Writing equations

An **equation** is a mathematical sentence that tells you that the quantities on either side of the = sign are equal. You use a letter to represent an unknown quantity or **variable**. You can use an equation to describe a word problem. The equation $3x - 6 = 12$ has one variable (x). It tells you that if you multiply x by 3 and then subtract 6, the answer is 12.

$$3x - 6 = 12$$

Golden rules

- ✓ Read the question carefully and choose a letter to represent the unknown.
- ✓ Write an expression that describes the situation.

  Three times a number subtract one → $3n - 1$
- ✓ Use the information in the question to put your expression equal to a known number.

  Three times a number subtract one equals 29 → $3n - 1 = 29$

For a reminder on writing expressions turn to page 29.

Worked example

Posters cost £3 each plus a one-off charge for postage of £2. Max spends £17 buying posters. Write an equation for this word problem.

Cost of posters is $3 \times p = 3p$

Add the postage charge of £2 → $3p + 2$

$3p + 2 = 17$

Choose a letter to represent the unknown (p).

Write an expression for the cost of the posters.

Add the postage charge. Make your expression equal to the amount Max spends.

Worked example

The perimeter of a regular hexagon is 30 cm. Write an equation for this word problem using s for the length of one side.

$s + s + s + s + s + s = 6s$

$6s = 30$

Problem solved!

A regular hexagon has six equal sides. The perimeter is the distance around the hexagon, so is the sum of all the sides. Use the information that the perimeter = 30 cm to write the equation.

You'll need brilliant problem-solving skills to succeed in GCSE – get practising now!

Worked example

Write each statement as an equation. Use n for the unknown number.

(a) Half of a number is eight.

$\frac{n}{2} = 8$

(b) Double a number, add seven, is eleven.

$2n + 7 = 11$

(c) Six times a number, subtract nine, is three.

$6n - 9 = 3$

(d) The square of a number is 144

$n^2 = 144$

- Use real numbers or empty boxes if you are not sure.
- Half a number is written as a fraction with denominator 2.
- Double a number is 2 × the number, written as $2n$.
- The square of a number is written as n^2.

Now try this

Write an equation for each word problem. Use the letter in brackets for the unknown number.

(a) Five times a number (n) subtract seven is eighteen.

(b) The perimeter of a regular octagon of side length (s) is 32 cm.

(c) My family bought cinema tickets (t) at £9 each, and there was a separate booking fee of £1. The total cost was £64.

Algebra problem-solving

You can use algebra to solve problems in other areas of maths.

Worked example

A chef uses a formula to calculate the length of time needed to cook a turkey. The cooking time t (in minutes) is 40 minutes for every kilogram plus an extra 50 minutes.

(a) Write a formula for the time t it takes to cook a turkey weighing w kg.

$t = 40w + 50$

(b) Use the formula to work out the time needed to cook a 4.5 kg turkey.

$t = 40 \times 4.5 + 50 = 230$ minutes

The time is 3 hours 50 minutes

(c) The chef cooks a turkey for 3 hours. What was its weight?

3 hours = 180 minutes

$180 = 40w + 50$

$40w = 130$

$w = 3.25$ kg

Problem solved!

(a) The subject of the formula is the time, so the formula begins $t = \ldots$
Each kilogram needs 40 minutes ($40w$), then add the extra 50 minutes.

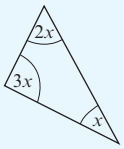
(b) Substitute $w = 4.5$ into the formula.

(c) First convert the time in hours to minutes. Then substitute the values you know and rearrange the formula to solve for w.

You'll need brilliant problem-solving skills to succeed in GCSE – get practising now!

Worked example

(a) Write an equation for the sum of the angles in this triangle.



$x + 2x + 3x = 6x$

$6x = 180°$

(b) Solve the equation to find the value of x.

$x = 180° \div 6 = 30°$

(c) Work out the size of each angle.

$x = 30°$

$2x = 2 \times 30° = 60°$

$3x = 3 \times 60° = 90°$

Angles are 30°, 60°, 90°

(d) What type of triangle is this?

One angle is 90°, so it is a right-angled triangle.

