decimal place are 72.4° and 74.8°, but when the side lengths are rounded to 1 significant figure the bounds are 75.9° and 67.7°, which is a wider range.

5 26.3 cm²

Exam practice

1 $\sqrt{7.65^2 - 4.15^2} = 6.43$ cm to 2 d.p.

13.2 Graph of the sine function

Purposeful practice 1

1 0.7 2 −0.6

Purposeful practice 2

1 0.5 2 −0.5 3 0.5 4 0.0872
5 0.9962 6 −0.0872 7 −0.9962 8 0.0872

Purposeful practice 3

1 **a** 36.9°, 143.1°, 396.9° and 503.1° **b** 36.9°, 143.1°, 396.9° and 503.1°
c 23.6°, 156.4°, 383.6° and 516.4°

2 **a** 0 **b** 0 **c** 0

Problem-solving practice

Students' own answers for **Q1–6** (multiple answers possible), for example:

1 30° and 210°, 170° and 190°

2 Hypotenuse length 3 cm and opposite side length 1.8 cm (any pair of lengths in the ratio 5 to 3)

3 431°, 425.5°, 474° any angles in the range $(424.1° < x < 475.9°)$

4 $-180° < x < 0°$ (any range of the form $(360n - 180)° < x < 360n°$)

5 $-30° < x < 30°$ and $330° < x < 390°$ (any ranges of the form $(180n - 30)° < x < (180n + 30)°$)

6 $0° < x < 53.1°$

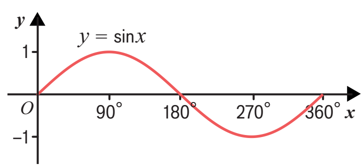
7 A (90, 2) B (180, −2) C (270, 1) D (540, −1)

8 1.75

9 (0, 0)

Exam practice

1



13.3 Graph of the cosine function

Purposeful practice 1

1 −0.83 2 0.43

Purposeful practice 2

1 0.87 2 −0.87 3 0.87 4 0.996
5 −0.0872 6 −0.996 7 0.0872 8 0.996

Purposeful practice 3

1 a 53.1°, 306.9° and 413.1° b 25.8°, 334.2° and 385.8°
c 113.6°, 246.4° and 473.6°

2 a 1 b 1 c 1

Problem-solving practice

Students' own answers for **Q1–6** (multiple answers possible), for example:

1 30° and 210°, 45° and 225°

2 Hypotenuse length 4 cm and adjacent side length 2.4 cm.

3 325°, 386°, 390° (angles in the range $323.2° < x < 334.1°$ or $385.9° < x < 396.8°$)

4 $-270° < x < -90°$ (any range of the form $(360n - 270)° < x < (360n - 90)°$)

5 $60° < x < 120°$ (any ranges of the form $(180n + 60)° < x < (180n + 120)°$)

6 $36.87° < x < 90°$

7 A (90, 1) B (180, 0) C (360, −1)

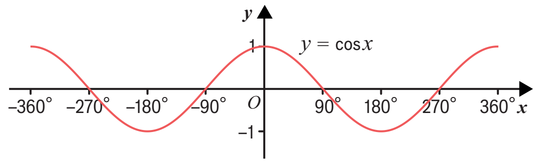
8 One

9 Four

10 Any line in the form $y = c$, where c is a constant and $c > 1$ or $c < -1$

11 Because the maximum value of cosine is 1.

Exam practice

1


13.4 The tangent function

Purposeful practice 1

1 −1.3 2 0.7

Purposeful practice 2

1 0.5774	2 0.5774	3 0.5774	4 0.0875
5 −11.4301	6 0.0875	7 −11.4301	8 0.0875

Purposeful practice 3

1 a 31.0°, 211.0° and 391.0° b 82.4°, 262.4° and 442.4°
c 91.4°, 271.4° and 451.4°

2 a 1 b 1 c 1

Problem-solving practice

Students' own answers for **Q1–7** (multiple answers possible), for example:

1 30° and 150°

2 Adjacent side length 13 cm and opposite side length 18 cm (any pair of lengths in the ratio 13 to 18)

3 263°, 443°, 443.5° (angles in the range $262.9° < x < 263.6°$ or $442.9° < x < 443.6°$)

4 $-90° < x < 0°$ (any range of the form $(180n - 90)° < x < 180n°$)

5 $-26.56° < x < 26.56°$ (any range of the form $(180n - 25.56)° < x < (180n + 25.56)°$)

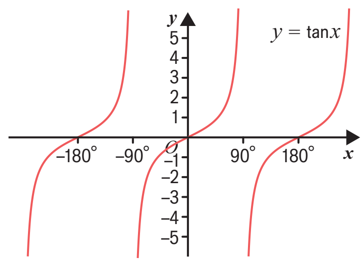
6 $0° < x < 38.6°$ (any range of the form $180n° < x < (180n + 38.6)°$)

7 a 90°, 270°, 450°, 630° (any angles of the form $(180n + 90)°$)
b 1, −1, 1, −1 (corresponding to students' own answers to **Q7a**)
c 0, 0, 0, 0

8 a four b four

9 Any equation describing an asymptote of the tan graph, for example, $x = 90$, (any line of the form $x = (180n + 90)°$)

Exam practice

1


13.5 Calculating areas and the sine rule

Purposeful practice 1

1 12 cm² 2 20.8 cm² 3 24 cm²

Purposeful practice 2

1 a 2.13 cm b 2.03 cm c 9.40 cm

2 a 23.6° b 93.7° c 65.8°

Problem-solving practice

1 21.6 cm 2 16.6 cm² 3 9.2 cm

4 a 25 cm, 36.6 cm and 18.3 cm b 213.1 cm²

5 10 cm

Exam practice

1 Area of triangle $= \frac{1}{2}ab \sin C$, so $\frac{1}{2}(x + 2)(x - 5)\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$$\left(\frac{\sqrt{3}}{4}\right)(x^2 - 3x - 10) = 2\sqrt{3}$$
$$x^2 - 3x - 10 = 8$$
$$x^2 - 3x - 18 = 0$$
$$(x - 6)(x + 3) = 0$$
$$x = 6$$

13.6 The cosine rule and 2D trigonometric problems

Purposeful practice 1

1 1.61 cm 2 2.05 cm 3 5.70 cm

Purposeful practice 2

1 $x = 82.8°$ 2 $x = 90°$ 3 $x = 34.0°$ and $y = 44.4°$

Purposeful practice 3

1 5.87 cm 2 5.73 cm 3 5.23 cm

Problem-solving practice

1 No, the perimeter is 45.0 cm

2 Let angle at centre of unshaded sector be x.

Using cosine rule: $\cos x = \frac{(8^2 + 8^2 - 10^2)}{(2 \times 8 \times 8)}$

So, $x = \cos^{-1}\left(\frac{7}{32}\right) = 77.4°$ (1 d.p.)

Therefore angle of shaded sector is $360° - 77.4° = 282.6°$.

This is $\frac{(282.6 \times 100)}{360}$ percent of the circle i.e. 78.5%.

3 81.3 cm 4 20.5 miles

Exam practice

1 $\frac{15}{\sin 103°} = \frac{BD}{\sin 27°}$

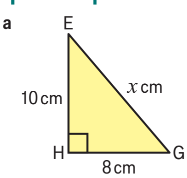
$BD = \frac{15 \times \sin 27°}{\sin 103°} = 6.99$

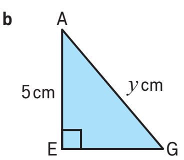
$AD^2 = 12^2 \times 6.99^2 - 2 \times 12 \times 6.99 \times \cos 74° = 146.62$

$AD = 12.1$ cm (3 s.f.)

13.7 Solving problems in 3D

Purposeful practice 1

1 a


b


2 a $x = 12.8$ cm b $y = 13.7$ cm

Purposeful practice 2

1 $\theta = 51.3°$ 2 $\alpha = 21.3°$

Problem-solving practice

1 647.4 cm³ 2 7.07 cm

3 No, the diagonal length of the pot is only 11.0 cm. 4 60.9°

5 No, the diameter of the base is only 179 cm.

Exam practice

1 $\sin 34^\circ = \dfrac{6.2}{AC}$

$AC = \dfrac{6.2}{\sin 34^\circ} = 11.08740...$

$x = \tan^{-1}\left(\dfrac{7.5}{11.08740}\right)$

$= 34.1^\circ$

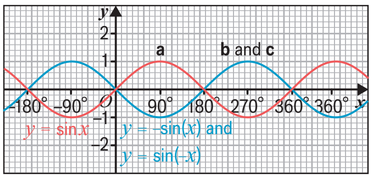
13.8 Transforming trigonometric graphs 1

Purposeful practice 1

1

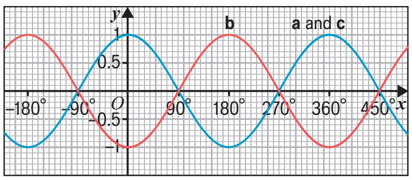
x	$\sin(x)$	$-\sin(x)$	$\sin(-x)$
0°	0	0	0
45°	0.71	−0.71	−0.71
90°	1	−1	−1
135°	0.71	−0.71	−0.71
180°	0	0	0
270°	−1	1	1
360°	0	0	0

2

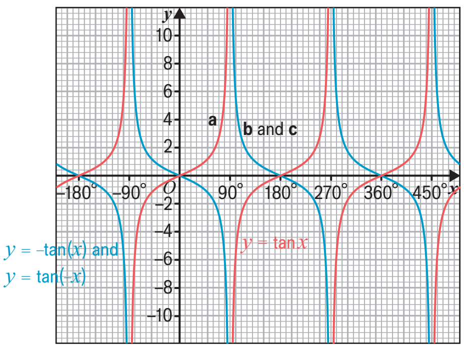


Purposeful practice 2

1



2



Problem-solving practice

1 180° and 360°

2 $y = \sin x$

3 $y = \tan(-x)$ or $-\tan(x)$

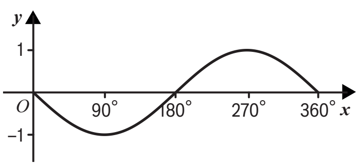
4 Students' own transformations, for example, a reflection in the y-axis followed by a reflection in the x-axis.

5 $y = -\cos(x)$

6 (−180, −1)

Exam practice

1



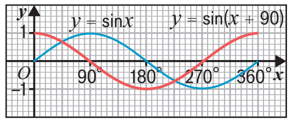
13.9 Transforming trigonometric graphs 2

Purposeful practice 1

1

x	$y = \sin(x)$	$y = \sin(x + 90)$	$y = \sin(x) + 90$
0°	0	1	90
90°	1	0	91
180°	0	−1	90
270°	−1	0	89
360°	0	1	90

2



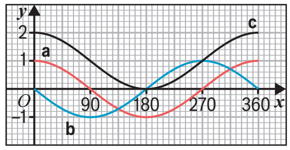
a The maximum and minimum values of both graphs are 1 and −1.

b $y = \sin x$: 0, 180°, 360°

$y = \sin(x + 90)$: 90°, 270°

Purposeful practice 2

1

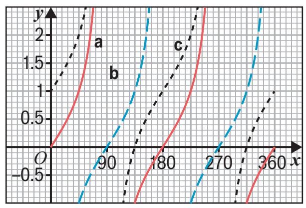


a i 1 and −1 ii 90° and 270°

b i 1 and −1 ii 180° and 360°

c i 2 and 0 ii 180°

2

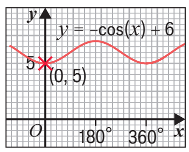


a 0, 180°, 360° b 90°, 180° c 135°, 315°

Problem-solving practice

1 A translation of $\begin{pmatrix} -90 \\ 0 \end{pmatrix}$

2



3 a (87, 1) b (93, 1) c (90, 4) d (90, −2)

4 Students' own answer, for example, (71.3°, 2.9) (multiple answers possible)

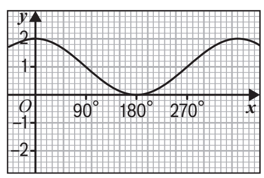
5 Students' own transformations, for example, translation of $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ (multiple answers possibile)

6 $y = \tan(x - 5) + 6$

7 Students' own answers, for example, $y = \sin(-x) + 1$ (multiple answers possible)

Exam practice

1



14 Further statistics

14.1 Sampling

Purposeful practice 1

1 a $\frac{50}{N}$	**b** $\frac{1}{10}$	**c** $\frac{50}{N} = \frac{1}{10}$	**d** 500
2 a $\frac{50}{N}$	**b** $\frac{1}{5}$	**c** $\frac{50}{N} = \frac{1}{5}$	**d** 250
3 a $\frac{50}{N}$	**b** $\frac{1}{4}$	**c** $\frac{50}{N} = \frac{1}{4}$	**d** 200
4 a $\frac{30}{N}$	**b** $\frac{1}{8}$	**c** $\frac{30}{N} = \frac{1}{8}$	**d** 240

Purposeful practice 2

1 400 **2** 250 **3** 280

Problem-solving practice

1 A is $N = 900$, B is $N = 1000$, C is $N = 540$, D is $N = 1680$

2 9000, the assumptions are that the mouse population has not changed between Saturday and Sunday, the chance of being captured is the same for all mice and the marks on the mice have not disappeared.

3 2000, the assumptions are that the rabbit population has not changed between Monday and Tuesday, the chance of being captured is the same for all rabbits and the tags on the rabbits have not come off.

