Collecting like terms

Expressions can be simplified by collecting together any terms that are made up of the same letters.

5 Simplifying expressions

Like terms are terms that have the same letter(s) with the same powers but can have different numerical coefficients.

Terms with + or − in front of them can be simplified by collecting like terms.

$t + 3t$ can be combined to give $4t$

$t^2 + 3t^2$ can be combined to give $4t^2$

$t + 3t^2$ cannot be combined as t and t^2 are not like terms even though they have the same letter. They have different powers.

Combine the like terms. There are four *t*s so write this as $4t$.

Add or subtract the coefficients on like terms. $2 + 4 - 3 = 3$

There are two different sets of like terms in this expression. Collect all the *d* terms together and all the *c* terms together.

Combine x^2 terms in the same way you would combine x. Add or subtract the coefficients on the like terms.

10 Worked example — Grades 1–2

1 Circle the expression that can be written as $3a$.

a^3 $\quad a \times a \times a$ $\quad 3 + a$ $\quad a + a + a$

2 Simplify

(a) $t + t + t + t$

$= 4t$

(b) $u + u + u - u$

$= 2u$

3 Simplify

(a) $2a + 4a - 3a$

$= 3a$

(b) $d + 6d - 4c$

$= 7d - 4c$

4 Simplify

(a) $4x^2 + 2x^2 + 6x^2$

$= 12x^2$

(b) $7y^2 - 9y^2 + z - y^2$

$= -3y^2 + z$

Collect the like terms and then combine them. Combine the terms for y^2 and then add z.

5 Simplify

(a) $5e + 7f - 6e + 4f$

$= 5e - 6e + 7f + 4f$

$= -e + 11f$

(b) $x + 4x^2 + 3x - 5x^2$

$= x + 3x + 4x^2 - 5x^2$

$= 4x - x^2$

Collect the x and the x^2 terms separately. x and $4x^2$ are not like terms because the powers of x are different.

2 Expressions with different terms

Sometimes expressions have two or more terms which are not like terms. Arrange the expression so that all of the like terms are next to each other. Then you can collect together all the like terms to give a simplified expression.

For example, to simplify the expression $r + 4p + 2r - 9p$:

1. Rearrange the expression so the like terms are together:
   $r + 2r + 4p - 9p$
2. Add or subtract the coefficients to combine the terms.
   r terms: $1 + 2 = 3$
   p terms: $4 - 9 = -5$
3. Collect the like terms: $3r - 5p$

2 Checklist

- Keep each term together with the + or − in front of it.
- Like terms have the same letter(s) and the same power.
- x by itself is same as $1x$.

10 Exam-style practice — Grades 1–2

1 Simplify

(a) $m + m + m - m + m + m - m$ **[1 mark]**

(b) $5x - 3y + 4x - 2y$ **[2 marks]**

2 Simplify

(a) $3c^2 + 5c^2 - c^2$ **[1 mark]**

(b) $9x - 3y - 6x - 7y$ **[2 marks]**

3 Simplify $6t - 3 - 8t + 7$ **[2 marks]**

4 Simplify $7a + 5b - 2a - 9b$ **[2 marks]**

Simplifying expressions

You need to be able to simplify algebraic expressions that include multiplication signs and division signs.

Multiplying expressions

To simplify an algebraic expression that includes a multiplication sign, follow these rules.

1. Multiply all the numbers, including coefficients.
2. Use the laws of indices to simplify the powers of the letters. Go to page 25 to revise laws of indices.

A **coefficient** is the number in front of a letter in an algebraic expression. It means how many lots of that letter there are. For example: 3c means 3 × *c*

a is multiplied by itself 6 times so, using the laws of indices ($a^m \times a^n = a^{m+n}$), you can write it as a^6

5*x* means 5 × *x*. You can multiply in any order and get the same answer, so multiply the numbers and then the letters.

Write the division as a fraction.

Divide the numbers as much as possible, then cancel any common letters on the top and bottom of the fractions.

Letters in algebra can be simplified so that they are written next to each other in alphabetical order.