Problem solved!

(a) Add up all the angles. The angles in a triangle add to 180° so use this to form an equation.

(b) Divide both sides by 6.

(c) Substitute $x = 30°$ to find the other angles.

You'll need brilliant problem-solving skills to succeed in GCSE – get practising now!

Now try this

This graph converts approximately between ounces and grams.

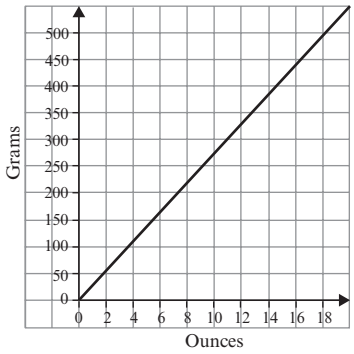
(a) Convert 10 ounces to grams.

(b) Convert 125 grams to ounces.

(c) Convert 1 kg to pounds and ounces.

(d) 16 ounces (oz) = 1 pound (lb). Convert 1 lb 10 oz to grams.

(e) Explain why it is important to use either ounces or grams in a recipe and not a mixture of both.



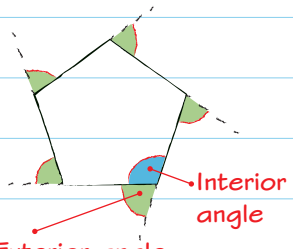
For part (c), the vertical scale doesn't go up to 1 kg, so pick a number on the scale that you can easily scale up to 1 kg.
1 kg = 1000 g = 2 × 500 g

Angles in polygons

Polygons are 2D shapes with straight sides. You need to be able to identify interior and exterior angles in polygons.

Exterior angle + interior angle = 180°

Angles on a straight line add up to 180°



Formulae for angles in polygons

Learn these formulae for a polygon with n sides.

1. Sum of interior angles $= (n - 2) \times 180°$
2. Sum of exterior angles $= 360°$

Worked example

(a) What is the sum of the interior angles of a heptagon?

A heptagon has 7 sides so $n = 7$

Sum of interior angles $= (n - 2) \times 180°$

$= (7 - 2) \times 180°$

$= 5 \times 180° = 900°$

(b) The sum of the interior angles of a polygon is 1080°. How many sides does it have?

$(n - 2) \times 180° = 1080°$

$n - 2 = 1080 \div 180$

$n - 2 = 6$

$n = 8$

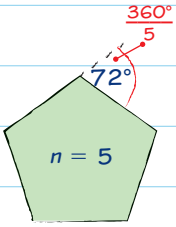
The shape has 8 sides.

For a reminder on solving equations turn to pages 39 and 40.

Regular polygons

A regular polygon has equal sides and equal angles.

This regular pentagon has 5 equal sides and 5 equal exterior angles.



The sum of the exterior angles of any polygon is 360°.

So each exterior angle of a regular polygon of n sides is $\frac{360°}{n}$

Each exterior angle of this pentagon is $\frac{360°}{n} = 72°$

Each interior angle is $180° - 72° = 108°$

Names of polygons

Learn the names of these polygons:

Quadrilateral – 4 sides

Pentagon – 5 sides

Hexagon – 6 sides

Heptagon – 7 sides

Octagon – 8 sides

Nonagon – 9 sides

Decagon – 10 sides

Worked example

(a) Work out the exterior angle of a regular decagon.

Sum of exterior angles is 360°.

So exterior angle $= 360° \div 10 = 36°$

(b) Calculate one of the interior angles.

Exterior angle + interior angle = 180°

Interior angle $= 180 - 36 = 144$

Interior angle is 144°

Now try this

1 Find the sum of the interior angles of a 12-sided shape.

Use the formula Sum $= (n - 2) \times 180°$

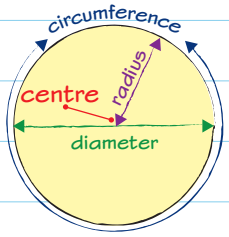
2 (a) Calculate one of the exterior angles in a regular nonagon.

(b) Calculate one of the interior angles.

