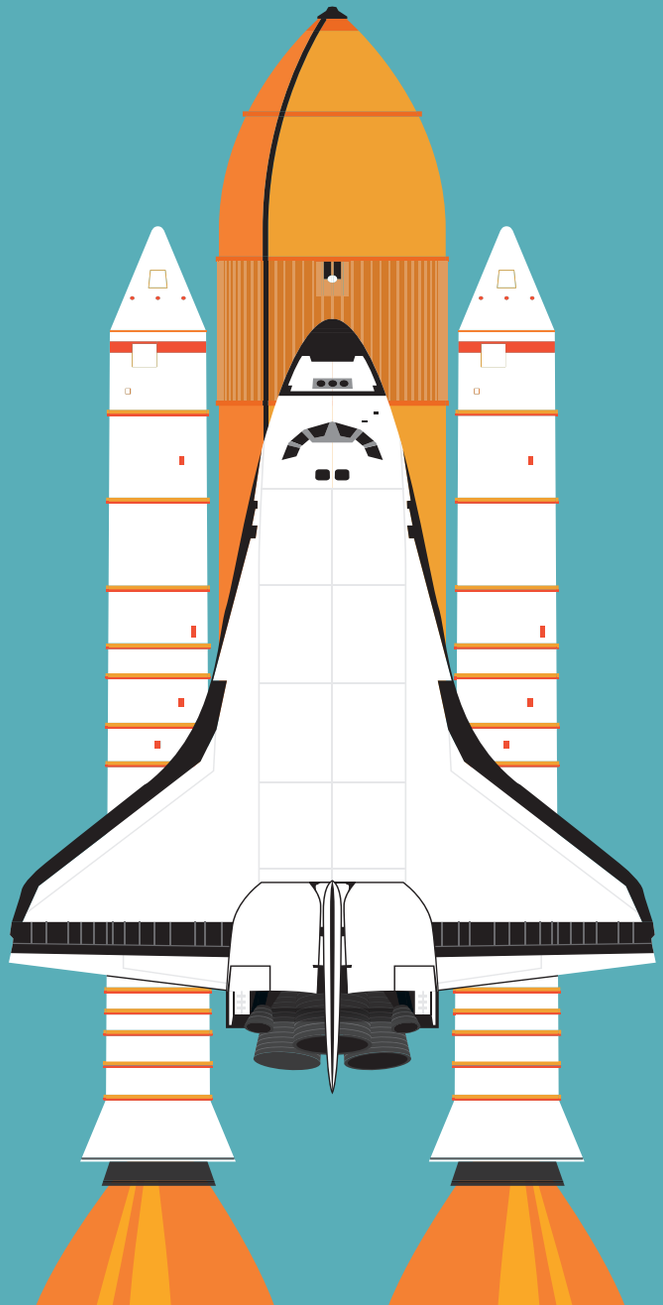
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Bitesize Pearson Edexcel GCSE (9–1) MATHEMATICS

REVISION WORKBOOK

HIGHER



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Contents

Number

- Fractions, decimals and percentages 1
- Manipulating fractions 2
- Percentage change 3
- Reverse percentages 4
- Growth and decay 5
- Estimation and counting 6
- Upper and lower bounds 7
- Accuracy and error 8
- Factors and primes 9
- Standard form 10
- Surds 11
- Exam skills: Number 12

Algebra

- Algebraic expressions 13
- Algebraic formulae 14
- Laws of indices 15
- Combining indices 16
- Linear equations 17
- Linear equations and fractions 18
- Simultaneous equations 19
- Quadratic equations 20
- Mixed simultaneous equations 21
- Completing the square 22
- The quadratic formula 23
- Linear inequalities 24
- Quadratic inequalities 25
- Arithmetic sequences 26
- Quadratic sequences 27
- Sequence problems 28
- Drawing straight-line graphs 29
- Equations of straight lines 30
- Parallel and perpendicular lines 31
- Quadratic graphs 32
- Cubic and reciprocal graphs 33
- Real-life graphs 34
- Trigonometric graphs 35
- Inequalities on graphs 36
- Using quadratic graphs 37
- Turning points 38
- Sketching graphs 39
- Exponential graphs 40
- Gradients of curves 41
- Velocity–time graphs 42
- Areas under curves 43
- Transforming graphs 44
- Algebraic fractions 45
- Quadratics and fractions 46
- Function notation 47
- Inverse functions 48
- Equation of a circle 49
- Iteration 50
- Exam skills: Algebra 51

Ratio, proportion and rates of change

- Ratio 52
- Proportion 53
- Compound measures 54
- Speed 55
- Density 56
- Proportion and graphs 57
- Proportionality formulae 58
- Harder relationships 59
- Exam skills: Ratio and proportion 60

Geometry and measures

- Angle properties 61
- Angle problems 62
- Angles in polygons 63
- Constructing perpendiculars 64
- Constructions, plans and elevations 65
- Loci 66
- Perimeter and area 67
- Volumes of 3D shapes 68
- Surface area 69
- Prisms 70
- Circles and cylinders 71
- Circles, sectors and arcs 72
- Circle facts 73
- Circle theorems 74
- Transformations 75
- Enlargement 76
- Combining transformations 77
- Similar shapes 78
- Congruent triangles 79
- Pythagoras' theorem 80
- Pythagoras' theorem in 3D 81
- Units of length, area and volume 82
- Trigonometry: lengths 83
- Trigonometry: angles 84
- Exact values 85
- Trigonometry in 3D 86
- The sine rule 87
- The cosine rule 88
- Triangles and segments 89
- Vectors 90
- Vector proof 91
- Line segments 92
- Exam skills: Geometry and measures 93

Probability

- Probability 94
- Relative frequency 95
- Venn diagrams 96
- Conditional probability 97
- Tree diagrams 98
- Exam skills: Probability 99

Statistics

- Sampling 100
- Stratified sampling 101
- Capture–recapture 102
- Mean, median and mode 103
- Frequency tables 104
- Interquartile range 105
- Line graphs 106
- Scatter graphs 107
- Cumulative frequency 108
- Box plots 109
- Histograms 110
- Frequency polygons 111
- Analysing data 112
- Exam skills: Statistics 113

Problem solving

- Problem solving strategies 114
- Solving number problems 115
- Solving proof problems 116
- Solving geometric problems 117
- Solving algebraic problems 118
- Solving statistical problems 119

- Non-calculator practice paper 120
- Calculator practice paper 123

Answers 126

Tick off each topic as you go.

