Scalars and vectors

Many physical quantities have a direction as well as a magnitude. You must take the direction into account when the quantities are combined.

Scalars and vectors

Scalars have magnitude but no direction.

Scalars include:

- mass
- temperature
- distance
- speed
- energy
- power.

Distance is the magnitude of displacement, and speed is the magnitude of velocity.

Vectors have magnitude and direction.

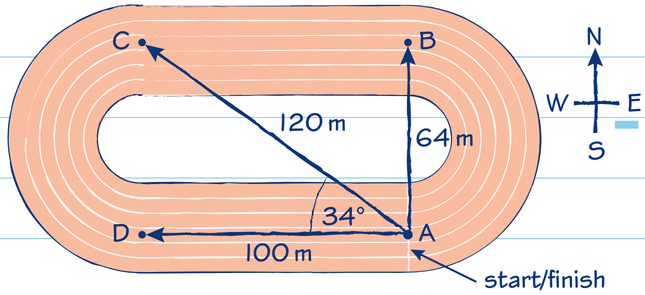
Vectors include:

- displacement
- velocity
- force
- momentum.

Scalars and vectors on a running track

Most athletics tracks are ovals with a distance of 400 m per lap. The straights are the same length as the curves (100 m).

A child runs one lap in 80 s. Consider the distances and displacements of the child as he runs from A to B, C and D.



From A to …	Distance	Displacement
B	100 m	64 m north (0°)
C	200 m	120 m 34°
D	300 m	100 m west
A	400 m	0 m

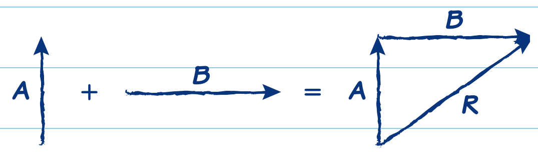
See how taking the direction into account changes things!

Adding vectors

Vectors can be represented by arrows whose…

- length represents the magnitude of the vector
- direction represents the direction of the vector.

Find the sum (resultant, ***R***) of two vectors ***A*** and ***B*** by putting them end to end.



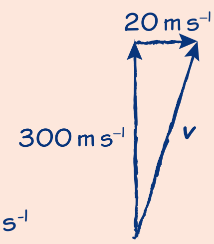
When two vectors are perpendicular we can calculate the magnitude of the resultant using Pythagoras' theorem. If the magnitudes of ***A*** and ***B*** above are 3 and 4 then the magnitude of ***R*** will be $5 = \sqrt{(3^2+4^2)}$.

Worked example

An aircraft is flying due north at 300 m s^{-1} and the wind is moving the air due east at 20 m s^{-1}.

(a) What is the speed of the aircraft relative to the ground? **(3 marks)**

Use Pythagoras' theorem to find the magnitude of the resultant vector.



Drawing a vector diagram makes it much easier to solve the problem!

$v = \sqrt{(300^2 + 20^2)} = 301\text{ m s}^{-1}$

(b) In what direction does the aircraft travel? **(2 marks)**

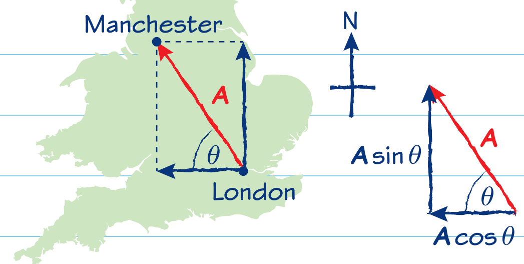
The tangent of the angle to north is

$\tan\theta = \dfrac{20}{300} = 0.0667$. Therefore

$\theta = 3.8°$ E of N

Resolving vectors

Manchester is north and west of London. The displacement vector from London to Manchester can be resolved into two perpendicular components, one pointing north and one pointing west, with magnitudes $A\cos q$ and $A\sin q$.



