Mixed simultaneous equations

5 Quick quiz

Write each equation in the form $ax^2 + bx + c = 0$.

(a) $x^2 - 3x = 7x - 6$ ……

(b) $2x^2 - 8 = x^2 + 5x$ ……

(c) $-3x^2 - x = -2x^2 + 3x - 10$ ……

15 Algebraic method — Grade 8–9

1. Solve the simultaneous equations $y = x^2 - 3x + 9$ and $y = 2x + 3$. **[5 marks]**

$x^2 - 3x + 9 = 2x + 3$

$x^2 - \ldots\ldots x + \ldots\ldots = 0$

$(x - \ldots\ldots)(x - \ldots\ldots) = 0$

$x = \ldots\ldots$ or $x = \ldots\ldots$

$y = 2(\ldots\ldots) + 3$ or $y = 2(\ldots\ldots) + 3$

$y = \ldots\ldots$ or $y = \ldots\ldots$

As both equations have y as the subject, set the RHSs equal.

Solve the quadratic equation and then substitute the x-values into the linear equation to find the values of y.

$x = \ldots\ldots, y = \ldots\ldots$ or $x = \ldots\ldots, y = \ldots\ldots$

2. Solve the simultaneous equations $x + 2y = -3$ and $x^2 - 2xy = 20$. **[5 marks]**

Always rearrange the linear equation and then substitute into the quadratic equation.

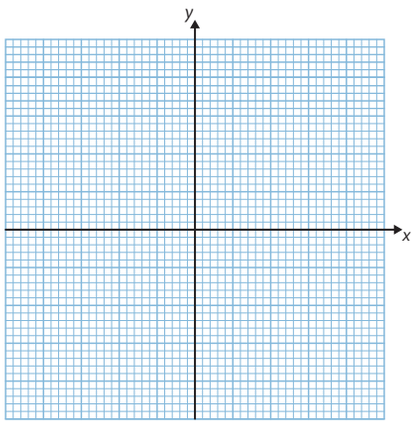
$x = \ldots\ldots, y = \ldots\ldots$ or $x = \ldots\ldots, y = \ldots\ldots$

3. Solve the simultaneous equations $x + 2y = 4$ and $x^2 - 2y^2 = 2$. **[5 marks]**

$x = \ldots\ldots, y = \ldots\ldots$ or $x = \ldots\ldots, y = \ldots\ldots$

10 Graphical method — Grade 8

4. Use a graphical method to solve the simultaneous equations $x^2 + y^2 = 25$ and $x + y = 1$. **[3 marks]**



$x = \ldots\ldots, y = \ldots\ldots$ or $x = \ldots\ldots, y = \ldots\ldots$

Completing the square

Quick quiz

Evaluate each expression.

(a) $-3^2 + 10$ .....

(b) $-5^2 - 15$ .....

(c) $-\left(\frac{3}{2}\right)^2 + 5$ .....

(d) $-\left(\frac{7}{2}\right)^2 - 2$ .....

Completing the square — Grade 8

1. The expression $x^2 + 10x - 8$ can be written in the form $(x + a)^2 + b$ for all values of x.

Find the value of a and the value of b. **[2 marks]**

$x^2 + 10x - 8 = (x + \ldots\ldots)^2 - \ldots\ldots^2 - 8$

$a = \ldots\ldots$, $b = \ldots\ldots$

Use this formula to complete the square:

$x^2 \pm 2bx + c = (x \pm b)^2 - b^2 + c$

Exam focus

If the question includes $(x - a)^2 + b$, you need to complete the square.

2. The expression $x^2 - 4x + 5$ can be written in the form $(x - p)^2 + q$ for all values of x.

Find the value of p and the value of q. **[2 marks]**

$p = \ldots\ldots$, $q = \ldots\ldots$

3. The expression $x^2 - 5x + 3$ can be written in the form $(x - p)^2 + q$ for all values of x.

Find the value of p and the value of q. **[2 marks]**

$p = \ldots\ldots$, $q = \ldots\ldots$

Finding turning points — Grade 8

4. The expression $x^2 - 8x + 18$ can be written in the form $(x - p)^2 + q$ for all values of x.

(a) Find the value of p and the value of q. **[2 marks]**

$p = \ldots\ldots$, $q = \ldots\ldots$

The graph of $y = x^2 - 8x + 18$ has a minimum point.