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Fractions, decimals and percentages

Quick quiz

1. Complete the table.

Decimal	Percentage	Fraction
0.25		
	50%	
		$\frac{3}{4}$

2. Find the answers to these calculations.

(a) 45% of 70 ..... **(b)** 32% of 180 ..... **(c)** $\frac{4}{5}$ of 90 ..... **(d)** $\frac{5}{8}$ of 160 .....

Working from a whole

Grade 4

1. There are 300 counters in a bag. Each counter is black, white or grey.
30% of the counters are black and $\frac{1}{4}$ of the counters are white.

Work out the number of grey counters in the bag. **[3 marks]**

Convert the fraction into a percentage.

The whole is 100%.

$30\% + \frac{1}{4} = 30\% + \ldots\ldots\% = \ldots\ldots\%$

$100\% - \ldots\ldots\% = \ldots\ldots\%$

$\ldots\ldots\%$ of $300 = \frac{\square}{100} \times 300 =$

.....

Exam focus
Always write out the percentage as a fraction to make the process clear.

2. All the pupils in Year 11 were asked to choose which sport they preferred out of football, rugby, hockey, tennis or badminton. The table shows the results, as proportions.

Football	Rugby	Hockey	Tennis	Badminton
24%	0.16	$\frac{1}{4}$	20%	

(a) What percentage of the pupils prefer:
(i) rugby or hockey **[1 mark]** **(ii)** badminton? **[1 mark]**

..... % ..... %

There are 150 pupils in Year 11.
(b) How many chose football? **[2 marks]**

.....

Recurring decimals

Grade 8

3. Show that $0.1\dot{7}\dot{8}$ can be written as the fraction $\frac{59}{330}$. **[3 marks]**

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Manipulating fractions

Quick quiz

Match the operations to the methods.

Adding or subtracting fractions	Multiply the numerators and multiply the denominators
Multiplying fractions	Invert the second fraction then multiply
Dividing fractions	Find equivalent fractions with the same denominator

The four operations — Grades 4–5

1. Find the answers to these calculations. Give your answers in their simplest form.

(a) $\frac{\cancel{12}^{6}}{7} \times \frac{5}{\cancel{6}_{2}}$ **[2 marks]**

$= \frac{1 \times 5}{7 \times 2}$

(b) $\frac{3}{10} \div \frac{11}{20}$ **[2 marks]**

$= \frac{3}{10} \times \frac{\square}{\square}$

$= \frac{3 \times \ldots\ldots}{\ldots\ldots \times \ldots\ldots}$

When multiplying fractions, always look to cancel first.

(c) $3\frac{1}{2} - 1\frac{3}{5}$ **[3 marks]**

$= \frac{(3 \times 2) + 1}{2} - \frac{(5 \times 1) + 3}{5}$

$= \frac{\square}{2} - \frac{\square}{5} = \frac{\square}{\square} - \frac{\square}{\square}$

....

Exam focus
Make sure you clearly show your working out when dealing with fractions.

Always convert mixed numbers to improper fractions first.

2. Find the answers to these calculations, giving your answers in their simplest form.

(a) $3\frac{3}{8} + 4\frac{2}{5}$ **[3 marks]**

(b) $4\frac{4}{7} - 2\frac{2}{3}$ **[3 marks]**

(c) $1\frac{1}{3} \times \frac{7}{10} \div \frac{2}{5}$ **[3 marks]**

....

Working with fractions in context — Grade 5

3. 120 children compete in a charity race. $\frac{1}{4}$ of the children don't complete the run.
Work out the number of children who complete the race. **[3 marks]**

.....

4. A headmaster collected information about 225 Year 11 students.
$\frac{1}{3}$ of the students wanted to go to college and $\frac{2}{5}$ of the students wanted to do apprenticeships.
The rest of the students wanted to stay on at school.
Work out the number of students who wanted to stay on at school. **[3 marks]**

.....

5. Mischa is cutting a cake. She gives $\frac{1}{3}$ of the cake to Tom and gives $\frac{1}{4}$ of the cake to Olivia.
Mischa keeps the rest of the cake. Olivia only wants half her slice and gives the rest of it back to Mischa.
What fraction of the whole cake does Mischa now have in total? **[4 marks]**

.....

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Percentage change

Quick quiz

1. 'Per cent' means out of ..... **3.** 25% of 80 = ...

2. 10% of 60 = ... **4.** 50% of 90 = ...

Percentage change — Grade 4

1. Find the percentage increase or decrease.

To find the percentage increase or decrease, find the difference, divide it by the original amount and multiply by 100.

(a) 36.4 g to 41.3 g **[2 marks]**

$41.3 - 36.4 = 4.9$

$\frac{4.9}{36.4} \times 100 =$ ..... %

(b) 83.4 m to 69.5 m **[2 marks]**

..... %

(c) 16.38 kg to 24.56 kg **[2 marks]**

..... %

2. (a) Increase 840 by 28%. **[2 marks]**

$\frac{28}{100} \times 840 =$ .....

.....

(b) Decrease 238 by 18%. **[2 marks]**

$238 \times 0.82 =$ .....

.....

(c) Increase £63.50 by 45%. **[2 marks]**

.....

To increase or decrease an amount by a percentage, find the percentage of the original amount, then add or subtract it to or from the original amount. Alternatively, use the multiplier method.

Everyday percentages — Grade 4

3. Gary buys and sells antique clocks. He wants to reach a target of at least 30% profit on each antique clock he sells.
In January, Gary buys an antique clock for £2500. In February, Gary sells the antique clock for £3250.
Does Gary reach his target for this antique clock? **[3 marks]**

.....

4. The table shows the costs, per person, of a holiday at two different hotels. It shows the cost for 5 nights per adult and the cost for each extra night. It also shows the discount for each child, which is applied to the adult price.

