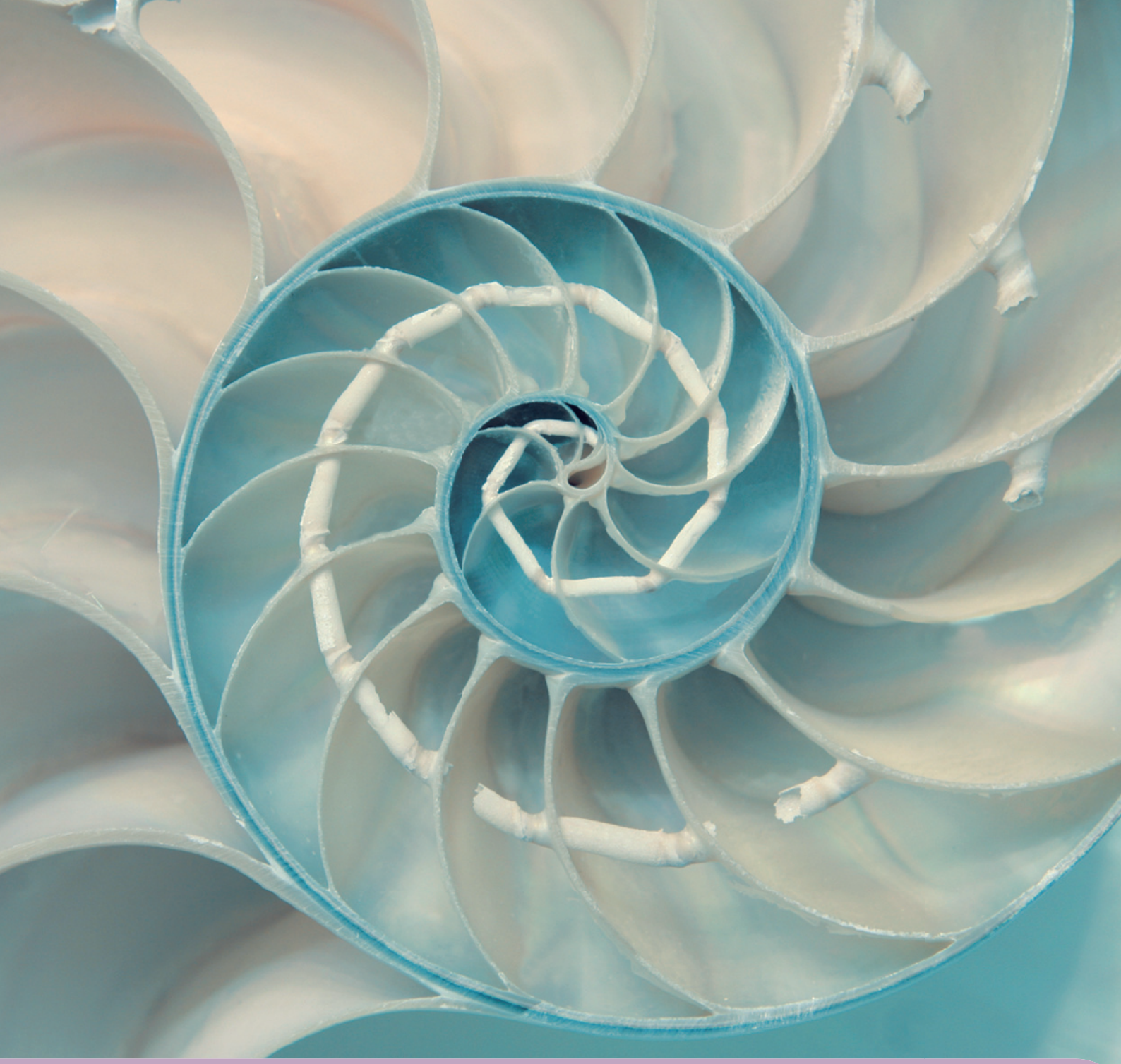


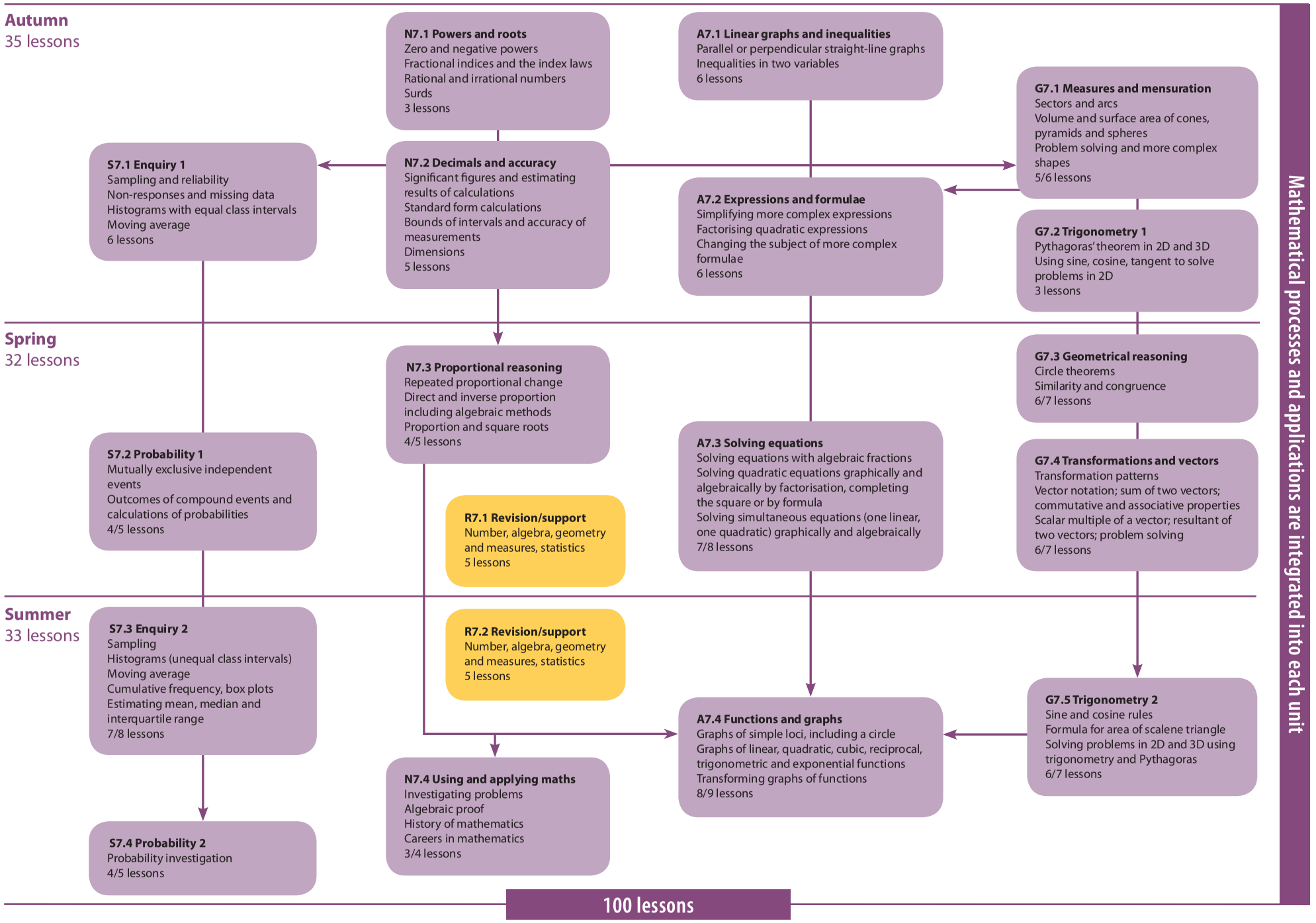
Exploring maths

Teacher's Book 7

Anita Straker, Tony Fisher, Rosalyn Hyde,
Sue Jennings and Jonathan Longstaffe

PEARSON
Longman

Exploring maths: Tier 7 National Curriculum levels 7, 8 and EP



Introduction

The materials

The *Exploring maths* scheme has seven tiers, indicated by the seven colours in the table below. Each tier has:

- a class book for pupils;
- a home book for pupils;
- a teacher's book, organised in units, with lesson notes, mental tests (for number and revision units), facsimiles of resource sheets, and answers to questions in the class and home books;
- a CD with interactive books for display, either when lessons are being prepared or in class, and ICT resources for use in lessons.

Content, structure and differentiation

The tiers are linked to National Curriculum levels so that they have the maximum flexibility. Tier 7 is for very able pupils working at National Curriculum levels 7 and 8 who have previously completed Tier 6 and who are likely to achieve Grade A* when they take GCSE.

The tiers take full account of the 2007 Programme of Study for Key Stage 3 and the Secondary Strategy's renewed *Framework for teaching mathematics in Years 7 to 11,* published in 2008. The standards for functional skills for level 2 are developed and embedded in Tiers 5, 6 and 7.

Labels such as 'Year 9' do not appear on the covers of books but are used in the table below to explain how the materials might be used.

