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# Bitesize AQA GCSE (9-1) MATHEMATICS

## REVISION GUIDE

## HIGHER



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Tick off each topic as you go.

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# How to use this book

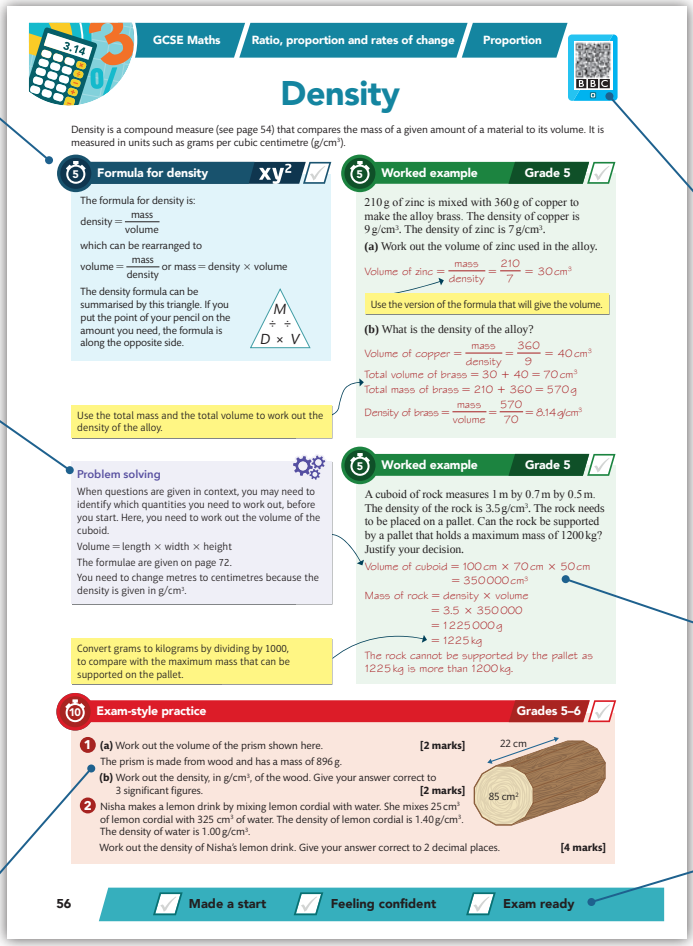
Use the features in this book to focus your revision, track your progress through the topics and practise your exam skills.

## Features to help you revise

Each bite-sized chunk has a **timer** to indicate how long it will take. Use them to plan your revision sessions.

Questions that test **problem-solving** skills are explained in callouts and in the *Problem solving* section at the back.

Test yourself with **exam-style practice** at the end of each page and check your answers at the back of the book.




Scan the **QR codes** to visit the BBC Bitesize website. It will link straight through to more revision resources on that subject.



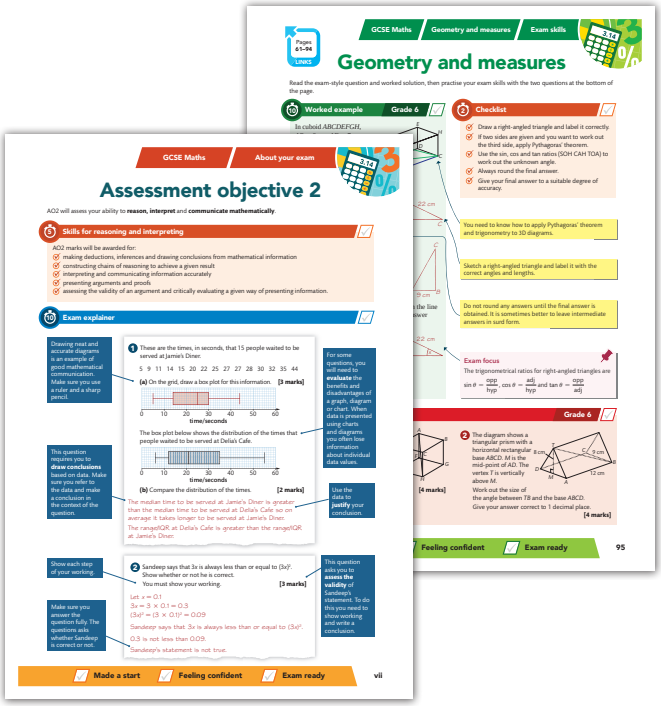
Complete **worked examples** demonstrate how to approach exam-style questions.

**Tick boxes** allow you to track the sections you've revised. Revisit each page to embed your knowledge.

## Exam focus features

The *About your exam* section at the start of the book gives you all the key information about your exams, as well as showing you how to identify the different questions.

Throughout the topic pages you will also find green *Exam skills* pages. These work through an extended exam-style question and provide further opportunities to practise your skills.




## ActiveBook and app

This Revision Guide comes with a **free online edition**. Follow the instructions from inside the front cover to access your ActiveBook.

You can also download the **free BBC Bitesize app** to access revision flash cards and quizzes.

If you do not have a QR code scanner, you can access all the links in this book from your ActiveBook or visit **www.pearsonschools.co.uk/BBCBitesizeLinks**.

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# Your Maths GCSE

This page will tell you everything you need to know about the structure of your upcoming AQA Higher GCSE Maths exam.

## About the exam papers

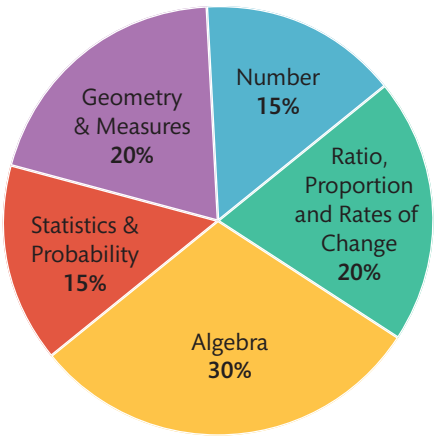
You will have to take **three papers** as part of your GCSE Maths qualification.

| Paper 1 | Paper 2 | Paper 3 |
|---|---|---|
| 1 hour 30 minutes<br>80 marks in total | 1 hour 30 minutes<br>80 marks in total | 1 hour 30 minutes<br>80 marks in total |
|  |  |  |

You could be tested on any of the topics you have studied in any of the three written papers. There will be a mixture of question styles on each paper. Papers will usually start with shorter and easier questions and will progress towards harder questions worth more marks at the end of the paper.

## Topics

Your AQA GCSE Maths Higher specification is divided into five topics. This pie chart shows the five topics and the proportion of marks that will be allocated to each one:




## Assessment objectives

In your exams, marks will be allocated based on three assessment objectives.

**Assessment objective 1 (AO1)** is about applying and using standard mathematical techniques. About 40% of the marks in your exam will be AO1 marks.

**Assessment objective 2 (AO2)** is about reasoning, interpreting and communicating mathematically. About 30% of the marks in your exam will be AO2 marks.

**Assessment objective 3 (AO3)** is about solving unfamiliar problems, and solving problems involving real-life contexts. About 30% of the marks in your exam will be AO3 marks.

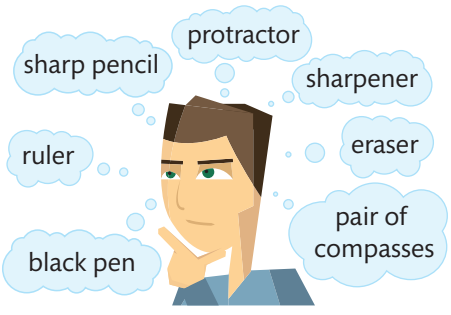
You can see some examples of the different assessment objectives on pages vi, vii and viii.

## My exam dates

Find out the date and time of each of your GCSE Maths papers and write them in this table.

| | Date | AM or PM? |
|---|---|---|
| **Paper 1** | | |
| **Paper 2** | | |
| **Paper 3** | | |

Watch out – no calculators allowed on this paper.






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# Exam strategies

In your exam you will need to demonstrate your **problem-solving skills**. This page gives some top tips for answering problem-solving questions and how to approach your exam.

## 5 Problem solving

Lots of questions in your GCSE Maths exam will have a problem-solving element. In your exam, marks will be awarded specifically for:

- following through mathematical processes clearly and correctly
- presenting mathematical proofs
- showing your methods clearly
- solving problems in unfamiliar contexts
- combining techniques in an unfamiliar way.

The **top tip** here is **don't be scared** if a question doesn't look familiar, or if it looks like it requires a lot of steps. You will definitely have covered the techniques needed in your course, so take a deep breath and have a go!

## 5 Revision advice

- Make a list of all the topics you need to revise.
- Create a realistic schedule – you can work backwards from the date of your exam!
- Start early – don't wait until a few days before your exam!
- Revise in small chunks and plan to revisit topics again later.
- Take regular short breaks.
- Drink plenty of water and eat healthy snacks like fruit or vegetables.
- Make sure your notes are easy to read – but remember, they don't have to be works of art.
- Don't work too late at night.
- Minimise TV and video game time in the run-up to your exams – it will help with concentration.

## 5 Exam advice

### Exam top tips

- Read all the instructions carefully.
- Check that you haven't missed any pages or questions at the end.
- Answer all the questions on each paper.
- Keep explanations short and use correct mathematical language.
- Make sure your answers look sensible.
- Show all your working.
- Read each question carefully before starting to answer it.
- Check your working if you have any spare time at the end.
- Write down some working even if you can't finish a question.
- 1 mark = about 1 minute.
- Write down an answer even if you're not sure it's right.
- Write down all the figures from your calculator display before rounding your answer.

## 2 Exam language

Sometimes the wording of a question gives you a clue about how to tackle it:

| You must show your working. | Explain... | You must give a reason for your answer. |
|---|---|---|
| This means you **have** to show your method and working clearly. If you just write down the correct answer, you might not get the marks. | You need to give a **written** answer. Make sure you use the correct mathematical language. You can back your answer up with data or calculations. | Either explain your answer in words, or make sure you have **shown enough working** to justify how you reached the conclusion. Or both! |

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# Assessment objective 1

AO1 will assess your ability to **use and apply standard techniques**.

## 5 Standard mathematical techniques

AO1 marks will be awarded for:

- accurately recalling facts, terminology and definitions
- using and interpreting notation correctly
- accurately carrying out routine procedures or set tasks requiring multi-step solutions.

## 10 Exam explainer

Multiplying out brackets is an example of a **routine** task. Questions like this should be familiar from your course and your revision.