	Hotel 1		Hotel 2	
	Cost for 5 nights	**Cost for each extra night**	**Cost for 5 nights**	**Cost for each extra night**
1 Apr – 31 May	£1269	£160	£949	£95
1 Jun – 16 Jul	£1329	£160	£1319	£105
17 Jul – 31 Aug	£1720	£170	£1950	£300
Discount for each child	10% off		25% off	

There are 2 adults and 2 children in the Marwaha family. The family want a holiday for 7 nights, starting on 1 August.
One hotel will be cheaper than the other hotel for them. Work out which hotel offers the cheaper stay.
You must show your working. **[5 marks]**

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Reverse percentages

Quick quiz

Match the multipliers to the change.

0.972	An increase of 12%
1.12	A decrease of 15%
1.105	An increase of 10.5%
0.85	A decrease of 2.8%

Working out the normal price

Grade 5

1. The normal price of a television is reduced by 35% in a sale.
The sale price of the television is £780.
Work out the normal price of the television. **[3 marks]**

The normal price is always 100%.

Method 1

Subtract the percentage reduction from 100% and equate this value with the sale price.

Sale price = 100% − 35%

= ..... %

£780 = ... %

$1\% = \frac{780}{\square}$

$100\% = \frac{780}{\square} \times 100$

£.....

Method 2

To find the multiplier, subtract from 100% and then divide by 100 to convert into a decimal.

100% − 35% = ... %

..... ÷ 100 = ...

Normal price = £780 ÷ .....

£.....

Exam focus
Choose the method that you find easiest.

2. In a sale, normal prices are reduced by 12.5%.
The normal price of a tablet is reduced by £18.
Work out the normal price of the tablet. **[3 marks]**

Work out the multiplier: 100% − 12.5%, then convert into a decimal.

£.....

3. Each year, Sol records how long it takes him, on average, to get to work each day. In 2017, he took 8% more time than in 2016. In 2017, the average time he took to get to work was 34 minutes. Work out the average time he took to get to work in 2016.
Give your answer to the nearest minute. **[3 marks]**

..... minutes

4. Ravina has two investments: a fixed bond and an ISA. During a 5-year period, the value of the fixed bond increased by 5% to £40 530. In the same period, the value of the ISA increased by 7.5% to £27 004.
In this 5-year period, which investment has grown more in terms of monetary value? **[4 marks]**

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Growth and decay

Quick quiz

Write down the multiplier for:

(a) an increase of 15% ..... **(b)** a decrease of 18% ..... **(c)** an increase of 2.35% ..... **(d)** a decrease of 6.9% .....

Growth — Grade 5

1. Alan invests £1800 at 2.75% per annum compound interest.
Work out the value of Alan's investment after 3 years. **[3 marks]**

As it is an increase, add the percentage.

100% + ...% = ...%

Multiplier = ...% ÷ 100 = ...

Value of investment = 1800 × (.....)^3

£.....

Exam focus
Make sure you give your answer to 2 decimal places.

Decay — Grade 5

2. John buys a new car for £32 400. The value of the car depreciates at the rate of 15% per year. After n years, the value of the car is £19 897.65
Find the value of n. **[2 marks]**

As it is a decrease, subtract the percentage.

100% − .....% = ...%

Multiplier = ...% ÷ 100 = ...

Value = 32 400 × (.....)^n = £19 897.65

Try different values of n.

n = ...

Money — Grade 5

3. Anna invests £6000 in a savings account for 3 years. The account pays compound interest at an annual rate of 5% for the first year, 4% for the second year and x% for the third year.
There is a total amount of £6715.80 in the savings account at the end of 3 years.
Work out the rate of interest in the third year. **[4 marks]**

Set up an equation. Write the multiplier for an increase by x% as $1 + \frac{x}{100}$.

.....%

4. Tori goes to college by bus. In March, the cost of her monthly bus pass increases by 8.5% to £69.44.
Work out the cost of her monthly bus pass before this increase. **[2 marks]**

£.....

Populations — Grade 5

5. A scientist studies a rabbit population. She finds that the population has been decreasing due to disease. The scientist assumes that it is decreasing at a rate of 12% each year. At the beginning of 2012, the rabbit population was 4800. Using the scientist's assumption, work out the size of this rabbit population at the start of 2017. **[2 marks]**

.....

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Estimation and counting

Quick quiz

Round each number to 1 significant figure.

(a) 82.6 ..... **(b)** 195.5 ..... **(c)** 0.198 ..... **(d)** 0.523 .....

Counting — Grade 5

1. There are 18 men and 25 women at a dancing club. One of the men and one of the women will pair up to perform the first dance.

(a) Work out the number of different pairs that can be chosen. **[2 marks]**

.....

Two women are to be chosen to perform the second dance.

Ashley thinks the number of different pairs that can be chosen is 600. Lisa thinks the number of different pairs that can be chosen is 300.

(b) Who is correct, Ashley or Lisa? Give a reason for your answer. **[2 marks]**

.....

2. There are 18 girls and 15 boys in a club. One girl and one boy are going to be chosen to go to a conference. Work out the total number of ways of choosing a girl and a boy. **[2 marks]**

.....

Estimating an answer — Grades 5–6

3. Work out an estimate for

(a) $\dfrac{204 \times 9.95}{0.509}$ **[2 marks]**

$= \dfrac{200 \times .....}{0.5} = \dfrac{\square}{0.5}$

.....

(b) $\sqrt{4.99 \times 2.05 + 7.24}$ **[3 marks]**

$= \sqrt{5 \times .....}$

.....

4. The radius of a circle is 5.4 cm.

(a) Work out an estimate for the area of the circle. **[2 marks]**

..... cm^2

(b) Without further calculation, explain whether your method gives you an overestimate or an underestimate for the area of the circle. **[1 mark]**

.....

5. Kathy has some cows. The cows produce an average total of 22.3 litres of milk per day for 191 days. Kathy sells the milk in half-litre cartons. Estimate the total number of cartons that Kathy will be able to fill with the milk produced over the 191 days. You must show clearly how you got your estimate. **[3 marks]**

.....

Exam focus
Write each number correct to 1 significant figure. Make it clear how you have rounded each individual number.

Remember BIDMAS.

The numerical value of π is approximately 3.14.

