

## Mechanics 1

# Edexcel Modular Mathematics for AS and A-level

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# About this book

This book is designed to provide you with the best preparation possible for your Edexcel M1 unit examination:

- This is Edexcel's own course for the GCE specification.
- Written by a senior examining team at Edexcel: the chair of examiners, chief examiners and principal examiners.
- The LiveText CD-ROM in the back of the book contains even more resources to support you through the unit.
- A matching M1 revision guide is also available.

Brief chapter overview and 'links' to underline the importance of mathematics: to the real world, to your study of further units and to your career

## Finding your way around the book

Detailed contents list shows which parts of the M1 specification are covered in each section

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Each section begins with a statement of what is covered in the section

Concise learning points

Step-by-step worked examples – they are model solutions and include examiners hints

Each chapter has a different colour scheme, to help you find the right chapter quickly

Each chapter ends with a mixed exercise and a summary of key points.

At the end of the book there is an examination-style paper.

Every few chapters, a review exercise helps you consolidate your learning

## 2

### Kinematics of a particle moving in a straight line

After completing this chapter you should be able to

- solve problems involving motion in a straight line with constant acceleration
- model an object moving vertically under gravity
- understand distance-time graphs and speed-time graphs.

There are many situations which can be modelled as motion in a straight line with constant acceleration.

- The acceleration due to gravity is constant. You can find a diver's vertical speed using the constant acceleration formulae.
- You can calculate the depth of a well by dropping a stone and timing how long it takes to reach the bottom.
- If you are modelling motion under gravity you often ignore air resistance. This skydiver isn't accelerating. The force of the air resistance equals the force of gravity. The speed at which this happens is called terminal velocity.

## 1

### Review Exercise

- 1 A train decelerates uniformly from  $15 \text{ m s}^{-1}$  to  $2 \text{ m s}^{-1}$  in a distance of  $350 \text{ m}$ . Calculate
  - a the deceleration,
  - b the total time taken, under this deceleration, to come to rest from a speed of  $15 \text{ m s}^{-1}$ .
- 2 In taking off, an aircraft moves on a straight runway AB of length  $1.2 \text{ km}$ . The aircraft moves from A with initial speed  $2 \text{ m s}^{-1}$ . It moves with constant acceleration and  $20 \text{ s}$  later it leaves the runway at C with speed  $74 \text{ m s}^{-1}$ . Find
  - a the acceleration of the aircraft,
  - b the distance BC.
- 3 A car is moving along a straight horizontal road at a constant speed of  $30 \text{ m s}^{-1}$ . At the instant when the car passes a lay-by, a motorcycle leaves the lay-by, starting from rest, and moves with constant acceleration  $2.5 \text{ m s}^{-2}$  in pursuit of the car. Given that the motorcycle overtakes the car  $T$  seconds after leaving the lay-by, calculate
  - a the value of  $T$ ,
  - b the speed of the motorcycle at the instant of passing the car.
- 4 A particle moves with constant acceleration along the straight line OAM and passes through the points O, L and M at times  $0$ ,  $4$  and  $10 \text{ s}$  respectively. Given that  $OL = 14 \text{ m}$  and  $OM = 50 \text{ m}$ , find
  - a the acceleration of the particle,
  - b the speed of the particle at M.
- 5 A particle P moves in a straight line with constant retardation. At the instant when P passes through the points A, B and C, it is moving with speeds  $10 \text{ m s}^{-1}$ ,  $7 \text{ m s}^{-1}$  and  $3 \text{ m s}^{-1}$  respectively. Show that  $\frac{AB}{BC} = \frac{51}{40}$ .
- 6 A car moving with uniform acceleration along a straight level road, passed points A and B when moving with speed  $9 \text{ m s}^{-1}$  and  $14 \text{ m s}^{-1}$  respectively. Find the speed of the car at the instant that it passed C, the mid-point of AB.

Past examination questions are marked 'E'

Each section ends with an exercise – the questions are carefully graded so they increase in difficulty and gradually bring you up to standard

### 5.1 You can find the moment of a force acting on a body.

The moment of a force measures the turning effect of the force on the body on which it is acting.

The moment of a force depends on the magnitude of the force and its distance from the axis of rotation.