Write out $(1 + \sqrt{5})^2$ in the form $a + b\sqrt{5}$. **[3 marks]**

$(1 + \sqrt{5})^2 = (1 + \sqrt{5})(1 + \sqrt{5})$

$= 1 + \sqrt{5} + \sqrt{5} + \sqrt{5}\sqrt{5}$

$= 1 + \sqrt{5} + \sqrt{5} + 5 = 6 + 2\sqrt{5}$

Hence, $a = 6$ and $b = 2$

You can check you have written the answer in the correct form by giving the values of $a$ and $b$.

**1** Given that $A = 2^4 \times 3^3 \times 5$ and $B = 2^3 \times 3^2 \times 5^2$ write down, as a product of powers of prime factors:

**(a)** the highest common factor (HCF) of $A$ and $B$ **[1 mark]**

**(b)** the lowest common multiple (LCM) of $A$ and $B$. **[1 mark]**

'Write down' questions can usually be answered quickly, using familiar techniques. You don't necessarily need to show a lot of working for these questions.

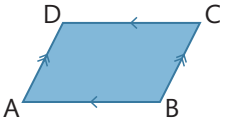
**2** Ken rounds a number, $x$, to one decimal place. The result is 9.4.

Write down the error interval for $x$. **[2 marks]**

## 10 Exam explainer

You need to be able to correctly interpret notation like the arrows which show parallel lines on this diagram.

ABCD is a parallelogram.




This is information you should be able to recall.

Prove that triangle ABD is congruent to triangle CDB. **[3 marks]**

BD is common.

∠ABD = ∠BDC (alternate angles are equal)

AB = CD (opposite sides of a parallelogram)

Triangles ABD and CDB are congruent (SAS).

In harder questions you still need to **recall** standard facts and information, such as angle facts about parallel lines.

Make sure you are confident with standard techniques and processes.

**3** Solve these simultaneous equations.

$3x + 5y = 4$

$2x - y = 7$ **[3 marks]**

Your exam will begin with **multiple choice questions**. Read them really carefully. In this question three of the expressions should be equivalent to $p^6$. You need to circle the one that is not. If you want to change your answer, cross it out then circle the correct answer.

**4** Circle the expression that is **not** equivalent to $p^6$.

$(p^3)^2$ $\quad \frac{p^7}{p}$ $\quad \sqrt{p^8}$ $\quad p \times p^5$ **[1 mark]**

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# Assessment objective 2

AO2 will assess your ability to **reason, interpret** and **communicate mathematically**.

## 5 Skills for reasoning and interpreting

AO2 marks will be awarded for:

- making deductions, inferences and drawing conclusions from mathematical information
- constructing chains of reasoning to achieve a given result
- interpreting and communicating information accurately
- presenting arguments and proofs
- assessing the validity of an argument and critically evaluating a given way of presenting information.

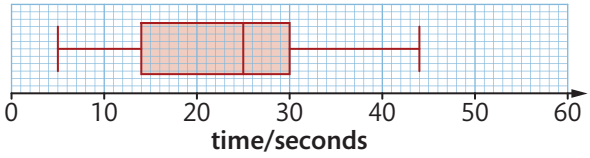
## 10 Exam explainer

Drawing neat and accurate diagrams is an example of good mathematical communication. Make sure you use a ruler and a sharp pencil.

**1** These are the times, in seconds, that 15 people waited to be served at Jamie's Diner.

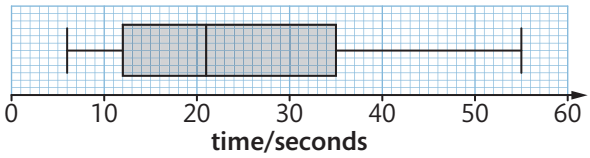
5 9 11 14 15 20 22 25 27 27 28 30 32 35 44

**(a)** On the grid, draw a box plot for this information. **[3 marks]**




For some questions, you will need to **evaluate** the benefits and disadvantages of a graph, diagram or chart. When data is presented using charts and diagrams you often lose information about individual data values.

The box plot below shows the distribution of the times that people waited to be served at Delia's Cafe.




**(b)** Compare the distribution of the times. **[2 marks]**

This question requires you to **draw conclusions** based on data. Make sure you refer to the data and make a conclusion in the context of the question.

Use the data to **justify** your conclusion.

The median time to be served at Jamie's Diner is greater than the median time to be served at Delia's Cafe so on average it takes longer to be served at Jamie's Diner.

The range/IQR at Delia's Cafe is greater than the range/IQR at Jamie's Diner.

Show each step of your working.

**2** Sandeep says that $3x$ is always less than or equal to $(3x)^2$.
Show whether or not he is correct.
You must show your working. **[3 marks]**

This question asks you to **assess the validity** of Sandeep's statement. To do this you need to show working and write a conclusion.

Let $x = 0.1$

$3x = 3 \times 0.1 = 0.3$

$(3x)^2 = (3 \times 0.1)^2 = 0.09$

Sandeep says that $3x$ is always less than or equal to $(3x)^2$.

0.3 is not less than 0.09.

Sandeep's statement is not true.

Make sure you answer the question fully. The question asks whether Sandeep is correct or not.

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# Assessment objective 3

AO3 will assess your ability to **solve problems within mathematics and in other contexts**.

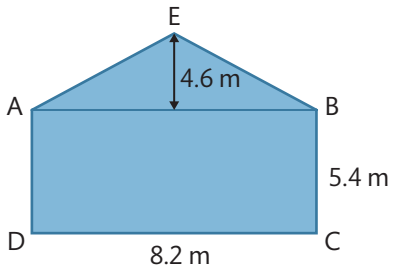
## 5 Skills for problem solving

AO3 marks will be awarded for:

- translating problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
- making and using connections between different parts of mathematics
- interpreting results in the context of the given problem
- evaluating methods used and results obtained
- evaluating solutions to identify how they may have been affected by assumptions made.

## 10 Exam explainer

A garden is in the shape of a rectangle, ABCD, and a triangle, ABE.




Joan is going to cover the garden with fertiliser.

A bag of fertiliser costs £7.99. One bag of fertiliser will cover an area of 9.4 $m^2$.

Work out the cost of buying enough fertiliser to cover the garden completely.

**[5 marks]**

Area of rectangle $= 8.2 \times 5.4 = 44.28\,m^2$

Area of triangle $= 0.5 \times 8.2 \times 4.6 = 18.86\,m^2$

Total area of garden $= 44.28 + 18.86 = 63.14\,m^2$

Number of bags $= 63.14 \div 9.4 = 6.71...$

Number of bags needed $= 7$

Cost of the bags $= 7 \times £7.99 = £55.93$

This question is given in a **non-mathematical context**. You need to select the most appropriate mathematical techniques to solve the real-life problem.

Questions that require you to work through a long answer will probably award you marks for using a correct method, so show all of your working.

You need to **evaluate** your solution and give an appropriate answer in context. In real life, you would need to buy a whole number of bags of fertiliser. So round up to the nearest whole number to work out the number of bags needed.

When there are a lot of steps in a question it is a good idea to write down your working at each stage.

Make sure you use the correct units in your answer, and check that it makes sense in the context of the question.

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# Fractions, decimals and percentages

Fractions, decimals and percentages are different ways of expressing a proportion of a quantity. You should be able to convert between them and use them interchangeably.

## 2 Converting

- To convert a fraction into a decimal, divide the numerator by the denominator.
  $\frac{3}{5} = 3 \div 5 = 0.6$
- To convert a fraction or a decimal into a percentage, multiply by 100.
  $\frac{5}{8} \times 100 = 62.5\%$ $\quad 0.785 \times 100 = 78.5\%$
- To convert a percentage into a fraction, divide by 100.
  $75\% = \frac{75}{100} = \frac{3}{4}$

## 2 Recurring decimals

$\frac{3}{20} = 0.15$ is a **terminating decimal**.

In a **recurring decimal**, a digit or a group of digits is repeated forever. You can use dots to indicate recurring digits.

$0.\dot{7} = 0.777\ 777\ 77$

$0.5\dot{7}2\dot{4} = 0.572\ 472\ 472\ 472\ 4\ \ldots$

### Problem solving

Multiply by 100 because there are two recurring digits. If there is one recurring digit multiply by 10, and if there are three, multiply by 1000.

## 5 Worked example — Grade 5

Here are four numbers.

Which one is the largest number?

Circle your answer.

(2/3) 65% 3/5 0.62

0.6666667 0.65 0.6 0.62

Largest is 0.6666667.

Convert each number to a decimal.

## 5 Worked example — Grade 8

Prove that $0.\dot{3}\dot{9} = \frac{13}{33}$.

$x = 0.393939$

$100x = 39.393939$

$100x = 39.393939$

$x = 0.393939$

$99x = 39$

$x = \frac{39}{99} = \frac{13}{33}$

Write the recurring decimal as $x$. Multiply it by 100 and subtract $x$ to remove the recurring part.

## 5 Worked example — Grade 4

This pie chart shows how Tony spent his money last month.

How much money did Tony save?

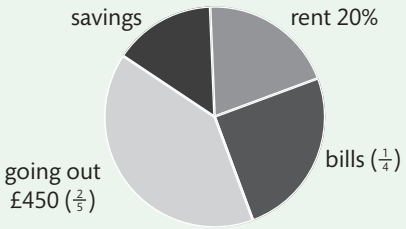
$20\% + \frac{1}{4} + \frac{2}{5} = 20\% + 25\% + 40\% = 85\%$

Savings $= 100\% - 85\% = 15\%$

$40\% = £450$, so $1\% = £11.25$

$15\% = £11.25 \times 15 = £168.75$

The percentages must add up to 100%.




## 10 Exam-style practice — Grades 4–8

1. Andrew, Ben and Carina bought a present for their father. Andrew paid 32% of the total, Ben paid $\frac{2}{5}$ of the total and Carina paid the rest. Carina paid £252. How much did the present cost? **[4 marks]**
2. $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{9}$ are four fractions. How many of these fractions convert to a recurring decimal? Circle your answer.
   0 1 2 3 4 **[1 mark]**
3. Show that $0.8\dot{0}3\dot{6} = \frac{446}{555}$. **[3 marks]**

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# Manipulating fractions

You should be able to add, subtract, multiply and divide fractions and mixed numbers without using a calculator.

## 2 Adding and subtracting fractions

Before you can add or subtract fractions you must make sure they have the same denominator. If not, start by finding a common denominator.