4 a 80

b The assumptions are that the frog population has not changed between the capture and recapture, the chance of being captured is the same for all frogs and the marks on the frogs have not come off.

5 Jonathan should not have added 10 and 20, he should have multiplied them to give 200 and then divided by 5 to give 40, instead of multiplying by 5.

Exam practice

1 a 400

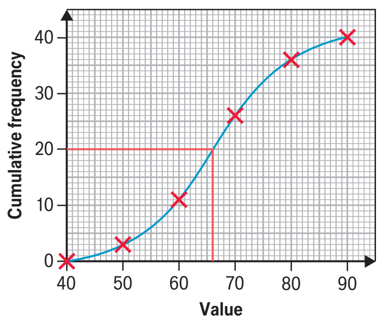
b If some of the tags had fallen off, then more than ten of the chickens which had originally been tagged may have been recaptured. This means that the estimate would be less than 400 as you would be dividing by a bigger number.

14.2 Cumulative frequency

Purposeful practice

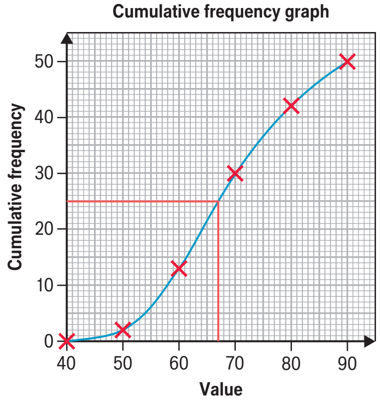
1 b Estimate of median = 26

2 a Cumulative frequency graph



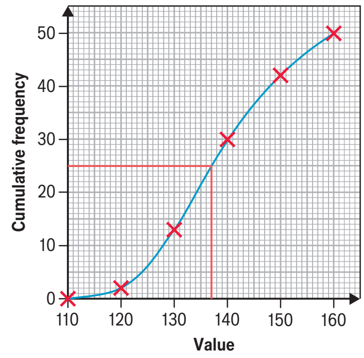
b Estimate of median = 66

3 a Cumulative frequency graph



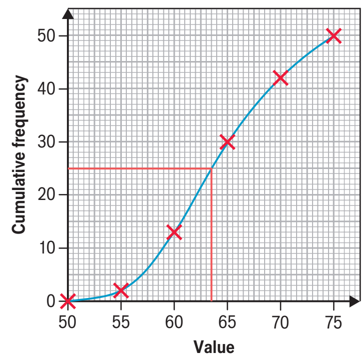
b Estimate of median = 67

4 a Cumulative frequency graph



b Estimate of median = 137

5 a Cumulative frequency graph

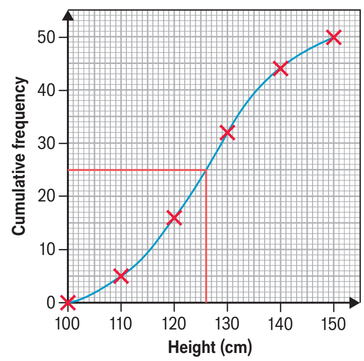


b Estimate of median = 63.5

Problem-solving practice

1 Ewan has plotted the points at the midpoints of the class intervals instead of the upper class boundaries and he has used a ruler to join the points instead of a smooth curve.

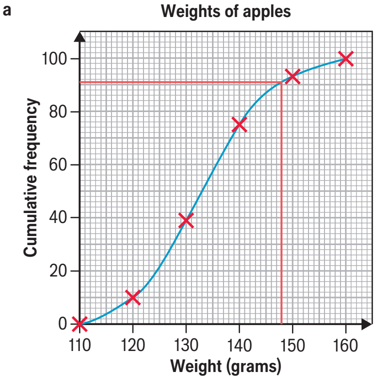
2 a Heights of children



b Louise has halved the cumulative frequency but not read the value of this median piece of data from the height axis. The median height is 126 cm.

c The estimate of the median gives you an estimate for the middle value of the data. Here the median height is 126 cm. 50% of the children are shorter than 126 cm and 50% are taller than 126 cm.

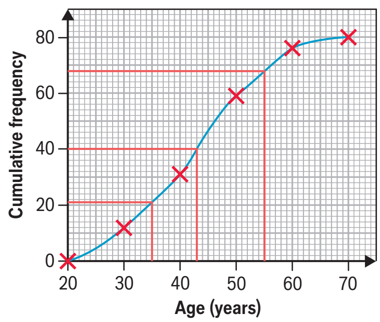
3 a Weights of apples



b 9 apples

Exam practice

1 a Ages of people



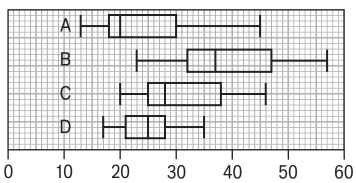
b 43 years old

c No, 68 − 21 = 47, 47 ÷ 80 × 100 = 58.75%

14.3 Box plots

Purposeful practice 1

1 Exam scores in different schools



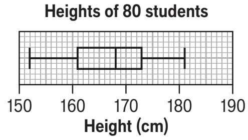
Purposeful practice 2

1 20 students 2 50 students 3 30 students 4 10 students
5 40 students 6 60 students 7 100 students 8 30 students

Problem-solving practice

1 The median is plotted incorrectly at 30 kg and not 32 kg. The box plot shows a maximum value of 52 kg, however this is the range of the data not the maximum value. The lightest weight is 5 kg and the range is 52 kg, so the maximum value should be 57 kg.

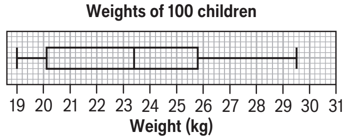
2 a i Heights of 80 students



ii Lower quartile = 161 cm, range = 29 cm

b 60 students

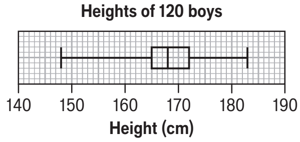
3 a Weights of 100 children



b 75 children

Exam practice

1 a Heights of 120 boys



b 90 boys

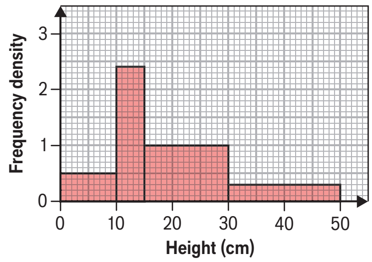
14.4 Drawing histograms

Purposeful practice

1 a

Height, x (cm)	Frequency	Class width	Frequency density
$0 < x \leqslant 10$	5	10	5 ÷ 10 = 0.5
$10 < x \leqslant 15$	12	**5**	**2.4**
$15 < x \leqslant 30$	15	**15**	**1**
$30 < x \leqslant 50$	6	**20**	**0.3**

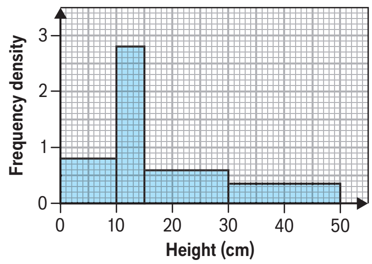
b Histogram showing heights



c The area of each bar should match its frequency.

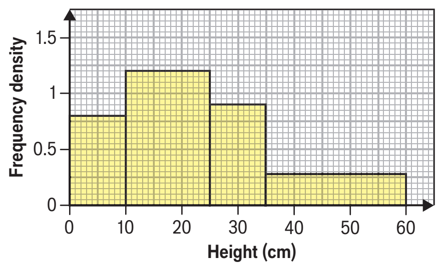
2 a 0.8, 2.8, 0.6, 0.35

b Histogram showing heights



c The area of each bar should match its frequency.

3 a Histogram showing heights



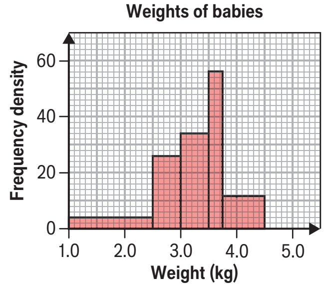
b The area of each bar should match its frequency.

Problem-solving practice

1 Megan has drawn the bars to the height of the frequency, not the frequency density. The last bar is too wide; it is from 20 to 50 but should only be from 20 to 40.

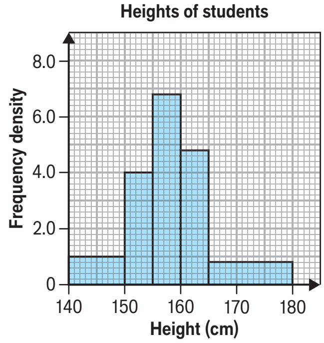
2 Frequency densities are 4, 26, 34, 56 and 12.

Weights of babies



3 a Frequency densities are 1, 4, 6.8, 4.8, 0.8.

Heights of students

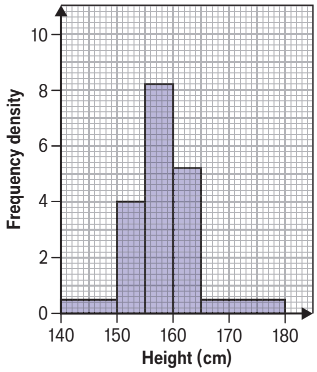


b The final bar would need to be extended to 190, and its height would decrease to show the new frequency density of the bar, which is 0.48.

Exam practice

1 Frequency densities are 0.5, 4, 8.2, 5.2 and 0.533.

Heights of students



14.5 Interpreting histograms

Purposeful practice

1 a

Height, x (cm)	Frequency density	Class width	Frequency
$0 < x \leqslant 10$	0.7	10	$0.7 \times 10 = 7$
$10 < x \leqslant 15$	**2.6**	**5**	$\mathbf{2.6 \times 5 = 13}$
$15 < x \leqslant 20$	**3.2**	**5**	$\mathbf{3.2 \times 5 = 16}$
$20 < x \leqslant 40$	**0.2**	**20**	$\mathbf{0.2 \times 20 = 4}$

b 40

2 a

Height, x (cm)	Frequency density	Class width	Frequency
$0 < x \leqslant 10$	**0.6**	**10**	**6**
$10 < x \leqslant 15$	**2.8**	**5**	**14**
$15 < x \leqslant 25$	**1.5**	**10**	**15**
$25 < x \leqslant 40$	**0.4**	**15**	**6**

b 35

3 a 47 b 6 c 53

Problem-solving practice

1 105 houses

2 a 8

b The data is grouped so we know that there are 16 people who took between 70 and 90 seconds, but we don't know if half of these took over 80 seconds, which is why 8 is an estimate.

Exam practice

1 7

14.6 Comparing and describing populations

Purposeful practice

1 a B b B c B, B

2 a A b A c higher, greater

3 a A b B

c On average, students in class A are heavier and students in class B have a greater spread of weights.

4 On average, students in class B are taller and students in class A have a greater spread of heights.

5 On average, students in class C are taller and students in class D have a greater spread of heights.

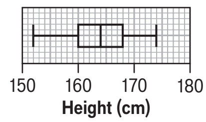
6 On average, students in class E are taller and have a greater spread of heights.

Problem-solving practice

1 On average, Ben's potato plants yield a greater mass of potatoes than Jordan's and have a greater spread of weights.

Exam practice

1 a Heights of Year 11 girls



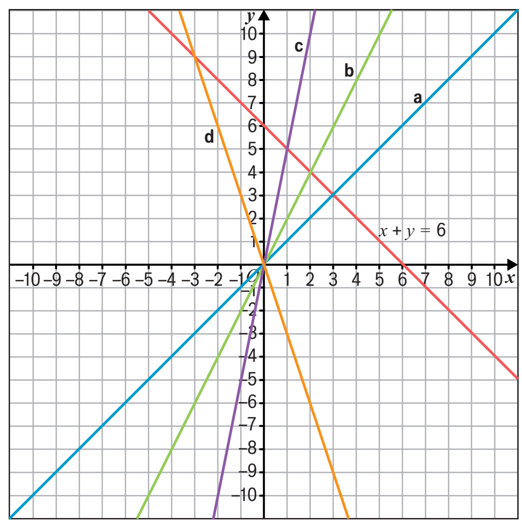
b On average, the Year 11 girls are taller and the Year 7 girls have a greater spread of heights.

15 Equations and graphs

15.1 Solving simultaneous equations graphically

Purposeful practice 1

1



a (3, 3) b (2, 4) c (1, 5) d (−3, 9)

2 a $x = 1, y = 3$ b $x = 3, y = 1$

3 a $x = 2, y = 6$ b $x = 6, y = 2$

4 a $x = 3, y = 9$ b $x = 9, y = 3$

Purposeful practice 2

1 $x = 1, y = 4$ 2 $x = 6, y = -1$ 3 $x = -1, y = -2$

Problem-solving practice

1 a $x = 1, y = 2$ and $x = -4, y = 7$ b $x = 1, y = 2$ and $x = 0, y = 3$

c $x = 4, y = 3$ and $x = -4, y = -3$ d $x = 2, y = 1$ and $x = -1, y = -2$

2 After 10 months, each method would have cost a total of £150.

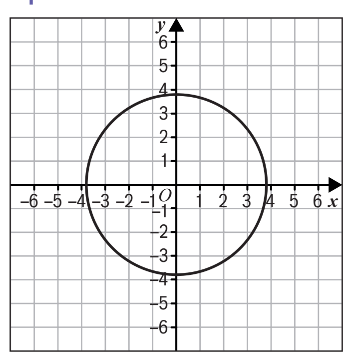
3 Roughly $x = 0.4, y = 1.4$ and $x = 4.6, y = 5.6$.