Interior angle + exterior angle = 180°

Circumference

You need to know these definitions of parts of a circle.



Circumference is the perimeter of a circle.

Diameter is the distance across the circle through the centre.

Radius is the distance from the centre to any point on the circumference. It is half of the diameter.

Radius $= \frac{1}{2} \times$ diameter $\quad r = \frac{d}{2}$

Diameter $= 2 \times$ radius $\quad d = 2r$

They are all distances, so are measured in units such as mm, cm, m and km.

Here are two formulae you can use to calculate the circumference.

1. Circumference $= \pi \times$ diameter
   $C = \pi d$
2. Circumference $= 2 \times \pi \times$ radius
   $C = 2\pi r$

π

π is the Greek letter 'pi'.

$\pi = 3.1415926...$

You can round π to 3.142 in calculations.

A scientific calculator will have a button for π. You might need to press the SHIFT key first.

If your calculator leaves π in the answer, press the S⇔D button to get your answer as a decimal.

Golden rule

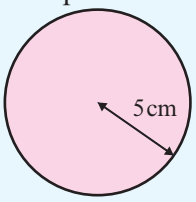
Write whether you have the radius or the diameter first so that you use the correct formula.

You may be asked to write your answer in terms of π. Just calculate with the numbers and leave π in your answer.

$C = 2 \times \pi \times 5 = 10\pi$

Worked example

Work out the circumference of this circle.
Use $\pi = 3.142$.
Give your answer to 1 decimal place.



$r = 5$ cm

Circumference $= 2\pi r$
$= 2 \times 3.142 \times 5$
$= 31.42$
$= 31.4$ cm (1 d.p.)

Worked example

Calculate the diameter of a circle with circumference 15 m. Write your answer to 1 decimal place.

$C = \pi d$
$15 = \pi d$
$d = 15 \div \pi = 4.7746... = 4.8$ m (1 d.p.)

You need to rearrange the equation $C = \pi d$ to make d the subject.

$C = \pi d \ (\div \pi)$

$d = C \div \pi$

For a reminder about rearranging formulae turn to page 43.

You need to find the radius – use the correct formula.

Now try this

1 Work out the circumference of a circle with
(a) radius 3 cm
(b) diameter 7 cm.

2 A circle has a circumference of 20 cm. Work out the radius of the circle.

Whole numbers

When you compare and order **whole numbers**, remember that the more digits a number has, the larger it is. When two numbers have the same number of digits, compare the digits, one by one, starting from the left.

3497 and 3502 both have **4 digits**. They both have **3 thousands**. 3497 has **4 hundreds** but 3502 has **5 hundreds**, so 3502 is larger.

Worked example

(a) Write the number 34 507 in words.

Thirty-four thousand, five hundred and seven

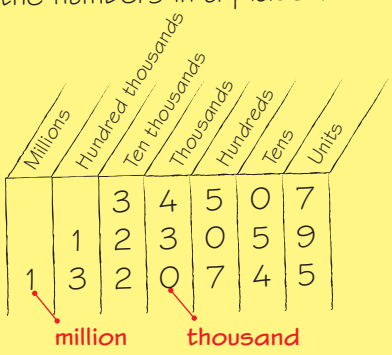
(b) Write the number 1 320 045 in words.

One million, three hundred and twenty thousand and forty-five

(c) Write five hundred and nine thousand and four in figures.

509 004

Write the numbers in a place value diagram:



The value of a digit depends on its position in the number.

The thousands digits are the same, so compare the hundreds.

There is only 5-digit number, so it is the largest.

There is only one 3-digit number, so it is the smallest.

Worked example

(a) Write the value of the digit 4 in
(i) 2411 (ii) 43 850 (iii) 994 (iv) 4 987 653

(i) 400 (ii) 40 000 (iii) 4 (iv) 4 000 000

(b) Write these numbers in order of size.
Start with the **smallest** number.
2408 2954 43 850 2411 944

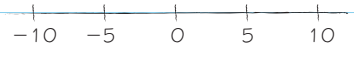
944 2408 2411 2954 43 850

(c) Write these numbers in order of size.
Start with the **largest** number.
375 890 380 001 2 000 000 99 999

7 digit > 6 digit and 380 000 > 375 000
2 000 000 380 001 375 890 99 999

Negative numbers

Numbers less than zero are negative numbers. On this number line, the negative numbers are to the left of 0. The further the number is to the left, the less its value.