	Year 7	Year 8	Year 9
Extra support For pupils who achieved level 2 or a weak level 3 at KS2 and who are likely to achieve Grade F–G at GCSE.	**Tier 1** NC levels 2–3 (mainly level 3)	**Tier 2** NC levels 3–4 (mainly level 4)	**Tier 3** NC levels 4–5 (both levels 4 and 5)
Support For pupils who achieved a good level 3 or weak level 4 at KS2 and who are likely to achieve Grade D–E at GCSE.	**Tier 2** NC levels 3–4 (mainly level 4)	**Tier 3** NC levels 4–5 (both levels 4 and 5)	**Tier 4** NC levels 5–6 (mainly level 5)
Core For pupils who achieved a secure level 4 at KS2 and who are likely to achieve B–C at GCSE.	**Tier 3** NC levels 4–5 (both levels 4 and 5)	**Tier 4** NC levels 5–6 (mainly level 5)	**Tier 5** NC levels 5–6 (mainly level 6)
Extension For pupils who achieved level 5 at KS2 and who are likely to achieve A or A* at GCSE.	**Tier 4** NC levels 5–6 (mainly level 5)	**Tier 5** NC levels 5–6 (mainly level 6)	**Tier 6** NC levels 6–7 (mainly level 7)
Gifted and talented For gifted pupils who achieved a strong level 5 at KS2 and who are likely to achieve A* at GCSE.	**Tier 5** NC levels 5–6 (mainly level 6)	**Tier 6** NC levels 6–7 (mainly level 7)	**Tier 7** NC levels 7–8+ (mainly level 8)

The *Exploring maths* scheme as a whole offers an exceptional degree of differentiation, so that the mathematics curriculum can be tailored to the needs of individual schools, classes and pupils.

Schools who like to keep track of pupils' progress by relating their assessments to National Curriculum levels will find the tiered structure of *Exploring maths* is ideally suited to their needs.

There are at least five tiers available for each of the year groups 7, 8 and 9. The range of tiers to be used in each year can be chosen by the school to match the progress and attainment of the pupils and their class organisation. Teachers of mixed-ability classes can select units from different tiers covering related topics (see Related units, p. xii).

The *Results Plus Progress* computer assessments, published separately, guide teachers on placing pupils in an appropriate tier at the start of Year 7 and monitoring their progress in each year thereafter. Each assessment indicates which topics in that tier may need special emphasis (see Computer-mediated assessments, p. viii).

Pupils can progress to the next tier as soon as they are ready, since the books are not labelled Year 7, Year 8 or Year 9. Alternatively, work on any tier could take more than a year where pupils need longer to consolidate their learning.

If teachers feel that pupils need extra support, one or more lessons in a unit can be replaced with or supplemented by lessons from revision units.

Each exercise in the class book offers differentiated questions, so that teachers can direct individual pupils to particular sections. Each exercise starts with easier questions and moves on to harder questions, identified by underscored question numbers. Pupils who are relatively more able in their class or set can tackle the extension problems.

Organisation of the units

Each tier is based on 100 lessons of 50 to 60 minutes, plus 10 extra lessons to use for revision or further support, either instead of or in addition to the main lessons.

Lessons are grouped into units, varying in length from three to nine lessons. The average number of lessons in a unit increases slightly through the tiers so that there are fewer but slightly longer units for the higher tiers.

Each unit is identified by a code: N for number, A for algebra, G for geometry and measures, S for statistics and R for revision. For example, Unit N7.2 is the second number unit for Tier 7, while Unit G5.3 is the third geometry and measures unit for Tier 5. Mathematical processes and applications are integrated throughout the units.

The units are shown in a flowchart giving an overview for the year (see p. ii). Some units need to be taught before others but schools can determine the precise order.

Schools with mixed-ability classes can align units from different tiers covering related topics. For more information on related units, see p. xii.

Revision units

Each optional revision unit consists of five stand-alone lessons on different topics. These lessons include questions mainly at levels 8 and EP (Exceptional performance) from the former Key Stage 3 tests, and occasional questions from GCSE examination papers.

The revision lessons can be taught in any order at any point of the year when they would be useful. They could be used with a whole class or part of a class.

The revision lessons can either replace lessons on more difficult topics or be taught in addition to lessons in the main units. Units where the indicative number of lessons is given as, say, 5/6 lessons, are units where a lesson could be replaced by a revision lesson if teachers wish.

Balance between aspects of mathematics

In Tiers 1 and 2 there is a strong emphasis on number and measures. The time dedicated to number then decreases throughout the tiers, with corresponding increases in the time for algebra, geometry and statistics. Mathematical processes and applications, and using and applying mathematics, are integrated into the content strands in each tier.

The lessons for each tier are distributed as follows.

	Number	Algebra	Geometry and measures	Statistics
Tier 1	54	1	30	15
Tier 2	39	19	23	19
Tier 3	34	23	24	19
Tier 4	26	28	27	19
Tier 5	20	29	29	22
Tier 6	19	28	30	23
Tier 7	17	29	29	25
TOTAL	**209**	**157**	**192**	**142**
	30%	23%	27%	20%

The teacher's book, class book and home book

Teacher's book

Each unit starts with a two-page overview of the unit. This includes:

- the necessary previous learning and the objectives for the unit, with the process skills and applications listed first for greater emphasis;
- the titles of the lessons in the unit;
- a brief statement on the key ideas in the unit and why it is important;
- brief details of the assessments integrated into the unit;
- common errors and misconceptions for teachers to look out for;
- the key mathematical terms and notation used in the unit;
- the practical resources required (equipment, materials, paper, and so on);
- the linked resources: relevant pages in the class book and home book, resource sheets, assessment resources, ICT resources, and so on;
- references to useful websites (these were checked at the time of writing but the changing nature of the Internet means that some may alter at a later date).

The overview is followed by lesson notes. Each lesson is described on a two-page spread. There is enough detail so that non-specialist teachers could if they wish follow the notes as they stand whereas specialist mathematics teachers will probably adapt them or use them as a source of ideas for teaching.

Each lesson identifies the main learning points for the lesson. A warm-up starter is followed by the main teaching activity and a plenary review.

The lesson notes refer to work with the whole class, unless stated otherwise. For example, where pupils are to work in pairs, the notes make this clear.

All the number and revision units include an optional mental test for teachers to read out to the class, with answers on the same sheet.

All units in the teacher's book include answers to questions in the class book, home book, check ups and resource sheets.

Class book

The class book parallels the teacher's book and is organised in units. The overall objectives for the unit, in pupil-friendly language, are shown at the start of the unit, and the main objective for each individual lesson is identified.

Interesting information to stimulate discussion on the cultural and historical roots of mathematics is shown throughout the units in panels headed 'Did you know that…?'

The exercises include practical work, activities, games or investigations for groups or individuals, practice questions and problems to solve. Questions are differentiated, with easier questions at the beginning of each exercise. Harder questions are shown by underlining of the question number. More challenging problems are identified as extension problems. The exercises for each lesson conclude with a summary of the learning points for pupils to remember.

Answers to exercises in the class book are given in the teacher's book.

Each unit ends with a self-assessment section for pupils called 'How well are you doing?' to help them to judge for themselves their grasp of the work. Answers to these self-assessment questions are at the back of the class book for pupils to refer to.

Home book

Each lesson has an optional corresponding homework task. Homework tasks in Tiers 6 and 7 are designed to take most pupils about 30 minutes.

Homework is normally consolidation of class work. It is assumed that teachers will select from the homework tasks and will set, mark and follow up homework in accordance with the school's timetable. Because each school's arrangements for homework are different, feedback and follow-up to homework is not included in the lesson notes. It is assumed that teachers will add this as appropriate.

Occasionally, the homework is other than consolidation (e.g. Internet research, collecting data for use in class). When this is the case, the next lesson refers to the homework and explains how it is to be used. Supplementary resource sheets are provided for teachers to copy for any pupils who missed the homework.

Answers to the homework tasks are given in the teacher's book.

The ActiveTeach CD-ROM

ActiveTeach

ActiveTeach contains interactive versions of the **teacher's book**, **class book**, **home book**, and a variety of ICT resources. Full notes on how to use *ActiveTeach* are included on the CD-ROM in the Help tab.

Teachers can use the interactive version of the teacher's book when they are planning or teaching lessons.

From the contents page of the teacher's book, teachers can navigate to the lesson notes for the relevant unit, which are then displayed in a series of double-page spreads.

Clicking on the thumbnail of the PowerPoint slide or the triangular icon shown on the edge of the page allows teachers to view ICT resources, resource sheets, and other Microsoft Office program files.

All these resources, as well as exercises in the class book and tasks in the home book, can also be accessed by clicking on the reference in bold to the resource in the main text.

There is also an option for teachers to use a resource palette to put together their own set of resources ready for a particular lesson, choosing from any of the *Exploring maths* resources in any tier, and adding their own if they wish. This option is proving to be especially useful for teachers of mixed-ability classes.

Interactive versions of the class book and home book can be displayed in class. From the contents page, teachers can go to the relevant unit, which is then shown in a series of double-page spreads. It is possible to zoom in and enlarge particular worked examples, diagrams or photographs, points to remember, homework tasks, and so on. Just as in the teacher's book, clicking on the triangular icon at the side of the page launches the relevant resource.

ICT resources

Each tier has a full range of ICT resources, including: a custom-built toolkit with over 60 tools, Flash animations, games and quizzes, spreadsheets and slides.

The different resources are coded as follows.