Dividing expressions

To simplify an algebraic expression that includes a division sign, follow these rules.

1. Write the expression as a fraction.
2. Cancel the numbers. Write any numbers that are not whole as fractions instead of decimals.
3. Use index rules (page 25) to simplify the powers of the letters.

Worked example — Grade 2

Simplify $\dfrac{5x^2 \times 4x^4}{6x^3}$

$$= \frac{5 \times x^2 \times 4 \times x^4}{6 \times x^3} = \frac{5 \times 4 \times x^2 \times x^4}{6 \times x^3}$$

$$= \frac{20x^6}{6x^3}$$

$$= \frac{10x^3}{3}$$

Simplify the expression in the numerator as much as possible. Then cancel a factor of 2 and a factor of x^3 from the top and bottom of the fraction.

Worked example — Grades 1–2

1 Simplify

(a) $a \times a \times a \times a \times a \times a$

$= a^6$

(b) $5x \times 3x$

$= 5 \times x \times 3 \times x$

$= 5 \times 3 \times x \times x = 15x^2$

2 Simplify

(a) $10x \div 2$

$= \dfrac{10 \times x}{2} = 5x$

(b) $20xy \div y$

$= \dfrac{20 \times x \times \cancel{y}}{\cancel{y}} = 20x$

3 Simplify

(a) $3b \times 4b \times 2b$

$= 3 \times b \times 4 \times b \times 2 \times b$

$= 3 \times 4 \times 2 \times b \times b \times b = 24b^3$

(b) $4x \times 5y$

$= 4 \times x \times 5 \times y$

$= 4 \times 5 \times x \times y = 20xy$

4 Simplify $25x^3 \div 5x$.

$= \dfrac{25 \times x^{\cancel{3}2}}{5 \times \cancel{x}}$

$= 5 \times x^2 = 5x^2$

Remember that *x* is the same as x^1. Use the rule $a^m \div a^n = a^{m-n}$ to simplify the expression.

Checklist

- $a \times a = a^2$ (not 2*a*)
- $a \times a \times a = a^3$ (not 3*a*)
- $a \times b = ab$ or ba
- $1a = a$

Exam-style practice — Grades 1–2

1. Simplify $c \times d \times 5$ **[1 mark]**
2. Simplify $3 \times w \times 2$ **[1 mark]**
3. Simplify $3g \times 5h$ **[1 mark]**
4. Simplify $24x \div 6$ **[1 mark]**
5. Simplify $48xy \div 8y$ **[1 mark]**

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Writing expressions

It is important to be able to interpret information and then write it in terms of algebraic expressions.

5 Interpreting information

Instructions or rules can be written as algebraic expressions.

Jess wants to put an advert for her school play in the local paper.

The cost is £2 for each line of text, plus a £10 fee.

To write this as an expression, use a letter to represent the number of lines of text.

For example, the cost is:

£2 × number of lines + £10

Or $2n + 10$

where n is the number of lines of text.

If there are 25 crayons in the tub to start with, then the number left must be p crayons less than this.

Remember that $5 \times x$ is written as $5x$.

The order of operations means that multiplication comes before addition.

In order for this expression to be correct, $d + 4$ must happen before $\times$ 15. Place brackets around the $d + 4$ expression to make sure this part of the formula is calculated first.

10 Worked example — Grade 3

1 Crayons are sold in cartons and in tubs.
There are 5 crayons in a carton.
There are 25 crayons in a tub.
Asha buys one tub of crayons.
She takes p crayons out of the tub.

(a) Write down an expression, in terms of p, for the number of crayons left in the tub.

$25 - p$

Poppy buys x cartons of crayons and y tubs of crayons.

(b) Write down an expression, in terms of x and y, for the total number of crayons Poppy buys.

$5x + 25y$

2 The cost of hiring a bike for d days can be worked out using this rule.

Add 4 to the number of days' hire.

Multiply your answer by 15.

Write down an expression, in terms of d, for the total cost for hiring a bike for d days.