−10 is less than −5

Worked example

Write these numbers in order of size.
Start with the **smallest** number.

−3	6	0	−9	10	−2	4
−9	−3	−2	0	4	6	10

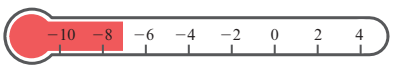
Now try this

1 The population of a town is 95 364.
(a) Write the number 95 364 in words.
(b) What is the value of the digit 5 in 95 364?

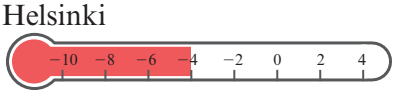
2 Write these numbers in order of size. Start with the **smallest** number.
(a) 21 089 20 098 1 000 010 3756 21 465 3765
(b) −3 6 −11 4 9 −1

3 Look at the thermometers. Which city has the lower temperature?

Moscow



Helsinki



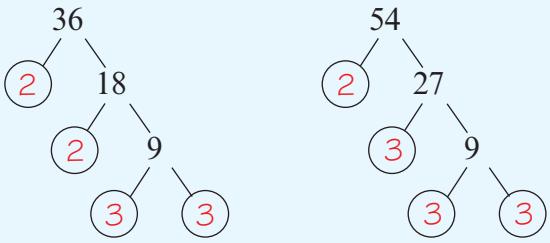
HCF and LCM

You can use prime factor decomposition to find the HCF and LCM of sets of numbers.

For a reminder about HCF and LCM turn to page 11, and for prime factor decomposition turn to page 15.

Worked example

Complete the factor trees and write the prime factor decompositions for 36 and 54. Use these to work out the HCF and LCM of 36 and 54.



$36 = \underline{2} \times 2 \times \underline{3} \times \underline{3}$
$54 = \underline{2} \times \underline{3} \times \underline{3} \times 3$
HCF $= 2 \times 3 \times 3 = 18$
LCM $= 18 \times 2 \times 3 = 108$

1. Complete the prime factor trees.
2. Write 36 and 54 as products **without index notation**.
3. Underline the common factors.
4. To find the **HCF** of 36 and 54, multiply the common factors together.
5. To find the **LCM**, multiply the HCF by the remaining factors.

For an alternative method of finding the HCF and LCM, turn to page 11.

Problem solved!

The total number of brownies must be a multiple of 30. The number of flapjacks must be a multiple of 45. You can answer this question by finding the LCM.

- Write each number as a product of its prime factors, without indices.
- Multiply together the factors common to both products to find the HCF.
- Multiply the HCF by the remaining factors to find the LCM.
- Work out how many batches of each type of product the bakery needs to make.

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Worked example

A bakery makes brownies in batches of 30 and flapjacks in batches of 45.

What is the lowest number of batches of each type of product the bakery should make to have the same number of each type?

$30 = 2 \times \underline{3} \times \underline{5}$
$45 = 3 \times \underline{3} \times \underline{5}$
HCF $= 3 \times 5 = 15$
LCM $= 15 \times 2 \times 3 = 90$
They need 90 of each type.
$90 \div 30 = 3$, $90 \div 45 = 2$
3 batches of brownies and 2 batches of flapjacks

Now try this

1 $32 = 2^5$ and $36 = 2^2 \times 3^2$. Work out the HCF and LCM of 32 and 36.

2 Find the HCF and LCM of 72 and 120.

3 Sue has two rolls of coloured tape. One is 48 m long and the other is 32 m. She wants to cut both of them into pieces of the same length so that no tape is left over. What is the longest length she can cut them into?

Work out the HCF of 32 and 48.

Work out the LCM of 24 and 18.

4 A baker makes large and small bread rolls. He can fit 24 small rolls and 18 large rolls on his baking trays. He wants to make the same number of each size. What is the minimum number of trays he can make of each size?