Any vector can be resolved into components along perpendicular axes in this way.

Now try this

1 Look at the example of the 400 m runner above.
(a) What is his average speed for one lap? **(1 mark)**
(b) What is his average velocity for one lap? **(1 mark)**

2 A river is flowing due east at 1.2 m s^{-1}. I can swim at a steady speed of 2.0 m s^{-1} relative to the water. I set off from the river bank swimming due north. What is my velocity vector relative to the bank? **(3 marks)**

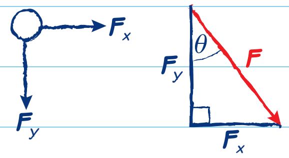
Vector triangles

Problems involving three vectors can often be solved by drawing a triangle and finding the lengths of its sides or the sizes of its angles.

Finding the resultant of two coplanar vectors

When two vectors are added the resultant can be found from the third side of the vector triangle.

The object right has two forces acting on it. The resultant is found by solving the right-angled triangle.



Vector $\boldsymbol{F} = \boldsymbol{F}_x + \boldsymbol{F}_y$

Magnitude $F^2 = F_x^2 + F_y^2$ (Pythagoras)

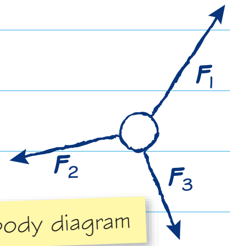
so $F = \sqrt{(F_x^2 + F_y^2)}$

$\tan\theta = \dfrac{F_x}{F_y}$

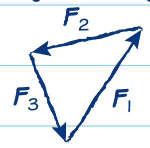
Equilibrium of three coplanar forces

If all the forces acting on a body are in equilibrium then their resultant is zero. This means that when drawn end to end they must form a closed polygon. If there are just three forces this will form a **triangle of forces**.

An unknown force can be found by solving the triangle.



Free-body diagram



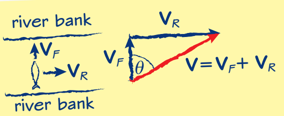
Triangle of forces

If it is a right-angled triangle then Pythagoras' theorem can be used to solve for the magnitude or angle of the unknown vector.

Maths skills: Relative velocity problems (addition of vectors)

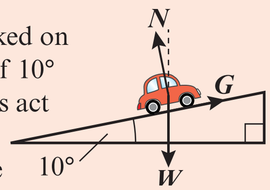
The velocity vector relative to the bank, $\boldsymbol{v}$, of a fish swimming in a moving river is the sum of the fish's velocity relative to the river, $\boldsymbol{v}_F$, and the velocity of the river relative to the bank, $\boldsymbol{v}_R$.

Solve problems like this using a velocity vector triangle.

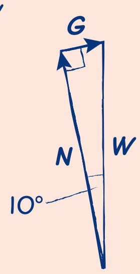


Worked example

The diagram shows a car parked on a hill. The hill is at an angle of 10° to the horizontal. Three forces act on the car: its weight of 12 kN, a normal contact force N at 90° to the road and a frictional force G parallel to the road. Use a triangle of forces to find the magnitudes of G and N. **(2 marks)**



The forces are in equilibrium so they form a triangle. G and N are perpendicular so it is a right-angled triangle.



$N = W\cos 10° = 12\,000\cos 10°$
$= 11\,800\text{ N}$

$G = W\sin 10° = 12\,000\sin 10°$
$= 2100\text{ N}$

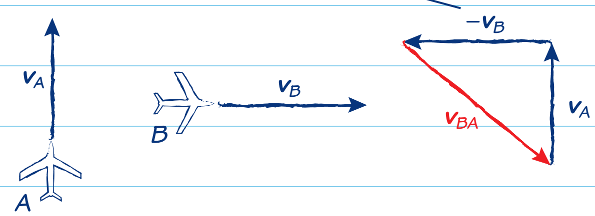
G could also be found using W and N and applying Pythagoras' theorem, or both N and G could be found by scale drawing.