If your answer is an **improper fraction**, change it to a mixed number.

$$\frac{3}{4}+\frac{3}{5}=\frac{15}{20}+\frac{12}{20}=\frac{27}{20}=1\tfrac{7}{20}$$

When adding or subtracting **mixed numbers** you can either:

- add or subtract the whole number parts and fraction parts separately
- convert both mixed numbers to improper fractions before you add or subtract.

## 2 Worked example — Grade 4

Work out $4\frac{1}{3}-2\frac{4}{5}$.

$$4\tfrac{1}{3}-2\tfrac{4}{5}=\frac{13}{3}-\frac{14}{5}$$

In this case, $\frac{4}{5}>\frac{1}{3}$ so it will be easier to change the mixed numbers into improper fractions first.

$$=\frac{65}{15}-\frac{42}{15}$$

$$=\frac{23}{15}$$

$$=1\tfrac{8}{15}$$

For your final answer, remember to convert the improper fraction to a mixed number.

## 5 Multiplying and dividing fractions

When **multiplying** fractions, multiply the numerators and multiply the denominators. Then cancel if you can.

$$\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}\overset{\div 6}{=}\frac{1}{2}$$

To **divide** one fraction by another fraction, turn the second fraction upside down and then multiply.

$$\frac{2}{5}\div\frac{4}{6}=\frac{2}{5}\times\frac{6}{4}=\frac{12}{20}\overset{\div 4}{=}\frac{3}{5}$$

## 10 Worked example — Grade 5

**1** Work out

**(a)** $\frac{4}{5}\times\frac{7}{10}$

Circle your answer.

$\frac{11}{15}$ $\quad$ (circled) $\frac{14}{25}$ $\quad$ $\frac{28}{15}$ $\quad$ $\frac{11}{50}$

$$\frac{4}{5}\times\frac{7}{10}=\frac{28}{50}=\frac{14}{25}$$

Change to improper fractions.

**(b)** $3\frac{3}{5}\div 2\frac{1}{4}$

Turn $\frac{9}{4}$ upside down, to get $\frac{4}{9}$, swap the ÷ sign for a × sign and multiply.

$$3\tfrac{3}{5}\div 2\tfrac{1}{4}=\frac{18}{5}\div\frac{9}{4}=\frac{\overset{2}{\cancel{18}}}{5}\times\frac{4}{\underset{1}{\cancel{9}}}=\frac{8}{5}=1\tfrac{3}{5}$$

**Problem solving**

Write down what you are working out at each stage of your answer.

**2** Yesterday a canteen used 120 potatoes.
$\frac{1}{4}$ of them were used for jacket potatoes.
$\frac{1}{3}$ of them were used for roast potatoes.
The rest of them were made into chips.
How many potatoes were used for chips?

Number of jacket potatoes $=\frac{1}{4}\times 120=30$

Number of roast potatoes $=\frac{1}{3}\times 120=40$

Potatoes used so far: $30+40=70$

Number of potatoes left for chips: $120-70=50$

## 10 Exam-style practice — Grades 4–5

**1** There are 240 counters in a box. The counters are black or red or green. $\frac{3}{8}$ of the counters are black and $\frac{2}{5}$ of the counters are red. Work out the number of green counters in the box.

**[4 marks]**

**2** A rope is $12\frac{3}{4}$ m long. How many lengths of rope each of $2\frac{1}{8}$ m can be cut from this rope?

**[3 marks]**

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# Percentage change

You may be asked to increase or decrease an amount by a percentage of the original value. For instance, in a sale the price of an item is decreased by 10%. There are two methods you can use and you need to know them both.

## 2 Calculating percentage change

To work out a percentage increase or decrease, you need to write the amount of the increase or decrease as a percentage of the **original amount**. Suppose that, in one year, a plant grew from 80 cm to 92 cm. How can you work out the percentage increase in its height?

Work out the amount of the increase.

92 cm – 80 cm = 12 cm

Write this as a percentage of the original amount.

$\frac{12}{80} \times 100 = 15\%$

## 5 Worked example — Grade 4

David's weight decreases from 74.5 kg to 69.3 kg.

Work out the percentage decrease in David's weight. Give your answer correct to 3 significant figures.

Decrease = 74.5 kg – 69.3 kg
= 5.2 kg

Percentage decrease
$= \frac{5.2}{74.5} \times 100$
= 6.98%

Give the decrease as a percentage of the original. The original weight is 74.5 kg.

## 5 Changing by a percentage

You can increase or decrease an amount by a given percentage either by finding the amount of the change, or by using a multiplier.

How would you work out the total of this bill?

**HAYES GARAGE**

**BILL**

| | |
|---|---|
| Repairs and oil change | £ 265 |
| Service charge @18% of total bill | £ |
| Total to pay | £ |

### Calculating the amount of the change

The amount you need to add on is 18% of £265.

$\frac{18}{100} \times £265 = £47.70$

Add this to the original amount.

£265 + £47.70 = £312.70

### Using a multiplier

The total to pay is (100% + 18%) of £265.

118% ÷ 100 = 1.18

To convert the percentage to a decimal divide by 100.

The multiplier is 1.18.

£265 × 1.18 = £312.70

## 5 Worked example — Grade 4

A set of tyres normally costs £450. In a sale there is a 35% discount.

Work out the sale price of the set of tyres.

35% of £450 $= \frac{35}{100} \times £450$
= £157.50

Work out the amount of the change.

£450 – £157.50 = £292.50

Subtract this from the original amount.

Or use a multiplier. Start by subtracting 35% from 100% and then convert the answer into a decimal.

100% – 35% = 65%
65 ÷ 100 = 0.65
Multiplier is 0.65
£450 × 0.65 = £292.50

You could choose either of these methods in your exam.

## 10 Exam-style practice — Grade 4

**1** A shop sells mobile phones. In January, the shop sold 160 mobile phones. In February, the shop sold 180 mobile phones.

Work out the percentage increase in the number of mobile phones sold from January to February.

**[3 marks]**

**2** Alan needs to buy some oil for heating at his farm. The capacity of the oil tank is 2300 litres. The tank is already 20% full. Alan wants to fill the tank completely. The price of oil is 73.04 pence per litre. Alan gets 8% off the price of oil. How much does he pay for the oil he needs to buy? **[5 marks]**

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# Reverse percentages

If you are given an amount after a percentage change, you need to be able to work out the original amount. There are two methods: the **unitary method** and the **multiplier method**.

## 10 Calculating a reverse percentage




You need to calculate a reverse percentage when you are given the final amount **after** a percentage change, and you want to find the original amount.

### Using the unitary method

Taking the original price as 100%, after a reduction of 15% the sale price is 100% – 15% = 85%.

The sale price is £204. So 85% is £204.

You need to work out 1%.

1% = £204 ÷ 85 = £2.40

To work out the original price multiply this by 100.

100% = £2.40 × 100 = £240

### Using the multiplier method

To work out the multiplier:

100% – 15% = 85%

85% ÷ 100 = 0.85

Divide by 100 to convert the percentage to a decimal.

The multiplier is 0.85.

So final price = original price × 0.85

To work out the original price, divide the final price by the multiplier.

£204 ÷ 0.85 = £240

## 5 Worked example — Grade 5

The price of all plane tickets to Birmingham airport increased by 6%.

**(a)** The price of a plane ticket from New Delhi to Birmingham increased by £35.28. Work out the price before this increase.

6% = £35.28

$1\% = \frac{£35.28}{6}$

$100\% = \frac{£35.28}{6} \times 100 = £588$

**(b)** After the increase, the price of a plane ticket from Shanghai to Birmingham was £466.93. Work out the price before this increase.

100% + 6% = 106% = 1.06

£466.93 ÷ 1.06 = £440.50

Part **(a)** is best answered by the unitary method.

You can use either method to answer part **(b)**.

## 2 Checklist

- To work out the multiplier for an increase, add the percentage increase to 100 and then divide by 100.

$\text{Multiplier} = \frac{100 + \%\ \text{increase}}{100}$

- To work out the multiplier for a decrease, subtract the percentage decrease from 100 and then divide by 100.

$\text{Multiplier} = \frac{100 - \%\ \text{decrease}}{100}$

## 10 Exam-style practice — Grade 5

**1** Kim is baking a cake. The cake loses 12% of its mass when it is baked. After the cake is baked its mass is 2.2 kg. Work out the mass of the cake before it is baked. **[3 marks]**

**2** Zak and Zoe record their commissions over two years. The table shows their commissions in 2016 and the percentage increase between 2015 and 2016.

Work out whose commission was greater in 2015. **[4 marks]**

| | Commission in 2016 | Percentage increase since 2015 |
|---|---|---|
| Zak | £33 450 | 4% |
| Zoe | £34 815 | 9% |

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# Growth and decay

You can use repeated percentages to model problems involving growth and decay. Typical examples of these are compound interest, population change and depreciation.

## 10 Compound interest

Most bank accounts pay **compound interest**. This means that the amount paid in interest is added to the balance of the account. The next time interest is calculated, the balance will be higher so the amount of interest will be higher. This is an example of **exponential growth**.

Suppose Anjali invests £1600 at 4.2% per annum compound interest. What is the value of Anjali's investment after 4 years?

### Using a table of values

100% + 4.2% = 104.2%

104.2 ÷ 100 = 1.042 so the multiplier is 1.042.

| End of year | Value of investment (£) |
|---|---|
| 1 | 1600 × 1.042 = 1667.20 |
| 2 | 1667.20 × 1.042 = 1737.22 |
| 3 | 1737.22 × 1.042 = 1810.19 |
| 4 | 1810.19 × 1.042 = 1886.21 |

### Using indices

$1600 \times 1.042 \times 1.042 \times 1.042 \times 1.042 = £1600 \times (1.042)^4$

$£1600 \times (1.042)^4 = £1886.21$

## 2 Depreciation

Something that **depreciates** loses value over time. This is an example of **exponential decay**. Suppose Tina bought a car that cost £15 600. Each year the value of the car depreciates by 16%. What is the value of the car at the end of 3 years?

### Using indices

100% – 16% = 84%

The multiplier is 84 ÷ 100 = 0.84.

The value after 3 years is $£15\,600 \times (0.84)^3 = £9246.18$.

## 1 Revision tips

- For growth, such as compound interest, the multiplier is more than 1.
- For decay, such as depreciation, the multiplier is less than 1.

## 10 Worked example — Grade 6

**1** Rita needs £55 000 to place a deposit on her new flat. She invests £50 000 at 3.5% compound interest. Does she have enough at the end of three years?

$£50\,000 \times (1.035)^3 = £55\,436$

This is more than £55 000 so she does have enough.

The multiplier for a 3.5% increase is × 1.035.