4 a 49p b £2.93

5 James' graph intersection would give a negative y, which is not possible. The blue line is incorrect. James has drawn the line $x - 5y = 7$ instead of the line $x + 5y = 7$

Exam practice

1 a

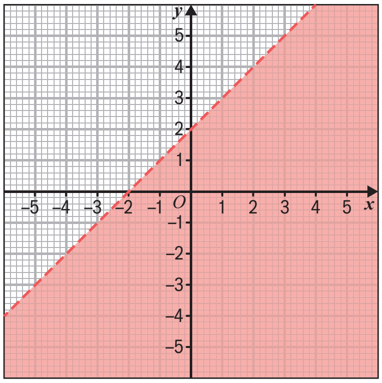


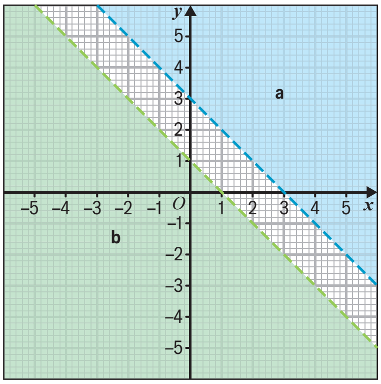
b Approximate answers $x = 2.5, y = 3$

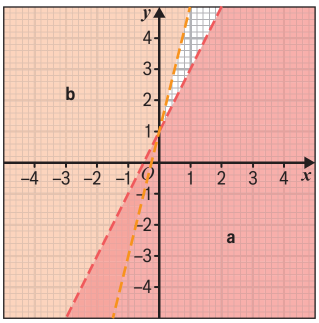
$x = -0.9, y = -3.8$

15.2 Representing inequalities graphically

Purposeful practice 1

1


2


3


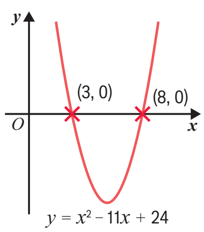
4 a No b Yes

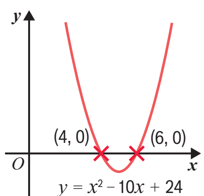
Purposeful practice 2

1 a $\{x : x < 5\}$ b $\{x : x < 6\}$ c $\{x : x < 4\}$
d $\{x : x > 8\}$ e $\{x : x > 10\}$ f $\{x : x > 12\}$

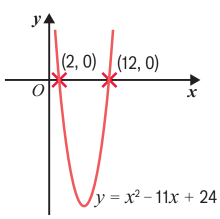
2 a $\{x : -4 \leqslant x \leqslant 4\}$ b $\{x : -9 < x < 9\}$
c $\{x : x < -16\} \cup \{x : x > 16\}$ d $\{x : x \leqslant -8\} \cup \{x : x \geqslant 8\}$

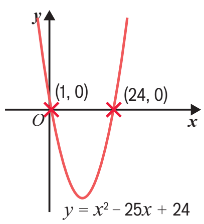
3 a $\{x : 3 \leqslant x \leqslant 8\}$ b $\{x : 4 \leqslant x \leqslant 6\}$



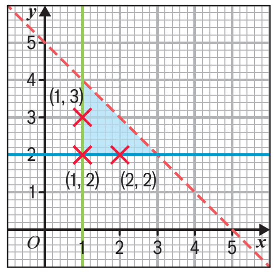


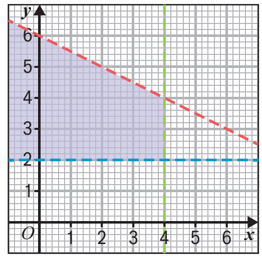
c $\{x : x \leqslant 2\} \cup \{x : x \geqslant 12\}$ d $\{x : x \leqslant 1\} \cup \{x : x \geqslant 24\}$





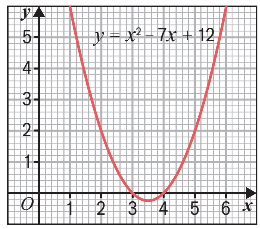
Problem-solving practice

1


2


The largest value of $x + y$ will be 7 when $x = 3$ and $y = 4$

3 a $y \leqslant 2x + 6, x + y \leqslant 6, y \geqslant 4$ b $y \geqslant -1, y \leqslant 3 - x, y \leqslant 2x + 4$
c $y \geqslant \frac{x}{2} - 4, x \geqslant 2, y \leqslant 6 - 2x$

4


The graph is below the x-axis for $\{x : 3 < x < 4\}$

Exam practice

1 $y \leqslant 2x + 3$
$y \leqslant 4 - x$
$y \geqslant -1$

15.3 Graphs of quadratic functions

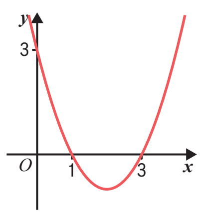
Purposeful practice 1

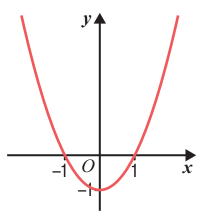
1 a $y = (x - 1)(x - 3)$; roots are $x = 1$ and $x = 3$
b $y = (x - 1)(x + 1)$; roots are $x = 1$ and $x = -1$
c $y = (x - 1)(x + 3)$; roots are $x = 1$ and $x = -3$

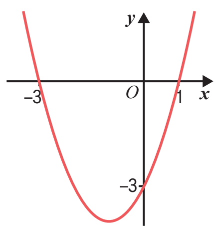
2 a $y = (4x - 1)(x - 1)$; $\left(\frac{1}{4}, 0\right)$ and $(1, 0)$
b $y = (3x - 1)(3x + 4)$; $\left(\frac{1}{3}, 0\right)$ and $\left(-\frac{4}{3}, 0\right)$
c $y = \left(\frac{1}{4}x - \frac{1}{2}\right)(x + 6)$; $(-6, 0)$ and $(2, 0)$

3 From **Q1**
a $(x - 2)^2 - 1$ turning point $(2, -1)$ b $x^2 - 1$ turning point $(0, -1)$
c $(x + 1)^2 - 4$ turning point $(-1, -4)$
From **Q2**
a $\left(2x - \frac{5}{4}\right)^2 - \frac{9}{16}$ turning point $\left(\frac{5}{8}, -\frac{9}{16}\right)$
b $\left(3x + \frac{3}{2}\right)^2 - \frac{25}{4}$ turning point $\left(-\frac{1}{2}, -\frac{25}{4}\right)$
c $y = \left(\frac{1}{2}x + 1\right)^2 - 4$ turning point $(-2, -4)$

4 From **Q1**

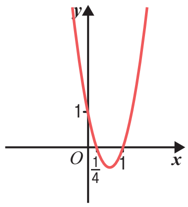
a


b


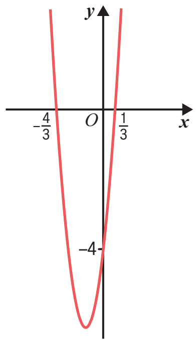
c


From **Q2**

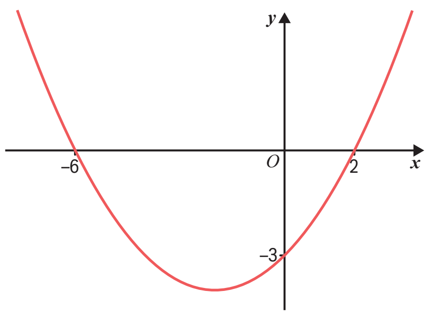
a



b



c



Purposeful practice 2

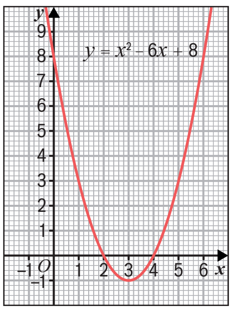
1 **a** completing the square: $y = 2(x + 1)^2 + 4$, so turning point is $(-1, 4)$

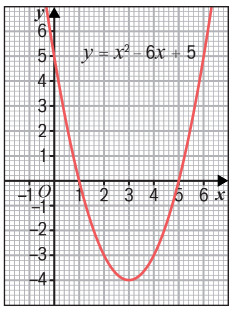
b completing the square: $y = 3(x + 1)^2 + 4$, so turning point is $(-1, 4)$

c completing the square: $y = 5(x + 1)^2 + 4$, so turning point is $(-1, 4)$

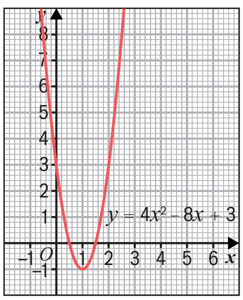
2 All three are minimums because the graphs will be ∪ shaped.

3 **a** $x = 3$ **b** $x = 3$





c $x = 1$



Problem-solving practice

1 **a** matches graph **ii** because it is ∪-shaped (x^2) and has roots at (1, 0) and (6, 0) since $y = (x - 1)(x - 6)$

b matches graph **i** because it is an upside down ∪ ($-x^2$) and has roots at (−3, 0) and (2, 0) since $y = (x + 3)(2 - x)$

c matches graph **iv** because it is ∪-shaped (x^2) and has roots at (−3, 0) and (2, 0) since $y = (x + 3)(x - 2)$

d matches graph **iii** because it is an upside down ∪ ($-x^2$) and has roots at (3, 0) and (−2, 0) since $y = (x + 2)(3 - x)$

2 The graph is the wrong way up. The negative x^2 in the equation would give a ∩ shape.

The graph has roots at $x = -3$ and $x = 2$, but if these values of x are substituted into the equation, the corresponding y-values are not 0.

The equation would intersect the y-axis at $y = 4$, but the graph has an intersection at $y = -6$

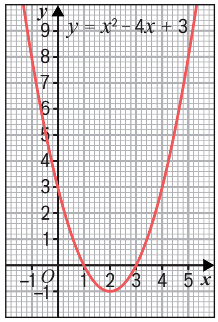
3 $y = x^2 - 2x - 3$

Exam practice

1 **a**

x	−1	0	1	2	3	4	5
y	**8**	3	**0**	−**1**	**0**	**3**	8

b



15.4 Solving quadratic equations graphically

Purposeful practice 1

1 **a** One repeated root **b** Two roots

c Two roots **d** No real roots

2 **a** One repeated root **b** No roots

c Two roots **d** Two roots

e One repeated root **f** No roots

3 Roughly $x = 0.3$ and $x = 3.7$

Purposeful practice 2

1 **a** The iterations are 1.63, 1.93, 2.05

b The iterations are 4.47, 4.63, 4.68

2 **a** 3.72 **b** 2.65 **c** 1.14

Problem-solving practice

1 **a** is graph **iii** **b** is graph **i** **c** is graph **iv** **d** is graph **ii**

Solutions **a** $x = 0.5$ and $x = -0.7$ **b** $x = -2$ and $x = 1$

c $x = -1$ and $x = 2$ **d** $x = 2$ (repeated)

2 **a** $x = 4 \pm \sqrt{14}$

b The quadratic formula leads to $\sqrt{(-3)^2 - 16} = \sqrt{-7}$ so no real roots.

3 **a** 0.73 **b** $16x^2 + 2x - 10 = 0$

Exam practice

a $2x^2 = 5 - x$

$x^2 = \frac{5 - x}{2}$

$x = \sqrt{\frac{5 - x}{2}}$

b $x_1 = 1.414213562$ ...

$x_2 = 1.338989626$ ...

$x_3 = 1.352961635$ ...

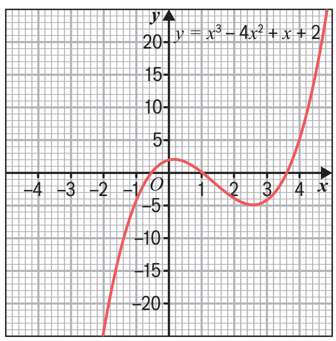
15.5 Graphs of cubic functions

Purposeful practice 1

1 **a, c** and **d** are cubic equations.

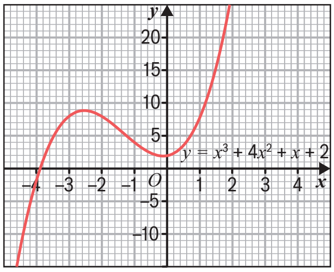
2 **a**

x	−2	−1	0	1	2	3	4
$x^3 - 4x^2 + x + 2$	−24	−4	2	0	−4	−4	6



b

x	-4	-3	-2	-1	0	1	2
$x^3 + 4x^2 + x + 2$	-2	8	8	4	2	8	28



3 a 3 **b** 2 **c** 2 **d** 1 **e** 1

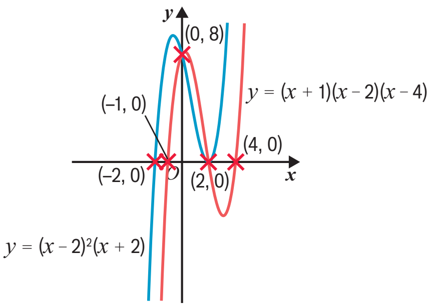
Purposeful practice 2

1 a 1, 2, 3 **b** 1, -2, 3
c 1 (repeated) and 2 **d** 1 and 2 (repeated)

2 a graph **ii** **b** graph **iii** **c** graph **iv** **d** graph **i**

Problem-solving practice

1



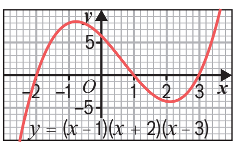
Similarities: students' own answers, for example, both cross the y-axis at $y = 8$; both have a root (2, 0).