- **Check ups (CU)**
  Each unit is supplemented by an optional check up for pupils in the form of a PDF file to print and copy (see also the section on Assessment for learning).
- **Resource sheets (RS)**
  Some units have PDF files of resource sheets to print and copy for pupils to use in class. Where possible, pupils are asked not to write on the sheets so that these can be collected and reused.
- **Tools (TO)**
  These general-purpose teaching tools can be used in many different lessons. Examples are:
  - squared paper and dotty paper;
  - an interactive scientific calculator, similar to an OHP calculator;
  - a function graph plotter;
  - simulated dice and spinners;
  - tools to draw a range of statistical graphs;
  - tools to draw and transform shapes;
  - drawing tools such as a protractor, ruler and compasses.
- **Simulations (SIM)**
  Some of these are animations to play and pause like a video film. Others are interactive and are designed to generate discussion; for example, the teacher may ask pupils to predict an outcome on the screen.
- **Quizzes (QZ)**
  These are quizzes of short questions for pupils to answer, e.g. on their individual whiteboards, often at the start or end of a lesson.
- **PowerPoint presentations (thumbnails)**
  These are slides to show in lessons. Projected slides can be annotated, either with a whiteboard pen or with the pen tool on an interactive whiteboard. Teachers without access to computer and data projector in their classrooms can print the slides as overhead projector transparencies and annotate them with an OHP pen.
- **Excel files (XL)**
  These are spreadsheets for optional use in particular lessons.
- **Geometer's Sketchpad files (GSP)**
  These are dynamic geometry files for optional use in particular lessons.

Other ICT resources, such as calculators, are referred to throughout the units.

The table on pp. x–xi identifies the main lessons where pupils have an opportunity to use ICT for themselves.

Assessment for learning

There is a strong emphasis on assessment for learning throughout *Exploring maths*.

- Learning objectives for units as a whole and for individual lessons are shown on slides and in the class book for discussion with pupils.
- Potential misconceptions are listed for teachers in the overview pages of each unit.
- Key questions for teachers to ask informally are identified in the lesson notes.
- The review that concludes every lesson allows the teacher to judge the effectiveness of the learning and to stress the learning points that pupils should remember.
- The points to remember are repeated in the class book and home book.
- A self-assessment section for pupils, 'How well are you doing?', is included in each unit in the class book to help pupils to judge for themselves their grasp of the work. Exemplar answers are provided at the back of the class book for pupils to refer to.
- Optional revision lessons provide extra support in those areas where pupils commonly have difficulty.
- Each unit on the CD-ROM, apart from the revision units, includes an optional check up of written questions.
- Each number and revision unit of the teacher's book includes an optional mental test of 12 questions for teachers to read to the class.

The mental test could be used as an alternative to part of the last lesson of the unit. About 20 minutes of lesson time is needed to give the test and for pupils to mark it. Answers are on the same sheet.

The written check ups include occasional questions at levels 6 and 7 from the former Key Stage 3 national tests, and questions from GCSE examinations. Teachers could use some or all of the check up questions, not necessarily on the same occasion, and pupils could complete them in class, at home, or as part of an informal test. For example, some written questions could be substituted for the final homework of a unit. Answers to the written check ups are given in the teacher's book.

Computer-mediated assessments

Exploring maths is complemented by *Results Plus Progress*, a series of stimulating online computer-mediated assessments supporting Key Stage 3 mathematics, available separately, see www.resultsplusprogress.com.

There is an entry test for Year 7 to guide teachers on placing pupils in an appropriate tier when they start *Exploring maths*. For Years 7, 8 and 9, there are end-of-term assessments for the autumn and spring terms, and an end-of-year assessment.

Each product offers sets of interactive test questions that pupils answer on computers, either in school or on home computers with Internet access. Because the tests are taken electronically, the products offer instant marking and analysis tools to identify strengths and weaknesses of individuals or groups of pupils. Future units from *Exploring math*s that are dependent on the same skills are identified so that teachers are aware of the units that they may need to adapt, perhaps by adding in extra revision or support lessons.

Results Plus Progress has been developed by the Test Development Team at Edexcel, who have had considerable experience in producing the former Key Stage 3 test and the optional tests for Years 7 and 8.

Where can I find…?

Historical and cultural references

N7.1	Descartes, Sir Isaac Newton and integer powers	Class book p.1
N7.1	The origins of the square root symbol	Class book p.3
N7.1	Al-Khwarizmi and irrational numbers	Class book p.6
N7.1	Irrational numbers	Home book p.2
N7.1	Hippasus of Metapontum and irrational numbers	Home book p.3
A7.1	Descartes and coordinate grids	Class book p.16
A7.1	Leonid Kantorovich and linear programming	Class book p.24
A7.1	George B. Dantzig, John von Neumann and mathematical modelling	Class book p.26
N7.2	The Sitka spruce and significant figures	Class book p.35
N7.2	Large numbers in the Universe	Class book p.36
N7.2	Le Grand K, the standard definition of the kilogram	Class book p.39
N7.2	Archimedes and upper and lower bounds	Class book p.42
N7.2	Dimensions	Class book p.45
N7.2	Avogadro's constant	Home book p.10
N7.2	Metric and imperial units	Home book p.12
N7.2	The definition of the length of one metre	Home book p.13
S7.1	Opinion polls	Class book p.51
S7.1	Cleaning statistical data in criminal investigations	Class book p.55
S7.1	Karl Pearson and histograms	Class book p.61
G7.1	Archimedes and the volumes of cones and spheres	Class book p.77
G7.1	Archimedes and the volumes of cylinders, cones and spheres	Class book p.81
G7.1	The Moscow Papyrus and the volume of a frustum	Class book p.84
G7.1	Benjamin Franklin and the volume of oil experiment	Class book p.91
G7.1	Mathematical facts about the Earth	Home book p.23
G7.1	The relationship between the surface area and volume of a sphere	Home book p.27
A7.2	Heron and mathematical formulae	Class book p.104
A7.2	Magic squares and the Chinese mathematician Yang Hui	Class book p.106
G7.2	Pythagoras' theorem and the distance between two numbers	Class book p.115
G7.2	Trigonometry and surveying	Class book p.119
N7.3	Percentages and Roman taxes	Class book p.127
N7.3	William Emerson and the proportionality symbol	Class book p.131
G7.3	Euclid of Alexandria, the father of geometry	Class book p.150
G7.3	Congruent triangles in architecture	Class book p.164
G7.3	The Pyramids of Giza and similar solids	Class book p.171
S7.2	Pierre de Fermat, Blaise Pascal and the scientific study of probability	Class book p.180
S7.2	Rock, paper, scissors and probability	Class book p.186
A7.3	The history of solving quadratic equations by completing the square	Class book p.210
A7.3	Brahmagupta and solving quadratic equations	Class book p.212
G7.4	The origins of the word 'vector'	Class book p.222

G7.4	Sir William Hamilton and the multiplication of vectors	Class book p.227
S7.3	Census	Class book p.240
S7.3	Sir Francis Galton and the interquartile range	Class book p.242
S7.3	John Tukey and box plots	Class book p.252
G7.5	Uses of trigonometry	Class book p.264
G7.5	Heron's formula for the area of a triangle	Class book p.268
G7.5	Trigonometric graphs and their uses in science and engineering	Class book p.271
G7.5	Claudius Ptolemy and his use of the sine rule in astronomy	Class book p.278
S7.4	Estimating a population using the capture-recapture method	Class book p.289
S7.4	Richard von Mises and the birthday problem	Class book p.292
S7.4	Games of chance and probability	Class book p.296
S7.4	A fair draw	Class book p.298
S7.4	John Scarne and the jeopardy dice game, 'Pig'	Class book p.298
S7.4	Pascal's triangle	Class book p.300
S7.4	Sir Francis Galton and the quincunx	Class book p.301
S7.4	Buffon's Needle, one of the oldest probability experiments	Home book p.95
A7.4	Thomas Harriott and polynomial functions	Class book p.311
A7.4	Hipparchus, Hérigone and trigonometric functions	Class book p.319
N7.4	The Platonic solids	Class book p.338
N7.4	A rhombicuboctahedron and J.C.P. Miller	Class book p.340
N7.4	Leonardo Fibonacci, Eduoard Lucas and sequences	Class book p.341
N7.4	Pixar and mathematicians	Class book p.344
N7.4	Richard Buckminster Fuller and the geodesic dome	Home book p.106
R7.1	The symbol for per cent	Class book p.348
R7.1	The history of graphics calculators	Class book p.353
R7.1	Harold Scott MacDonald Coxeter and geometrical reasoning	Class book p.362
R7.1	Jerome Cardan and the first book on probability	Class book p.367
R7.2	Graphics calculators and solving equations	Home book p.119

ICT lessons: hands-on for pupils

Exploring maths expects pupils to make significant use of ICT beyond the incidental use of calculators. Some of the main opportunities are shown below.