The area of a circle is πr^2.

When estimating, always round to 1 significant figure.

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Upper and lower bounds

Quick quiz

Choose the correct word in each sentence by circling it.

(a) To find the upper bound, you **add** / **subtract** half of the unit to which you are rounding.

(b) To find the lower bound you **add** / **subtract** half of the unit to which you are rounding.

Finding upper and lower bounds — Grade 8

1. $T = \frac{x}{y + z}$, where $x = 98.3$ correct to 1 decimal place, $y = 21$ correct to 2 significant figures and $z = 15.23$ correct to 2 decimal places.
Work out the upper bound for the value of T. Give your answer correct to 1 decimal place. Show your working clearly. **[4 marks]**

	x	y	z
UB	98.35	21.5	15.235
LB	98.25	20.5	15.225

Set up a table of upper and lower bounds for each value.

$T = \frac{x}{y + z} = \frac{\square}{20.5 + \ldots\ldots}$

.....

For the upper bound of T, the numerator will be an upper bound and the denominator will be a lower bound.

The lower bound of the denominator must be the sum of the lower bounds of the two numbers.

Exam focus
Read the question carefully.
The final answer must be correct to 1 decimal place.

2. Each of these values has been rounded to 1 significant figure. Write down the upper and lower bound for each value.

(a) 2000 m **[1 mark]**
LB: ..... m
UB: ..... m

(b) 0.6 kg **[1 mark]**
LB: ..... kg
UB: ..... kg

(c) 5 seconds **[1 mark]**
LB: ..... s
UB: ..... s

(d) 10 miles **[1 mark]**
LB: ..... miles
UB: ..... miles

Using upper and lower bounds — Grade 8

3. Mark runs 200 metres, correct to the nearest metre. He takes 25.3 seconds, correct to the nearest 0.1 of a second.
Work out the lower bound of Mark's average speed. Give your answer correct to 3 significant figures. **[3 marks]**

If the distance has been rounded to the nearest metre, the actual value could be ±0.5 m.

..... m/s

4. A soda machine makes fizzy drinks. The machine puts 220 ml fizzy water (measured correct to the nearest ml) and 29 ml cordial (measured correct to the nearest ml) into each bottle.
The volume of a bottle is 250 ml, correct to the nearest 0.1 ml.
Tom thinks the total volume of the drink could be greater than the capacity of the bottle. Is Tom correct?
You must show how you get your answer. **[3 marks]**

First work out the bounds for each liquid and then find the upper bound of the capacity.

.....

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Accuracy and error

Quick quiz

1. A rope is 24.3 m long, correct to 1 decimal place. Write down the:

(a) upper bound ..... m

(b) lower bound ..... m

2. A wall is 18 m long, correct to 2 significant figures. Write down the error interval for the length, l cm, of the wall.

..... $\leqslant l <$ .....

Finding appropriate degree of accuracy

Grade 8

1. The length of the side of a metal cube is 113 mm correct to the nearest mm. The mass of the cube is 3.48 kg correct to 2 decimal places. Find the value of the density of the metal to an appropriate degree of accuracy. Give your answer in g/cm^3. You must explain why your answer is to an appropriate degree of accuracy. **[5 marks]**

	Mass	Length
UB	3.485 kg	
LB		112.5 mm

Set up a table of upper and lower bound values.

Convert kilograms to grams and millimetres to centimetres.

To find the upper bound of density, you must find the upper bound of mass and the lower bound of volume.

Upper bound $= \dfrac{\text{mass}}{\text{volume}}$

$= \dfrac{\ldots\ldots\ldots \times 1000}{(\ldots\ldots\ldots \div 10)^3}$ g/cm^3

= ..... g/cm^3

Lower bound $= \dfrac{\text{mass}}{\text{volume}}$

$= \dfrac{\ldots\ldots\ldots \times 1000}{(\ldots\ldots\ldots \div 10)^3}$ g/cm^3

= ..... g/cm^3

Density = ... g/cm^3 to ..... decimal place(s)

This is the appropriate degree of accuracy because both values round to the same number when rounded to ..... decimal place(s).

2. $P = \dfrac{E}{10b}$, $E = 68.65$ correct to 2 decimal places and $b = 2.243$ correct to 3 decimal places.

By considering bounds, work out the value of P to a suitable degree of accuracy.

You must show your working and give a reason for your final answer. **[5 marks]**

Work out the upper and lower bounds for E and b and use these to find the upper and lower bounds for P.

3. Hannah drops a ball from a height of h metres onto the ground.

The time, t seconds, that the ball takes to reach the ground is given by $t = \sqrt{\dfrac{2h}{g}}$, where g m/s^2 is the acceleration due to gravity, $h = 84.9$ correct to 1 decimal place and $g = 9.8$ correct to 2 significant figures. By considering bounds, work out the value of t to a suitable degree of accuracy. You must show your working and give a reason for your final answer.

[5 marks]

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Factors and primes

Quick quiz

Write in the missing powers.

(a) $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^{\square} \times 3^{\square} \times 5$

(b) $5 \times 5 \times 7 \times 11 \times 11 \times 11 = 5^{\square} \times 7 \times 11^{\square}$

Product of prime factors — Grade 5

1. Express each number as a product of its prime factors.

(a) 72 **[2 marks]**

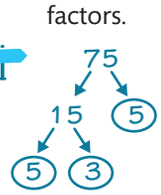
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(b) 180 **[2 marks]**

.....

Finding highest common factors and lowest common multiples — Grade 5

2. (a) Express 75 as a product of powers of its prime factors. **[2 marks]**



Use a factor tree to write 75 as a product of its prime numbers.

$75 = 3 \times \ldots\ldots \times \ldots\ldots$

.....

(b) Find the highest common factor of 75 and 90. **[2 marks]**

$75 = (3) \times (5) \times 5$

$90 = 2 \times (3) \times 3 \times (5)$

Circle the numbers that appear in **both** lists.

$HCF = 3 \times \ldots\ldots$

.....

(c) Find the lowest common multiple of 75 and 90. **[1 mark]**

$75 = 3 \times 5 \times 5$

$90 = 2 \times 3 \times 3 \times 5$

Look for the numbers that appear in **either** list.