Differences: students' own answers, for example $y = (x-2)^2(x+2)$ has two roots while $y = (x+1)(x-2)(x-4)$ has three roots; $y = (x-2)^2(x+2)$ has a turning point in the second quadrant while $y = (x+1)(x-2)(x-4)$ has a turning point in the fourth quadrant.

2 a $y = x^3 - 4x^2 + x + 6$ **b** $y = x^3 - 7x^2 + 16x - 12$
c $y = 2x^3 - 7x^2 - 68x - 32$

3 a First error $-1 \times 2 \times -3 = -6$ should give $+6$
Second error $(x-1)(x+2)(x-3) = 0$ should give $x = 1$, and then -2 and then 3

b

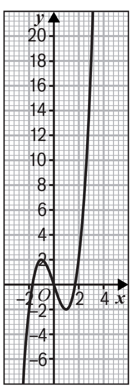


Exam practice

1 a

x	-3	-2	-1	0	1	2	3
y	**-18**	**-2**	2	0	**-2**	**2**	18

b



16 Circle theorems

16.1 Radii and chords

Purposeful practice 1

1 OPQ, OPR, OEF

2 $a = 40°$ $b = 100°$ $c = 30°$ $d = 60°$
$e = 60°$ $f = 80°$ $g = 48°$

Purposeful practice 2

1 6 cm **2** 16 cm **3** 5 cm **4** 12 cm

Problem-solving practice

1 $3\sqrt{7}$ cm

2 OA = OB (radii)
OBA = 32° (base angles in an isosceles triangle are equal)
AOB = 116° (the angles in a triangle add to 180°)
$m = 64°$ (angles on a straight line add to 180°)

3 For N to be the midpoint, ORN must be a right-angled triangle and RN must be 8 cm. Using Pythagoras, assuming it is a right-angled triangle, RN $= \sqrt{13^2 - 11^2} = 6.9$ cm (1 d.p.). This is not 8 cm, so N is not the midpoint.

4 OQP = 20° (angles on a straight line add to 180°)
OQ = OP = 7 cm (radii)
Distance between midpoint of PQ and Q = 7 cos 20° = 6.5778...
PQ = 2 × 6.5778... = 13.2 cm (1 d.p.)

Exam practice

1 OBA = 40° (angles on a straight line add to 180°)
OB = OA = 5cm (radii)
$\left(\frac{1}{2}\text{AB}\right) = 5 \cos 40° = 3.8302...$ cm
AB = 2 × 3.8302... cm = 7.7 cm (1 d.p.)

16.2 Tangents

Purposeful practice 1

1 Yes **2** Yes **3** No

Purposeful practice 2

1 a $a = 64°, b = 26°$ **b** $c = 120°$ **c** $d = 46°$
2 a $a = 30°, b = 75°$ **b** $c = 40°, d = 140°$ **c** $e = 38°$ $f = 19°$

Problem-solving practice

1 Triangle OMN is right-angled (the angle between a tangent and the radius is 90°)
ON² = 10² + 24² (Pythagoras' theorem)
ON² = 676
ON = 26 cm

2 OBC = 90° (the angle between a tangent and the radius is 90°)
BOC = 52° (the angles in a triangle add to 180°)
BOA = 128° (the angles on a straight line add to 180°)
OB = OA (radii)
$x = (180° - 128°) \div 2 = 26°$ (the angles in a triangle add to 180° and the base angles of an isosceles triangle are equal)

3 ORP = 32° (angles on a straight line add to 180°)
OPR = 90° (the angle between a tangent and the radius is 90°)
POR = 58° (the angles in a triangle add to 180°)
QOP = 122° (the angles on a straight line add to 180°)
$x = (180° - 122°) \div 2 = 29°$ (the angles in a triangle add to 180° and the base angles of an isosceles triangle are equal)
$y = 180 - (90 + 29) = 61°$ (the angles on a straight line add to 180°)

4 ABO = 90° (the angle between a tangent and the radius is 90°)
Sin OAB $= \frac{4}{8} = \frac{1}{2}$ (opposite over hypotenuse for a right-angled triangle)
OAB = 30° (known fact that sin 30° $= \frac{1}{2}$)

Exam practice

1 Angle OBC = 90° as OB is a radius of the circle and the angle between the tangent and a radius is 90°.
Angle BOC $= (90 - x)°$ because angles in a triangle add to 180°.
Angle AOB $= (90 + x)°$ because angles on a straight line add to 180°.
Angle OAB $= (180 - (90 + x))° \div 2 = \frac{(90 - x)}{2} = \left(45 - \frac{x}{2}\right)°$ as triangle AOB is an isosceles triangle.

16.3 Angles in circles 1

Purposeful practice 1

1 $a = 50°$ **2** $b = 140°$ **3** $c = 43°$
4 $d = 112°$ **5** $e = 121°$ **6** $f = 264°$

Purposeful practice 2

1 $a = 90°$ **2** $b = 45°$ **3** $c = 29°, d = 48°$
4 $e = 51°$ **5** $f = 50°, g = 65°$

Problem-solving practice

1 $a = 118°$ (the angle at the centre of a circle is twice the angle at the circumference)
AOC = 124° (the angles at a point add to 360°)
$b = 62°$ (the angle at the centre of a circle is twice the angle at the circumference)

2 QOR = 30° (the angle at the centre of a circle is twice the angle at the circumference)
$x = 15°$ (the angle at the centre of a circle is twice the angle at the circumference)

3 OYZ = 90° and OWZ = 90° (the angle between a tangent and the radius is 90°)
WOY = 96° (the angles in a quadrilateral add to 360°)
$x = 48°$ (the angle at the centre of a circle is twice the angle at the circumference)

4 FGH = $\frac{n}{2}$ (the angle at the centre of a circle is twice the angle at the circumference)
$m = 180 - \frac{n}{2}$ (the angles on a straight line add to 180°)
$\frac{n}{2} = 180 - m$
$n = 2(180 - m)$ or $n = 360 - 2m$

Exam practice

1 Students' own proof, for example,
let angle OYZ be labelled a.
Angle OZY = a (base angles in an isosceles triangle are equal)
Angle YOZ = $180° - 2a$ (angles in a triangle add to 180°)
Angle XOZ = $2a$ (angles on a straight line add to 180°)
Angle OZX = $\frac{(180° - 2a)}{2} = 90° - a$ (angles in a triangle add to 180° and base angles of an isosceles triangle are equal)
Angle XZY = Angle OZY + Angle OZX = $a + (90° - a) = 90°$

16.4 Angles in circles 2

Purposeful practice 1

1 $a = 41°$ **2** $b = 37°, c = 37°$ **3** $d = 83°, e = 28°$

Purposeful practice 2

1 $a = 88°$ **2** $b = 85°, c = 82°$ **3** $d = 88°, e = 89°$

Purposeful practice 3

1 $a = 75°$ **2** $b = 72°, c = 78°$ **3** $d = 126°$

Problem-solving practice

1 $x = 43°$ (alternate segment theorem)
BAC = 43° (angles subtended by the same arc are equal)
$y = 137°$ (angles on a straight line add to 180°)

2 $a = 38°$ (the angle on a semicircle is 90°)
$b = 52°$ (angles subtended by the same arc are equal)
$c = 128°$ (opposite angles in a cyclic quadrilateral add to 180°)

3 Yes. Angle ADC = 50° (angles on a straight line add to 180°)
So, angle ABE = 50° (angles subtended by the same arc are equal)
Therefore, angle BAD = 50° (angles in a triangle add to 180°).
Angle BAD = angle ADC, so AB and CD are parallel (alternate angles are equal for parallel lines)

4 Students own reasoning, for example,
Angle ADC + $x + y = 180°$ (angles on a straight line add to 180°)
Angle ADC = $180° - z$ (opposite angles in a quadrilateral add to 180°)
Therefore, $180° - z + x + y = 180°$
So, $z = x + y$

Exam practice

1 Angle BDF + angle BDO = 90° (the angle between the radius and the tangent is 90°)
So, angle BDF = $90 - x$.
Angle BCD = angle BDF (alternate segment theorem)
Therefore, angle BCD = $90 - x$.

16.5 Applying circle theorems

Purposeful practice 1

1 a 2 **b** -0.5
2 a $-\frac{1}{3}$ **b** 3

Purposeful practice 2

1 $y = \frac{2}{3}x + \frac{13}{3}$ **2** $y = -\frac{2}{3}x - \frac{13}{3}$

Problem-solving practice

1 a $y = -\frac{3}{4}x + \frac{25}{4}$ **b** $y = -\frac{4}{3}x + \frac{25}{3}$

2 $(0, -\frac{10}{3})$

3 $\left(\frac{34}{3}, 0\right)$

4 $T = \left(0, \frac{41}{5}\right), R = \left(\frac{41}{4}, 0\right)$
$\frac{1681}{40} = 42\frac{1}{40}$ units^2

5 $P = (-10, 0), Q = (0, 5)$
$PQ = 5\sqrt{5}$

Exam practice

1 $y = -\sqrt{3}x + 2\sqrt{3}$

Mixed exercises D

Mixed problem-solving practice D

1 a i B **ii** E **iii** C **iv** H **v** I **vi** F
b A: $y = -\sin x$ or $y = \sin(-x)$ D: $y = -(x + 2)^2(x - 2)$
G: $y = -\tan x$ or $y = \tan(-x)$

2 a 169.5 cm
b No, Jane may not be correct as the minimum height could be less than 160 cm since the graph is not completed to show the data for the shortest 4 students.
c 14 students have a height greater than 175 cm. 25% of 60 = 15 and 14 is less than 15, so less than 25% of the students have a height greater than 175 cm.

3 a $a = 5$ and $b = 3$ **b** (5, 3)

4 35.4°

5 3.60 m

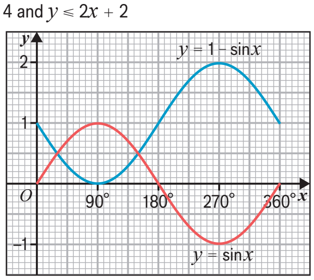
6 a

Time, m (minutes)	Frequency
$0 < m \leqslant 10$	35
$10 < m \leqslant 15$	47
$15 < m \leqslant 20$	59
$20 < m \leqslant 30$	18
$30 < m \leqslant 50$	2

b 10.6 (1 d.p.)

7 $x < 1, y \geqslant -x - 4$ and $y \leqslant 2x + 2$

8 a and **b**

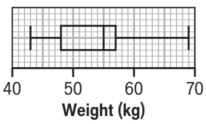


9 (3, −11)

Exam practice

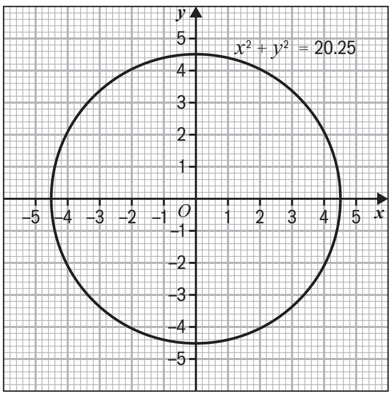
10 a 11 kg

b Weight of leopards



c Yes, Tom is correct as the median cougar weight is 58 kg and the median leopard weight is 55 kg.

11 a



b Approx $x = 1.7$ and $y = -4.2$ or $x = -1.1$ and $y = 4.4$

12 $x < -3, x > \frac{1}{2}$

13 a $2^3 + 2 = 10$, $3^3 + 3 = 30$, 17 is between 10 and 30 so the equation $x^3 + x = 17$ has a solution between 2 and 3

b $x^3 + x = 17$, so $x^3 = 17 - x$ and therefore $x = \sqrt[3]{17 - x}$

c $x = 2.44$ (2 d.p.)