N7.1	Lesson 1	Using power and root keys on calculators	Teacher's book p.4
N7.1	Lesson 2	Calculating fractional powers using calculators	Teacher's book p.7
A7.1	Lesson 1	Using calculators to find the hypotenuse	Teacher's book p.18
N7.2	Lesson 2	Entering numbers in standard form into calculators	Teacher's book p.43
S7.1	Lesson 1	Using random number generators on calculators	Teacher's book p.59
S7.1	Lesson 2	Exploring class intervals using the Internet	Teacher's book p.60
S7.1	Lesson 3	Using ICT to explore frequency diagrams	Teacher's book p.62
S7.1	Lesson 6	Using ICT to explore moving averages	Teacher's book p.69

G7.2	All lessons	Using a calculator to solve trigonometric problems	Class book pp.115-126
A7.3	Lesson 2	Using graph-plotting software or graphics calculators to solve quadratic equations	Teacher's book p.188
A7.3	Lesson 7	Using graph-plotting software or graphics calculators to solve simultaneous linear and quadratic equations	Teacher's book p.198
A7.3	Lesson 8	Using graph-plotting software or graphics calculators to solve simultaneous linear and non-linear equations	Teacher's book p.200
G7.4	Lesson 1	Using the Internet to research symmetry patterns	Teacher's book p.210
S7.3	Lesson 1	Using Excel to explore population samples	Teacher's book p.228
S7.3	Lesson 1	Using the statistical functions on calculators	Teacher's book p.229
S7.3	Lesson 5	Using the Internet to explore statistics for grouped data	Teacher's book p.236
S7.3	Lesson 6	Using ICT to explore box plots	Teacher's book p.239
G7.5	All lessons	Using a calculator to solve trigonometric problems	Class book pp.264-288
S7.4	Lesson 4	Using ICT to explore probability in games of chance	Teacher's book p.286
A7.4	Lesson 1	Using graph-plotting software or graphics calculators to explore quadratic and cubic functions	Teacher's book p.298
A7.4	Lesson 2	Using graph-plotting software or graphics calculators to explore properties of polynomial functions	Teacher's book p.300
A7.4	Lesson 3	Using graph-plotting software or graphics calculators to explore reciprocal functions	Teacher's book p.302
A7.4	Lesson 4	Using graph-plotting software or graphics calculators to explore exponential functions	Teacher's book p.304
A7.4	Lesson 4	Using the exponential key on calculators	Teacher's book p.304
A7.4	Lesson 5	Using graph-plotting software or graphics calculators to generate trigonometric functions	Teacher's book p.306
A7.4	Lesson 6	Using graph-plotting software or graphics calculators to explore trigonometric functions	Teacher's book p.308
A7.4	Lesson 7	Using graph-plotting software or graphics calculators to transform functions	Teacher's book p.310
A7.4	Lesson 8	Using graph-plotting software or graphics calculators to explore loci	Teacher's book p.312
A7.4	Lesson 9	Using graph-plotting software or graphics calculators and spreadsheet software to solve problems	Teacher's book p.314
N7.4	Lesson 1	Using the Internet to explore the history of convex polyhedra	Teacher's book p.332
N7.4	Lesson 3	Using ICT to explore algebraic proof	Teacher's book p.334
N7.4	Lesson 4	Using the Internet to explore careers in mathematics	Teacher's book p.336
N7.4	Lesson 4	Using PowerPoint to explore careers in mathematics	Class book p.344
R7.1	Lesson 4	Using ICT to explore geometrical reasoning	Teacher's book p.352

Functional skills

Tiers 3 and 4 begin to lay the groundwork for the content and process skills for **functional skills at level 2**. These are developed further in Tiers 5, 6 and 7.

Activities to encourage the development of functional skills are integrated throughout the Tier 7 class book. In addition, there are eight specific activities. These can be tackled at any point in the year, including the beginnings and ends of terms. They are all group activities which lend themselves to further development and follow-up. Many of the questions are open ended. These activities are:

FS1	Can you post it?	Class book p.30
FS2	Where is the mathematics?	Class book p.70
FS3	Sports injuries	Class book p.148
FS4	Designing an aquarium	Class book p.178
FS5	Being a scientist	Class book p.218
FS6	Where is the mathematics?	Class book p.238
FS7	Free range organic chickens	Class book p.306
FS8	Marketing new designer jeans	Class book p.336

The activities focus on these **process skills**:

- identifying the mathematics in a situation and mathematical questions to ask;
- recognising that aspects of a situation can be represented using mathematics;
- making an initial model of a situation using suitable forms of representation;
- deciding on the information, methods, operations and tools to use, including ICT;
- examining patterns and relationships;
- changing values and assumptions or adjusting relationships to see the effects on answers in the model;
- finding and interpreting results and drawing conclusions;
- considering how appropriate and accurate results and conclusions are;
- choosing appropriate language and forms of presentation to communicate results and solutions.

Suggestions, solutions and answers for the functional skills activities are on p.36, p.77, p.143, p.165, p.206, p.225, p.294 and p.328 of this teacher's book.

Related units

Units from Tiers 6 and 7 can be aligned if necessary

For example, Unit N7.1 *Powers and roots* in Tier 7 can be used alongside the Tier 6 Unit N6.1 *Powers and roots*

Tier 6	Tier 7
N6.1 Powers and roots	**N7.1** Powers and roots
N6.3 Decimals and accuracy	**N7.2** Decimals and accuracy
N6.2 Proportional reasoning	**N7.3** Proportional reasoning
N6.4 Using and applying maths	**N7.4** Using and applying maths
A6.2 Linear graphs and inequalities	**A7.1** Linear graphs and inequalities
A6.1 Expressions and formulae	**A7.2** Expressions and formulae
A6.3 Expressions, equations and graphs	**A7.3** Equations

A6.4 Using algebra	**A7.4** Functions and graphs
G6.1 Geometrical reasoning	**G7.3** Geometrical reasoning
G6.3 Transformations and loci	**G7.4** Transformations and vectors
G6.4 Measures and mensuration	**G7.1** Measures and mensuration
G6.2 Trigonometry 1 **G6.5** Trigonometry 2	**G7.2** Trigonometry 1 **G7.5** Trigonometry 2
S6.1 Enquiry 1	**S7.1** Enquiry 1
S6.3 Enquiry 2	**S7.3** Enquiry 2
S6.2 Probability 1	**S7.2** Probability 1
S6.4 Probability 2	**S7.4** Probability 2
R6.1 Revision unit 1	**R7.1** Revision unit 1
R6.2 Revision unit 2	**R7.2** Revision unit 2

Tier 7

Contents

Schools planning a shortened two-year programme for Key Stage 3 may not have time to teach all the lessons. The lessons in black cover the essential material for pupils taking this route. The lessons in purple provide useful consolidation and enrichment opportunities. These lessons should be included wherever possible.

N
7.1

Powers and roots

Previous learning

Before they start, pupils should be able to:

- recognise that a recurring decimal is an exact fraction
- use the index laws to multiply and divide positive and negative integer powers
- use the power and root keys of a calculator
- estimate square roots and cube roots.

Objectives based on NC levels 7, 8 and EP (mainly level 8)

In this unit, pupils learn to:

- **model contexts or problems through precise use of symbols and representations**
- **select and apply a range of mathematics and mathematical techniques to find solutions**
- show insight into the mathematical connections in the context or problem
- use accurate notation
- **calculate accurately, using mental methods or calculating devices as appropriate**
- extend generalisations
- examine critically strategies adopted and arguments presented
- **communicate solutions to problems in familiar and unfamiliar contexts**

and to:

- understand and use rational and irrational numbers
- use the index laws for fractional values
- use inverse operations, understanding that the inverse of raising a positive number to power n is raising the result of this operation to power $\frac{1}{n}$
- use surds and π in exact calculations, without a calculator, and rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

Objectives in colour lay the groundwork for Functional Skills at level 2.

Lessons

1 **Negative powers**

2 **Fractional indices**

3 **Surds**

About this unit

A sound understanding of powers and roots of numbers helps pupils to generalise the principles in their work in algebra.

This unit builds on work on powers and roots in Unit N6.1. Pupils use the index laws for negative and fractional powers, and carry out exact calculations involving surds, without a calculator. They consider rational and irrational numbers and how these together form the system of real numbers.

Assessment

This unit includes:

- an optional mental test which could replace part of a lesson (p. 10);
- a self-assessment section for pupils (*N7.1 How well are you doing?* class book p. 10);
- a set of questions to replace or supplement questions in the exercises or homework tasks, or to use as an informal test (*N7.1 Check up*, CD-ROM).

Common errors and misconceptions

Look out for pupils who:

- think that $a^0 = 0$ or $2^4 = 2 \times 4$;
- work out $27^{\frac{2}{3}}$ by finding the cube root of 27 and then doubling rather than squaring;
- write $\sqrt{8} = 4\sqrt{2}$ rather than $\sqrt{8} = 2\sqrt{2}$;
- mis-apply the laws of indices, e.g. $a^3 + a^4 = a^7$, or $a^3 \times a^4 = a^{12}$;
- confuse the exponent and power keys on their calculators;
- think that when the denominator of a fraction is rationalised, the answer must always be a fraction;
- think that $\sqrt{m} + \sqrt{n} = \sqrt{m+n}$, or that $\sqrt{m} - \sqrt{n} = \sqrt{m-n}$, when m and n are not zero.

Key terms and notation

problem, solution, method, pattern, relationship, generalise, explain, verify, prove, justify

calculate, calculation, calculator, operation, multiply, divide, divisible, product, quotient, fraction, decimal, reciprocal, rational, irrational

positive, negative, integer, factor, power, root, square, cube, square root, cube root, index, indices and notation $a^{\frac{1}{n}}$

Practical resources

scientific calculators for pupils
individual whiteboards
squared dotty paper

Exploring maths

Tier 7 teacher's book
N7.1 Mental test, p. 10
Answers for Unit N7.1, pp. 12–15

Tier 7 CD-ROM
PowerPoint files
- N7.1 Slides for lessons 1 to 3
- N7.1 Abu Kamil

Tools and prepared toolsheets
- Calculator tool

Tier 7 class book
N7.1, pp. 1–11
N7.1 How well are you doing? p. 10

Tier 7 home book
N7.1, pp. 1–3

Tier 7 CD-ROM
N7.1 Check up

Useful websites

Topic B: Indices: simplifying
www.mathsnet.net/algebra/index.html

Powers and roots
nrich.maths.org/public/freesearch.php?search=Powers+and+roots

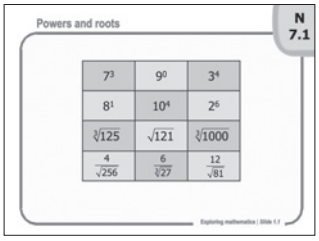
Surds
nrich.maths.org/public/freesearch.php?search=surds

1 Negative powers

Learning points

- To multiply two numbers in index form, add the indices, so $a^m \times a^n = a^{m+n}$.
- To divide two numbers in index form, subtract the indices, so $a^m \div a^n = a^{m-n}$.
- To raise the power of a number to a power, multiply the indices, so $(a^m)^n = a^{m \times n}$.
- These rules work with both positive and negative integer powers.