Relative velocity problems (subtraction of vectors)

Imagine two aircraft, A and B, travelling in different directions. The relative velocity, $\boldsymbol{v}_{AB}$, of A as it appears to B is found by subtracting the velocity of B relative to the air, $\boldsymbol{v}_B$, from the velocity of A relative to the air, $\boldsymbol{v}_A$.

To subtract a vector, simply add a vector of the same magnitude pointing in the opposite direction.

$\boldsymbol{v}_{AB} = \boldsymbol{v}_A - \boldsymbol{v}_B$



Now try this

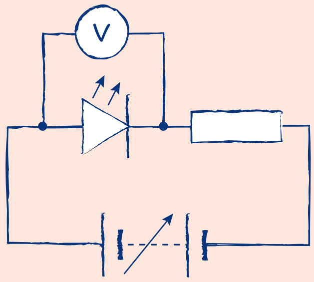
1 Look at the diagram of the car on the hill. Find the magnitudes of N and G if the road makes an angle of 20° to the horizontal. **(2 marks)**

2 A ferry boat sails between two jetties on opposite sides of a wide river that flows due east. The boat has a speed relative to the water of 5.0 m s^{-1} and the river is flowing at 3.0 m s^{-1}. In what direction must the boat set sail from the southern side in order to travel directly to the jetty on the northern side? **(3 marks)**

Exam skills

This exam-style question uses knowledge and skills you have already revised. Have a look at pages 76–80 for a reminder about the quantum nature of light.

Worked example



A red LED emits light of wavelength 630 nm. It is connected into a circuit like the one shown, and the supply voltage is slowly increased until the LED is just seen to glow. The reading on the voltmeter at this moment is 1.96 V.

(a) Calculate the work done (in joules) on a single electron as it moves through the LED. **(2 marks)**

$W = eV = 1.60 \times 10^{-19} \times 1.96$
$= 3.14 \times 10^{-19}$ J.

(b) Assume that all of the energy supplied to the electron as it moves through the LED is transferred to one photon of red light of wavelength 630 nm. Use this assumption to calculate a value for the Planck constant. **(4 marks)**

Photon energy $E = hf = \frac{hc}{\lambda}$.

If all of the work done on the electron is transferred to one photon, then $E = 3.14 \times 10^{-19}$ J.

Rearranging to make h the subject:

$$h = \frac{E\lambda}{c} = \frac{3.14 \times 10^{-19} \times 630 \times 10^{-9}}{3.0 \times 10^{8}}$$
$$= 6.59 \times 10^{-34} \text{ J s}$$

(c) When the supply voltage is increased, the LED glows more brightly but the colour of the light it emits is unchanged. Explain this in terms of photons. **(4 marks)**

Increasing the supply voltage will increase the current flowing through the LED. More electrons per second will pass through the LED, so more photons per second will be emitted. This accounts for the increase in brightness. The fact that the colour of the light does not change means that the wavelength is still the same, so the photons have the same energy as before. Therefore each electron is transferring the same amount of energy to a photon as it passes through the LED.

(d) Suggest what the observations in part (c) imply about how the potential differences across the LED and resistor change as the supply voltage is increased. **(3 marks)**

If the energy transferred by each electron is unchanged, then the voltage across the LED is unchanged. The sum of the p.d. across the LED and the p.d. across the resistor must equal the supply voltage, so the p.d. across the resistor must increase as the supply voltage increases.

Command word: Calculate

When you are asked to calculate something you should always show your working.

It is good practice to start with the algebra (rearranging equations if necessary), then substitute values and finally calculate your answer.

Don't forget to include the units! If you get stuck, you can work out the units from the equation.

Command word: Explain

If a question asks you to 'explain' something, make sure you:

- Address all parts of the question: in part (c) there are **two** things to explain.
- Respond to any directions in the question: in part (c) you are asked to explain **in terms of photons**, so make sure that is what you do.
- Think first and write your answer in a clear logical sequence.