Probability tree diagrams

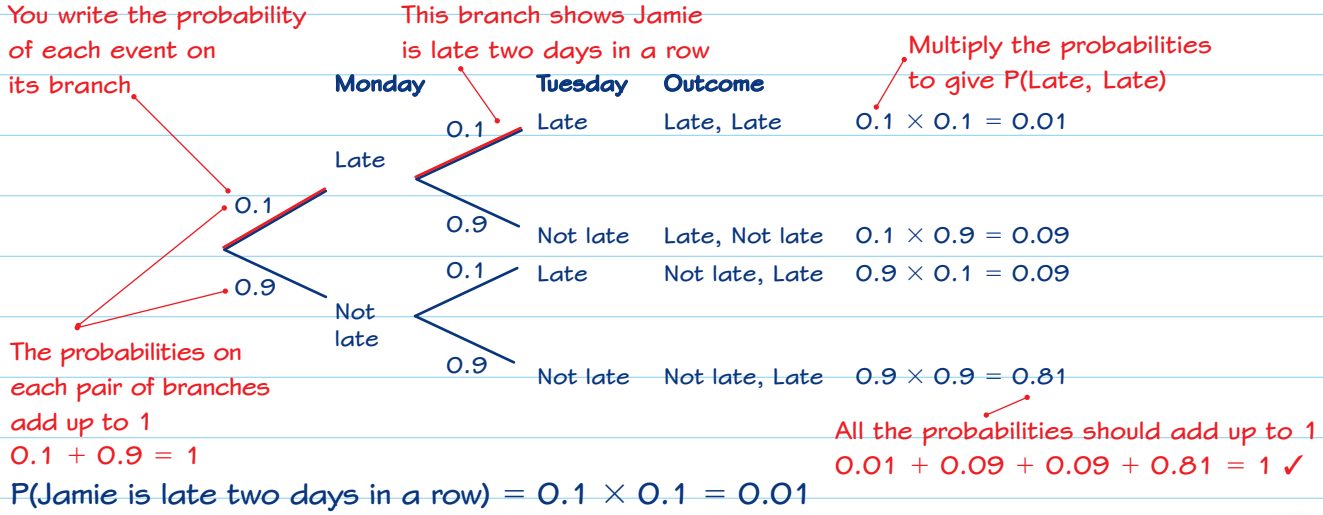
A **tree diagram** shows all the outcomes of two or more events.

You work out the probability of different outcomes by multiplying along the branches.

Probabilities on tree diagrams

The probability of Jamie being late for school on any day is 0.1

The tree diagram shows the probability of Jamie being late for school two days in a row.



P(Jamie is late two days in a row) = 0.1 × 0.1 = 0.01

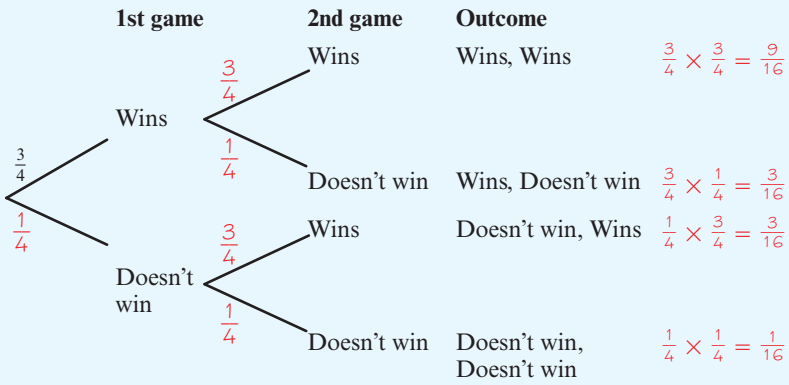
Worked example

Marcus plays tennis. He either wins a game or doesn't win.
The probability that Marcus wins a game is $\frac{3}{4}$.
He plays two games of tennis.

(a) What is the probability Marcus doesn't win a game?

P(Doesn't win) = 1 − P(Wins)
= $1 - \frac{3}{4} = \frac{1}{4}$

(b) Complete this probability tree diagram to show the probabilities of all the possible outcomes.