$(d + 4) \times 15$

$15(d + 4)$

5 Worked example — Grade 3

Sandeep, Pavan and Jake sell toy cars.

Sandeep sells x cars. Pavan sells 6 more cars than Sandeep. Jake sells twice as many cars as Sandeep.

Write an expression, in terms of x, for the total number of toy cars sold by Sandeep, Pavan and Jake.

Sandeep	Pavan	Jake
x	$x + 6$	$2x$

The total number of toy cars is $x + x + 6 + 2x$

$= x + x + 2x + 6$

$= 4x + 6$

Exam focus

When you have finished working out an expression, make sure that you have collected all of the like terms and cancelled the number parts or the indices to simplify the equation.

'6 more than' means adding 6 and 'twice as many' means multiply by 2, or double it.

10 Exam-style practice — Grade 3

1 Ben has x cats.
Jenny has twice as many cats as Ben.
Kathy has 2 more cats than Ben.
Write an expression, in terms of x, for the total number of pets that Ben, Jenny and Kathy have. **[2 marks]**

2 Blank revision cards are sold in packets and in boxes.
There are 8 revision cards in a packet.
There are 27 revision cards in a box.
Avi buys p packets of revision cards and b boxes of revision cards.
Write an expression for the number of revision cards Avi buys, in terms of p and b. **[3 marks]**

Algebraic formulae

A formula is a mathematical rule. You use algebra to write a formula (the plural of formula is formulae). A formula is similar to an algebraic expression, but it has an equals sign, and more than one variable. You need to be able to substitute numbers into formulae to solve them.

5 Writing a formula

Peter advertises his business in the local magazine.

To write this as an algebraic formula, substitute the variables for letters.

For example, if the total cost is £T, and the number of hours is n, then the formula Peter can use is:

total cost = (number of hours × £3) + £5

$$T = 3n + 5$$

When you define your variables, you must give their units. If you have a value for n you can now solve this formula. If Peter takes his neighbour's dog on a two-hour walk, how much will he charge?

$T = 3 \times 2 + 5$

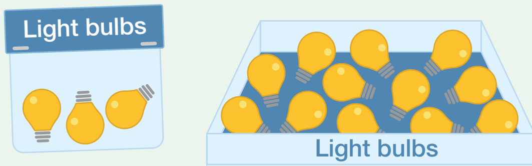
$= 6 + 5 = 11$

He will charge £11

5 Worked example — Grade 3

Bulbs are sold in packets and in boxes. There are 3 bulbs in a packet. There are 12 bulbs in a box.

Kamran buys x packets of bulbs and y boxes of bulbs.



(a) Write down a formula, in terms of x and y, for the total number, N, of bulbs Kamran buys.

$N = 3 \times x + 12 \times y$

$= 3x + 12y$

(b) Kamran buys 4 packets and 2 boxes of bulbs. How many bulbs does he buy?

$N = 3 \times 4 + 12 \times 2$

$= 12 + 24$

$= 36$ He buys 36 bulbs.

The variables are N (the total number of bulbs), x (the number of packets) and y (the number of boxes).

Substitute the values given in the question into the formula you worked out in part **(a)**.

5 Worked example — Grade 5

This formula gives you the distance, s metres, travelled by an object in t seconds.

$s = 10t + 5t^2$

Work out the value of s when $t = 3$

$s = 10 \times 3 + 5 \times 3^2$

$= 30 + 5 \times 9$

$= 30 + 45$

$= 75$

Substitute the value of t into the formula.

When substituting, you might use brackets. You could write $10t$ as $10(t)$ or $10(3)$. If there are numbers or letters outside brackets, without an operation in between, this means that you multiply the term outside the brackets with whatever is inside the brackets.

For example: $10(3)$ means 10×3

Order of operations is very important when you are evaluating formulae. Remember to use BIDMAS.