14 $\text{MN}^2 = \text{AC}^2 = x^2 + x^2 - 2 \times x \times x \times \cos 45°$ (cosine rule on triangle ABC)

$$= 2x^2 - 2x^2 \times \frac{\sqrt{2}}{2}$$

$$= x^2(2 - \sqrt{2})$$

$$\cos \text{MBN} = \frac{5^2 + 5^2 - x^2(2 - \sqrt{2})}{2 \times 5 \times 5} = 1 - \frac{x^2(2 - \sqrt{2})}{50}$$

(cosine on triangle MBN)

17 More algebra

17.1 Rearranging formula

Purposeful practice 1

1 $k = \frac{5bct}{2a}$ **2** $k = 5bt - 2a$ **3** $k = \frac{5bt - 2a}{2}$

4 $k = \sqrt{\frac{5bt}{2a}}$ **5** $k = \sqrt{5bt - 2a}$ **6** $k = \sqrt{\frac{5bt - 2a}{3}}$

7 $k = t - \frac{2a}{5b}$ **8** $k = 4\left(t - \frac{2a}{5b}\right)$ **9** $k = \sqrt{t - \frac{2a}{5b}}$

10 $k = \sqrt[3]{t - \frac{2a}{5b}}$ **11** $k = \left(\frac{t}{2a}\right)^2$ **12** $k = \left(\frac{5t}{2a}\right)^2$

13 $k = \left(\frac{5bct}{2a}\right)^2$ **14** $k = \left(t - \frac{2a}{5b}\right)^2$ **15** $k = \left(t - \frac{2a}{5b}\right)^3$

16 $k = t^2 - \frac{2a}{5b}$ **17** $k = (5bt)^2 - 2a$ **18** $k = \frac{(5bt)^2 - 2a}{7}$

Purposeful practice 2

1 $a = \frac{3}{1 - b}$ **2** $a = \frac{3}{4 - b}$ **3** $a = \frac{3}{4 - 6b}$

4 $a = \frac{3 + c}{4 - 6b}$ **5** $a = \frac{3b + c}{4 - 6b}$ **6** $a = \frac{3b + c}{4 - 6b}$

7 $a = \frac{3b + c}{4 - 6b}$ **8** $a = \frac{bc}{c - b}$ **9** $a = \frac{2bc}{c - b}$

Problem-solving practice

1 $h = \sqrt{\frac{p - a}{a}} = \sqrt{\frac{p}{a} - \frac{\cancel{a}}{\cancel{a}}} = \sqrt{\frac{p}{a} - 1}$

2 a $h = \frac{6}{\pi}$ **b** $r = \sqrt{\frac{210}{\pi}}$

3

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

$$a^2 + 2bc\cos\theta = b^2 + c^2 \cancel{- 2bc\cos\theta} \cancel{+ 2bc\cos\theta}$$

$$2bc\cos\theta + \cancel{a^2} - \cancel{a^2} = b^2 + c^2 - a^2$$

$$\frac{\cancel{2bc}\cos\theta}{\cancel{2bc}} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$$

4 $h = \frac{A - 2\pi r^2}{2\pi r}$ **5** $g = \frac{3}{h^2 - 1}$

6 John should multiply by $\frac{-1}{-1}$

7 Hannah has incorrectly expanded the bracket on the third line. It should say $xV + x = V$.

On the fourth line the right-hand side should be $V - xV$ not $V + xV$.

Exam practice

1 $t = \sqrt{3k^2 + 1}$

17.2 Algebraic fractions

Purposeful practice 1

1 $\frac{5x}{6}$ **2** $\frac{11x}{4}$ **3** $\frac{15x - 2}{6}$

4 $\frac{5x - 4}{2}$ **5** $\frac{x}{2}$ **6** $\frac{13x}{6}$

7 $\frac{5x + 4}{6}$ **8** $\frac{5x - 8}{6}$ **9** $\frac{-7x + 16}{6}$

10 $\frac{-3x + 10}{4}$ **11** $\frac{5}{6x}$ **12** $\frac{16}{9x}$

13 $\frac{2}{9x}$ **14** $\frac{-14}{9x}$ **15** $\frac{28}{15x}$

Purposeful practice 2

1 $\frac{xy}{6}$ **2** $\frac{x^2}{6}$ **3** $\frac{3x^2}{2}$ **4** $\frac{9x^2}{10}$

5 $\frac{9xy^2}{10}$ **6** $\frac{3xy^2}{2}$ **7** $\frac{3x}{2y^3}$ **8** $\frac{3}{2x\ 4y^3}$

9 $\frac{3(y + 6)}{2x\ 4y^5}$ **10** $\frac{9(y + 6)}{2x\ 5y^5}$ **11** $\frac{27x\ 2}{y^3}$ **12** $\frac{y}{x}$

13 $\frac{x}{y}$ **14** $\frac{3}{2}$ **15** $\frac{9x}{2}$ **16** $\frac{3x}{20}$

17 $\frac{20}{3x}$ **18** $\frac{10x}{3(x + 5)}$

Problem-solving practice

1 $T = \frac{600x - 3000}{x(x - 10)}$

2 $\frac{\mathbf{4}x + 3}{\mathbf{2}} + \frac{x + 2}{4} = \frac{9x + 8}{4}$

3 a $\frac{20y}{x}$ **b** $\frac{2x}{3y^2}$

4 a $\frac{7}{2x} + \frac{5}{\mathbf{3x}} = \frac{\mathbf{31}}{6x}$ **b** $\frac{\mathbf{2}x + 2}{3} - \frac{x - 5}{4} = \frac{5x + \mathbf{23}}{12}$

c $\frac{25x^2}{7\mathbf{y^3}} \div \frac{\mathbf{10x^2}}{21y^4} = \frac{15y}{2}$

Exam practice

1 $\frac{3x + 10}{10}$ **2** $\frac{y^5x^2}{9}$

17.3 Simplifying algebraic fractions

Purposeful practice 1

1 3 **2** $3(x - 6)$ **3** $\frac{3}{x - 6}$

4 $3x$ **5** $3x$ **6** $x - 2$

7 $\frac{x + 3}{x - 2}$ **8** $\frac{x + 4}{x - 2}$ **9** $\frac{x - 4}{x + 4}$

Purposeful practice 2

1 $\frac{x + 1}{x + 4}$ **2** $\frac{x + 3}{x + 2}$ **3** $\frac{x + 3}{x - 1}$

4 $\frac{x + 1}{x + 5}$ **5** $\frac{x + 3}{x + 4}$ **6** $\frac{x - 1}{x + 1}$

7 $\frac{x - 1}{x + 3}$ **8** $\frac{x + 3}{x - 2}$ **9** $\frac{x - 1}{x - 2}$

10 $\frac{2(x - 2)}{x + 1}$ **11** $\frac{x + 1}{x + 3}$ **12** $\frac{2(x + 2)}{x - 5}$

13 $\frac{2x - 1}{2x + 1}$ **14** $\frac{3x + 2}{x - 5}$ **15** $\frac{5x + 2}{x - 4}$

Problem-solving practice

1 $x - y$; 12

2 $\frac{6x^2 + 10x + 4}{4x^2 - 2x - 6}$
$= \frac{3x^2 + 5x + 2}{2x^2 - x - 3}$
$= \frac{(3x + 2)(x + 1)}{(2x - 3)(x + 1)}$
$= \frac{3x + 2}{2x - 3}$

3 Students' own answers, for example,
$\frac{x^2 + 7x + 12}{x^2 + 2x - 8} = \frac{\cancel{(x + 4)}(x + 3)}{\cancel{(x + 4)}(x - 2)} = \frac{x + 3}{x - 2}$

4 $\frac{x^2 - 5x - \mathbf{14}}{x^2 - \mathbf{49}} = \frac{\cancel{(x - 7)}(x + 2)}{\cancel{(x - 7)}(x + 7)} = \frac{(x + 2)}{(x + 7)}$

5 -2

6 a $\frac{x - 5}{x + 1}$ b $\frac{3}{x + 6}$

7 $3 - x$

8 $= \frac{(9 - x^2)(x^2 - 3x - 10)(2x^2 + 14x + 24)}{(14x + 42)(x^2 - 2x - 15)(x + 4)}$
$= \frac{(3 - x)(3 + x)(x + 2)(x - 5)2(x + 4)(x + 3)}{14(x + 3)(x - 5)(x + 3)(x + 4)}$
$= \frac{(3 - x)(x + 2)}{7}$

Exam practice

1 $\frac{5x + 1}{3x - 2}$

17.4 More algebraic fractions

Purposeful practice 1

1 $\frac{10x + 7}{6}$ 2 $\frac{3}{2x}$ 3 $\frac{2x + 1}{2x\ 2}$ 4 $\frac{(2x - 1)}{(x + 2)(x - 4)}$

5 $\frac{3x - 24}{(x + 2)(x - 4)}$ 6 $\frac{-1}{3x - 12}$ or $\frac{-1}{3x - 4}$ or $\frac{1}{12 - 3x}$ or $\frac{1}{3(4 - x)}$

7 $\frac{9x - 4}{4x}$ 8 $\frac{2x + 9}{(x + 4)(x + 1)}$ 9 $\frac{9x - 19}{(x + 4)(x - 3)(x + 1)}$

10 $\frac{3x + 13}{(x + 2)(x + 3)}$ 11 $\frac{x - 13}{(2x - 1)(x + 2)(x - 3)}$

12 $\frac{5x + 7}{(x + 1)(x - 1)(x + 3)}$

Purposeful practice 2

1 $\frac{x + 2}{x - 3}$ 2 $\frac{(x + 2)(x + 1)}{2(x - 3)}$ 3 $\frac{4(x + 2)}{5(x - 3)}$

4 $\frac{6}{x + 1}$ 5 $(x + 4)(x - 3)$ 6 $\frac{(2x - 3)(x - 4)}{(x - 5)(3x + 1)}$

Problem-solving practice

1 a $\frac{12x + 12}{x(x + 2)}$ b $\frac{42x - 78}{3x - 6}$

2 a $\frac{x + 1}{x + 4}$ b $\frac{1}{4}$

3 a $\frac{2x + 3}{(2x + 4)(x - 8)}$ b $\frac{2x + 3}{(x - 1)(x + 3)(x + 4)}$

4 $\frac{1}{x^2 - \mathbf{25}} + \frac{2}{x + 5} = \frac{2x - 9}{x^2 - \mathbf{25}}$

5 $x + 1$ and $x - 1$

6 $\frac{1}{3(x - 3)} + \frac{1}{(x - 3)(x + 6)}$
$= \frac{1}{3(x - 3)} \times \frac{x + 6}{x + 6} + \frac{1}{(x - 3)(x + 6)} \times \frac{3}{3}$
$= \frac{x + 6}{3(x - 3)(x + 6)} + \frac{3}{3(x - 3)(x + 6)}$
$= \frac{x + 9}{3(x - 3)(x + 6)}$
$A = 9, B = 3$

Exam practice

1 $\frac{x + 10}{x(x - 5)}$

17.5 Surds

Purposeful practice 1

1 $3(\sqrt{2} + 2)$ 2 $2(\sqrt{3} + 3)$ 3 $6(\sqrt{2} + 1)$
4 $6(\sqrt{2} + 1)$ 5 $2(2\sqrt{3} + 3\sqrt{2})$ 6 $2(2\sqrt{6} + 3\sqrt{2})$

Purposeful practice 2

1 $4\sqrt{3} + 8$ 2 $\sqrt{6} + 2\sqrt{2}$ 3 $\sqrt{6} - 8\sqrt{2}$
4 $6\sqrt{2} - 4$ 5 $30\sqrt{2} - 20$ 6 $30\sqrt{2} - 80$
7 $7 + 4\sqrt{3}$ 8 $79 + 20\sqrt{3}$ 9 $31 + 10\sqrt{6}$
10 40 11 $292 - 160\sqrt{3}$ 12 -92

Purposeful practice 3

1 $\frac{1 + \sqrt{3}}{2}$ 2 $\frac{\sqrt{3} - 1}{2}$ 3 $\sqrt{2} - 1$
4 $\frac{4 + \sqrt{2}}{14}$ 5 $\frac{2 + \sqrt{2}}{4}$ 6 $\frac{6 + 3\sqrt{2}}{2}$
7 $\frac{6 + 5\sqrt{2}}{14}$ 8 $\frac{4 + 3\sqrt{2}}{4}$ 9 $3 - 2\sqrt{2}$

Problem-solving practice

1 a $8\sqrt{2}$ b $\frac{(-25\sqrt{2})}{2} + 2\sqrt{5}$

2 $16 + 17\sqrt{2}$

3 $8 + 5\sqrt{2}$

4 a $-1 - \sqrt{6}, -1 + \sqrt{6}$ b $\frac{2 + \sqrt{13}}{3}, \frac{2 - \sqrt{13}}{3}$

5 $\frac{4 + 3\sqrt{2}}{2}$

6 a $\frac{3 + \sqrt{3}}{3}$

7 a In the third line Andrew has multiplied $\sqrt{2} \times \sqrt{3}$ to give $\sqrt{5}$. It should be $\sqrt{6}$. In the fourth line he has cancelled the 2s. We can only cancel if all the terms in the numerator and the denominator have a common factor.
b $\frac{2 - \sqrt{2} - 2\sqrt{3} + \sqrt{6}}{2}$

Exam practice

1 $\frac{23 + 17\sqrt{2}}{49}$

17.6 Solving algebraic fraction equations

Purposeful practice 1

1 $x = 4$ 2 $x = \frac{3}{4}$ 3 $x = \frac{3}{4}$
4 $x = 11$ 5 $x = \frac{3}{10}$ 6 $x = 2$
7 $x = -3$ 8 $x = 3$ 9 $x = \frac{-11}{3}$

Purposeful practice 2

1 a $x = 5, x = -5$ b $x = 2\sqrt{2}, x = -2\sqrt{2}$
c $x = 7, x = -7$ d $x = 4, x = -4$

2 a $x = 3.85, x = -2.85$
b $x = 3.46, x = -2.46$ c $x = 4.39, x = -2.39$ d $x = 4.13, x = -3.63$
e $x = 5.50, x = -6.00$ f $x = 4.90, x = -4.90$ g $x = -0.87, x = 6.87$
h $x = 0.00, x = 9.00$ i $x = -19.00$

Problem-solving practice

1 2, 3

2 a $\frac{x}{3} + \frac{3x}{4} = 26$ b 24

3 $R_1 = 3000$ Ohms, $R_2 = 6000$ Ohms

4 a 0.5, 8
b If $x = 0.5$ the 5 cm² rectangle would have a side of -2.5 cm, which is impossible.