(c) Work out the probability that Marcus wins both games.

P(Wins, Wins) = $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

Now try this

The probability that Peter passes a maths test is $\frac{1}{2}$

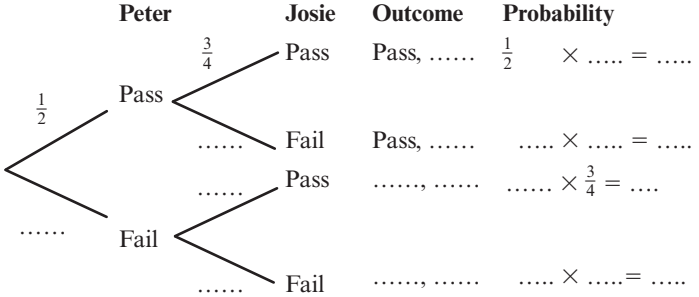
The probability that Josie passes the test is $\frac{3}{4}$

(a) What is the probability Peter does not pass the test?

(b) What is the probability that Josie does not pass the test?

(c) Complete this tree diagram.

(d) Work out the probability that both Peter and Josie pass the maths test.



Proportion

A proportion is a relationship between two numbers. You can write a proportion as a ratio, a percentage or a fraction.

2 : 3

$\frac{2}{5} + \frac{3}{5} = 1$

40% + 60% = 100%

3 : 1

$\frac{3}{4} + \frac{1}{4} = 1$

75% + 25% = 100%

5 : 3

$\frac{3}{8} + \frac{5}{8} = 1$

62.5% + 37.5% = 100%

For a reminder about ratios turn to page 57 and for equivalence turn to page 24.

Worked example

(a) The ratio of carrots to onions in a vegetable plot is 3:2. What fraction are carrots?

$\frac{3}{3+2} = \frac{3}{5}$

(b) In a class $\frac{1}{6}$ of students are left-handed. What is the ratio of left- to right-handed students in this class?

$1 - \frac{1}{6} = \frac{5}{6}$ are right-handed

left	:	right
$\frac{1}{6}$	:	$\frac{5}{6}$
1	:	5

(a) Add the parts of the ratio to find the total, to give the denominator of the fraction.

(b) Work out how many are right-handed. Then write the ratio in the order asked for.

Worked example

These ingredients make a pizza that serves 8 people:

400 g bread dough
120 ml tomato puree
240 g grated cheese

(a) How much bread dough is needed for 12 people?

400 g × $1\frac{1}{2}$ = 400 g + 200 g = 600 g

(b) Maisie has 120 g cheese. How many people can she make pizza for if she has enough of the other ingredients?

120 is half of 240

So halve the number of people: 8 ÷ 2 = 4 people

(c) Yasdi has 500 g dough, 160 ml tomato puree and 160 g cheese. Is that enough ingredients to serve 10 people?

400 ÷ 8 = 50 g dough per person

50 g × 10 = 500 g. Enough dough.

120 ÷ 8 = 15 ml tomato puree per person

10 × 15 = 150 ml. Enough tomato puree

240 ÷ 8 = 30 g cheese per person

30 g × 10 = 300 g. Not enough cheese.

No, he does not have enough cheese to make pizzas to serve 10 people.

(a) 12 is $1\frac{1}{2}$ × 8 so multiply the recipe quantity of bread dough by $1\frac{1}{2}$

(c) Work out how much he needs of each ingredient for one person and then multiply this by 10.

Now try this

1 60% of a group are female. What proportion of the group are male? Give your answer as a simplified fraction.

Convert the hours and minutes to minutes: 2.5 × 60 = 150 minutes and 1 hour 50 = 60 + 50 minutes. Divide the time by the number of components made to work out how long each worker takes to make 1 component.

2 Factory workers are making components. Which worker is the fastest, which two workers work at the same rate, and which worker is the slowest?

Worker	Number made	Time taken
A	15	2.5 h
B	20	3.5 h
C	18	3 h
D	12	1 h 50 min

Maps and scales

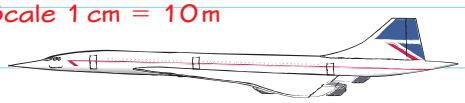
You can use the scales on maps and scale drawings to work out distances in real life.