10 Exam-style practice — Grades 4–5

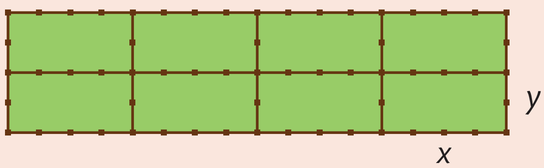
1 $L = \dfrac{2x + 3y}{x}$

Work out the value of L when $x = 8$ and $y = 12$

Give your answer as a fraction in its simplest form.

[3 marks]

2 A farmer uses 200 metres of fencing to make an enclosure divided into eight equal rectangular pens.



The length of each pen is x metres and the width of each pen is y metres.

(a) Show that $y = 20 - 1.2x$

The total area of the enclosure is A m^2.

(b) Show that $A = 160x - 9.6x^2$ **[3 marks]**

Algebraic indices

Indices are also called powers. They represent how many times a number has been multiplied by itself. Examples include squaring and cubing numbers.

5 Basic rules of indices

Learn the basic rules of indices.

$a^m \times a^n = a^{m+n}$ $\quad x^4 \times x^6 = x^{4+6} = x^{10}$

$a^m \div a^n = a^{m-n}$ $\quad x^4 \div x^6 = x^{4-6} = x^{-2}$

$(a^m)^n = a^{mn}$ $\quad (x^4)^6 = x^{4\times6} = x^{24}$

$a^{-n} = \dfrac{1}{a^n}$ $\quad x^{-4} = \dfrac{1}{x^4}$

$a^0 = 1$ $\quad x^0 = 1$

Go to page 15 to revise how indices work.

2 Indices checklist

- Only combine powers (indices) when the base numbers are the same.
- When you multiply, add the powers.
- When you divide, subtract the powers.
- When you raise a power to a power, multiply the powers together.

10 Worked example — Grades 4–5

1 Simplify

(a) $p^7 \times p^4$

$= p^{7+4} = p^{11}$

The base is the same, p, so you can combine the indices.

(b) $p^9 \div p^5$

$= p^{9-5} = p^4$

(c) $(p^2)^4$

$= p^{2\times4} = p^8$

2 Simplify

(a) $\dfrac{x^5 \times x^7}{x^3}$

$= \dfrac{x^{5+7}}{x^3} = \dfrac{x^{12}}{x^3} = x^9$

(b) $\left(\dfrac{x^8}{x^5}\right)^2$

$= (x^{8-5})^2 = (x^3)^2 = x^6$

3 Simplify

Deal with each base letter separately.

(a) $5x^4y^3 \times 2x^3y^2$

$= 5x^4 \times 2x^3 \times y^3 \times y^2 = 10x^7y^5$

(b) $\dfrac{24x^4y^3}{8x^2y}$

Treat the numbers and each base separately.
$24 \div 8 = 3$
$x^4 \div x^2 = x^2$
$y^3 \div y = y^2$

$= \dfrac{24x^4}{8x^2} \times \dfrac{y^3}{y} = 3x^2y^2$

4 Work out 2^{-3}

$= \dfrac{1}{2^3} = \dfrac{1}{8}$

A negative power shows that the value is a reciprocal and can be written as a fraction, $a^{-n} = \dfrac{1}{a^n}$

Substitute 2^{-3} into $a^{-n} = \dfrac{1}{a^n}$

5 Worked example — Grade 5

1 Work out the value of n given that

$p^4 \times p^n = p^{10}$

$4 + n = 10$

Add the indices then set this equal to 10.

$n = 6$

2 Work out the value of t given that

$(5^4)^t = 5^{12}$

$4t = 12$

$t = 3$

Since the base is the same on both sides, the powers must be equal, so you can form an equation and solve it.

10 Exam-style practice — Grades 4–5

1 Simplify

(a) $p^2 \times p^9$ **[1 mark]**

(b) $\dfrac{x^4 \times x^6}{x^2}$ **[2 marks]**

(c) $4x^2y^4 \times 3xy$ **[2 marks]**

2 Simplify

(a) $m^8 \div m^2$ **[2 marks]**

(b) $(m^5)^3$ **[2 marks]**

(c) $3w^2y^3 \times 4w^6y$ **[2 marks]**

(d) $\dfrac{32x^6y^8}{4x^2y}$ **[2 marks]**

3 Solve

$x^{15} = x^n \times x^8$ **[1 mark]**

4 $1000^a \times 100^b = 10^x$

Show that $x = 3a + 2b$ **[2 marks]**

Expanding brackets

Sometimes mathematical expressions include terms written in brackets. You can remove the brackets by expanding them.