5 a 2nd line: Edmund has made an error expanding $-2(x + 1)$. It should be $-2x - 2$
Final Line: He has added 5 to both sides but he should have subtracted 5 from both sides.
b $x = 0$ or $\frac{1}{2}$

6 $x = +\sqrt{3}, -\sqrt{3}$

Exam practice

1 $x = \frac{1}{41}$

17.7 Functions

Purposeful practice 1

1 a 0 b 12 c 12 d 0.75 e $\frac{4}{9}$ f 8.16

2 a 4 b 3.4 c −12 d 3.3 e $\frac{73}{21}$
f $-\frac{489}{35} = -13.97$ (2 d.p.)

Purposeful practice 2

1 32	2 86	3 32	4 30	5 132	6 240
7 20	8 6	9 132	10 −5	11 −23	12 −5

Purposeful practice 3

1 $f^{-1}(x) = \frac{x}{3}$ 2 $f^{-1}(x) = \frac{x-2}{3}$ 3 $f^{-1}(x) = 3(x-2)$

4 $f^{-1}(x) = \frac{3(x-2)}{5}$ 5 $f^{-1}(x) = 3x - 2$ 6 $f^{-1}(x) = \frac{3x-2}{4}$

7 $f^{-1}(x) = \frac{x}{4} - 3$ 8 $f^{-1}(x) = \frac{\frac{x}{4} - 3}{5} = \frac{x}{20} - \frac{3}{5}$

9 $f^{-1}(x) = \frac{\frac{7x}{4} - 3}{5} = \frac{7x}{20} - \frac{3}{5}$

Problem-solving practice

1 a i $\frac{2}{2x+3}$ ii $\frac{2}{3-x}$ iii $\frac{3x+11}{x+3}$
b $x = -3$ would mean that the denominator would be 0. Dividing by zero is undefined.

2 a $d^{-1}(x) = \frac{x+4}{5}$ b $x = 1$ 3 B, D

4 a −5, 4 b −4, 10 5 −31.5

6 $a = -1 + \sqrt{6}$, $a = -1 - \sqrt{6}$ 7 −1, −7

8 95 9 $a = 4, b = 3$

Exam practice

1 $a = 2, b = 14$

17.8 Proof

Purposeful practice 1

1 Students' own answers, for example, $\frac{3}{4} \times \frac{8}{3} = \frac{24}{12} = 2$

2 The answer will be the same, zero, when the numbers in the calculation are both the same.

3 Any number between −1 and 1, for example, $\left(\frac{1}{2}\right)^2 < \frac{1}{2}$

4 Students' own answers, for example, $3^2 + 4^2$ or $6^2 + 7^2$

Purposeful practice 2

1 Any number squared is positive or zero. When you add 4 to a positive number or zero, the answer will always be positive.

2 a 2
b When a is a non-zero integer, $2a$ is a multiple of 2 and so is an even number.

3 $m + n$ is **even**. $2(m + n)$ is **even** because **it is a multiple of 2**.

4 $2(2mn + n + m)$ is **even** because **it is a multiple of 2**.
Adding 1 to the previous expression will therefore be **odd** because **any number which is 1 more than a multiple of 2 is odd**.

5 $2n$ is a multiple of 2, and so is an even number. An even number plus an odd number (3) is always odd.

6 a Students' own answers, for example, $n, n + 1, n + 2$
b $(2n + 1) + (2n + 3) + (2n + 5) = 6n + 9 = (6n + 8) + 1$
$6n + 8$ can be written as $2(3n + 4)$ so is even. Adding 1 to it will make the result odd.

7 In any two consecutive numbers, one number will be even and one will be odd.
The even number can be written as $2n$, where n is an integer.
Let us call the odd number m.
Therefore the product will be $2nm$, which is a multiple of 2.
So, the product of any two consecutive numbers is even.

Problem-solving practice

1 a 5 b The answer is still 5.
c $\frac{x + (x+1) + 9}{2} - x = \frac{2x + 10}{2} - x$
$= (x + 5) - x$
$= 5$

2 a 2 is the 5th term in the sequence.
b Students' own answers, for example, $3^2 + 1 = 10$
c Students' own answers, for example, $3^3 + 4^3 = 91$
d Students' own answers, for example, $(-2)^3 = -8$ and $-8 < -2$. Any negative integer

3 $(2m)^2 + (2m + 2)^2 = 4m^2 + (4m^2 + 8m + 4)$
$= 8m^2 + 8m + 4$
$= 4(2m^2 + 2m + 1)$
$2m^2 + 2m + 1$ is an integer, so $4(2m^2 + 2m + 1)$ is a multiple of 4 and therefore in the 4 times table.

4 $4n^2 + 4n + 1 = 4(n^2 + n) + 1$
$n^2 + n$ is an integer, therefore $4(n^2 + n)$ is even. Adding 1 to an even number will always give an odd number.

5 $x^2 + (x + 1)^2 = x^2 + (x^2 + 2x + 1) = 2x\ 2 + 2x + 1 = 2(x^2 + x) + 1$
$(x^2 + x)$ is an integer, so $2(x^2 + x)$ is even. Therefore $2(x^2 + x) + 1$ will always be odd.

6 $A = 1$

7 $3x - a = 15$
$3x = 15 + a$
$x = \frac{15}{3} + \frac{a}{3}$
$x = 5 + \frac{a}{3}$
For x to be an integer $\frac{a}{3}$ must be an integer. For $\frac{a}{3}$ to be an integer a must be divisible exactly by 3 and so is in the 3 times table.

8 Height and length of base of large triangle $= x$, so its area is $\frac{1}{2} \times x \times x = \frac{x^2}{2}$
Height and length of base of small triangle $= \frac{x}{2}$, so its area is $\frac{1}{2} \times \frac{x}{2} \times \frac{x}{2} = \frac{x^2}{8}$
So, the area of the shape which is left is $= \frac{x^2}{2} - \frac{x^2}{8}$
$= \frac{4x^2}{8} - \frac{x^2}{8}$
$= \frac{3x^2}{8}$

9 $(2n - 2)^2 - (4n - 4)(n - 2) = (4n^2 - 8n + 4) - (4n^2 - 12n + 8) = 4n - 4$
$= 4(n - 1)$
Since n is an integer, $(n - 1)$ is also an integer and therefore $4(n - 1)$ is a multiple of 4.

Exam practice

1 $a^2 - b^2$ is the difference between two square numbers.
$a^2 - b^2 = (a + b)(a - b)$
$a + b$ is the sum of the two numbers. $a - b$ is the difference between the numbers. Therefore the sum of the two numbers multiplied by the difference between the two numbers will always be equal to the difference between the square of the numbers.

18 Vectors and geometric proof

18.1 Vectors and vector notation

Purposeful practice 1

1 a (8, 8) b (2, 8) c (8, 0) d (2, 0)
e (9, 7) f (9, 8) g (5, 8) h (9, 4)

2 a $\begin{pmatrix}3\\4\end{pmatrix}$ b $\begin{pmatrix}3\\4\end{pmatrix}$ c $\begin{pmatrix}-3\\-4\end{pmatrix}$ d $\begin{pmatrix}-2\\-6\end{pmatrix}$ e $\begin{pmatrix}-5\\-3\end{pmatrix}$ f $\begin{pmatrix}-6\\-2\end{pmatrix}$

Purposeful practice 2

1 5	2 5	3 5	4 10	5 10	6 $\sqrt{73}$
7 $\sqrt{68}$	8 $\sqrt{65}$	9 8	10 $\sqrt{65}$	11 $\sqrt{68}$	12 $\sqrt{68}$

Problem-solving practice

1 a $\begin{pmatrix}4\\-3\end{pmatrix}$ b 12 c A right-angled triangle

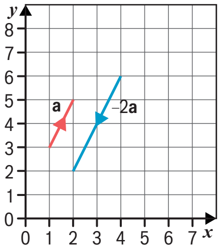
2 a $\begin{pmatrix}-2\\-7\end{pmatrix}$ b (15, 13)

3 Students' own answers, for example, $\begin{pmatrix}2\\9\end{pmatrix}$

4 (3, 15) or (3, −9)

5 Students' own answers, for example, $\begin{pmatrix}7\\24\end{pmatrix}$

Exam practice

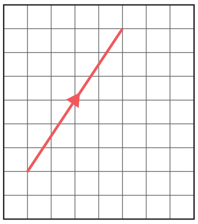
1 

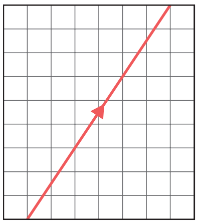
18.2 Vector arithmetic

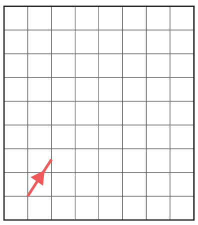
Purposeful practice 1

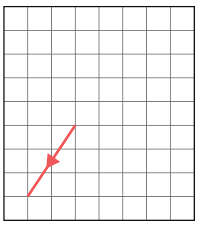
1 a $\begin{pmatrix}4\\6\end{pmatrix}$ b $\begin{pmatrix}6\\9\end{pmatrix}$ c $\begin{pmatrix}1\\1.5\end{pmatrix}$ d $\begin{pmatrix}-2\\-3\end{pmatrix}$ e $\begin{pmatrix}-4\\-6\end{pmatrix}$

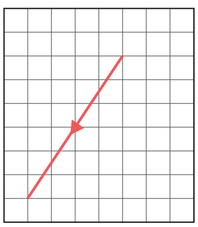
f $\begin{pmatrix}0\\-6\end{pmatrix}$ g $\begin{pmatrix}0\\-9\end{pmatrix}$ h $\begin{pmatrix}0\\9\end{pmatrix}$ i $\begin{pmatrix}0\\3\end{pmatrix}$ j $\begin{pmatrix}1\\-2\end{pmatrix}$

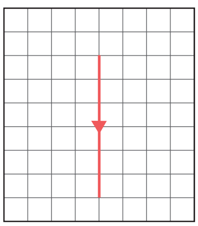
2 a 

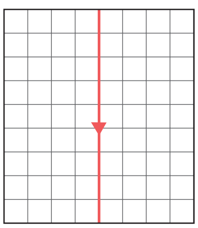
b 

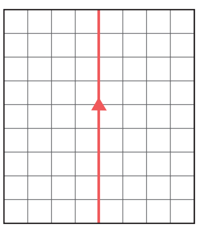
c 

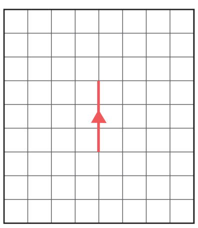
d 

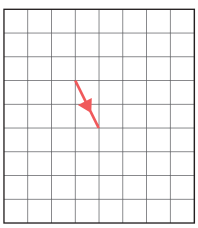
e 

f 

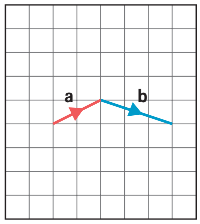
g 

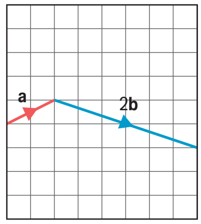
h 

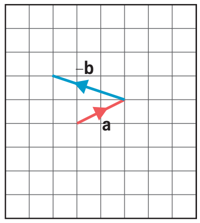
i 

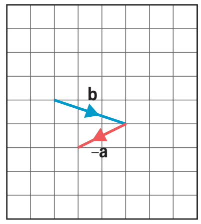
j 

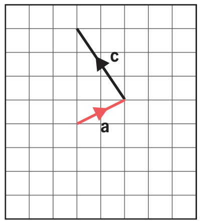
Purposeful practice 2

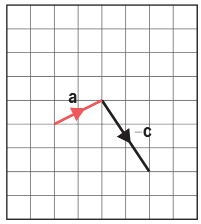
1 a 

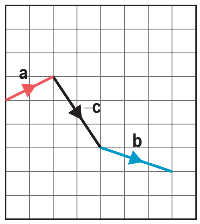
b 

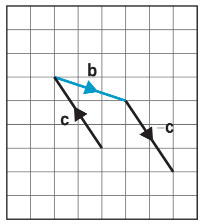
c 

d 

e 

f 

g 

h 

2 a $\begin{pmatrix}5\\0\end{pmatrix}$ b $\begin{pmatrix}8\\-1\end{pmatrix}$ c $\begin{pmatrix}-1\\2\end{pmatrix}$ d $\begin{pmatrix}1\\-2\end{pmatrix}$

e $\begin{pmatrix}0\\4\end{pmatrix}$ f $\begin{pmatrix}4\\-2\end{pmatrix}$ g $\begin{pmatrix}7\\-3\end{pmatrix}$ h $\begin{pmatrix}3\\-1\end{pmatrix}$

3 Students' own answers, for example, $\begin{pmatrix}4\\-3\end{pmatrix}$

4 $\begin{pmatrix}-4\\3\end{pmatrix}$

Problem-solving practice

1 a $\begin{pmatrix}6\\10\end{pmatrix}$ b $\begin{pmatrix}9\\15\end{pmatrix}$ and $\begin{pmatrix}3\\5\end{pmatrix}$

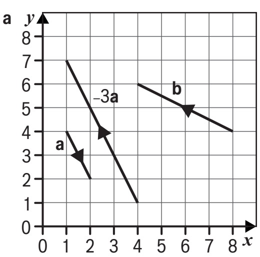
2 **a** = **b**

3 $\overrightarrow{SQ}$ is parallel to $\overrightarrow{TP}$. $\overrightarrow{SQ}$ = **b** − **a** and $\overrightarrow{TP}$ = **a** + **b** − **a** − **a** = **b** − **a**

4 17.66

5 Students' own answers, for example, $\begin{pmatrix}2\\-12\end{pmatrix}$

Exam practice

1 a 

b $\begin{pmatrix}-11\\4\end{pmatrix}$

18.3 More vector arithmetic

Purposeful practice 1

1 $\mathbf{b} + \mathbf{a}$ 2 $-\mathbf{b}$ 3 $-\mathbf{a}$ 4 $-\mathbf{a} - \mathbf{b}$ 5 $-\mathbf{a} - \mathbf{b}$

6 $\frac{1}{2}(-\mathbf{a} - \mathbf{b})$ 7 $\frac{1}{2}(\mathbf{a} - \mathbf{b})$ 8 $\frac{1}{2}(\mathbf{a} - \mathbf{b})$ 9 $\frac{1}{2}(\mathbf{b} - \mathbf{a})$

Purposeful practice 2

1 $\mathbf{h}$ 2 $\mathbf{k}$ 3 $\mathbf{h} + \mathbf{k}$ 4 $-\mathbf{k} - \mathbf{h}$ 5 $\mathbf{k} - \mathbf{h}$

Problem-solving practice

1 $\overrightarrow{AD} = 3\mathbf{b}$

$\overrightarrow{AD} = \mathbf{a} + \mathbf{b} + \mathbf{c}$

$3\mathbf{b} = \mathbf{a} + \mathbf{b} + \mathbf{c}$

$2\mathbf{b} = \mathbf{a} + \mathbf{c}$

$\mathbf{b} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$

2 a $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ b Yes, sides are 13, 9.19 and 9.19.