(b) Write down the coordinates of this point. **[2 marks]**

.....

5. $x^2 - 12x + 25 \equiv (x - a)^2 + b$

(a) Work out the value of a and the value of b. **[3 marks]**

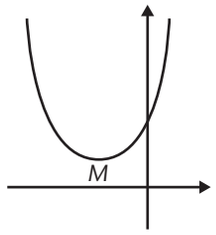
$a = \ldots\ldots$, $b = \ldots\ldots$

(b) What is the minimum value of $x^2 - 12x + 25$? **[2 marks]**

.....

Finding equations from turning points — Grade 8

6. The equation of a curve is $f(x) = x^2 + ax + b$. The diagram shows a sketch of part of the graph of $y = f(x)$.
The coordinates of the turning point M are $(-3, 5)$.
Work out the equation of the curve $y = f(x)$ where a and b are integers. **[3 marks]**



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The quadratic formula

Quick quiz

Work out the value of $b^2 - 4ac$ when:

(a) $a = 2$, $b = 3$ and $c = 4$ **(b)** $a = 3$, $b = -4$ and $c = -8$ **(c)** $a = 4$, $b = -6$ and $c = 5$ **(d)** $a = 1$, $b = 5$ and $c = -10$

....

Using the formula

Grade 8

1. Solve $x^2 + 7x + 8 = 0$.
Give your solutions correct to 2 decimal places.
[3 marks]

$a = 1, b = 7, c = 8$

$$x = \frac{-7 \pm \sqrt{7^2 - (4 \times 1 \times 8)}}{2 \times 1}$$

$x =$ ..... or $x =$ .....

Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
where a, b and c are the constants in the general form $ax^2 + bx + c = 0$.

Exam focus
When you are asked to solve a quadratic equation to a specified degree of accuracy, always use the quadratic formula.

2. Solve the equation $3x^2 + 6x = 2$.
Give your solutions correct to 2 decimal places.
[3 marks]

$x =$ ..... or $x =$ .....

3. Solve the equation $x^2 - 2x - 3 = x - 1$.
Give your solutions correct to 2 decimal places.
[3 marks]

$x =$ ..... or $x =$ .....

4. Solve $3x(2x - 1) = (x - 3)^2$.
Give your solutions correct to 3 significant figures.
[3 marks]

$x =$ ..... or $x =$ .....

5. Solve the equation $2(x - 4)^2 - 10(x - 3) = 14$.
Give your solutions correct to 2 decimal places.
[3 marks]

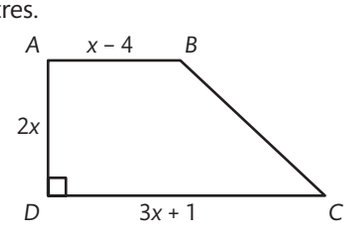
$x =$ ..... or $x =$ .....

Using the formula in context

Grade 8

6. The diagram shows a trapezium. All measurements are in centimetres.

The area of the trapezium is 64 cm^2. Work out the value of x.
Give your answer correct to 3 significant figures.
Show your working clearly.



[5 marks]

Area of trapezium $= \frac{1}{2}(a + b)h$

$x =$ .....

Linear inequalities

Quick quiz

Match each sign to the correct meaning.

(a) $<$	**(b)** $>$	**(c)** $\leqslant$	**(d)** $\geqslant$

greater than or equal to	less than	less than or equal to	greater than

Simple inequalities — Grade 5

1. Solve

(a) $4x - 5 > 19$ **[2 marks]**

$4x > 19 + 5$

$4x > \ldots\ldots$

.....

(b) $6x \leqslant 2x - 18$ **[2 marks]**

$6x - \ldots\ldots \leqslant -18$

$\ldots\ldots x \leqslant \ldots\ldots$

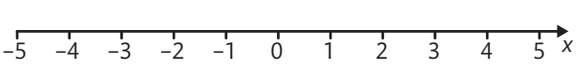
.....