Starter

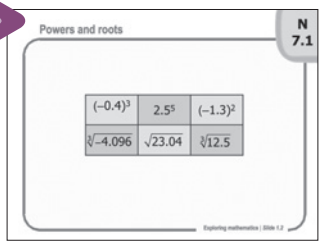


Slide 1.1

Tell the class that in this unit they will be working with powers and roots. They will extend their knowledge of the index laws to negative and fractional values of the indices, and work out exact calculations involving roots, without a calculator.

Show the grid on **slide 1.1**. Point to different expressions on the grid. Ask pupils to work out the values mentally, writing their answers on their whiteboards. Stress that any number raised to the power zero is 1.

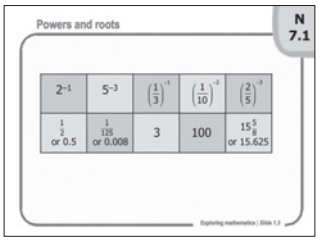
TO



Slide 1.2

Use the **Calculator tool** to remind pupils how to use the power and root keys on their calculators. Then show **slide 1.2**. Point to different expressions on the grid. Ask pupils to use their calculators and to write their answers on their whiteboards.

Main activity



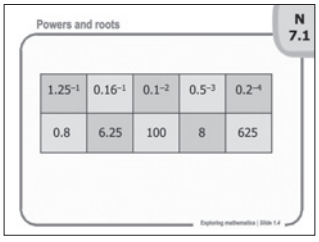
Slide 1.3

Show the negative powers of small numbers on **slide 1.3**. Point to the first box in the top row. Ask pupils to use the power key to find the decimal value, then ask them for the equivalent fraction. Click on the slide to reveal it.

Repeat with the rest of the numbers, then ask:

What do you notice? $\left[a^{-n} = \frac{1}{a^n}\right]$

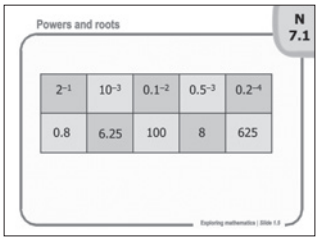
Explain that this means that the reciprocal of a^n is a^{-n}, and vice versa.



Slide 1.4

Show the negative powers of decimals on **slide 1.4**. As before, point to each expression in turn, asking pupils to use their calculators to find the value, then clicking on the slide to reveal it.

Now discuss how to calculate the value of each expression mentally, by changing the number to a fraction then finding the reciprocal.



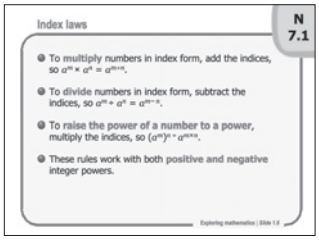
Slide 1.5

Show **slide 1.5**. Ask pupils to do these without their calculators.

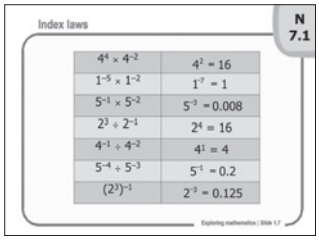
Remind the class of the index laws on **slide 1.6**. Explain that these apply to both positive and negative powers.

Show the expressions on **slide 1.7**. Point to each expression in turn. Ask pupils to simplify it without using their calculators.

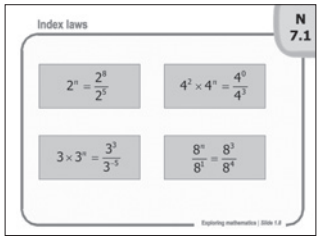
Finally, show the expressions on **slide 1.8**. Ask pupils to find the value of n, again without using their calculators. [3, 7, −5, 0]



Slide 1.6



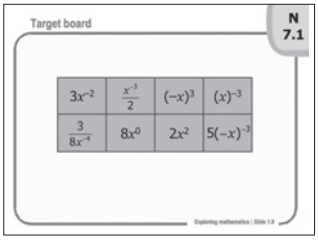
Slide 1.7



Slide 1.8

Ask pupils to do **N7.1 Exercise 1** in the class book (p. 1).

Review



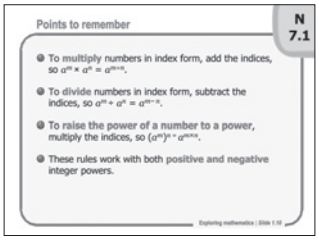
Slide 1.9

Show **slide 1.9**. Tell pupils that $x = 4$.

Point to different expressions and ask pupils to evaluate them mentally as integers or fractions.

$[\frac{3}{16}, \frac{1}{132}, -64, \frac{1}{64}, \frac{3}{2048}, 8, 32, -\frac{5}{64}]$

Then ask them to evaluate the expressions using their calculators, giving their answers in decimal form.



Slide 1.10

Use **slide 1.10** to sum up the lesson with reminders about the index laws.

Homework

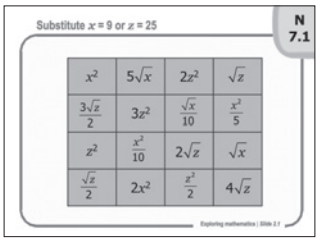
Ask pupils to do **N7.1 Task 1** in the home book (p. 1).

2 Fractional indices

Learning points

- $a^{\frac{1}{2}}$ is the same as the square root of a.
- $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ means the nth root of a, e.g. $\sqrt[3]{a}$ or $a^{\frac{1}{3}}$ is the cube root of a.
- The law of indices $(a^m)^n = a^{m\times n}$, which holds for integer values of m and n, also holds for fractional values of m and n. So:
  $(a^{\frac{1}{m}})^n = a^{\frac{1}{m}\times n} = a^{\frac{n}{m}}$ and $(a^m)^{\frac{1}{n}} = a^{m\times\frac{1}{n}} = a^{\frac{m}{n}}$

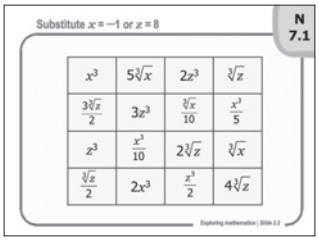
Starter



Slide 2.1

Say that this lesson is about working with powers of numbers when they are fractions and that the index laws hold not only for positive and negative integer powers but also for fractions.

Show the expressions involving squares and square roots on **slide 2.1**. Point to different expressions on the grid. Ask pupils to substitute the values $x = 9$ or $z = 25$, to work out the answers mentally and write them on their whiteboards. Each time, invite someone to explain how they calculated their answer.



Slide 2.2

Remind the class that $\sqrt[3]{a}$ means the cube root of a and repeat with the expressions involving cubes and cube roots on **slide 2.2**, this time substituting the values $x = -1$ or $z = 8$.

Main activity

Explain that

$$a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1 = a$$

This means that $a^{\frac{1}{2}}$ multiplied by itself gives a, so $a^{\frac{1}{2}}$ is the same as the square root of a, that is, $a^{\frac{1}{2}} = \sqrt{a}$.

Stress that the answer to 'Find the value of $25^{\frac{1}{2}}$' is 5, not $\sqrt{5}$.

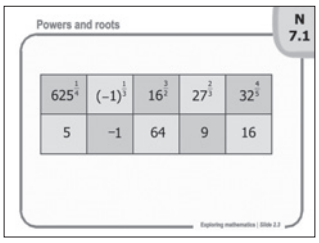
Explain that $\sqrt[n]{a}$ and $a^{\frac{1}{n}}$ both mean the nth root of a, so $\sqrt[3]{a}$ and $a^{\frac{1}{3}}$ both mean the cube root of a.

Develop the identities:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\left(a^{\frac{1}{m}}\right)^n = a^{\frac{1}{m}\times n} = a^{\frac{n}{m}}$$

$$(a^m)^{\frac{1}{n}} = a^{m\times\frac{1}{n}} = a^{\frac{m}{n}}$$



Slide 2.3

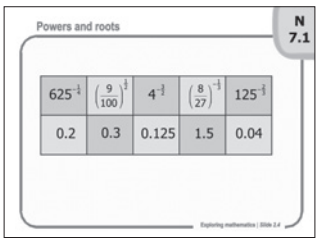
Explain that to work out, say, $16^{\frac{3}{4}}$ you can first cube 16, then find the fourth root, or find the fourth root of 16 and cube it. Either way the answer is 8.

Show **slide 2.3**. Point to the first box in the top row. Ask pupils to work it out mentally and to write it on their whiteboards. Click to reveal the answer. Ask someone to explain how they did it.

Use the **Calculator tool** to demonstrate how to work out the same values using the power and fraction keys. For example, to calculate $625^{\frac{1}{4}}$, press:

6 2 5 x^y 1 $a^b/_c$ 4 =

giving 5 in the display.