3 $\overrightarrow{JK} = \mathbf{b} + \mathbf{a}$

$\overrightarrow{IL} = \mathbf{a} + \mathbf{b} + \mathbf{a} + \mathbf{b}$

$= 2(\mathbf{a} + \mathbf{b})$

$\overrightarrow{IL} = 2\overrightarrow{JK}$, therefore parallel

Exam practice

1 a $\overrightarrow{QR} = 2\mathbf{a} + 5\mathbf{b}$ b $\overrightarrow{PR} = -2\mathbf{a} + 5\mathbf{b}$

18.4 Parallel vectors and collinear points

Purposeful practice 1

1 a $\begin{pmatrix}3\\4\end{pmatrix}$ b $\begin{pmatrix}4\\5\end{pmatrix}$ c $\begin{pmatrix}3\\5\end{pmatrix}$ d $\begin{pmatrix}4\\4\end{pmatrix}$ e $\begin{pmatrix}1\\0\end{pmatrix}$

2 a $\begin{pmatrix}6\\10\end{pmatrix}$ b $\overrightarrow{OY}$ because they both pass through the origin and are parallel.

Purposeful practice 2

1 a $\begin{pmatrix}4\\8\end{pmatrix}$ b $\begin{pmatrix}4\\8\end{pmatrix}$ c $\begin{pmatrix}3\\7\end{pmatrix}$ d $\begin{pmatrix}2\\4\end{pmatrix}$ e $\begin{pmatrix}3\\5\end{pmatrix}$ f $\begin{pmatrix}2\\6\end{pmatrix}$

2 a $\begin{pmatrix}2\\4\end{pmatrix}$ b $\begin{pmatrix}2\\4\end{pmatrix}$ c $\begin{pmatrix}2\\4\end{pmatrix}$ d $\begin{pmatrix}1\\1\end{pmatrix}$ e $\begin{pmatrix}1\\1\end{pmatrix}$ f $\begin{pmatrix}1\\3\end{pmatrix}$

3 a Yes b Yes c No d No e No f Yes

4 Yes **a** and **d**. No **b**, **c**, **e** and **f**.

5 **a**, **b** and **d**

Problem-solving practice

1 a Yes, LM is parallel to KL and they both go through point L.

b $(-12, 14.5)$

2 $g = 60$ $h = 2$

3 Students' own answers, for example, $\begin{pmatrix}1.25\\-1.5\end{pmatrix}$

4 a Yes, because it will be a scalar multiple of **a**.

b Yes, because it will be a scalar multiple of **b**.

c Yes, because it will be a scalar multiple of **a**.

5 $x = 4$

6 Two possible student answers: (4, 4) with the vector $\overrightarrow{PR} = \begin{pmatrix}1\\4\end{pmatrix}$ and the vector $\overrightarrow{RQ} = \begin{pmatrix}1\\4\end{pmatrix}$

Or R (6, 12) with $\overrightarrow{PQ} = \begin{pmatrix}2\\8\end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix}1\\4\end{pmatrix}$ meaning $\overrightarrow{PQ}$ is a scalar multiple and so is parallel.

7 a 10 b Multiple answers possible, for example, $\mathbf{a} = 11$, $\mathbf{b} = 22$

Exam practice

1 $\mathbf{a} + \mathbf{b} = \begin{pmatrix}2\\5\end{pmatrix}$

$3(\mathbf{a} + \mathbf{b}) = \begin{pmatrix}6\\15\end{pmatrix}$

18.5 Solving geometric problems

Purposeful practice 1

1 a $\begin{pmatrix}5\\5\end{pmatrix}$ b $\begin{pmatrix}15\\15\end{pmatrix}$ c $\begin{pmatrix}-15\\-15\end{pmatrix}$

2 a $\begin{pmatrix}12\\12\end{pmatrix}$ b $\begin{pmatrix}8\\8\end{pmatrix}$ c $\begin{pmatrix}-8\\-8\end{pmatrix}$

Purposeful practice 2

1 $\mathbf{c}$ 2 $\mathbf{a} + \mathbf{b} + \mathbf{c}$ 3 $\mathbf{a} + \mathbf{c}$ 4 $\mathbf{a}$

5 $-\mathbf{a}$ 6 $-3\mathbf{a}$ 7 $-4\mathbf{a}$ 8 $\mathbf{c} - 4\mathbf{a}$

9 $\mathbf{b} + \mathbf{c} - 4\mathbf{a}$ 10 $\mathbf{b} + \mathbf{c} - 3\mathbf{a}$

Problem-solving practice

1 $\frac{2}{5}\mathbf{b} - 3\mathbf{a}$

2 a $\overrightarrow{AD} = \frac{5}{2}\mathbf{a} + 15\mathbf{b}$ b $(-28.5, 75.5)$

3 a $8\mathbf{a} - 3\mathbf{b}$ b 109.56

Exam practice

1 $\overrightarrow{MP} = -\frac{1}{2}\mathbf{b} + \mathbf{a} + k\mathbf{b} = \mathbf{a} + (k - \frac{1}{2})\mathbf{b}$

$\overrightarrow{PN} = (1 - k)\mathbf{b} + 2\mathbf{a}$

Comparing $\overrightarrow{MP}$ and $\overrightarrow{PN}$

$2\overrightarrow{MP} = \overrightarrow{PN}$

$2\left(k - \frac{1}{2}\right) = 1 - k$

$2k - 1 = 1 - k$

$3k = 2$

$k = \frac{2}{3}$

19 Proportion and graphs

19.1 Direct proportion

Purposeful practice 1

1 $y \propto x$ — y is directly proportional to x — $y = kx$

$x \propto y$ — x is directly proportional to y — $x = ky$

$a \propto b$ — a is directly proportional to b — $a = kb$

$b \propto a$ — b is directly proportional to a — $b = ka$

Purposeful practice 2

1 $y = 2x$ 2 $y = \frac{1}{2}x$ 3 $y = -\frac{1}{2}x$ 4 $y = -2x$

5 $a = 0.6b$, $a = \frac{6}{10}b$ or $a = \frac{3}{5}b$

6 $a = -0.6$, $a = -\frac{6}{10}b$ or $a = -\frac{3}{5}b$

7 $a = 0.6$, $a = \frac{6}{10}b$ or $a = \frac{3}{5}b$ 8 $a = 1.6b$, $a = \frac{10}{6}b$ or $a = \frac{5}{3}b$

9 $p = 0.3125q$ or $p = \frac{5}{16}q$ 10 $p = \frac{5}{74}q$

11 $h = \frac{1}{6}d$ 12 $h = \frac{5}{6}d$

Problem-solving practice

1 $m = 9.6$

2

x	−5	−2	0	7
y	7.5	3	0	−10.5

3 $r = -0.78z$ 4 £11.60 5 $y = 28$

6 $\frac{t}{m} = \frac{2.66}{0.7} = \frac{3.42}{0.9} = \frac{6.08}{1.6} = 3.8$

So $t = 3.8m$, which means t is directly proportional to m.

Exam practice

1 $s = 6$

19.2 More direct proportion

Purposeful practice 1

1 $y = \frac{1}{2}x^2$ 2 $y = \frac{1}{8}x^3$ 3 $y = 4\sqrt{x}$ 4 $y = -5a^2$

5 $y = -5a^3$ 6 $y = -5\sqrt{a}$ 7 $f = 0.09375g^2$ or $f = \frac{3}{32}g^2$

8 $f = 0.0234375g^3$ or $f = \frac{3}{128}g^3$ 9 $f = 0.75\sqrt{g}$ 10 $w = \frac{1}{9}t^2$

11 $w = \frac{1}{27}t^3$ 12 $w = \frac{1}{\sqrt{3}}\sqrt{t}$

Purposeful practice 2

1 $y = 0$ 2 $y = 0$ 3 $y = 0$

Problem-solving practice

1 $d = 10t^2$ 2 $V = 4.16r^3$ 3 $C = 36\sqrt{A}$

4 a 372 m b 161 seconds

5 £27 6 $y = 1\frac{4}{5}$ or $y = 1.8$

7 4.47 m 8 400 J 9 $y = 56$

Exam practice

1 $y = \frac{8}{9}$

19.3 Inverse proportion

Purposeful practice 1

1 a 400 b −400 c 4000 d −4000
2 a 16 000 b 16 000 c 1 600 000 d 1 600 000
3 a 640 000 b −640 000 c 640 000 000 d −640 000 000
4 a $20\sqrt{10}$ b $-20\sqrt{10}$ c 200 d −200

Purposeful practice 2

1 a 0.4 b −0.4 c 0.04 d −0.04
2 a 0.016 b 0.016 c 0.000 16 d 0.000 16
3 a 0.000 64 b −0.000 64 c 0.000 000 64 d −0.000 000 64
4 a 2 b −2 c $0.2\sqrt{10}$ d $-0.2\sqrt{10}$

Problem-solving practice

1 $x = 2$ 2 $x = \frac{4}{9}$
3 a $y = \frac{5}{\sqrt{x}}$ b $y = \frac{5}{8}$ c $x = \frac{25}{49}$

Exam practice

1 a Graph D b Graph C c Graph B d Graph A
2 $y = \frac{9}{x^2}$

19.4 Exponential functions

Purposeful practice 1

1 a

x	−5	−2	−1	0	1	2
y	0.031 25	0.25	0.5	1	2	4

b

x	−5	−2	−1	0	1	2
y	0.004 11...	0.111...	0.333...	1	3	9

c

x	−5	−2	−1	0	1	2
y	0.000 976...	0.0625	0.25	1	4	16

2 a

x	−2	−1	0	1	2	5
y	4	2	1	0.5	0.25	0.031 25

b

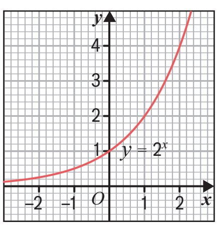
x	−2	−1	0	1	2	5
y	9	3	1	0.333...	0.111...	0.004 11...

c

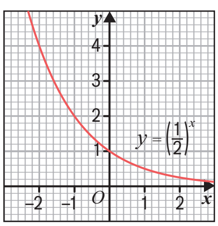
x	−2	−1	0	1	2	5
y	16	4	1	0.25	0.0625	0.000 976...

Purposeful practice 2

1 a

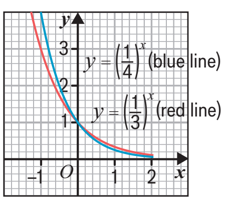


b



2 a In the graph of $y = 2^x$, y doubles every time x increases by 1.
b In the graph of $y = \left(\frac{1}{2}\right)^x$, y halves every time x increases by 1.

3 a, c



b y values decrease fastest in the graph of $\left(\frac{1}{3}\right)^x$

Problem-solving practice

1 a 25 years b $\frac{1}{8}$

2 a

Time, t (minutes)	0	1	2	3	4	5
Number of cells, n	1	2	**4**	**8**	**16**	**32**

b The number of cells is $n = 2^t$, which is an exponential function, and the number of cells is growing.

3 a In each match one player wins and one loses, so half the players lose and leave each round.

b

Round, x	1	2	3	4	5
Number of players, y	$32 = 32\left(\frac{1}{2}\right)^0$	$16 = 32\left(\frac{1}{2}\right)^1$	$8 = 32\left(\frac{1}{2}\right)^2$	$4 = 32\left(\frac{1}{2}\right)^3$	$2 = 32\left(\frac{1}{2}\right)^4$

The numbers are halving each time so the exponential function is of the form $y = ka^{x-1}$

When $x = 1$, $y = 32$, so $y = ka^0 = 32$, and $k = 32$

c At the end of round 5 (after the match is played), there will only be one player. So 5 rounds are needed.