**2** The value of a car depreciates at the rate of 20% per year. Hanna buys a new car for £36 500. After $n$ years the value of the car is £14 950.40. What is the value of $n$?

After 2 years: $£36\,500 \times (0.80)^2 = £23\,360$

After 3 years: $£36\,500 \times (0.80)^3 = £18\,688$

After 4 years: $£36\,500 \times (0.80)^4 = £14\,950.40$

The value of $n$ is 4.

Try different values of $n$.

## 10 Exam-style practice — Grade 6

**1** Sandra invested £3250 in a savings account. She was paid 3.2% per annum compound interest. Work out the number of years it took her to save more than £3650. **[2 marks]**

**2** A ball was dropped from a height of 3 m onto horizontal ground. The ball hit the ground and bounced up. Each time the ball bounced, it rose to 65% of its previous height. Work out the height the ball reached after the third bounce. Give your answer correct to 2 decimal places. **[3 marks]**

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# Estimation and counting

You can estimate the answer to a calculation by rounding each number to 1 significant figure and then doing the calculation.

## Estimation

To estimate an answer to a calculation, round all the numbers to 1 significant figure, then write out the calculation with the rounded values and work out your estimate.

$5.36 \times 19.47 \approx 5 \times 20 = 100$

≈ is the symbol for approximately equal.

The answer is approximately equal to 100. You might have to make estimates like this on your non-calculator paper.

Multiply top and bottom by 100 to remove the decimal 0.02.

$$\frac{3500}{0.02} \overset{\times 100}{=} \frac{350\,000}{2}$$

## Worked example — Grade 5

Work out an estimate for

**(a)** $\dfrac{67 \times 402}{97}$

Round each number to 1 significant figure.

Circle your answer.

140 2800 (280) 28

$$\frac{67 \times 402}{97} \approx \frac{70 \times 400}{100} = \frac{28\,000}{100} = 280$$

**(b)** $\dfrac{509 \times 6.89}{0.021}$

$$\frac{509 \times 6.89}{0.021} \approx \frac{500 \times 7}{0.02} = \frac{3500}{0.02} = \frac{350\,000}{2} = 175\,000$$

## Estimation checklist

- Always round each number to 1 significant figure.
- To remove a decimal from the denominator multiply numerator and denominator by 10 or 100 or 1000.
- Estimate square roots or cube roots by finding nearest square number or cube number.

## Product rule for counting

If there are $m$ ways of choosing one item, and $n$ ways of choosing a second item, then there are a total of $m \times n$ ways of choosing both items. If you have six different t-shirts and four different pairs of jeans, then there are 24 different ways of choosing a t-shirt and a pair of jeans.




$= 24$

Use the product rule. You could simplify your working using indices: $26^5 \times 10^2$.

## Worked example — Grade 7

The diagram shows a UK number plate. It consists of two letters, followed by two numbers, followed by a further three letters.




If the letters can be chosen from any letters in the alphabet and the numbers can be any digit from 0 to 9, work out the total number of combinations that are possible. Give your answer in standard form to 3 significant figures.

$26 \times 26 \times 10 \times 10 \times 26 \times 26 \times 26$
$= 1.19 \times 10^9$

## Exam-style practice — Grades 5–7

**1** Work out an estimate for

**(a)** $\dfrac{765}{3.9 \times 9.7}$ **[2 marks]**

**(b)** $\dfrac{5.78 \times 324}{0.521}$ **[2 marks]**

**(c)** $\dfrac{9.82 + 19.54}{0.183}$ **[2 marks]**

**2** A combination lock on a suitcase uses three numbers. Work out the number of possible combinations if

**(a)** all three numbers can be any digit from 0 to 9 **[2 marks]**

**(b)** the first and last numbers must be prime numbers. **[2 marks]**

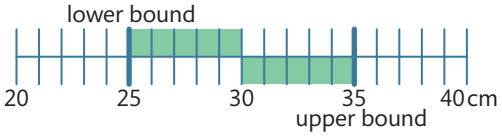
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# Upper and lower bounds

When a quantity is rounded, the actual value could be either higher or lower than the rounded value. You need to be able find an upper bound (UB) and a lower bound (LB) for the actual value.

## 5 Rules for upper and lower bounds

An exercise book has a width of 30 cm, rounded to the nearest 10 cm. The actual width could be between 25 and 35 cm.




The lower bound for the width is 25 cm and the upper bound is 35 cm.

If you are using upper or lower bounds in a calculation, use these rules to work out the maximum and minimum possible values of the answer.

| | Maximum value | Minimum value |
|---|---|---|
| **Addition** | UB + UB | LB + LB |
| **Subtraction** | UB – LB | LB – UB |
| **Multiplication** | UB × UB | LB × LB |
| **Division** | UB ÷ LB | LB ÷ UB |

## 5 Worked example — Grade 8

The length of a rectangular field is 450 m, to the nearest metre. The width of the field is 243 m, to the nearest metre.

**(a)** Work out the lower bound for the perimeter of the field.

450 m → UB = 450.5 m, LB = 449.5 m

243 m → UB = 243.5 m, LB = 242.5 m

LB of perimeter = 449.5 + 449.5 + 242.5 + 242.5
= 1384 m

The UB is 0.5 m greater than the rounded measure. The LB is 0.5 m lower.

**(b)** Work out the upper bound for the area of the field.

UB of area = 450.5 × 243.5
= 109 696.75 m²

Use the rules in the table to find upper and lower bounds in your calculations.

## 10 Worked example — Grade 8

**1** A ball is thrown vertically upwards with speed $v$ m/s. The height, $h$ metres, to which it rises is given by $h = \frac{v^2}{2g}$ where $g$ m/s² is the acceleration due to gravity.

$v = 12.6$ correct to 3 significant figures.

$g = 9.8$ correct to 1 decimal place.

Work out the maximum value of the height, $h$.
Give your answer correct to 3 significant figures.

12.6 → UB = 12.65, LB = 12.55

9.8 → UB = 9.85, LB = 9.75

UB of $h = \frac{12.65^2}{2 \times 9.75} = 8.21$ m

Divide the upper bound by the lower bound to find the maximum possible value of $h$.

**2** A metal bar has a mass of 1340 g (to the nearest gram) and a volume of 9.4 cm³ (to 1 decimal place).

Given that density $= \frac{\text{mass}}{\text{volume}}$, work out the lower bound for the density of the metal.
Give your answer, in g/cm³, to 3 significant figures.

1340 g → UB = 1340.5 g, LB = 1339.5 g

9.4 cm³ → UB = 9.45 cm³, LB = 9.35 cm³

LB of density $= \frac{\text{mass}}{\text{volume}} = \frac{1339.5}{9.45} = 142$ g/cm³

## 15 Exam-style practice — Grade 8

**1** $I = \frac{P}{V}$ where $V = 240$ correct to the nearest 5 and $P = 4200$ correct to the nearest 100. Work out the minimum value of $I$. Give your answer correct to 3 decimal places. You must show your working. **[3 marks]**

**2** Samuel is driving along a French motorway at a constant speed. He has driven 2270 metres, correct to the nearest 10 metres. This took 105 seconds, correct to the nearest 5 seconds. The speed limit for the motorway is 80 km/h. Determine whether Samuel could have exceeded the speed limit. You must show your working. **[4 marks]**

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# Accuracy and error

When a question involves upper and lower bounds, you might need to give an answer to an appropriate degree of accuracy.

## Appropriate degree of accuracy

To find an appropriate degree of accuracy, start by finding the upper and lower bounds. For an appropriate degree of accuracy, both the upper and lower bounds must round to the **same number**.

Suppose a rectangle is 10.8 cm long, correct to 3 significant figures, and it is 5.37 cm wide, correct to 3 significant figures. By considering bounds, how can you work out the area of the rectangle to a suitable degree of accuracy?

UB of area is $10.85 \times 5.375 = 58.318\,75\ \text{cm}^2$

LB of area is $10.75 \times 5.365 = 57.673\,75\ \text{cm}^2$

When the two numbers are rounded they must be equal. Rounded to 2 significant figures:

UB $= 58\ \text{cm}^2$ and LB $= 58\ \text{cm}^2$

So the area is $58\ \text{cm}^2$ correct to 2 significant figures.

If you rounded to 3 significant figures you would get:
UB $= 58.3\ \text{cm}^2$ LB $= 57.6\ \text{cm}^2$
These are different, so this is not an appropriate degree of accuracy.

Use the rules given in the table on page 7 to find upper and lower bounds.

## Worked example Grade 8

$t = \frac{\sqrt{a}}{b}$

$a = 6.94$ correct to 2 decimal places.

$b = 16.264$ correct to 3 decimal places.

By considering bounds, work out the value of $t$ to a suitable degree of accuracy.

You must show all your working and give a reason for your answer.

6.94 → UB = 6.945, LB = 6.935

16.264 → UB = 16.2645, LB = 16.2635

UB of $t$ is $\frac{\sqrt{6.945}}{16.2635} = 0.162\,039\,95$

LB of $t$ is $\frac{\sqrt{6.935}}{16.2645} = 0.161\,913\,29$

So $t = 0.162$ to 3 decimal places, because the LB and UB are equal when rounded to that accuracy.

## Worked example Grade 8

Bo runs 400 metres, correct to the nearest metre. She takes 50.5 seconds, correct to the nearest 0.1 seconds.

**(a)** Write down the error interval for the distance, $d$, run by Bo.

$399.5 \leqslant d < 400.5$

**(b)** Work out the upper and lower bounds of Bo's average speed. Give your answers correct to 5 decimal places.

400 → UB = 400.5, LB = 399.5

50.5 → UB = 50.55, LB = 50.45

UB speed $= \frac{400.5}{50.45} = 7.938\,55$ m/s

LB speed $= \frac{399.5}{50.55} = 7.903\,07$ m/s

**(c)** Use your answers to part (a) to write down the value of Bo's average speed to a suitable degree of accuracy. You must explain your answer.

Average speed = 7.9 m/s because the UB and the LB both round to 7.9 m/s when rounded to 2 significant figures.

An **error interval** is a way of writing upper and lower bounds using inequalities. Use $\leqslant$ for the lower bound and $<$ for the upper bound.

## Exam-style practice Grade 8

**1** The time Marcus took to complete a puzzle was 90 seconds.

This time, $t$, is to the nearest second.

Complete the error interval due to rounding.

____ $\leqslant t <$ ____ **[4 marks]**

**2** A lorry can carry a maximum safe load of 30 tonnes to the nearest tonne and delivers pallets of cement to a building site. A pallet of cement weighs 650 kg to the nearest 50 kg.
1 tonne = 1000 kg.