Slide 2.4

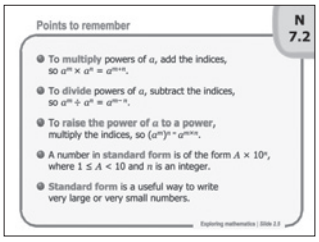
Show **slide 2.3** again. Repeat with the other examples.

Repeat with **slide 2.4**, asking pupils first to calculate mentally and then to confirm with their calculators, using the power, fraction and sign change keys.

Discuss how a power such as 2^{12} can be written as $(2^2)^6 = 4^6$, $(2^3)^4 = 8^4$, $(2^4)^3 = 16^3$ or $(2^6)^2 = 64^2$.

Demonstrate how to simplify and write in the form a^n an expression such as:

$$8^{\frac{1}{2}} \times (\sqrt{2})^{\frac{1}{3}} \left[= 2^{\frac{3}{2}} \times 2^{\frac{1}{6}} = 2^{\frac{3}{2}+\frac{1}{6}} = 2^{\frac{5}{3}}\right]$$



Slide 2.5

Explain that several steps may be needed. The aim is to express each part as powers of the same number so that they can be combined.

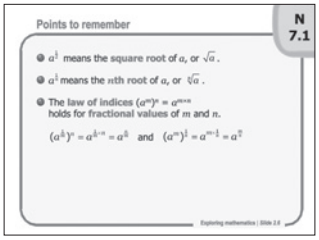
Show the expressions on **slide 2.5**. Ask pupils to simplify them, writing them in the form a^n. Invite pupils to explain how they arrived at their answers.

Select individual work from **N7.1 Exercise 2** in the class book (p. 4).

Review

Use the **Calculator tool** to show that the cube roots of numbers between 1 and 10 lie between 1 and 2.2, and of numbers between 10 and 100 between 2.2 and 4.7. Ask the class to discuss in pairs an estimate to one decimal place of the cube root of a number like 75.

Take feedback, asking for reasoning, e.g. since 75 is greater than 64, which is 4^3, but quite a bit less than 125, which is 5^3, an estimate of the cube root of 75 might be a little over 4, say 4.2.



Slide 2.6

Sum up the lesson with the points on **slide 2.6**.

Homework

Ask pupils to do **N7.1 Task 2** in the home book (p. 2).

3 Surds

Learning points

- A *surd* is a root that does not have an exact value.
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$.
- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$
- To *rationalise* $\dfrac{a}{\sqrt{b}}$, multiply numerator and denominator by $\sqrt{b}$.

Starter

Say that this lesson is about surds, which are roots that don't have an exact value. For example, $\sqrt{2}$ is a surd but $\sqrt{4} = \pm 2$ is not.

Explain the meaning of *rational* and *irrational* numbers, which together form the system of *real numbers*. Say that a surd like $\sqrt{2}$ is an example of an irrational number. Others are decimals like π that neither terminate nor recur, and expressions such as $\dfrac{3\pi}{4}$ or $4\sqrt{7}$.

Explain and illustrate with numeric examples that:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Stress that $\sqrt{a} + \sqrt{b} \neq \sqrt{a + b}$ and $\sqrt{a} - \sqrt{b} \neq \sqrt{a - b}$ (unless a or b are zero).

Main activity

Show how to simplify a square root by removing a factor that is a square number.

$$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$$

$$\sqrt{108} = \sqrt{9 \times 4 \times 3} = \sqrt{9} \times \sqrt{4} \times \sqrt{3} = 3 \times 2 \times \sqrt{3} = 6\sqrt{3}$$

$$\sqrt{\frac{32}{81}} = \frac{\sqrt{16 \times 2}}{\sqrt{81}} = \frac{\sqrt{16} \times \sqrt{2}}{\sqrt{81}} = \frac{4\sqrt{2}}{9}$$

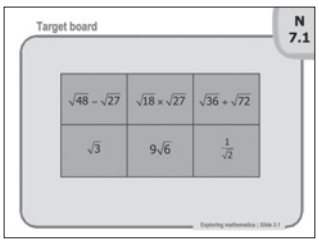
Show how to rationalise a fraction such as $\dfrac{2}{\sqrt{3}}$ by multiplying by $\dfrac{\sqrt{3}}{\sqrt{3}}$ to remove the surd from the denominator.

Move on to a sum or difference such as $\sqrt{27} - \sqrt{50}$. [$3\sqrt{3} - 5\sqrt{2}$]

Now show how to simplify an expression such as $\sqrt{20} + \sqrt{5}$ by writing $\sqrt{20}$ as $2\sqrt{5}$ then adding the multiples of $\sqrt{5}$ (or removing the common factor $\sqrt{5}$).

$$\sqrt{20} + \sqrt{5} = 2\sqrt{5} + \sqrt{5} = (2 + 1)\sqrt{5} = 3\sqrt{5}$$

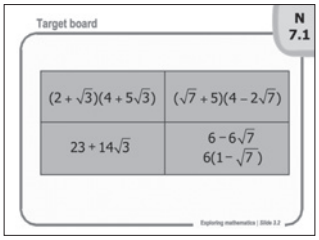
Show the expressions on **slide 3.1** and ask pupils to simplify them on their whiteboards. Click on the slide to reveal the answers.



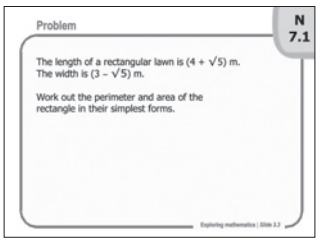
Slide 3.1

Discuss how to expand and simplify an expression such as $(4 + \sqrt{2})(5 - 3\sqrt{2})$, using a multiplication grid.

×	4	$+\sqrt{2}$	
5	20	$+5\sqrt{2}$	$20 + 5\sqrt{2}$
$-3\sqrt{2}$	$-12\sqrt{2}$	-6	$-6 - 12\sqrt{2}$
			$14 - 7\sqrt{2}$



Slide 3.2



Slide 3.3

Show how the answer of $14 - 7\sqrt{2}$ can be written as $7(2 - \sqrt{2})$.

Ask pupils to expand and simplify the expressions on **slide 3.2**.

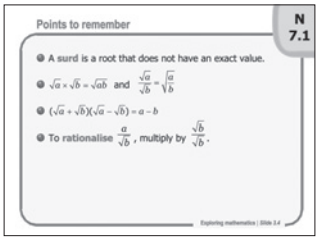
Now ask them to solve the problem on **slide 3.3**.

> **The length of a rectangular lawn is $(4 + \sqrt{5})$ m. The width is $(3 - \sqrt{5})$ m. Work out the perimeter and area of the lawn in their simplest forms.**
> [perimeter: 14 m, area: $(7 - \sqrt{5})$ m^2]

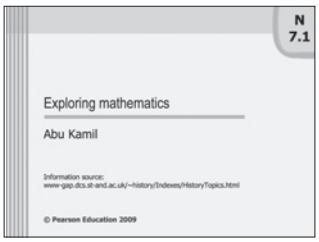
Select individual work from **N7.1 Exercise 3** in the class book (p. 6).

Review

Discuss how to use the identity $a - b = (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ to rationalise an expression such as $\frac{1}{\sqrt{3} - 1}$ by multiplying by the fraction $\frac{\sqrt{3} + 1}{\sqrt{3} + 1}$.



Slide 3.4



N7.1 Abu Kamil

Ask pupils to remember the points on **slide 3.4**.

Round off the unit by showing the PowerPoint presentation **N7.1 Abu Kamil**. Stress the significant contributions made to present-day mathematics by the mathematicians working in Baghdad in the 9th and 10th centuries.

You could ask pupils to verify the identity on the last slide by asking them to substitute values such as $a = 25$, $b = 16$.

Refer again to the objectives for the unit. Ask pupils to find time to see if they are achieving level 8 by trying the self-assessment problems in **N7.1 How well are you doing?** in the class book (p. 10). They will need squared dotty paper.

Homework

Ask pupils to do **N7.1 Task 3** in the home book (p. 3).

N7.1 Mental test

Read each question aloud twice.

Allow a suitable pause for pupils to write answers.

1 What is the square root of forty thousand? *2006 KS3*

2 m squared equals one hundred. *2006 KS3*
Write down the two possible values of m plus fifteen.

3 I write all the integers from one to one hundred. *2003 KS3*
How many of these integers contain a digit two?

4 Nine multiplied by nine has the same value as three to the power what? *2006 KS3*

5 Four to the power nine divided by four to the power three is four to the power what? *2007 KS3*

6 One hundred pet owners had a dog or a cat, or both. *2004 KS3*
Fifty-five of the hundred had a dog. Sixty-five had a cat. How many had both a dog and a cat?

7 What would be the last digit of one hundred and thirty-three to the power four? *2003 KS3*

8 Look at the equation. j and k are consecutive integers. *1999 KS3*
Write down the values of j and k.
[Write on board $2j + k = 22$*]*

9 Three to the power five divided by three to the power eight is three to the power what?

10 Work out the value of two to the power minus four multiplied by two squared.

11 Sixteen to the power one half is two to the power what?

12 What is the value of the cube root of seven to the power six?

Key:
KS3 Key Stage 3 Mental Test
Questions 1–8 are at level 7. Questions 9–12 are beyond level 7.