4 One of:
For exponential growth the price has to be multiplied by the same number every year, and the graph does not show this (with an example, between 2000 and 2005 the price was multiplied by 1.1 and between 2005 and 2010 the price was multiplied by 1.55 to 2 d.p.).
The graph is not the same shape as an exponential curve – it is not rising steeply enough and 2018 shows a reduction in chocolate price from 2017.

Exam practice

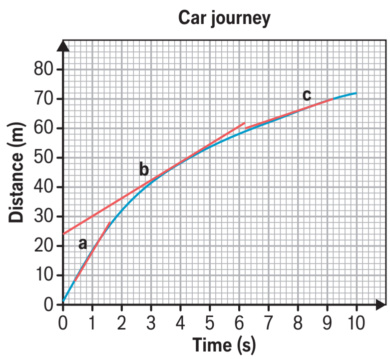
1 (0, 1)

19.5 Non-linear graphs

Purposeful practice 1

1 The speed gradually decreases as the time increases.

2



3 a 16.7 m/s (accept between 15 and 18)
b 6.1 m/s (accept between 5 and 8)
c 3.3 ms (accept between 2 and 5)
4 a 12 m/s (accept between 11 and 13)
b 8.25 m/s (accept between 7 and 9.5)

Purposeful practice 2

1 Car A: distance travelled is 92 m.
Car B: distance travelled is 230 m.

Problem-solving practice

1 Between 17 and 21 m/s
2 Fred has not drawn the tangent. A tangent does not have to go through zero. It should touch the graph only once and not intersect the graph. He has found the average speed in the first 4 seconds instead.
3 Gradient = −3

Exam practice

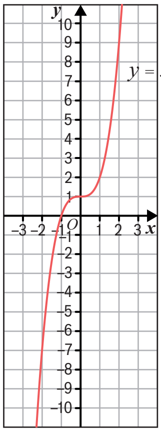
1 a 540 m (accept 520–560)
b The estimate is an underestimate as the strips do not include all the area under the graph.

19.6 Translating graphs of functions

Purposeful practice

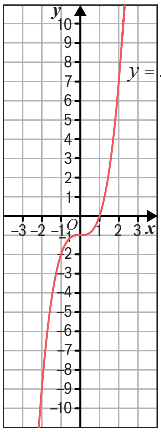
1

x	−2	−1	0	1	2
x^3	−8	−1	0	1	8
$y = f(x) + 1 = x^3 + 1$	**−7**	**0**	**1**	**2**	**9**



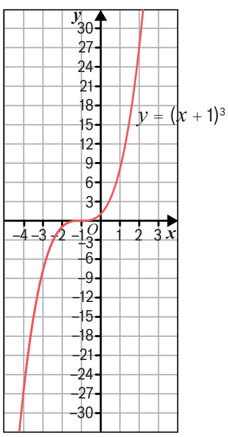
2

x	−2	−1	0	1	2
x^3	−8	−1	0	1	8
$y = f(x) - 1 = x^3 - 1$	**−9**	**−2**	**−1**	**0**	**7**



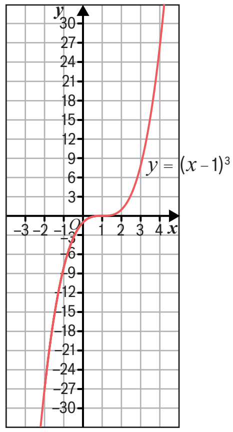
3

x	−2	−1	0	1	2
x^3	−8	−1	0	1	8
$y = f(x + 1) = (x + 1)^3$	**−1**	**0**	**1**	**8**	**27**



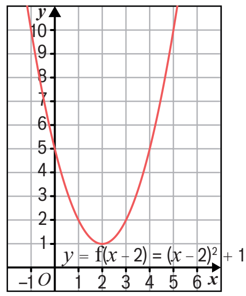
4

x	−2	−1	0	1	2
x^3	−8	−1	0	1	8
$y = f(x - 1) = (x - 1)^3$	**−27**	**−8**	**−1**	**0**	**1**



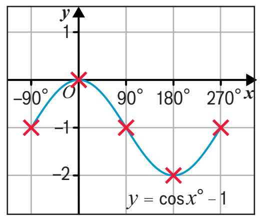
Problem-solving practice

1



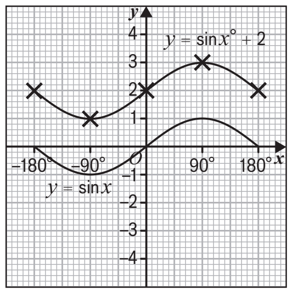
2 $y = (x + 2)^2 + 3(x + 2)$

3



Exam practice

1



19.7 Reflecting and stretching graphs of functions

Purposeful practice 1

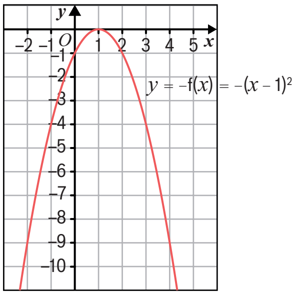
1

Point	Coordinates of reflection in x-axis	Coordinates of reflection in y-axis
A(3, 2)	**(3, −2)**	**(−3, 2)**
B(−2, 3)	**(−2, −3)**	**(2, 3)**
C(1, −4)	**(1, 4)**	**(−1, −4)**
D(−4, −1)	**(−4, 1)**	**(4, −1)**

Purposeful practice 2

1

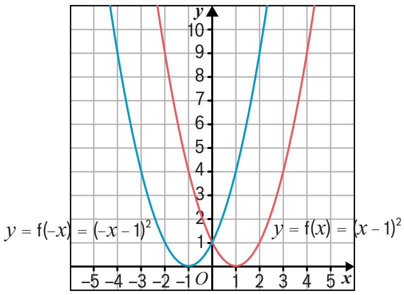
x	−2	−1	0	1	2
$y = f(x) = (x - 1)^2$	9	4	1	0	1
$y = -f(x) = -(x - 1)^2$	**−9**	**−4**	**−1**	**0**	**−1**



2

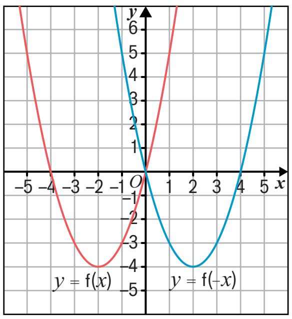
x	-2	-1	0	1	2
$y = f(x) = (x - 1)^2$	9	4	1	0	1
$y = f(-x) = (-x - 1)^2$	**1**	0	**1**	**4**	9

3

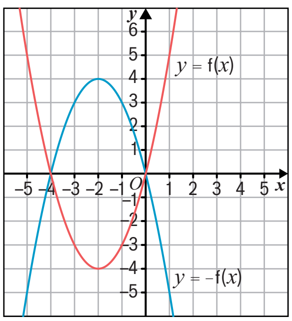


Problem-solving practice

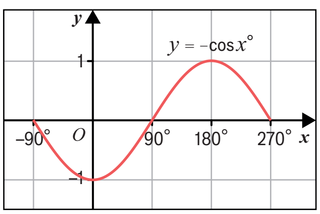
1 a



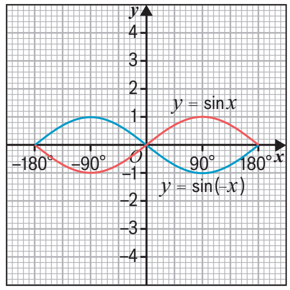
b

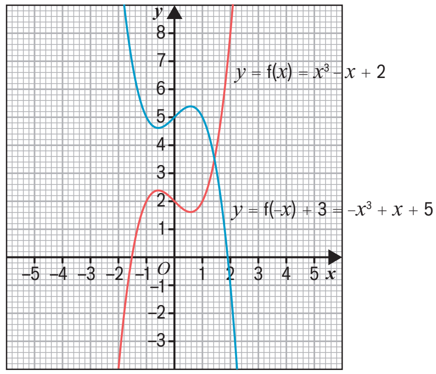


2



3





b (1, 5)

Exam practice

1 P is (4, −3)

Mixed exercises E

Mixed problem-solving practice E

1 Angle OAD = 90° because the angle between a tangent and the radius is 90°.
Angle AOC = 65° because the angle sum of a triangle is 180°.
The reflex angle AOC = 295° because the angle sum at a point is 360°.
Angle ABC = 147.5 because the angle at the centre of a circle is twice the angle at the circumference when subtended by the same arc.

2 $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

3 $\frac{1}{2}n(n + 1) + \frac{1}{2}(n + 2)(n + 9) = \frac{1}{2}(n^2 + n) + \frac{1}{2}(n^2 + 11n + 18)$

$$= \frac{1}{2}(2n^2 + 12n + 18)$$

$$= n^2 + 6n + 9 = (n + 3)^2$$

$(n + 3)^2$ is a square number so the sum of $\frac{1}{2}n(n + 1)$ and $\frac{1}{2}(n + 2)(n + 9)$ is always a square number.

4 OP = OS = OR as they are the radii, so triangles POR and OPS are isosceles. Angle POR = 120° as it is double angle PQR (the angle at the centre of a circle is twice the angle at the circumference). Angle POS = half of 120° because OS bisects side PR, so angle POS = 60°. Triangle OPS is isosceles so angle
OPS = angle OSR = 60°. Therefore, triangle OPS is equilateral.

5 Chantal should have written −5 not +5 on the denominator in the second line of working as $-\sqrt{5} \times +\sqrt{5} = -5$ not $+5$

6 ROP bisects angle ORS and angle OPS because it is the line of symmetry of the kite.
So, angle OPS = $\frac{x}{2}°$ and angle ORS = $y°$.
Angle PDO and angle RGO are 90° because the angle between the radius and the tangent is 90°.
Therefore, angle POD = $(180 - 90 - \frac{x}{2})° = (90 - \frac{x}{2})°$ and angle ROC = $(180 - 90 - y)° = (90 - y)°$ as angles in a triangle add to 180°.
So, angle COD = $180° - (90 - \frac{x}{2})° - (90 - y)° = (\frac{x}{2} + y)°$ since angles in a straight line add to 180°.

7 $x = \dfrac{y}{y+1}$

8 $y = \mathrm{f}(-x)$

9 $\overrightarrow{AB} = -2\mathbf{p} + 2\mathbf{q}$

$\overrightarrow{PQ} = -\mathbf{p} + \mathbf{q}$

$\overrightarrow{AB} = 2\overrightarrow{PQ}$ so AB is parallel to PQ.

10 $p = 3$ and $q = -18$

11 $a = 90$, $b = -1$

12 3

13 Length scale factor $= \dfrac{x^2 - 4}{3(x-2)} = \dfrac{(x+2)(x-2)}{3(x-2)} = \dfrac{x+2}{3}$

Area scale factor $= \dfrac{(x+2)^2}{3^2}$

$A = 9 \times \dfrac{(x+2)^2}{9} = (x+2)^2 = x^2 + 4x + 4$

14 $\dfrac{14\sqrt{3}}{3}$

Exam practice

15 **a** $\dfrac{5}{9}$ **b** $\dfrac{1}{20}$ or 0.05

16 $c = \dfrac{8b}{a - 24}$

17 **a** $q = \dfrac{16}{p^2}$ **b** $\dfrac{4}{5}$

18 Students identify two correct pairs of equal angles with correct reasons, for example, angle BAE = angle CDE because angles in the same segment are equal, and angle AEB = DEC because vertically opposite angles are equal. Therefore, the three pairs of angles are equal ABE = DCE, BEA = CED, EAB = EDC, as the angles in a triangle total 180°. So triangle ABE and triangle DCE are similar.

19 $y = \dfrac{12}{x^4}$

20 Substitute $x = 2y + 15$ into $x^2 + y^2 = 45$ to give $(2y + 15)^2 + y^2 = 45$

$4y^2 + 60y + 225 + y^2 = 45$

$5y^2 + 60y + 180 = 0$

$5(y + 6)^2 = 0$

There is only one solution of $y = -6$ and $x = 3$, so the straight line with equation $x - 2y = 15$ is a tangent to the circle with equation $x^2 + y^2 = 45$

21 $\overrightarrow{BA} = \mathbf{a} - \mathbf{b}$, $\overrightarrow{BP} = k(\mathbf{a} - \mathbf{b})$, $\overrightarrow{MN} = -\frac{1}{2}\mathbf{a} + 4\mathbf{b}$,

$\overrightarrow{PN} = -k(\mathbf{a} - \mathbf{b}) + 3\mathbf{b} = (-k)\mathbf{a} + (k + 3)\mathbf{b}$

PN is a line segment of MN so yPN = MN for some number y

$y[(-k)\mathbf{a} + (k + 3)\mathbf{b}] = -\frac{1}{2}\mathbf{a} + 4\mathbf{b}$

Equating coefficients of **a** gives $-yk = -\frac{1}{2}$, therefore $ky = \frac{1}{2}$

Equating coefficients of **b** gives $(k + 3)y = 4$ or $ky + 3y = 4$

So, $\frac{1}{2} + 3y = 4$ and therefore $y = \frac{7}{6}$

$ky = \frac{1}{2}$ so $k = \frac{1}{2} \div \frac{7}{6} = \frac{3}{7}$

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