= 650 kg

By considering bounds, work out the maximum number of pallets that can be loaded safely on to the lorry. **[4 marks]**

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# Factors and primes

Any integer that divides exactly into another given number is a factor of that number. A prime number has only two factors: 1 and itself. A factor that is a prime number is a **prime factor**.

## 2 Prime factors of a number

The numbers 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29 are the first ten **prime numbers**.

If a prime number is a factor of another number it is a **prime factor** of that number.

You can use factor trees to work out prime factors. You keep dividing by prime numbers until all the branches end in a prime factor.

## 5 Lowest common multiple (LCM)

The LCM of two numbers is the lowest number that is a multiple of both numbers.

How can you identify the LCM of 36 and 84?

List their prime factors.

$36 = 2 \times 2 \times 3 \times 3$ and $84 = 2 \times 2 \times 3 \times 7$

For the LCM, compare how many times each factor appears in the two numbers. Look for the maximum number of times that the factor appears in **either** list.

$36 = 2 \times 2 \times 3 \times 3$

$84 = 2 \times 2 \times 3 \times 7$

Both numbers have two 2s, so you need $2 \times 2$.

36 has two 3s and 84 has one, so you need $3 \times 3$.

84 has one 7 and 36 has none, so you need 7.

Then multiply these together, so the LCM of 36 and 84 is $2 \times 2 \times 3 \times 3 \times 7 = 252$.

## 5 Worked example Grade 5

**(a)** Express 120 as the product of its prime factors.

$120 = 2 \times 2 \times 2 \times 3 \times 5$

**(b)** What is the highest common factor of 120 and 150?

$150 = 2 \times 3 \times 5 \times 5$

$HCF = 2 \times 3 \times 5$

$= 30$

**(c)** What is the lowest common multiple of 120 and 150?

$LCM = 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$= 600$

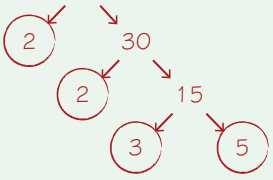
Look at all the factors that appear in **either** list. Take the higher number of each factor and multiply the values together. 120 has three 2s where 150 only has one 2, so you use $2 \times 2 \times 2$.

2, 3 and 5 appear in **both** lists.

## 2 Worked example Grade 5

Express the following numbers as the product of powers of their prime factors.

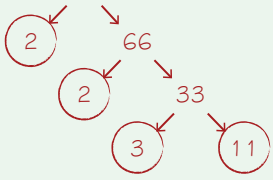
**(a)** 60




$60 = 2 \times 2 \times 3 \times 5$

$60 = 2^2 \times 3 \times 5$

**(b)** 132




$132 = 2 \times 2 \times 3 \times 11$

$132 = 2^2 \times 3 \times 11$

You can write the final answer in index form.

## 5 Highest common factor (HCF)

The HCF of two numbers is the highest number that is a factor of both numbers.

To identify the HCF of 36 and 84, list their prime factors.

$36 = 2 \times 2 \times 3 \times 3$ and $84 = 2 \times 2 \times 3 \times 7$

Circle the numbers that appear in **both** lists.

$36 = (2) \times (2) \times (3) \times 3$ and $84 = (2) \times (2) \times (3) \times 7$

Both lists include $2 \times 2 \times 3$.

HCF of 36 and 84 is $2 \times 2 \times 3 = 12$.

## 5 Exam-style practice Grade 5

**1** 24 and 108 are expressed as the product of powers of prime factors.

$24 = 2^3 \times 3$ $\quad$ $108 = 2^2 \times 3^3$

**(a)** Work out the highest common factor (HCF) of 108 and 24. Circle your answer.

2 $\quad$ 6 $\quad$ 12 $\quad$ 24 **[1 mark]**

**(b)** Work out the lowest common multiple (LCM) of 108 and 24. Circle your answer.

108 $\quad$ 432 $\quad$ 216 $\quad$ 2592 **[1 mark]**

**2** **(a)** Express 56 as the product of powers of its prime factors. **[3 marks]**

**(b)** What is the highest common factor (HCF) of 56 and 98? **[1 mark]**

**(c)** What is the lowest common multiple (LCM) of 56 and 98? **[1 mark]**

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# Standard form

You can use standard form to write very small and very large numbers in terms of a number between 1 and 10 and a power of 10.

## Numbers in standard form

### Notation

A number in standard form is written as:

$A \times 10^n$

where $1 \leqslant A < 10$ and $n$ is an integer.

Standard form is used for writing very small and very large numbers.

$856\,000\,000 = 8.56 \times 10^8$

$0.000\,000\,321 = 3.21 \times 10^{-7}$

For numbers less than 1, $n$ is negative.

### Writing numbers in standard form

Think of the number as two parts: a number, $A$, multiplied by a power of 10, $10^n$.

$34\,500\,000 = 3.45 \times 10\,000\,000$

$= 3.45 \times (10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10)$

$= 3.45 \times 10^7$

### Changing from standard form

To convert the number from standard form, work out the calculation.

$6.34 \times 10^6 = 6.34 \times 1\,000\,000$

$= 6\,340\,000$

## Multiplying and dividing

### Multiplying standard form numbers

Rearrange so that the powers of 10 are together.

$(6.4 \times 10^7) \times (2 \times 10^{-3})$

$= (6.4 \times 2) \times (10^7 \times 10^{-3})$

Multiply the number parts and **add** the powers.

$= 12.8 \times 10^4$

Rewrite your answer in standard form if necessary.

$= 1.28 \times 10^5$

### Dividing standard form numbers

Rearrange so that the powers of 10 are together.

$= (3 \times 10^{-8}) \div (6 \times 10^5)$

$= (3 \div 6) \times (10^{-8} \div 10^5)$

Divide the number parts and **subtract** the powers.

$= 0.5 \times 10^{-13}$

Rewrite your answer in standard form if necessary.

$= 5 \times 10^{-14}$

See page 15 for the rules of indices. You need to learn them.

## Worked example — Grade 5

**1** Write these numbers in standard form.

Divide by a power of 10, to leave a number between 1 and 10.

**(a)** 562 000

$562\,000 = 5.62 \times 100\,000$

$= 5.62 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 5.62 \times 10^5$

**(b)** 0.000 326

Take out a factor that is a negative power of 10, to leave a number between 1 and 10.

$0.000\,326 = 3.26 \times 0.0001$

$= 3.26 \times 10^{-4}$

**2** Write $8.29 \times 10^{-5}$ as an ordinary number.

$8.29 \times 10^{-5} = 8.29 \times 0.000\,01 = 0.000\,082\,9$

Work out the calculation.

## Use of a calculator

You can enter standard form into a calculator by using the $\times 10^x$ button.

Suppose you want to enter $3.2 \times 10^{12}$ into the calculator.

Enter 3 . 2 and then press $\times 10^x$ followed by 1 2.

It will show $3.2 \times 10^{12}$.

Then add, subtract, multiply or divide the number as usual.

## Worked example — Grade 5

Work these out, both using a calculator and without using a calculator.

**(a)** $(4.5 \times 10^8) \times (3 \times 10^{-6})$

$(4.5 \times 3) \times (10^8 \times 10^{-6}) = 13.5 \times 10^2$

$= 1.35 \times 10^3$

**(b)** $(3 \times 10^9) \div (4 \times 10^{12})$

$(3 \div 4) \times (10^9 \div 10^{12}) = 0.75 \times 10^{-3} = 7.5 \times 10^{-4}$

## Exam-style practice — Grade 5

**(a)** A satellite travels for $5 \times 10^2$ hours at a speed of $9 \times 10^4$ km/h.

Work out the distance travelled by the satellite. Give your answer in standard form. **[2 marks]**

**(b)** A second satellite travels $3 \times 10^4$ km one month and $4 \times 10^3$ km the next month.

Work out the distance travelled by the satellite in the two months. Give your answer in standard form. **[2 marks]**

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# Surds

A **rational number** can be expressed in the form $\frac{a}{b}$, where $a$ and $b$ are integers. Any number which cannot be expressed in this form is called an **irrational number**. Square roots of positive integers which are not perfect squares are irrational. **Surds** are irrational numbers that contain an irrational square root, such as $\sqrt{2}$, $3+\sqrt{5}$ or $\frac{7}{\sqrt{11}}$. You can use $\pi$ and surds to give exact answers.

## 5 Simplifying surds

You can use the following rules to simplify surds.

$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ — $\sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ — $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$

$\sqrt{a} \times \sqrt{a} = a$ — $\sqrt{5} \times \sqrt{5} = 5$

A surd is in its **simplest form** when the whole number under the square root sign is the smallest possible number.

Look for a factor of 12 that is a square number.

There is more about multiplying out brackets on page 13.

## 2 Rationalising the denominator

If your answer is a fraction, you might need to **rationalise** the denominator to remove any surd parts. You can do this by multiplying the top and bottom by the surd in the denominator.

$$\frac{21}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{21\sqrt{3}}{3} = 7\sqrt{3}$$

## 5 Worked example — Grade 9

Show that $\dfrac{\sqrt{5}}{\frac{1}{\sqrt{5}} + 1}$ can be written as $\dfrac{5\sqrt{5} - 5}{4}$.

Start by multiplying the top and bottom by $\sqrt{5}$ to remove the fraction in the denominator.

$$\frac{\sqrt{5}}{\frac{1}{\sqrt{5}} + 1} = \frac{5}{1 + \sqrt{5}}$$

$$= \frac{5}{1 + \sqrt{5}} \times \frac{1 - \sqrt{5}}{1 - \sqrt{5}}$$

$$= \frac{5 - 5\sqrt{5}}{-4} = \frac{5\sqrt{5} - 5}{4}$$

### Problem solving

If the denominator is in the form $a + \sqrt{b}$, you need to multiply the top and bottom by $a - \sqrt{b}$. So here multiply top and bottom by $1 - \sqrt{5}$.