$LCM = 2 \times 3 \times \ldots\ldots \times 5 \times \ldots\ldots$

.....

3. $X = 2^2 \times 3 \times 5^2$ and $Y = 2^3 \times 5$.

(a) Find the highest common factor (HCF) of X and Y. **[1 mark]**

.....

(b) Find the lowest common multiple (LCM) of X and Y. **[2 marks]**

.....

4. The highest common factor (HCF) of two numbers is 9. The lowest common multiple (LCM) of two numbers is a multiple of 15. Write down **two** possible numbers that satisfy these conditions. **[2 marks]**

.....

Real-life problem — Grade 5

5. Asha is planning a party. She wants to buy some naans and some kebabs. She is expecting more than 75 guests. A pack of 6 naans costs £1.75. A pack of 9 kebabs costs £3.40. Asha wants to buy more than 75 naans and more than 75 kebabs. She wants to buy exactly the same number of naans as kebabs. What is the smallest amount of money Asha could spend? **[5 marks]**

£.....

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Standard form

Quick quiz

Write each number as a power of 10.

(a) 1000 ..... **(b)** 100 000 ..... **(c)** 100 000 000 ..... **(d)** 1 000 000 000 .....

Standard form

Grade 5

1. (a) Write 7.8×10^{-4} as an ordinary number. **[1 mark]**

$7.8 \times 0.0001 =$ .....

.....

(b) Write 95 600 000 as a number in standard form. **[1 mark]**

$9.56 \times 10\,000\,000 = 9.56 \times 10 \times ..... \times ..... \times ..... \times ..... \times ..... \times .....$

.....

(c) Work out $(3 \times 10^8) \div (5 \times 10^6)$. Give your answer in standard form. **[2 marks]**

$(3 \div 5) \times (10^8 \div 10^6) =$ ..... $\times 10^{\square}$

.....

Exam focus

A number in standard form is written as $A \times 10^n$, where $1 \leqslant A < 10$ and n is an integer.

Take out a factor that is a power of 10 to leave a number in the range $1 \leqslant A < 10$.

Make sure the first number lies in the range $1 \leqslant A < 10$.

2. Write these numbers in order of size. Start with the lowest number. **[3 marks]**

0.0045×10^6 ..... 45×10^{-4} ..... -4.5×10^{-2} ..... 4.5×10^5 ..... 45×10^3 .....

Using standard form

Grade 5

3. One sheet of paper is 8×10^{-3} cm thick. Maria wants to put 1000 sheets of paper into the paper tray of her photocopier. The paper tray is 7.5 cm deep. Is the paper tray deep enough for 1000 sheets of this paper? You must explain your answer. **[3 marks]**

.....

4. A satellite is travelling at a speed of 3460 metres per second. How many seconds will the satellite take to travel a distance of 4.75×10^{12} metres? Give your answer in standard form, correct to 3 significant figures. **[3 marks]**

You may want to use this formula triangle:



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Surds

2 Quick quiz

Evaluate

(a) $\sqrt{49}$ ..... **(b)** $8 \times \sqrt{81}$ ..... **(c)** $\sqrt{64} \times \sqrt{36}$ ..... **(d)** $\sqrt{25} \times \sqrt{25}$ .....

10 Surds — Grades 7–8

1. (a) Express $4\sqrt{27}$ in the form $n\sqrt{3}$, where n is a positive integer. **[2 marks]**

$4\sqrt{27} = 4 \times \sqrt{9 \times \ldots}$

$= 4 \times \sqrt{\ldots} \times \sqrt{\ldots}$

$= 4 \times \ldots\ldots \times \sqrt{\ldots}$

.....

$\sqrt{27}$ is in its simplest form when the whole number under the square root sign is the smallest possible number.

Exam focus

To rationalise means to simplify a surd so that the denominator does not contain a surd.

(b) Rationalise the denominator of $\frac{28}{\sqrt{7}}$. **[2 marks]**

$\frac{28}{\sqrt{7}} \times \frac{\square}{\sqrt{7}} = \frac{\square}{\square} = \ldots\ldots$

.....

Multiply top and bottom by $\sqrt{7}$.

(c) Expand $(2 + \sqrt{3})(4 - \sqrt{3})$. Give your answer in the form $a + b\sqrt{3}$ where a and b are integers. **[3 marks]**

$(2 + \sqrt{3})(4 - \sqrt{3}) = 2 \times 4 - 2\sqrt{3} + 4\sqrt{\ } - \ldots\ldots$.

.....

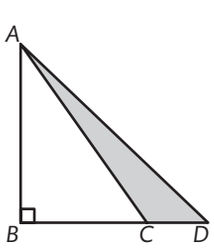
Multiply out the brackets.

15 Using surds — Grade 9

2. ABD is a right-angled triangle. All measurements are given in centimetres.

$AB = BD = \frac{\sqrt{3}}{3}$ and $BC = \frac{\sqrt{2}}{3}$.

Work out the exact area, in cm^2, of the shaded region. **[3 marks]**



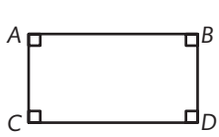
..... cm^2

3. The diagram shows a rectangle $ABCD$.

The area of the rectangle is 1 m^2. The length of AB is $(1 + \sqrt{2})$ m.

Work out the perimeter of $ABCD$.

Give your answer in the form $a\sqrt{b}$, where a and b are integers. **[4 marks]**



..... m

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Number

Factors and multiples — Grade 5

1. Rohan is planning his birthday party. He wants to buy some bottles of orange juice and bars of chocolate.

Orange juice	**Chocolate**
There are 8 bottles in a case. A case costs £4.50.	There are 6 bars in a packet. A packet costs £2.75.

He wants to buy exactly the same number of bottles of orange juice as bars of chocolate, but needs at least 70 of each.
Calculate the smallest amount that he will need to spend. **[5 marks]**

£.....

Percentages — Grade 6

2. Anjali invests £8400 for 4 years.
The investment gets compound interest of x% per annum. At the end of 4 years the investment is worth £10017.16.
Work out the value of x. **[3 marks]**

.....