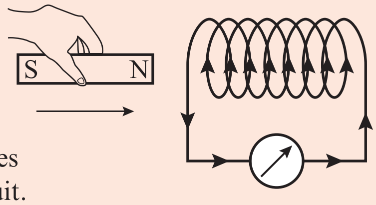
Notice how the answer to part (d) has used Kirchhoff's second law (see page 51 for an explanation of this very useful circuit law).

Exam skills

This exam-style question uses knowledge and skills you have already revised. Have a look at pages 135 and 136 for a reminder about magnetic flux linkage and Faraday's and Lenz's laws.

Worked example

The diagram shows a simple experiment used to demonstrate electromagnetic induction.



As the bar magnet is moved towards one end of the coil the ammeter indicates a small current flowing around the circuit.

(a) Explain how the current is generated. **(4 marks)**

The bar magnet creates a **magnetic field** in the space around it. As the bar magnet moves towards the coil, the **magnetic flux** through the coil changes.

Faraday's law states that there will be an **induced e.m.f.** in the coil equal to the **rate of change of magnetic flux linkage**.

The circuit is complete, so the **e.m.f.** results in a flow of **current**.

(b) State and explain the effect of reversing the direction of motion of the bar magnet and moving it away from the coil at a higher speed. **(4 marks)**

The direction of current in the circuit would reverse and the magnitude of the current would increase.

The current reverses because the magnet has changed direction. This changes the sign of the rate of change of flux linkage and so reverses the sign of the induced e.m.f.

The current increases in magnitude because the rate of change of flux linkage is greater.

(c) The current induced in the coil dissipates some energy in the circuit. Use Lenz's law to explain how energy is conserved in this experiment. **(5 marks)**

- The induced current makes the coil into an electromagnet that exerts forces on the bar magnet.
- Lenz's law states that the induced e.m.f. is in a direction such as to oppose the change that caused it.
- This means that the current is in such a direction that it exerts a repulsive force on the bar magnet when the magnet approaches and an attractive force on it when it is moved away.
- The person moving the magnet must do work against these forces.
- The energy transferred by this work is equal to the energy dissipated as heat in the circuit.

(d) When the bar magnet is moved towards the coil, the flux through the circuit changes by 8.0×10^{-4} Wb in 0.50 s. The coil has 20 turns and the total resistance of the circuit is 5.0 Ω. Calculate the average induced current during this time. **(4 marks)**

Change of flux linkage $= 8.0 \times 10^{-4} \times 20 = 0.016$ Wb

Rate of change of flux linkage $= \dfrac{0.016}{0.50} = 0.032\ \text{Wb s}^{-1}$

$= \text{induced e.m.f.}$

Average current $= \dfrac{\text{induced e.m.f.}}{\text{resistance}} = \dfrac{0.032}{5.0} = 6.4 \times 10^{-3}$ A

$= 6.4$ mA.

The introduction mentions electromagnetic induction – this should alert you to use Faraday's and Lenz's laws and to think about magnetic field strength and flux.

When explaining try to include relevant technical terms – this helps to show that you understand the important underlying physics.

Read the question carefully and make sure you answer all parts of it. In this question there are two things to state and two explanations to give.

An alternative approach to answering (b) would be to start by quoting Faraday's and Lenz's laws in the form:

$$E = -\frac{\Delta(N\phi)}{\Delta t}$$

and then indicating how the changes affect this.

This part asks you to use Lenz's law, so it is important to show how you have used it. The simplest way to do this is to state the law before you apply it.

Using bullet points for a question worth several marks helps to ensure that you are making separate points. It also sets up a logical structure to your answer.

Notice how this has been structured – whilst it would be acceptable to quote Faraday's law and substitute all the values straight into the equation, it is often helpful to break up the calculation into simple steps.