Answers

1 200 **2** 5 and 25 **3** 19 **4** 4

5 6 **6** 20 **7** 1 **8** 7 and 8

9 -3 **10** 0.25 or $\frac{1}{4}$ **11** 2 **12** 49

N7.1 Check up

Check up — N7.1

Write your answers in your book.

Powers and roots (no calculator)

1 *1999 level 7*

a Write the values of k and m.

$$64 = 8^2 = 4^k = 2^m$$

b Use the information below to work out the value of 2^{14}.

$$2^{15} = 32\,768$$

2 *GCSE 1387 November 2007*

a Simplify $(a^2)^4$.

b Work out the value of x.

$2^{30} \div 8^9 = 2^x$

3 *2007 level 8*

a Is 3^{100} even or odd? Explain your answer.

b Which of the numbers below is the same as $3^{100} \times 3^{100}$?

A 3^{200} B 6^{100} C 9^{200} D $3^{10\,000}$ E $9^{10\,000}$

4 *2000 level 8*

Look at the table.

$7^0 = 1$	$7^3 = 343$	$7^6 = 117\,649$
$7^1 = 7$	$7^4 = 2401$	$7^7 = 823\,543$
$7^2 = 49$	$7^5 = 16\,807$	$7^8 = 5\,764\,801$

a Explain how the table shows that $49 \times 343 = 16\,807$.

b Use the table to help you work out the value of $\frac{5\,764\,801}{823\,543}$.

c Use the table to help you work out the value of $\frac{117\,649}{2401}$.

d The units digit of 7^6 is 9. What is the units digit of 7^{12}?

5 *GCSE 1387 June 2007*

Expand and simplify: $(\sqrt{3} - 2)(\sqrt{3} + 2)$

N7.1 Check up [continued]

6 *2001 Exceptional performance*

a Show that $\sqrt{2\frac{2}{3}} = 2 \times \sqrt{\frac{2}{3}}$.

b Show that the equation $\sqrt{\frac{n+n}{n+1}} = n \times \sqrt{\frac{n}{n+1}}$ will simplify to $n^3 - n^2 = 2n$.

c Solve the equation $n^3 - n^2 = 2n$.

N7.1 Answers

Class book

Exercise 1

1 a $\frac{1}{2}$ b $\frac{1}{9}$
c $\frac{1}{125}$ d $\frac{1}{1000}$
e 1 f 3
g $\frac{9}{4}$ h $\frac{2}{5}$

2 a $2^2 \times 2^{-5} = 2^{-3} = \frac{1}{8}$
b $3^{-2} \times 3^{-3} = 3^{-5} = \frac{1}{243}$
c $10^{-2} \times 10^{-2} = 10^{-4} = \frac{1}{10\,000}$
d $9^5 \times 9^{-3} = 9^2 = 81$
e $5^{-3} \div 5^{-1} = 5^{-2} = \frac{1}{25}$
f $10^{-4} \div 10^{-3} = 10^{-1} = \frac{1}{10}$
g $4^{-2} \div 4^{-1} = 4^{-1} = \frac{1}{4}$
h $3^2 \div 3^{-1} = 3^3 = 27$
i $(2^{-3})^2 = (\frac{1}{8})^2 = \frac{1}{64}$
j $(5^2)^{-2} = 25^{-2} = \frac{1}{625}$
k $(3^4)^2 = (81)^2 = 6561$
l $(2^{-1})^{-4} = (\frac{1}{2})^{-4} = 16$

3 a $\frac{2^4 \times 2^2}{2^7} = \frac{2^6}{2^7} = 2^{-1} = \frac{1}{2}$
b $\frac{3^4 \times 3^{-2}}{3^5} = \frac{3^2}{3^5} = 3^{-3} = \frac{1}{27}$
c $\frac{10^{-2} \times 10^2}{10} = \frac{10^0}{10} = \frac{1}{10}$
d $\frac{2^{-4} \times 2^2}{2^{-7}} = \frac{2^{-2}}{2^{-7}} = 2^5 = 32$
e $\frac{2^4}{2^7 \times 2^{-2}} = \frac{2^4}{2^5} = 2^{-1} = \frac{1}{2}$
f $\frac{3^4 \times 3^2}{3 \times 3^7} = \frac{3^6}{3^8} = 3^{-2} = \frac{1}{9}$
g $\frac{4^4 \times 4^{-2}}{4^{-1}} = \frac{4^2}{4^{-1}} = 4^3 = 64$
h $\frac{2^4 \times 2^2}{2^7 \times 2^{-1}} = \frac{2^6}{2^6} = 1$

4 a $\frac{2^2}{2^5} = 2^{-3}$, so $n = -3$
b $\frac{3^4}{3^6} = 3^{-2}$
$3^{-2} \div 3^1 = 3^{-3}$, so $n = -3$
c $\frac{10^3}{10^5} = 10^{-2}$
$10^{-2} \times 10 = 10^{-1}$, so $n = -1$
d $\frac{4}{4^5} = 4^{-4}$
$4^{-4} \div 4^2 = 4^{-6}$, so $n = -6$

5 a 0.6 b 820
c 0.0029 d 0.087
e 1600 f 0.0004

6 a $n = 3$ b $n = 4$
c $n = -1$ d $n = 3$
e $n = -3$ f $n = -1$

7 a 0.8 b 6.25
c 6.25 d 0.064
e 0.0064 f 2.22
g 1.39 h 7.72

Extension problems

8 The last digit is 4.
5^{-55} is $(\frac{1}{5})^{55}$, or $(0.2)^{55}$.
When multiplied repeatedly by itself, any number with a last digit of 2 has a last digit cycling through 2, 4, 8, 6, so $(0.2)^{55}$ has a last digit of 4.

9 3 to the power 42, which is about
109 418 989 131 512 359 209
(calculated using Calculator in the Accessories of a PC, by putting 729 ($=3^6$) into the memory and multiplying it by itself seven times)

Exercise 2

1 a 3 b 11
c $\frac{1}{6}$ d 100
e 3 f 100
g $-\frac{1}{4}$ h $\frac{1}{6}$
i 243 j 8
k -9 l 125

2 a $\frac{1}{16}$ b $\frac{1}{9}$
c $\frac{16}{625}$ d $\frac{27}{64}$
e $\frac{1}{5}$ f $\frac{1}{9}$
g $\frac{1}{16}$ h $\frac{1}{256}$

3 a 2^1 b $3^{\frac{1}{6}}$
c $10^{\frac{7}{10}}$ d 8^0

4 a $n = 3$ b $n = \frac{8}{3}$
c $n = 4$ d $n = -\frac{11}{6}$

5 a 1.2 b 2.9
c 1.5 d 2.4

6 a $\sqrt[3]{50}$ b $\sqrt[4]{235}$
c $\sqrt[4]{20}$ d $35^{\frac{1}{2}}$
e $210^{\frac{1}{4}}$ f $40^{\frac{1}{4}}$

7 5 and 8 (or 8 and 5)

8 729

Extension problems

9 The square root must lie between $\sqrt{5}$ and $\sqrt{6}$, or between 2.24 and 2.45. For example:

numerator	12	16	23
denominator	5	7	10
square	$\frac{144}{25}$	$\frac{256}{49}$	$\frac{529}{100}$
	$=5\frac{19}{25}$	$=5\frac{11}{49}$	$=5\frac{29}{100}$

Other solutions are possible, such as $\frac{2209}{441} = 5\frac{4}{441}$, the square of $\frac{47}{21}$.

10 $\sqrt{30\,976} = 176$,
so D is 3, O is 0, Z is 9, E is 7, N is 6 and T is 1.

TEN must lie between 100 and 317, since 317 squared is a six-digit number.
Since N × N is a number with a units digit of N,
N is 5 or 6.
T and E are different digits.
Omitting numbers with repeated digits, and working systematically through the numbers
105, 106,
125, 126,
…,
305, 306,
315, 316 produces the result.

Exercise 3

1 **a** $n = 2$ **b** $n = 4$
c $n = \sqrt{6}$ **d** $n = 5$

2 **a** $\sqrt{3}(4 + \sqrt{3}) = 3 + 4\sqrt{3}$

b $(\sqrt{3} + 1)(2 + \sqrt{3}) = 3 + 2\sqrt{3} + 2 + \sqrt{3}$
$= 5 + 3\sqrt{3}$

c $(\sqrt{5} - 1)(2 + \sqrt{5}) = 5 + 2\sqrt{5} - 2 - \sqrt{5}$
$= 3 + \sqrt{5}$

d $(\sqrt{7} + 1)(2 - 2\sqrt{7}) = 2\sqrt{7} + 2 - 2\sqrt{7} - 14$
$= -12$

e $(3 - \sqrt{5})(3 - \sqrt{5}) = 5 - 3\sqrt{5} - 3\sqrt{5} + 9$
$= 14 - 6\sqrt{5}$

f $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = 5 + \sqrt{5}\sqrt{3} - \sqrt{5}\sqrt{3} - 3$
$= 2$

3 **a** $\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$

b $\frac{4}{\sqrt{5}} = \frac{4 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{4\sqrt{5}}{5}$

c $\frac{3}{\sqrt{7}} = \frac{3 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{3\sqrt{7}}{7}$

d $\frac{3}{\sqrt{2}} = \frac{3 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{2}$

e $\frac{5}{\sqrt{13}} = \frac{5 \times \sqrt{13}}{\sqrt{13} \times \sqrt{13}} = \frac{5\sqrt{13}}{13}$