Surds — Grade 7

3. Work out the value of A using the equations below. Give your answer in the form $n\sqrt{3}$ where n is an integer.

$x = \sqrt{12} + 5$

$y = \sqrt{12} - 5$

$A = x^2 - y^2$ **[4 marks]**

.....

Bounds — Grade 9

4. Cecily is carrying out an experiment to see how a sledge slides down a hill.
The sledge starts from point X. The acceleration, a m/s^{-2} of the sledge is given by $a = \frac{2s}{t^2}$ where s is the distance travelled in metres and t is the time taken in seconds.

She finds that $s = 44.7$ metres, correct to 1 decimal place and $t = 5.86$ seconds, correct to 3 significant figures.

By considering bounds, work out the value of a to a suitable degree of accuracy.
Give a reason for your answer. **[5 marks]**

Work out the upper bound and lower bound of s and t.

Exam focus

If an answer is given to a suitable degree of accuracy, the upper and lower bounds will both round to that value.

.....

.....

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Algebraic expressions

2 Quick quiz

1. Simplify

(a) $12x - 7x + 9x$ .....

(b) $4(x - 5)$ .....

2. Factorise

(a) $x^2 + 4x$ .....

(b) $x^2 - 9$ .....

20 Simplifying, expanding and factorising — Grade 5

1. (a) Simplify $4y + 3x - 5 + 6x + 8$. **[1 mark]**

$4y + 3x + 6x + 8 - 5$

Collect the x terms and collect the number terms.

.....

(b) Factorise $4x^2 - 6x$. **[2 marks]**

$4x^2 - 6x =$ .....$x($ ..... $x -$ .....$)$

Look for the HCF of $4x^2$ and $6x$.

.....

(c) Expand and simplify $12 - 4(2x + 5)$. **[2 marks]**

$12 -$ .....$x -$ ..... $=$ $-$ .....$x -$ .....

Multiply out brackets and then simplify.

.....

(d) Expand and simplify $(2x - 3)(3x + 7)$. **[2 marks]**

$(2x - 3)(3x + 7) =$ ..... $x^2 +$ ..... $x -$ ..... $x -$ .....

.....

2. Expand and simplify

(a) $7x + 4(x - 2y) - 2(3x - 4y)$ **[2 marks]**

.....

(b) $(x + 4)(x - 8)$ **[2 marks]**

.....

(c) $(n + 3)^2 + (n - 2)^2$ **[2 marks]**

.....

3. Factorise completely

(a) $15x^3y - 18xy^2$ **[2 marks]**

.....

(b) $15x^4y^5 - 35x^2y^3$ **[2 marks]**

.....

(c) $2ax - 2ay + bx - by$ **[3 marks]**

.....

5 Application — Grade 6

4. (a) Show that $(n + 4)^2 - (n - 4)^2$ is an even number for all positive integer values of n. **[3 marks]**

(b) Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers. **[4 marks]**

Two consecutive numbers are written as n and $n + 1$.

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Algebraic formulae

Quick quiz

1. Expand and simplify

(a) $x(x+6)$ **(b)** $(x+2)(x-3)$

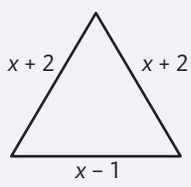
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2. Rearrange this equation so that the height, h, is the subject: $A = \frac{1}{2}bh$.

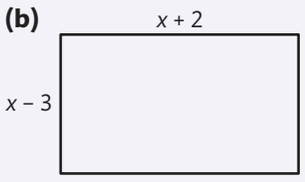
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3. Write down the perimeter of each shape in terms of x.

(a)



(b)



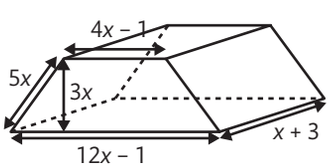
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Deriving expressions — Grades 5–6

1. A shop sells packets of cards in two sizes. There are 6 cards in a small packet and 15 cards in a large packet.
The total number of cards in x small packets and y large packets is T. Write down a formula for T in terms of x and y.
[3 marks]

$T =$ .....

2. This shape is a solid prism.
The cross-section of the prism is an isosceles trapezium.
All the measurements are given in cm.
Show that the total surface area of the prism is $74x^2 + 70x - 6\,\text{cm}^2$.

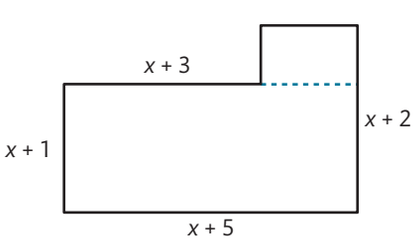


[4 marks]

Work out the area of each face, then add up the areas of all the faces to find the total surface area.

Finding area — Grade 6

3. In the diagram, all measurements are in centimetres.
All the corners are right angles.
The area of the whole shape is A cm^2.
Find a formula for A in terms of x.
Give your answer in its simplest form.



[4 marks]

Split the compound shape into two rectangles. Then, work out the area of each rectangle.

Height of small rectangle
= (x + 2) − (..... + ...)
= ..... + ... − ..... − .....
= .....

Width of small rectangle
= (x + 5) − (.....+ .....)
= ..... + ... − ..... − .....
= .....

Multiply out brackets.

Area of small rectangle = ... × ..... = ...

Area of large rectangle = (x + 1)(..... + ...)
= ..... + ... + ... + ...
= ..... + ... + ...

Area of whole shape = ... + ... + ... + ...
= ..... + ... + ...

Exam focus
Always leave your answer in its simplest form.

$A =$ .....

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Laws of indices

Quick quiz

Simplify fully

(a) $a \times a \times a \times a$ .....

(b) $\frac{a \times a \times a \times a \times a \times a}{a \times a}$ .....

(c) $6 \times a \times 3 \times a \times b$ .....

(d) $3 \times a \times a \times b \times b \times b$ .....

Applying laws of indices — Grade 5

1. (a) Simplify $x^7 \div x^3$. **[1 mark]**

.....

When you divide, subtract the powers.

(b) Simplify $6x^4y^3 \times 3x^2y$. **[2 marks]**

$= 6 \times \ldots\ldots \times x \ldots\ldots \times y \ldots\ldots$

.....

When you multiply, add the powers.