The nuclear atom

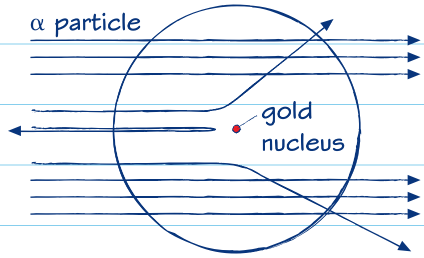
At the beginning of the twentieth century a series of experiments established the nuclear model of the atom.

Alpha particle scattering

The nuclear model of the atom was developed following the results of an experiment in which alpha particles were directed at thin gold foil.

The vast majority of the alpha particles experienced no or very little deflection as they passed through the foil, but a tiny fraction deflected through very large angles. Some alpha particles even rebounded back from the foil.

The nuclear model



The **nuclear model** of the atom explains the alpha particles' deflection: an atom is made mostly of empty space but has a tiny, dense nucleus containing all of the positive charge in the atom and most of its mass. Nuclei have a radius of a few femtometres (10^{-15} m) whereas atomic radii are measured in tens or hundreds of picometres (1 pm = 10^{-12} m). If the nucleus of an atom were enlarged to the size of a tennis ball, its electrons would be a kilometre away.

The nucleus consists of protons and neutrons (collectively called nucleons). The **proton number** (or **atomic number**), Z, identifies the element, and it is also equal to the number of electrons surrounding the nucleus in the neutral atom. The **mass number** or **nucleon number**, A, is the total number of nucleons in the nucleus.

Notation

We use symbols to represent the type of a nucleus (nuclide) as:

$$^{A}_{Z}\mathrm{X}$$

X is its chemical symbol.

A carbon nucleus normally consists of 6 protons and 6 neutrons, so we write

$$^{12}_{6}\mathrm{C}$$

for the major isotope of carbon.

Isotopes

Nuclides with the same number of protons but different numbers of neutrons are called **isotopes**. Isotopes have different mass numbers, A.
For example, carbon-14 is a radioactive isotope of carbon.

$$^{14}_{6}\mathrm{C}$$

Although it has 6 protons in its nucleus, there are 8 neutrons. The extra neutrons lead to this isotope being unstable.

Worked example

Write the symbols for the following nuclides: aluminium, with 13 protons and 14 neutrons; sulfur, with 16 neutrons and 16 electrons. **(2 marks)**

Al: $Z = 13$, $A = 13 + 14 = 27$ so $^{27}_{13}\mathrm{Al}$

S: number of electrons = number of protons in neutral atom so $Z = 16$;
$A = 16 + 16 = 32$ so $^{32}_{16}\mathrm{S}$

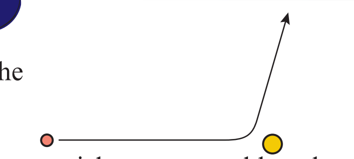
In the neutral atom there will always be the same number of electrons as there are protons in the nucleus.

Light nuclei have about the same number of neutrons as protons, but in heavy nuclei the ratio of neutrons to protons is about 3 : 2.

Remember: You do not have to memorise the chemical symbols for elements, or their proton numbers.

Now try this

1 The figure shows the path of an alpha particle passing near a nucleus.



(a) Add to the diagram to show the force on the alpha particle in the position where the force is a maximum.

(b) The nucleus is replaced with one that has a smaller proton number. Draw on the diagram the path of an alpha particle that starts with the same velocity and position as that of the alpha particle drawn. **(2 marks)**

2 Complete the table for the following nuclides:

Number of protons	Number of neutrons	Number of electrons	Symbol
2	2	2	$^{4}_{2}\mathrm{He}$
8	8		O
	138	88	Ra
			$^{238}_{92}\mathrm{U}$

(3 marks)

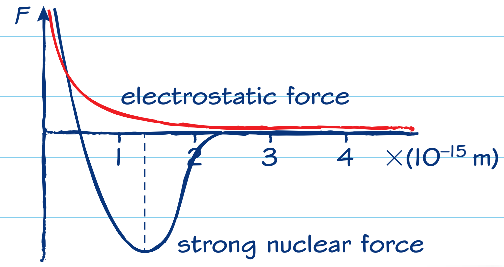
Nuclear forces

At the tiny distances in the nucleus, protons and neutrons experience the strong nuclear interaction.