4 **a** $\frac{2}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

b $\frac{3}{\sqrt{15}} = \frac{3\sqrt{15}}{15} = \frac{\sqrt{15}}{5}$

c $\frac{15}{\sqrt{20}} = \frac{15\sqrt{20}}{20} = \frac{3\sqrt{20}}{4}$

d $\frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

e $\frac{14}{\sqrt{7}} = \frac{14\sqrt{7}}{7} = 2\sqrt{7}$

5 **a** $\frac{10 + \sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}(10 + \sqrt{5})}{5}$
$= \frac{10\sqrt{5} + 5}{5} = 2\sqrt{5} + 1$

b $\frac{2 - \sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}(2 - \sqrt{2})}{2}$
$= \frac{2\sqrt{2} - 2}{2} = \sqrt{2} - 1$

c $\frac{22 + \sqrt{11}}{\sqrt{11}} = \frac{\sqrt{11}(22 + \sqrt{11})}{11}$
$= \frac{22\sqrt{11} + 11}{11} = 2\sqrt{11} + 1$

d $\frac{14 - \sqrt{14}}{\sqrt{14}} = \frac{\sqrt{14}(14 - \sqrt{14})}{14}$
$= \sqrt{14} - 1$

6 6 cm

7 **a** $\sqrt{40}$ cm **b** $4\sqrt{40}$ cm

8 **a** area $= \frac{1}{2} \times 2\sqrt{8} \times 2\sqrt{2} = 8\,\text{m}^2$

b $2\sqrt{8} = 2 \times \sqrt{4 \times 2} = 2 \times \sqrt{4} \times \sqrt{2} = 4\sqrt{2}$

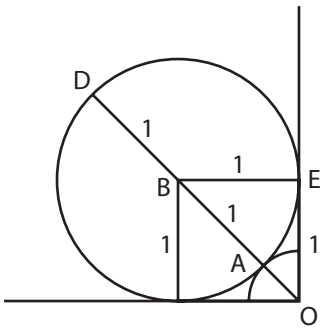
c hypotenuse $= \sqrt{(2\sqrt{2})^2 + (2\sqrt{8})^2}$
$= \sqrt{8 + 32} = \sqrt{40} = 2\sqrt{10}$

9 Perimeter $= 14$ cm
Area $= 7 + \sqrt{5}\,\text{cm}^2$

10 Let the radius of the largest circle be R.
Area A of largest circle is area of smallest circle plus area of nine rings:
So $A = \pi R^2 = \pi + 9\pi = 10\pi$, giving $R = \sqrt{10}$.

Extension problems

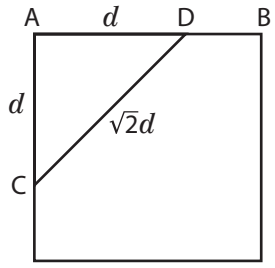
11 **a**



The line OBAO through the centres of the circles is a straight line.
OD is the radius of the large circle.
By Pythagoras, in triangle OBE, OB $= \sqrt{2}$,
so OD $=$ OB $+$ BD $= \sqrt{2} + 1$.

b OA $=$ OB $-$ AB $= \sqrt{2} - 1$

12



CD is the side of the regular octagon.
Let AD be d cm.
In right triangle ADC, CD is $\sqrt{2}d$ (Pythagoras).

DB is $\frac{\sqrt{2}d}{2} = \frac{d}{\sqrt{2}}$ (half the side of the octagon).

AD + DB = 5 (half the side of the square)

$d + \frac{d}{\sqrt{2}} = 5$

$d(\sqrt{2} + 1) = 5\sqrt{2}$

$d = \frac{5\sqrt{2}}{\sqrt{2}+1} = \frac{5\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 5(2 - \sqrt{2})$

So the cuts should be made $5(2 - \sqrt{2})$ cm from the vertices of the square.

How well are you doing?

1 **a** $a = 4, b = 3$ **b** $c = 7$

2 **a** 100 **b** 6

3 **a** $0.8n$ **b** n^2, $\sqrt{n}$ and $\frac{1}{n}$

c $0.8n$, $\sqrt{n}$ and $\frac{1}{n}$ **d** n^2, $0.8n$

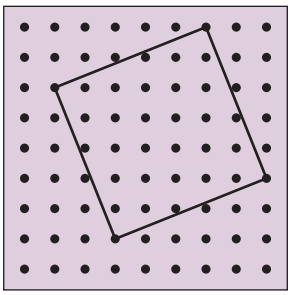
4 **a** 8 **b** $2\sqrt{2}$

c $5\sqrt{2}$ **d** $\frac{\sqrt{2}}{2} + 1$

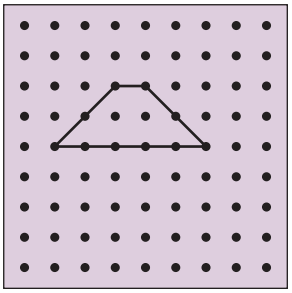
5 **a** Using Pythagoras, the two shorter sides of the rectangle are each:
$\sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$
Using Pythagoras, the two longer sides are each:
$\sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$
The perimeter is twice the shorter side plus twice the longer side, or:
$6\sqrt{2} + 8\sqrt{2} = 14\sqrt{2}$

b Using Pythagoras, the two shorter sides of the rectangle are each:
$\sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$
Using Pythagoras, the two longer sides are each:
$\sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$
The perimeter is twice the shorter side plus twice the longer side, or:
$4\sqrt{5} + 6\sqrt{5} = 10\sqrt{5}$

c Length of one side of the square is $\sqrt{29}$, so the square of one side is 29.
29 expressed as the sum of two integer squares is
$25 + 4 = 5^2 + 2^2$
So a possible square is:



d Assume the trapezium is isosceles. The perimeter is the sum of the two parallel sides, plus the sum of the two sloping sides, which are equal in length.
Assume that the sum of the two parallel sides is 6, and that the shorter side is 1 and the longer side is 5.
Assume that the sum of the two sloping sides is $4\sqrt{2}$, so one side is $2\sqrt{2}$. The square of $2\sqrt{2}$ is 8, and 8 is $2^2 + 2^2$.
So a possible trapezium is:



Home book

Task 1

1 **a** 3^2 **b** 2^{-3} **c** 5^{-3} **d** 8
e 5^{-2} **f** 10^4 **g** 2^{-6} **h** 5^4

2 **a** $n = -3$ **b** $n = -3$
c $n = -1$

3 4 to the power (3 to the power 2) is greater.
4 to the power (3 to the power 2) is
4 to the power 9, or 262 144.
(4 to the power 3) to the power 2 is
(4 to the power 3) × (4 to the power 3),
which is 4 to the power 6, or 4096.

4 The last digit is 5.
2^{-22} is $\left(\frac{1}{2}\right)^{22}$, or $(0.5)^{22}$.
When multiplied by itself, any number with a last digit of 5 has a last digit of 5, so $(0.5)^{22}$ has a last digit of 5.

Task 2

1 **a** 4 **b** 5
c 2 **d** 10

2 **a** 125 **b** 3
c 243 **d** 125
e 128 **f** 9
g $\frac{1}{8}$ **h** $\frac{1}{5}$

3 **a** $z^{\frac{5}{6}}$ **b** $x^{\frac{1}{12}}$

4 256 cm^2

5 27

Task 3

1 **a** $\frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$

b $\frac{3\sqrt{24}}{24} = \frac{\sqrt{24}}{8} = \frac{2\sqrt{6}}{8} = \frac{\sqrt{6}}{4}$

c $\frac{25\sqrt{35}}{35} = \frac{5\sqrt{35}}{7}$

d $\frac{6\sqrt{14}}{14} = \frac{3\sqrt{14}}{7}$

e $\frac{8\sqrt{8}}{8} = \sqrt{8}$

2 $\frac{22\sqrt{21}}{21}$

3 Area of triangle BCD
$= \frac{1}{2}$CD $\times$ BC
$= \frac{1}{2}(1 + \sqrt{3})(1 + \sqrt{3})$
$= \frac{1}{2}(1 + 2\sqrt{3} + 3)$
$= \frac{1}{2}(4 + 2\sqrt{3}) = (2 + \sqrt{3})$ cm^2

4 6 cm

CD-ROM

Check up

1 **a** $k = 3, m = 6$

b $2^{14} = 32\,768 \div 2 = 16\,384$

2 **a** a^8

b $x = 3$

3 **a** The product of odd numbers is always odd, so 3^{100}, which is 3 multiplied by itself 100 times, is odd.

b A

4 **a** $49 \times 343 = 7^2 \times 7^3 = 7^5 = 16\,807$

b 7

c 49

d 1

5 -1

6 **a** $\sqrt{2\frac{2}{3}} = \sqrt{\frac{8}{3}} = \sqrt{\frac{4 \times 2}{3}} = 2 \times \sqrt{\frac{2}{3}}$

b $\sqrt{n + \frac{n}{n+1}} = n \times \sqrt{\frac{n}{n+1}}$

$n + \frac{n}{n+1} = \frac{n^3}{n+1}$ (squaring both sides)
$n(n + 1) + n = n^3$ (multiplying by $n + 1$)
$n^2 + 2n = n^3$ or $n^3 - n^2 = 2n$

c $n^3 - n^2 = 2n$

One solution is $n = 0$, so dividing by n gives:

$n^2 - n = 2$

or $n^2 - n - 2 = 0$

Factorising:

$(n - 2)(n + 1) = 0$

so the solutions to the equation $n^3 - n^2 = 2n$ are $n = 0$, $n = -1$ or $n = 2$.