The moment of a force  $F$  about a point  $P$  is the product of the magnitude of the force and the perpendicular distance of the line of action of the force from the point  $P$ .

**Example 1**  
Find the moment of  $F$  about  $P$ .

Moment of  $F$  about  $P = F \times d$  anticlockwise

When describing a moment you need to give a direction of rotation.

**Example 2**  
Find the moment of  $F$  about  $P$ .

Moment of  $F$  about  $P = F \times d \sin \theta$  anticlockwise

Use trigonometry to find the perpendicular distance.

The magnitude of the force is measured in newtons (N) and the distance is measured in metres (m), so the moment of the force is measured in newton-metres (Nm).

**Example 3**  
The diagram shows two forces acting on a lamina. Find the moment of each of the forces about the point P.

The distance given on the diagram is the perpendicular distance, so you can substitute the given values directly into the formulae.

Moment of the  $5 \text{ N}$  force  
= magnitude of force  $\times$  perpendicular distance  
=  $5 \times 2 = 10 \text{ Nm}$  clockwise

Moment of the  $8 \text{ N}$  force  
= magnitude of force  $\times$  perpendicular distance  
=  $8 \times 2.5 \sin 50^\circ = 12.5 \text{ Nm}$  anticlockwise (3 s.f.)

Don't forget to include the sense of the rotation when you describe the moment of the force.

This time you need to start by finding the perpendicular distance  $2 \sin 50^\circ$ .

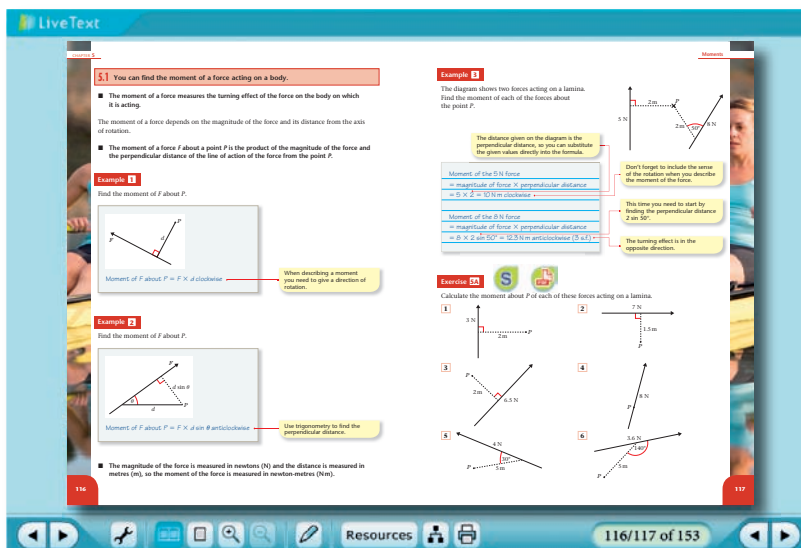
The turning effect is in the opposite direction.

**Exercise 5A**  
Calculate the moment about P of each of these forces acting on a lamina.

- 1
- 2
- 3
- 4
- 5
- 6

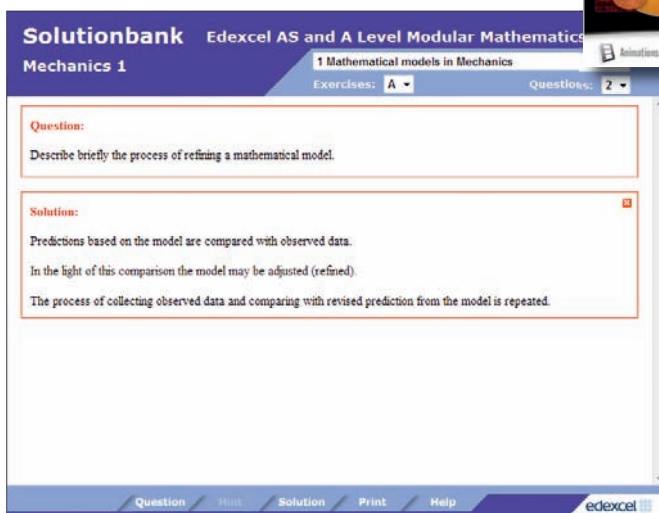
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