(c) Simplify $(3x^2y^3)^3$. **[2 marks]**

$= 3x^2y^3 \times 3x^2y^3 \times 3x^2y^3$

.....

Multiply or divide any numbers first, then use the laws of indices to work out the new powers.

(d) Simplify fully $\frac{15x^6y^4}{3x^4y}$. **[2 marks]**

$= (15 \div \ldots\ldots) \times x \ldots\ldots \times y \ldots\ldots$

.....

Exam focus
Make sure you learn the laws of indices.

Indices with algebra — Grade 5

2. Simplify each expression fully.

(a) $3x^2y \times 5xy^3$ **[2 marks]**

.....

Multiply the numbers first.

(b) $\frac{20x^5(x+1)^3}{12x(x+1)}$ **[2 marks]**

.....

Cancel the numbers first.

(c) $\frac{20x^7y^3}{4x^2y}$ **[2 marks]**

.....

(d) $(2x^6y^4)^4$ **[2 marks]**

.....

Indices with equations — Grade 5

3. Work out the value of n, given that $\frac{t^7 \times t^n}{t} = t^{10}$. **[2 marks]**

Use the laws of indices to set up an equation.

$n = \ldots\ldots$

4. Work out the value of x when

(a) $2^{20} \div 8^{12} = 2^x$ **[2 marks]**

$x = \ldots\ldots$

(b) $3^{10} \times 27^4 \div 9 = 3^x$ **[2 marks]**

$x = \ldots\ldots$

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Combining indices

Quick quiz

Evaluate

(a) $\sqrt{25} =$ ..... **(b)** $\sqrt{81} =$ ..... **(c)** $\sqrt[3]{27} =$ ..... **(d)** $\sqrt[3]{64} =$ .....

Negative and fractional indices — Grade 7

1. Evaluate

(a) 4^{-2} **[1 mark]** **(b)** 5^{-3} **[1 mark]** **(c)** 2^{-3} **[1 mark]**

$= \frac{1}{4^2}$

....

When the power is negative, write the number as a fraction and invert it. Change the negative power to a positive power.

2. Evaluate

(a) $16^{\frac{3}{2}}$ **[1 mark]** **(b)** $64^{\frac{2}{3}}$ **[1 mark]** **(c)** $27^{\frac{2}{3}}$ **[1 mark]**

$= \left(16^{\frac{1}{2}}\right)^3$

....

Split the power into the product of a unit fraction and an integer.

3. Evaluate

(a) $27^{-\frac{2}{3}}$ **[2 marks]** **(b)** $81^{-\frac{1}{4}}$ **[2 marks]** **(c)** $4^{-\frac{3}{2}}$ **[2 marks]**

$= \frac{1}{27^{\frac{2}{3}}} = \frac{1}{\left(27^{\frac{1}{3}}\right)^2}$

....

Always apply the laws of indices one step at a time.

4. Evaluate

(a) $\left(\frac{16}{81}\right)^{\frac{3}{4}}$ **[2 marks]** **(b)** $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$ **[2 marks]** **(c)** $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$ **[2 marks]**

$= \left(\frac{4}{25}\right)^{\frac{3}{2}} = \left(\frac{4^{\frac{1}{2}}}{25^{\frac{1}{2}}}\right)^3$

....

Exam focus

Always use brackets to make your working clear, especially when using negative powers.

Solving equations involving indices — Grade 9

5. Solve the equations.

(a) $5^n = 1$ **[1 mark]** **(b)** $5^n = \frac{1}{25}$ **[1 mark]** **(c)** $3^{2n} = \frac{1}{81}$ **[2 marks]** **(d)** $3^n = 9 \times \sqrt{27}$ **[2 marks]**

$n =$ ..... $n =$ ..... $n =$ ..... $n =$ .....

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Linear equations

5 Quick quiz

Solve

(a) $x + 8 = 13$ **(b)** $x - 6 = -3$ **(c)** $3x = 12$ **(d)** $\frac{x}{5} = -6$

....

15 Solving equations — Grades 4–5

1. Solve these equations.

(a) $6x + 7 = 18$ **[2 marks]**

$6x = 18 - \ldots\ldots$

$6x = \ldots\ldots$

Apply inverse operations to each step to collect all x terms on one side and numbers on the other.

$x = \ldots\ldots$

(b) $9x + 6 = 2x - 19$ **[3 marks]**

$9x - \ldots\ldots = -19 - \ldots\ldots$

Exam focus
Every line of working should include an equals sign.

$x = \ldots\ldots$

(c) $7(x + 2) = 21$ **[2 marks]**

$7x + \ldots\ldots = 21$

$x = \ldots\ldots$

(d) $4(y - 7) = 2(4 - y)$ **[3 marks]**

$4y - \ldots\ldots = \ldots\ldots - \ldots\ldots$

$y = \ldots\ldots$

(e) $4 - 3x = 11$ **[2 marks]**

$x = \ldots\ldots$

(f) $5x - 14 = 2(3 + 2x)$ **[3 marks]**

$x = \ldots\ldots$

2. (a) Solve $3(5 - 2x) - 2(2x + 3) = 2$. **[3 marks]**

$x = \ldots\ldots$

(b) Solve $2(3x + 2) - 5(x - 2) = 4(x - 1)$. **[3 marks]**

$x = \ldots\ldots$

10 Applying algebra to problems — Grade 5

3. Company A sent x employees to a diversity course.
Company B sent three times as many employees as Company A.
Company C sent 9 fewer employees than Company A. The cost for the diversity course was £25 for each employee. Altogether, the companies paid a total of £2150.
Work out how many employees were sent by each company to the diversity course.
You must show your working. **[5 marks]**

Write out an equation in terms of x.

.....

4. Arianne and Betty are selling plates in a shop. They sell boxes of plates and single plates.
Arianne sells 6 boxes of plates and 32 single plates. Betty sells 5 boxes of plates and 4 single plates. Altogether, Arianne sells twice as many plates as Betty.
Work out how many plates there are in a box. You must show your working. **[4 marks]**

Let y represent the number of plates in a box.

.....

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Linear equations and fractions

Quick quiz

Solve

(a) $2x - 1 = 9$

.....