$(1 + \sqrt{5})(1 - \sqrt{5}) = 1 + 1\sqrt{5} - 1\sqrt{5} - \sqrt{5}\sqrt{5}$

$= 1 - 5$

$= -4$

## 5 Worked example — Grade 7

**1** Simplify fully

**(a)** $\sqrt{12}$

$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

**(b)** $5\sqrt{32}$

$5\sqrt{32} = 5\sqrt{16 \times 2} = 5\sqrt{16} \times \sqrt{2}$

$= 5 \times 4 \times \sqrt{2}$

$= 20\sqrt{2}$

**2** Show that $(5 + \sqrt{7})(5 - \sqrt{7}) = 18$.

$(5 + \sqrt{7})(5 - \sqrt{7})$

$= 25 + 5\sqrt{7} - 5\sqrt{7} - (\sqrt{7} \times \sqrt{7})$

$= 25 - 7$

$= 18$

Remember that $\sqrt{7} \times \sqrt{7} = 7$

## 1 Simplified surds checklist

Make sure:

- ☑ the numbers under the square roots have no square numbers as factors
- ☑ you have expanded any brackets
- ☑ there are no surds in the denominators of fractions.

## 10 Exam-style practice — Grades 7–9

**1** Simplify fully:

**(a)** $\sqrt{18}$ **(b)** $\sqrt{50}$ **(c)** $\sqrt{45}$ **(d)** $4\sqrt{98}$ **[4 marks]**

**2** Show that $\dfrac{(6 - \sqrt{3})(6 + \sqrt{3})}{\sqrt{11}}$ can be written in the form $k\sqrt{11}$ where $k$ is an integer. **[3 marks]**

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# Number

Pages 1–11 LINKS

Read the exam-style question and worked solution, then practise your exam skills with the two questions at the bottom of the page.

## 10 Worked example Grade 8

**1** A jug of water holds 4.5 litres measured to the nearest 0.5 litres. Write down the error interval.

Circle your answer.

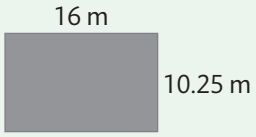
$4 < v \leqslant 4.5$ $4.25 \leqslant v < 4.75$

$4.25 < v \leqslant 4.75$ $4 \leqslant v < 4.5$

Least possible value = 4.5 – 0.25 = 4.25

Greatest possible value = 4.5 + 0.25 = 4.75

**2** Ravina measured the length and the width of her rectangular patio.




She measured the length to be 16 m, correct to the nearest metre, and the width to be 10.25 m, correct to the nearest centimetre.

**(a)** Write down the error interval for the length, $l$ metres.

The least possible value is 15.5 m.

The greatest possible value is 16.5 m.

The error interval is $15.5\text{ m} \leqslant l < 16.5\text{ m}$.

**(b)** Write down the error interval for the width, $w$ metres.

The least possible value is 10.245 m.

The greatest possible value is 10.255 m.

The error interval is

$10.245\text{ m} \leqslant w < 10.255\text{ m}$

**(c)** Hence, or otherwise, work out the error interval for the area, $A$ metres squared, of the rectangle.

Least possible area = 15.5 × 10.245

= 158.7975 m²

Greatest possible area = 16.5 × 10.255

= 169.2075 m²

Error interval for the area is

$158.7975\text{ m}^2 \leqslant A < 169.2075\text{ m}^2$

### Problem solving

You need to find the least possible area of the rectangle, and the greatest possible area. The least possible area is the product of the lower bounds for length and width, and the greatest possible area is the product of the upper bounds. Do not round your final answer.

## 2 Exam focus

- Make sure you know how to work out the lower bound and upper bound of a number given to a specified degree of accuracy.
- Learn the rules for bounds when applied to addition, subtraction, multiplication and division.
- Make sure you understand when the final answer must be given to a suitable degree of accuracy.

This is the lower bound, the smallest value that a number can take, to a specified degree of accuracy.

This is the upper bound, the largest value that a number can take, to a specified degree of accuracy.

An error interval of a number, $x$, can be written as lower bound $\leqslant x <$ upper bound.

For any number correct to a specified degree of accuracy, the exact value lies in a range from half a unit below to half a unit above the given number.

## 15 Exam-style practice Grade 8

**1** A solid sphere has mass 2340 g measured to the nearest gram and radius 7.1 cm measured to the nearest millimetre.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

complete the error interval for the density of the sphere, $s$, in g/cm³.

Give your answer to 3 significant figures.

____ $\leqslant s <$ ____ **[5 marks]**

**2** Govind drops a ball from a height of $h$ metres onto the ground. The time, $t$ seconds, that the ball takes to reach the ground is given by

$$t = \sqrt{\frac{2h}{g}}$$

where $g$ m/s² is the acceleration due to gravity.

$h = 26.1$ correct to 3 significant figures.

$g = 9.8$ correct to 2 significant figures.

By considering bounds, work out the value of $t$ to a suitable degree of accuracy. You must show all your working and give a reason for your final answer.

**[5 marks]**

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# Algebraic expressions

You need to be able to collect like terms, factorise, expand and simplify algebraic expressions. Always give your answers in their simplest form.

## 2 Algebraic expressions

An **algebraic term** is a number or variable, or a combination of numbers and variables, such as 2, $x$, $4xy$.

An **algebraic expression** is a collection of terms together with arithmetical operators (such as $\times$, $+$).

$2x$, $y^2$ and $17x - 6y$ are all algebraic expressions.

To **simplify** an expression you need to **collect the like terms**, for example, $3x + 6x = 9x$.

$3x + 6x = 9x$ — $3x$ and $6x$ are like terms so can be simplified

$2x + 3y$ — $2x$ and $3y$ are not like terms so this expression cannot be simplified

## 10 Worked example — Grade 5

**1** Expand and simplify

**(a)** $2(x - 4) + 3(x + 5)$

$= 2x - 8 + 3x + 15$

$= 2x + 3x - 8 + 15$

$= 5x + 7$

**(b)** $(2x - 3)(x + 4)$

$= 2x(x + 4) - 3(x + 4)$

$= 2x^2 + 8x - 3x - 12$

$= 2x^2 + 5x - 12$

**(c)** $x(x + 1)(x - 2)$

$= x[x(x - 2) + 1(x - 2)]$

$= x[x^2 - 2x + x - 2]$

$= x[x^2 - x - 2]$

$= x^3 - x^2 - 2x$

**2** Simplify

**(a)** $3y + 2x - 4 + 5x + 7$

$= 3y + 2x + 5x - 4 + 7$

$= 3y + 7x + 3$

**(b)** $5x^2 + 4y^3 + x^2 - 7y^3$

$= 5x^2 + x^2 + 4y^3 - 7y^3$

$= 6x^2 - 3y^3$

**3** Factorise

**(a)** $2x^2 - 4x$

$= 2x(x - 2)$

**(b)** $12x^3y - 18xy^2$

$= 6xy(2x^2 - 3y)$

**(c)** $6y^2 - 9xy$

$= 3y(2y - 3x)$

Use BIDMAS:
Brackets
Indices
Division
Multiplication
Addition
Subtraction

## 2 Brackets and factorising

You can **expand brackets** by multiplying the term outside the brackets by everything inside the brackets. The opposite process to expanding brackets is called **factorising**.

Expanding brackets

$2a(a^2 - 5b) = 2a \times a^2 - 2a \times 5b = 2a^3 - 10ab$

Factorising

You can expand a product of two brackets by multiplying each term in the first bracket by every term in the second bracket:

$(x + 3)(x - 2) = x^2 + 3x - 2x - 6 = x^2 + x - 6$

There are two terms in each bracket so there are four terms in the expanded expression before you simplify.

Multiply out the brackets, then collect like terms.

Multiply everything in the second bracket by everything in the first bracket.

Start by multiplying out $(x + 1)(x - 2)$. Then multiply everything in that expression by $x$.

Combine the $x$ terms and combine the number terms.

Combine the $x^2$ terms and combine the $y^3$ terms.

The HCF of 2 and 4 is 2 and the HCF of $x^2$ and $x$ is $x$, so take out a factor of $2x$.

## 15 Exam-style practice — Grades 5–6

**1** Simplify $6y^2 + 7x^3 - 11 + 8x^3 + 17$. **[2 marks]**

**2** Expand and simplify

**(a)** $4(y - 3) - 5(3y + 1)$ **[2 marks]**

**(b)** $x(x + 1)(x - 1)$ **[2 marks]**

**3** Factorise fully $20x^2 - 5$. **[2 marks]**

**4** Prove algebraically that $(2x + 1)^2 - (2x + 1)$ is an even number for all positive integer values of $x$. **[3 marks]**

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# Algebraic formulae

A formula is a mathematical rule. You use algebra to write a formula (the plural of formula is formulae). You should be able to rearrange a formula to **change the subject** and write a formula to describe a simple situation.

## 5 Writing a formula

Peter advertises his business in the local magazine.

> **Peter the plumber**
> **Call-out charge £45**
> **£15 per hour**

You can write the total cost as an algebraic formula. Start by defining the variables. For example, you can say the total cost of hiring Peter is $T$ pounds and the number of hours worked is $n$.

total cost = (number of hours × £15) + call-out charge

$T = 15n + 45$

When you define your variables, you must give their units.

Use the information given to write $y$ in terms of $x$. Then use the formula for the volume of a cuboid to find an expression for $V$ in terms of $x$.

## 10 Worked example — Grade 5

**1** Bulbs are sold in packets and in boxes. There are 3 bulbs in a packet. There are 12 bulbs in a box.

Kamran buys $x$ packets of bulbs and $y$ boxes of bulbs.




Write down a formula, in terms of $x$ and $y$, for the total number, $N$, of bulbs Kamran buys.

$N = 3 \times x + 12 \times y$

$N = 3x + 12y$

The variables are $N$ (the total number of bulbs), $x$ (the number of packets) and $y$ (the number of boxes).

**2** This formula gives you the distance ($s$ metres) travelled by an object moving with speed ($u$ m/s), in time ($t$ seconds).

$s = ut + 5t^2$

Work out the value of $s$, when $u = -4$ and $t = 3$.

$s = (-4)(3) + 5(3)^2$

$s = -12 + 45$

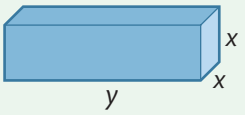
$s = 33$

Substitute the values of $u$ and $t$ into the formula.

Priority of operations is very important when you are evaluating formulae. Remember to use BIDMAS.

## 5 Worked example — Grade 6

The diagram shows a cuboid of volume $V$ cm$^3$. The length of the cuboid is $y$ cm. The width and height of the cuboid are both $x$ cm.




The total length of all the edges of the cuboid is 120 cm and its volume is $V$ cm$^3$.

Show that $V = 30x^2 - 2x^3$.

The sum of all the edges is 120.

$4y + 8x = 120$

$4y = 120 - 8x$

$y = 30 - 2x$

Area of cross-section $= x^2$

$V = x^2y$

volume of a cuboid = area of cross-section × length

Substitute $y$ into the formula for $V$.