(c) $10x + 4 > 3x + 25$. **[3 marks]**

.....

2. $-2 \leqslant x < 4$

Represent this inequality on the number line. **[2 marks]**



Solve the inequality as you would solve an equation.

A shaded circle means the number is included. An unshaded circle means the number is not included.

Solving inequalities — Grade 6

3. Solve $\frac{4x - 3}{3} < \frac{2x - 1}{2}$. **[2 marks]**

.....

4. Find all the integers, x, that satisfy the inequalities $-4 \leqslant x < 3$ and $-3 < x < 7$. **[2 marks]**

.....

5. Helena is going to the bakery. She has £10. She wants to buy some buns for 32p each and cakes for 45p each.

(a) Write down an inequality to show the number of buns and the number of cakes that Helena can buy. **[2 marks]**

.....

(b) Helena buys 14 buns. Find the greatest number of cakes she can buy. **[3 marks]**

.....

6. If $3x + 6 > 19$, find the smallest possible integer value of x. **[3 marks]**

.....

Quadratic inequalities

Quick quiz

Factorise each expression.

(a) $x^2 - 3x$ ..... **(b)** $x^2 + 6x + 8$ ..... **(c)** $2x^2 + 7x - 4$ .....

Simple quadratic inequalities — Grade 9

1. Solve $x^2 - 8x + 15 \geqslant 0$. **[3 marks]**

(.....) × (.....) = 15

(.....) + (.....) = −8

Factorise and solve the quadratic equation.

The x-values found are the critical values.

Exam focus
Your final answer(s) must be given with inequality signs.

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Harder quadratic inequalities — Grade 9

2. Solve $x^2 \leqslant 4(x + 8)$. **[3 marks]**

.....

3. Solve $(x - 2)^2 - 4(x + 1) > 0$.

Give your answer using set notation. **[3 marks]**

.....

Inequalities in context — Grade 9

4. A rectangular room has a width of x m. The room is 4 m longer than it is wide.

(a) Given that the perimeter of the room is greater than 12 m, show that $x > 1$. **[3 marks]**

(b) Given also that the area of the room is less than 32 m^2: **[4 marks]**

(i) Write down an inequality, in terms of x, for the area of the room.

.....

(ii) Solve this inequality.

.....

(c) Use your answers to parts (a) and (b) to find the range of possible values for x. **[1 mark]**

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Arithmetic sequences

Quick quiz

Write down the next three terms in each sequence.

(a) 2 5 8 .....

(b) 100 97 94 .....

(c) 16 21 26 .....

The *n*th term — Grade 5

1. Here are the first four terms of an arithmetic sequence: 8 13 18 23

(a) Write an expression, in terms of n, for the nth term of this sequence. **[2 marks]**

Work out the common difference and use this as the coefficient of n.

nth term = ... n + ...

(b) The nth term of another sequence is $4n + 7$. Is 206 a term of this sequence?

You must show your working. **[2 marks]**

Equate 206 with $4n + 7$ and then solve for n.

$4n + 7 =$ .....

.....

Exam focus

Once you have worked out the value of n, you can answer the question with 'yes' or 'no'.

2. Here are the first four terms of an arithmetic sequence: 5 12 19 26

(a) Write an expression, in terms of n, for the nth term of this sequence. **[2 marks]**

.....

(b) The nth term of another sequence is $3n - 8$. Find the fourth term of the sequence. **[2 marks]**

.....

3. The nth term of an arithmetic sequence is $3n + 4$, where n is a positive integer.

(a) Determine whether 110 is a term in this arithmetic sequence. **[2 marks]**

.....

(b) Find an expression for the sum of the nth term and the $(n - 1)$th terms of this sequence. Give your answer in its simplest form. **[2 marks]**

.....

The *n*th term for proofs — Grade 5

4. The nth term of sequence X is $4n - 3$. The nth term of sequence Y is $14 - 3n$.
Show that there is only one number that is in both sequences. You must explain your answer. **[3 marks]**

5. Here are the first five terms of an arithmetic sequence: 3 7 11 15 19

Prove that the difference between the squares of any two consecutive terms of the sequence is always a multiple of 8. **[5 marks]**