The strong nuclear force

As protons are brought together, the electrostatic (coulomb) repulsive force acting on them grows larger and larger.

Gravitation is a weak interaction and cannot hold the protons together. Therefore there must be another very strong force acting between protons at nuclear separations. This force acts between all nucleons and is called the **strong nuclear force**.



The variations in forces between protons over distance: positive values are repulsive.

Nuclear density

The strong nuclear force is very short range. At separations less than about 3×10^{-15} m it is attractive, but it becomes repulsive at separations less than about 0.5×10^{-15} m.

This force maintains the nucleons at an approximately constant separation in the nucleus, so nuclear density is about the same for all nuclides, about $2 \times 10^{17}\,\text{kg m}^{-3}$.

This compares with the largest density of a macroscopic material, which is about $2 \times 10^{4}\,\text{kg m}^{-3}$.

Nucleon number and nuclear radius

$\rho = \frac{m}{V}$ where ρ is the density

So for a spherical nucleus:

$\rho = \dfrac{A \times m_N}{\frac{4}{3}\pi R^3}$ where m_N is the nucleon mass and R is the nuclear unit radius.

$\therefore R^3 \propto A$ so $R = r_0 A^{\frac{1}{3}}$ where r_0 is a constant, the value for R when A is one (that is, the hydrogen nuclear radius).

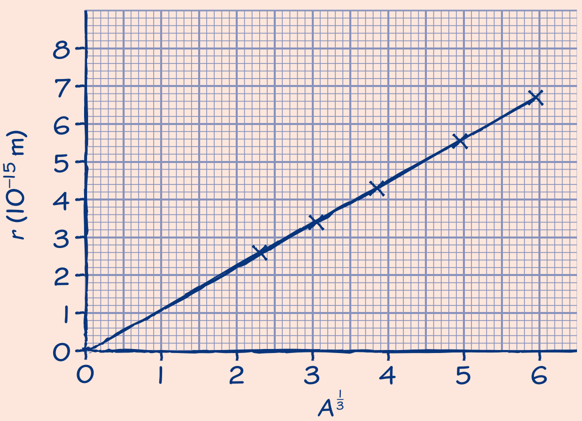
Experimental results confirm this relationship between nuclear radius and nucleon number.

Worked example

The table shows data for a range of nuclei.

Element	$r(10^{-15}$ m)	A
C	2.66	12
Si	3.43	28
Fe	4.35	56
Sn	5.49	120
Pb	6.66	208

Use the data to plot a straight-line graph and hence estimate the value of r_0. **(5 marks)**



Gradient $= r_0 = 1.1 \times 10^{-15}$ m

Maths skills Note that since we know that $r = r_0 A^{\frac{1}{3}}$ the easiest way to obtain a straight-line graph is to plot the nuclear radius against the cube root of the nucleon number.

An alternative method would be to find the log values of r and A.

If $r = r_0 A^{\frac{1}{3}}$ then $\log r = \frac{1}{3}\log A + \log r_0$

So a graph of $\log r$ against $\log A$ would yield a straight line with y-intercept equal to $\log r_0$.

A log–log plot can be used to reduce a power relationship to a linear relationship.

Now try this

It is known from electron diffraction experiments that the radius of a gold nucleus (Au-197) is about 7.0×10^{-15} m.

(a) Calculate the density of matter in the nucleus. **(4 marks)**

(b) Determine the radius of an iron nucleus, Fe-56. **(3 marks)**