$V = x^2(30 - 2x)$

$V = 30x^2 - 2x^3$

## 10 Exam-style practice — Grades 5–6

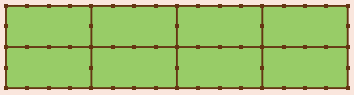
**1** Work out the value of $L$ given that

$$L = \frac{2(x^2 + xy + y^2)}{x + y}$$ when $x = 2.8$ and $y = 3.9$.

Give your answer correct to 3 significant figures.

**[3 marks]**

**2** A farmer makes an enclosure divided into eight equal, rectangular pens. He uses fencing for the perimeter of the enclosure and to divide the enclosure into pens. In total he uses 200 metres of fencing.



The length of each pen is $x$ metres and the width of each pen is $y$ metres.

**(a)** Show that $y = 20 - 1.2x$.

The total area of the enclosure is $A$ m$^2$.

**(b)** Show that $A = 160x - 9.6x^2$. **[3 marks]**

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# Laws of indices

You can use the laws of indices to simplify powers and algebraic expressions.

## 5 Basic laws of indices

Learn these laws of indices.

$a^m \times a^n = a^{m+n}$ $\quad x^4 \times x^6 = x^{4+6} = x^{10}$

$a^m \div a^n = a^{m-n}$ $\quad x^4 \div x^6 = x^{4-6} = x^{-2}$

$(a^m)^n = a^{mn}$ $\quad (x^4)^6 = x^{4\times 6} = x^{24}$

$a^0 = 1$ $\quad x^0 = 1$

## 5 Worked example Grades 5–7

Simplify

**(a)** $\dfrac{x^5 \times x^7}{x^3}$

$\dfrac{x^5 \times x^7}{x^3} = \dfrac{x^{12}}{x^3} = x^9$

**(b)** $\left(\dfrac{x^8}{x^5}\right)^2$

$\left(\dfrac{x^8}{x^5}\right)^2 = (x^3)^2 = x^6$

**(c)** $(2m^4n^2)^5$

$(2m^4n^2)^5 = 2^5 m^{4\times 5} n^{2\times 5} = 32m^{20}n^{10}$

Multiply or divide any numbers first and then use index rules to work out the new powers.

Use the laws of indices to simplify the powers.

Since the base is the same on both sides, the powers must be equal so form an equation and solve it.

## 2 Worked example Grade 4

Simplify

**(a)** $p^7 \times p^4$ ← Use $a^m \times a^n = a^{m+n}$

$p^7 \times p^4 = p^{7+4} = p^{11}$

**(b)** $p^6 \div p^9$

$p^6 \div p^9 = p^{6-9} = p^{-3}$

**(c)** $(p^2)^4$

$(p^2)^4 = p^{2\times 4} = p^8$

## 1 Indices checklist

- Only combine powers (indices) when the bases are the same.
- When you multiply, add the powers.
- When you divide, subtract the powers.
- When you raise a power to a power, multiply the powers.

## 2 Worked example Grade 7

Work out the value of $n$, given that

$\dfrac{y^4 \times y^n}{y} = y^8$

$\dfrac{y^{4+n}}{y} = y^8$

$y^{4+n-1} = y^8$

So

$4 + n - 1 = 8$

$n = 5$

## 15 Exam-style practice Grades 4–7

**1** Simplify $p^2 \times p^7$. Circle your answer. **[1 mark]**

$p^{14}$ $\quad 2p^{14}$ $\quad 2p^9$ $\quad p^9$

**2** Simplify

**(a)** $\dfrac{x^4 \times x^6}{x^2}$ **[2 marks]**

**(b)** $4x^2y^4 \times 3xy$ **[2 marks]**

**3** Simplify

**(a)** $m^8 \div m^2$ **[1 mark]** **(b)** $(m^5)^3$ **[1 mark]**

**(c)** $3w^2y^3 \times 4w^6y$ **[2 marks]** **(d)** $\dfrac{32x^6y^8}{4x^2y}$ **[2 marks]**

**(e)** $(4x^3y^4)^3$ **[2 marks]**

**4** Solve to work out the value of $n$.

$x^6 = \dfrac{x^n \times x^8}{x^5}$ **[3 marks]**

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# Combining indices

You can use the index laws to simplify expressions involving fractions and negative powers.

## 5 Further laws of indices

$a^{-n} = \frac{1}{a^n}$ $\qquad 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\qquad \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $\qquad \left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 = \frac{5^2}{2^2} = \frac{25}{4}$

$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$ $\qquad 16^{-\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^{-3} = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

Fractional indices can also be formed as a root: $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

## 2 Worked example — Grade 9

**1** Solve $4^n = 8^{n-3}$

$(2^2)^n = (2^3)^{n-3}$ ← Remember that $8 = 2^3$, so change the base number to 2.

$2^{2n} = 2^{3(n-3)}$

$2n = 3(n-3)$

$2n = 3n - 9$

$n = 9$

**2** Work out the value of $n$ when $3n^{\frac{3}{2}} = 81^{-\frac{1}{2}}$.

$3n^{\frac{3}{2}} = 81^{-\frac{1}{2}}$

$3n^{\frac{3}{2}} = \frac{1}{81^{\frac{1}{2}}}$ ← $81 = 9^2$, so you can find the square root straight away.

$3n^{\frac{3}{2}} = \frac{1}{9}$

$n^{\frac{3}{2}} = \frac{1}{27}$

$n = \left(\frac{1}{27}\right)^{\frac{2}{3}}$ ← To find $n$, invert the fraction in the power on the left-hand side and then apply it as a power to the number on the right-hand side.

$n = \left(\frac{1}{27^{\frac{1}{3}}}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

## 1 Indices checklist

- You can only combine indices if the base numbers are the same.
- If there is no index, the power is 1, $x = x^1$.
- Always use brackets to make the working clear, especially when using negative powers.

## 5 Worked example — Grades 7–8

Work out the value of

**(a)** $4^{-2}$ $\qquad$ **(b)** $49^{\frac{1}{2}}$

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ $\qquad 49^{\frac{1}{2}} = \sqrt{49} = \pm 7$

**(c)** $27^{-\frac{2}{3}}$

$27^{-\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^{-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

**(d)** $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

$\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}} = \left(\frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

## 2 Solving equations

When solving equations in index form, make sure that the base number is the same in each term.

For example, to work out the value of $n$ when $2^n = 16^{-\frac{1}{2}}$, make the base number on the right-hand side 2.

$2^n = 16^{-\frac{1}{2}}$

$2^n = (2^4)^{-\frac{1}{2}}$ ← Write 16 as a power of 2. Remember to use brackets.

$2^n = 2^{-2}$

$n = -2$

## 15 Exam-style practice — Grades 8–9

**1** Work out the value of

**(a)** $8^{\frac{2}{3}}$ **[1 mark]**

**(b)** $25^{-\frac{1}{2}}$ **[2 marks]**

**2** **(a)** Work out the value of $64^{-\frac{1}{2}}$. **[2 marks]**

**(b)** Write $\frac{\sqrt{3}}{9}$ as $3^n$. **[2 marks]**

**3** Work out the value of

**(a)** $(8 \times 4^{-3})^{\frac{1}{3}}$ **[2 marks]**

**(b)** $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$ **[2 marks]**

**4** Solve the equation $9^{n-1} = 27$. **[3 marks]**

**5** Work out the value of $y$ for which $2 \times 4^y = 64$. **[4 marks]**

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# Simple linear equations

When you solve a linear equation your aim is to find the value of the unknown. You can often do this by rearranging the equation and using inverse operations.

## Unknown on one side

You may be asked to solve an equation that involves an unknown on one side, such as

$6x + 10 = 34$

You will often need to use **inverse operations**, which are 'opposite' calculations.

The term $+10$ can be removed from the equation by subtracting 10 from each side. This gives:

$6x + 10 - 10 = 34 - 10$

$6x = 24$

To get $x$ by itself, divide both sides by 6.

$\frac{6x}{6} = \frac{24}{6}$

$x = 4$

Every line of working should include an equals (=) sign. You should align the equals signs underneath each other.

## Worked example Grades 4–5

**1** Solve

$5x + 7 = 11$

$5x + 7 - 7 = 11 - 7$

$5x = 4$

$x = \frac{4}{5}$

**2** Solve

$7x + 8 = 2x - 3$

$7x - 2x + 8 = 2x - 2x - 3$

$5x + 8 - 8 = -3 - 8$

$5x = -11$

$x = -\frac{11}{5}$

**3** Andy, Tom and Chris play hockey. Andy has scored 9 more goals than Chris, Tom has scored 6 more goals than Andy. The total number of goals scored by the three players is 90.

How many goals did they each score?

| Andy | Tom | Chris |
|---|---|---|
| $x + 9$ | $x + 9 + 6$ | $x$ |

$x + 9 + x + 9 + 6 + x = 90$

$3x + 24 = 90$

$3x = 90 - 24$

$3x = 66$

$x = \frac{66}{3} = 22$

| Andy | Tom | Chris |
|---|---|---|
| $22 + 9 = 31$ | $22 + 9 + 6 = 37$ | 22 |

## Unknown on both sides

Sometimes an equation has unknowns on both sides. To solve such an equation, rearrange it so that all the unknowns are on one side.

$3 - 4x = 15 - x$

$3 - 4x + 4x = 15 - x + 4x$ — Remove the term $-4x$ from the equation by adding $4x$ to each side.

$3 = 15 + 3x$

$3 - 15 = 15 - 15 + 3x$

$-12 = 3x$ — To get $x$ by itself, divide both sides by 3.

$\frac{-12}{3} = \frac{3x}{3}$

$-4 = x$ or $x = -4$

## Equations with brackets

Always multiply out brackets first and then collect like terms. You will be expected to solve an equation such as:

$2(2x + 5) - (3x + 4) = 9(2x + 5)$ — Multiply out the brackets.

$4x + 10 - 3x - 4 = 18x + 45$ — Collect like terms.

$x + 6 = 18x + 45$

$x - x + 6 = 18x - x + 45$

$6 - 45 = 17x + 45 - 45$

$-39 = 17x$

$-\frac{39}{17} = x$

$x = -\frac{39}{17}$

### Problem solving

Assign a letter to the unknown value and create an algebraic equation using this letter.

Set up the equation by adding all the expressions and equating them to the total number of goals.