Removing brackets

To remove brackets, you **expand** them. 'Expand' means **multiply**.

An expression such as $2(x + 4)$ can be expanded by multiplying 2 and x, and 2 and 4. There is an invisible multiplication sign between the 2 and the $(x + 4)$.

$$2(x + 4) = 2 \times (x + 4)$$
$$= 2 \times x + 2 \times 4$$
$$= 2x + 8$$

Negative terms

When the term outside is negative, you have to multiply the terms inside the bracket by a negative number.

For example:

$$-5(a - 2) = -5 \times (a - 2)$$
$$= -5 \times a + -5 \times -2$$
$$= -5a + 10$$

A negative multiplied by another negative gives a positive number: $-5 \times -2 = +10$.

Go to page 2 to revise multiplying negative numbers.

Problem solving

You can be asked to use multiple skills in one question. If you are asked to expand and simplify, you need to expand all the brackets and then collect like terms and combine them.

For example, to expand and simplify $5(e + 1) - 3e(4 - 6e)$:

$$5(e + 1) - 3e(4 - 6e)$$
$$= 5 \times e + 5 \times 1 - 3e \times 4 - 3e \times -6e$$
$$= 5e + 5 - 12e + 18e^2$$
$$= 18e^2 - 7e + 5$$

Worked example — Grades 2–4

1 Expand

(a) $4(x + 5)$

$= 4 \times x + 4 \times 5$

$= 4x + 20$

When multiplying out brackets, always multiply every term inside the bracket by the term outside.

(b) $3(x - 7)$

$= 3 \times x - 3 \times 7$

$= 3x - 21$

2 Expand

(a) $-x(x - 3)$

Draw arrows from the term outside the brackets to each term inside, so you know which terms you need to multiply.

$= -x \times x - x \times -3$

$= -x^2 + 3x$

(b) $-3x(x + 1)$

$= -3x \times x - 3x \times 1$

$= -3x^2 - 3x$

After multiplying out the brackets, collect the like terms and combine them.

3 Expand and simplify

(a) $5(x + 7) + 3(x - 2)$

$= 5 \times x + 5 \times 7 + 3 \times x - 3 \times 2$

$= 5x + 35 + 3x - 6$

$= 5x + 3x + 35 - 6$

$= 8x + 29$

(b) $3m(m + 4) - 2m(4m + 1)$

$= 3m \times m + 3m \times 4 - 2m \times 4m - 2m \times 1$

$= 3m^2 + 12m - 8m^2 - 2m$

$= 3m^2 - 8m^2 + 12m - 2m$

$= -5m^2 + 10m$ or $10m - 5m^2$

Exam focus

Write out the expression with each separate operation in it. Include any negative numbers.

This will help you make sure you haven't missed any terms or signs out.

Exam-style practice — Grades 2–4

1 Expand

(a) $5(m + 2)$ **[1 mark]** **(b)** $-3(n + 6)$ **[1 mark]** **(c)** $x(x - 5)$ **[1 mark]** **(d)** $3x(x - 2)$ **[1 mark]** **(e)** $-4x(x - 1)$ **[1 mark]**

2 Simplify

(a) $7a + 4(a - 2b)$ **[2 marks]** **(b)** $4(3 + 2g) + 2(5 - 3g)$ **[2 marks]**

(c) $4r(3 + 4p) + 3p(8 - r)$ **[2 marks]** **(d)** $t(3t + 4) + 3t(3 + 2t)$ **[2 marks]**

3 Expand $x(x - 5)$. Circle your answer. **[1 mark]**

$x^2 - 5$ $\quad$ $x^2 - 5x$ $\quad$ $2x - 5$ $\quad$ $-5x^2$

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