(b) $\frac{x}{3} - 1 = 5.$

.....

Solving equations with fractions — Grade 7

1. Solve $\frac{x+1}{2} + \frac{2x-1}{3} = \frac{5}{6}$. **[4 marks]**

Get rid of the fractions before you solve the equation.

$\frac{6(x+1)}{2} + \frac{6(2x-1)}{3} = \frac{6 \times 5}{6}$

$.....(x + 1) + ...(2x - 1) = ...$

$.....x + ... + ...x - ..... = ...$

$.....x + ... = ...$

$.....x = ... - .....$

$.....x = ...$

$x = ...$

$x = ...$

Multiply every term by the lowest common denominator, which is the lowest common multiple (LCM) of the denominators.

Exam focus

Every line of working should include an equals sign and must be simplified step-by-step.

2. Solve $x - 5 + \frac{2x-1}{4} = 3$. **[4 marks]**

$x = ...$

3. Solve $\frac{3}{2x-5} = \frac{2}{3x-2}$. **[3 marks]**

$x = ...$

4. Solve $\frac{5}{x+2} = \frac{9}{x-2}$. **[5 marks]**

$x = ...$

5. Solve $\frac{7}{x-2} - \frac{2}{x+1} = 0$. **[5 marks]**

$x = ...$

Solving algebraic equations in context — Grade 8

6. A group of people play the lottery in a pool. One week, they win a prize and decide to share it equally between the 205 people in the group.
Before the prize can be shared, they discover that 5 people in the group did not pay for their share of the tickets, so the group decides instead to share the prize equally between the remaining 200 people. They discover that this means they will get an extra £2 each.
Let £x be the amount of the prize. Form an equation and solve it to find x. **[6 marks]**

$x = ...$

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Simultaneous equations

Quick quiz

Solve

(a) $x = 6$ and $2x - 3y = 0$

.....

(b) $x = -1$ and $4x + 10y = 5$.

.....

Algebraic method

Grade 6

1. Solve these simultaneous equations: **[3 marks]**

$3x - 8y = 11$ (1)

$2x - 5y = 6$ (2)

.....x − .....y = ... (1) × 2

.....x − .....y = ... (2) × 3

..... (2) from (1)

.....y = ...

y = ...

Substitute y = ... into (1)

$3x - 8($.....$) = 11$

Rearrange for x

.....

x = ..., y = ...

Start by numbering each equation.

Multiply one or both of the equations so that the coefficients of one unknown are the same.

Substitute the value for this unknown into one of the original equations to find the other unknown.

Exam focus

Check the answer by substituting both unknowns into the unused original equation.

2. Solve $x - 15 = 5y$ and $3x + 8y = -1$. **[4 marks]**

x = ..., y = ...

Graphical method

Grade 6

3. Use a graphical method to solve the simultaneous equations $3x - y = 9$ and $x - 2y = -2$. **[3 marks]**

When drawing a straight line, always choose three values of x and work out the corresponding y values.

Plot the three points and draw a straight line through all of them.

x = ..., y = ...

Setting up equations

Grade 6

4. The total cost of 4 shirts and 3 pairs of trousers is £104.
The total cost of 5 shirts and 2 pairs of trousers is £98.50.
Find the cost of each shirt and each pair of trousers.

Set up two equations.

[4 marks]

.....

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Quadratic equations

Quick quiz

Fill in the brackets so that the same two numbers make the product and the sum.

(a) (.....) × (.....) = 20
(.....) + (.....) = 12

(b) (.....) × (.....) = −15
(.....) + (.....) = 2

(c) (.....) × (.....) = 4
(.....) + (.....) = −5

(d) (.....) × (.....) = −20
(.....) + (.....) = −1

Solving quadratic equations — Grade 7

1. Solve

(a) $2x^2 - 8x = 0$ **[2 marks]**

$2x(x - \ldots\ldots) = 0$

$x = \ldots\ldots$ or $x = \ldots\ldots$

(b) $x^2 - 11x + 10 = 0$ **[2 marks]**

(.....) x (.....) = 10

(.....) + (.....) = −11

$(x\ldots\ldots)(x\ldots\ldots) = 0$

$x = \ldots\ldots$ or $x = \ldots\ldots$

(c) $9x^2 + 18x - 7 = 0$ **[2 marks]**

$9x^2 + 21x - \ldots\ldots x - 7 = 0$

$3x(\ldots\ldots x + \ldots\ldots) - 1(\ldots\ldots x + \ldots\ldots) = 0$

$x = \ldots\ldots$ or $x = \ldots\ldots$

Solving quadratic equations — Grade 7–9

2. Solve

(a) $x^2 - 5x - 36 = 0$ **[2 marks]**

$x = \ldots\ldots$ or $x = \ldots\ldots$

(b) $x^2 - 8x + 15 = 0$ **[2 marks]**

$x = \ldots\ldots$ or $x = \ldots\ldots$

To answer this type of question, you have to rearrange the equation into the form $ax^2 + bx + c = 0$.

3. Solve

(a) $12x^2 - 5x - 7 = 5 + 2x$ **[3 marks]**

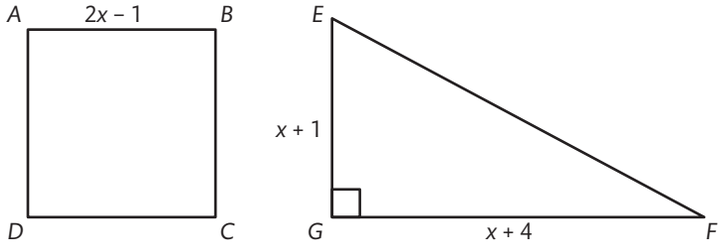
$x = \ldots\ldots$ or $x = \ldots\ldots$

(b) $(3x - 1)^2 = 64$ **[3 marks]**

$x = \ldots\ldots$ or $x = \ldots\ldots$

Using quadratic equations — Grade 8

4. *ABCD* is a square and *EFG* is a right-angled triangle. All measurements are in centimetres. **[5 marks]**



Work out the area of the square and the triangle, then set up an equation.

The area of the square *ABCD* is equal to the area of the triangle *EFG*.
Work out the value of *x*. You must show your working.

.....