## Exam-style practice Grades 4–5

**1** Solve

**(a)** $6x + 3 = 24$ **[2 marks]**

**(b)** $3(2x - 1) = 6$ **[2 marks]**

**(c)** $3x + 7 = 5x - 1$ **[3 marks]**

**2** Ann is $x$ years old, Ben is twice as old as Ann. Carl is 4 years younger than Ann. The total of their ages is 92 years.

Work out the age of each person. **[3 marks]**

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# Linear equations and fractions

If you are dealing with a linear equation that contains fractions, you need to rearrange the equation to remove them, before you can solve it.

## 5 Equations with fractions

You might need to solve an equation such as

$\frac{x}{5} + \frac{x+3}{4} = 3$

You need to remove any fractions before you can solve the equation.

To do this, multiply every term by the lowest common denominator, which is the lowest common multiple (LCM) of the denominators.

$\frac{20x}{5} + \frac{20(x+3)}{4} = 3 \times 20$ ← LCM of 5 and 4 is 20.

$4x + 5(x+3) = 60$ ← Cancel the fractions.

$4x + 5x + 15 = 60$

$9x + 15 = 60$

$9x + 15 - 15 = 60 - 15$

$9x = 45$

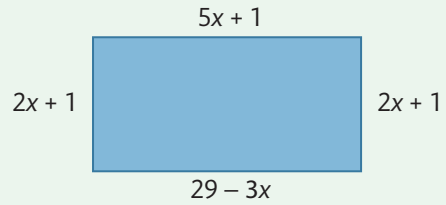
$x = 5$

Sometimes you need to multiply by an **expression** to remove a fraction.

In this case the equation is multiplied by the expression $x + 1$.

## 5 Worked example Grade 7

The diagram shows the length, in centimetres, of each side of a rectangle. The perimeter of the rectangle is $P$ cm.




Work out the value of $P$.

Opposite sides of a rectangle are equal.

$5x + 1 = 29 - 3x$

$8x = 28$

$x = 3.5$

The perimeter is the sum of all the sides.

$P = 2[(2 \times 3.5) + 1] + 2[(5 \times 3.5) + 1]$

$= 16 + 37$

$= 53$ cm

## 5 Worked example Grade 7

**1** Solve

$\frac{4x-1}{3} + 2(x+5) = \frac{3}{2}$

The LCM of the two denominators (2 and 3) is 6, so multiply all terms by 6.

$\frac{6(4x-1)}{3} + 12(x+5) = \frac{18}{2}$

$2(4x-1) + 12(x+5) = 9$

For a reminder about expanding brackets, have a look at page 13.

$8x - 2 + 12x + 60 = 9$

$20x + 58 = 9$

$20x = -49$

$x = -\frac{49}{20}$

**2** Solve

$\frac{-5-2x}{x+1} = 4$

$-5 - 2x = 4(x+1)$

$-5 - 2x = 4x + 4$

$-5 - 4 = 4x + 2x$ ← Collect all the $x$'s on one side.

$-9 = 6x$

$\frac{-9}{6} = x$ so $x = -1.5$

### Problem solving

To solve this problem you need to use the properties of a rectangle to set up and solve an equation. You know that opposite sides of a rectangle are equal. Choose the sides with different expressions and set them equal to each other to form an equation. Solve this equation to find $x$, then use your value of $x$ to find the perimeter.

You need to work out the value of $x$ so that you can substitute this into the expression for the perimeter.

## 10 Exam-style practice Grades 7–8

**1** Solve $\sqrt{2 - \frac{x+4}{2x+5}} = 3$ **[4 marks]**

**2** Solve $\frac{4}{2x+3} = \frac{3}{4-x}$ **[3 marks]**

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# Simultaneous equations

Simultaneous equations are equations that are written in terms of the same two unknowns. In this section, you will review solving two simultaneous linear equations.

## 5 Elimination method

Here are a pair of simultaneous equations

$3x + 4y = 5$ (1)

$2x - 3y = 9$ (2)

Start by numbering each equation, so you can refer to them in your working.

If necessary, multiply one or both of the equations so that the **coefficients** of one unknown are the same.

$6x + 8y = 10$ (1) × 2

$6x - 9y = 27$ (2) × 3

Subtract equation (2) from equation (1), to eliminate the $x$ terms.

$17y = -17$

$y = -1$

Now substitute this unknown into one of the original equations to find the other unknown.

$3x + 4(-1) = 5$

$3x = 9$

$x = 3$

Check the answer by substituting the unknown into the other (unused) original equation.

$2x - 3y = 9$

$2(3) - 3(-1) = 9$

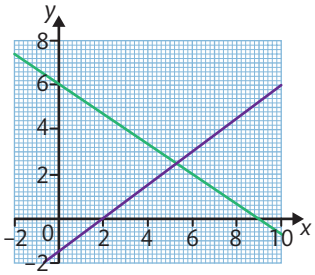
$9 = 9$

### Problem solving

You need to convert the word problem into a pair of simultaneous equations. Let the weight of one apple be $a$ grams, and the weight of one pear be $p$ grams.

## 2 Graphical method

You can solve simultaneous linear equations by drawing up a table of values and plotting a graph. For example, consider $2x + 3y = 18$ and $3x - 4y = 6$.




The coordinates of the point of intersection give the solution to the simultaneous equations.

The solution is $x = 5.2$ and $y = 2.5$. The lines intersect at (5.2, 2.5).

## 10 Worked example — Grade 6

**1** Solve the simultaneous equations

$5x + 2y = 11$ (1)

$4x - 3y = 18$ (2)

$15x + 6y = 33$ (1) × 3

$8x - 6y = 36$ (2) × 2

Adding will eliminate the $y$ terms.

$23x = 69$

$x = 3$

$5(3) + 2y = 11$

$2y = -4$

$y = -2$

### Problem solving

Always check your answer by substituting into the original equation that you did not use.

$4(3) - 3(-2) = 18$

**2** Five apples and four pears together weigh 760 g, while four apples and five pears weigh 770 g. Work out how much one apple and one pear weigh together.

$5a + 4p = 760$ (1)

$4a + 5p = 770$ (2)

$25a + 20p = 3800$ (1) × 5

$16a + 20p = 3080$ (2) × 4

$9a = 720$ so $a = 80$

Substitute in (1).

$5 \times 80 + 4p = 760$

$4p = 360$ so $p = 90$

$a + p = 170$

An apple and a pear together weigh 170 g.

## 10 Exam-style practice — Grade 6

**1** Use a graphical method to work out an approximate solution to the simultaneous equations $7x + 5y = 35$ and $2x - 3y = 2$. **[3 marks]**

**2** Solve the simultaneous equations

$3x - 4y = 8$

$9x + 5y = -1.5$ **[3 marks]**

**3** A theatre sells adult tickets and child tickets.

The total cost of 5 adult tickets and 1 child ticket is £40.50. The total cost of 2 adult tickets and 3 child tickets is £24. Work out the cost of an adult ticket and the cost of a child ticket. **[4 marks]**

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# Quadratic equations

Quadratic equations have the form $ax^2 + bx + c = 0$, $a \neq 0$. You will need to know how to solve a quadratic equation by factorisation, without using a calculator, by completing the square or by the quadratic formula.

## Solving $x^2 + bx + c = 0$

To factorise the left-hand side of a quadratic equation in the form $x^2 + bx + c = 0$, you need to find two numbers $p$ and $q$ such that $pq = c$ and $p + q = b$. The factorised equation is then $(x + p)(x + q) = 0$.

So the solutions are $x = -p$ and $x = -q$.

## Worked example — Grade 5

Solve $x^2 - x - 72 = 0$

$(x + 8)(x - 9) = 0$ ← 8 and −9 are a factor pair of −72 that add up to −1.

$x + 8 = 0$ or $x - 9 = 0$

$x = -8$ or $x = 9$

You can check your factorisation by expanding the brackets.

## Solving $ax^2 + bx + c = 0$

You can solve quadratic equations of the form $ax^2 + bx + c = 0$ by finding values $p$ and $q$ such that $p + q = b$ and $pq = ac$.

$3x^2 - 7x + 2 = 0$ ← $b = -7$ and $ac = 6$ so $p = -1$ and $q = -6$ are suitable values.

$3x^2 - x - 6x + 2 = 0$ ← Use your values of $p$ and $q$ to write $-7x$ as $-x - 6x$.

$x(3x - 1) - 2(3x - 1) = 0$ ← Factorise the first pair of terms and the last pair of terms.

$(3x - 1)(x - 2) = 0$ ← Factorise again then set each factor equal to 0 to solve $x = \frac{1}{3}$ or $x = 2$.

## Worked example — Grade 6

**1** Solve $3x^2 - 5x = 0$.

$3x^2 - 5x = 0$

$x(3x - 5) = 0$

$x = 0$ or $3x - 5 = 0$

$x = 0$ or $x = \frac{5}{3}$

You can take out a factor of $x$ from both terms. Remember there are still two solutions. $x = 0$ or $x = \frac{5}{3}$.

**2** Solve $4x^2 - 9 = 0$.

$4x^2 - 9 = 0$

$(2x - 3)(2x + 3) = 0$

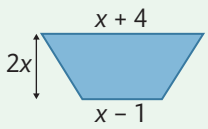
$2x - 3 = 0$ or $2x + 3 = 0$

$x = \frac{3}{2}$ or $x = -\frac{3}{2}$

This is the **difference of two squares**. Use $x^2 - a^2 = (x - a)(x + a)$.

## Worked example — Grade 7

The diagram shows a trapezium. All measurements are in centimetres. The area of the trapezium is 65 cm².




Work out the value of $x$.

Area of trapezium $= \frac{1}{2}(x + 4 + x - 1)(2x)$

$= \frac{1}{2}(2x + 3)(2x)$

$= 2x^2 + 3x$

Hence: $2x^2 + 3x = 65$

$2x^2 + 3x - 65 = 0$ ← Always write the equation in the form $ax^2 + bx + c = 0$.

$(2x + 13)(x - 5) = 0$

$2x + 13 = 0$ or $x - 5 = 0$

$x = -\frac{13}{2}$ or $x = 5$

$x = 5$ because it cannot be negative.

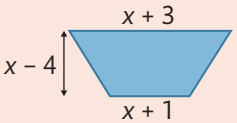
## Exam-style practice — Grade 7

**1** Solve

**(a)** $9x^2 - 25 = 0$ **[2 marks]**

**(b)** $3x^2 - 11x = 4$ **[3 marks]**

**2** The diagram shows a trapezium. All measurements are in centimetres. The area of the trapezium is 55 cm².




**(a)** Show that $x^2 - 2x - 63 = 0$. **[2 marks]**

**(b)** Work out the value of $x$. **[2 marks]**