This is a scale drawing of a plane. You can use the scale to find the length of the real plane. The scale drawing is 6 cm long so in real life the length of the plane is

	drawing	real life	
× 6	1 cm	10 m	× 6
	6 cm	60 m	

The plane is 60 metres long.

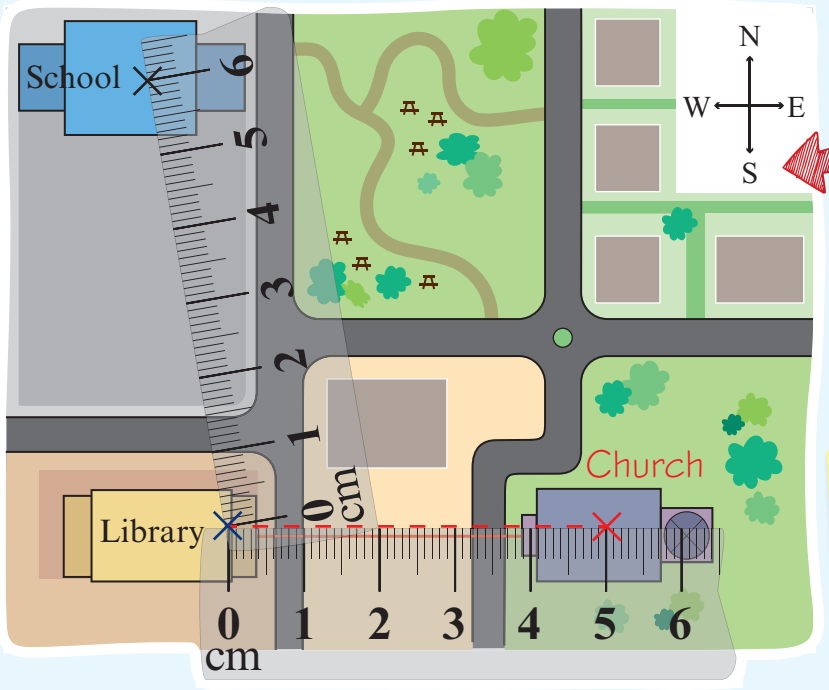
Scale 1 cm = 10 m



6 cm

Worked example

The diagram shows a scale drawing of a village.
Scale: 1 cm = 3 km



1. Use a ruler to measure the distance on the map from the library to the school. Distance on map = 6 cm
2. Now use the scale to work out the real distance.

	map	real	
× 6	1 cm	3 km	× 6
	6 cm	18 km	

	map	real	
× 5	1 cm	3 km	× 5
	5 cm	15 km	

(a) What is the distance between the school and the library? 3 × 6 = 18 km

(b) The church is 15 km east of the library. Mark its position with a cross.

Now try this

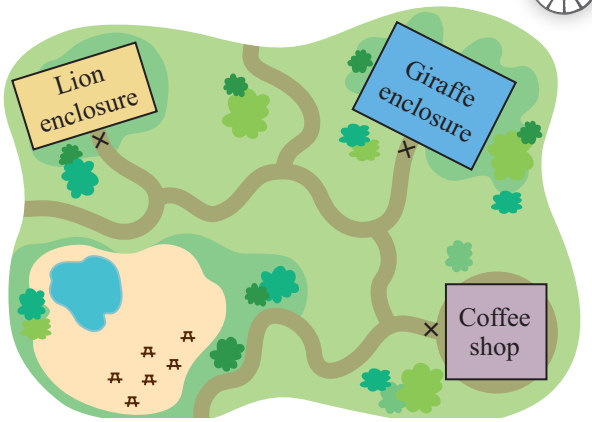
The map shows the attractions at a zoo.

(a) Work out the real distance between the coffee shop and the lion enclosure.

(b) Paula says the distance between the giraffe enclosure and the lion enclosure is less than 250 m. Use the map to show whether she is correct.

(c) The distance between the giraffe enclosure and the children's area is 400 m. What is the distance on the map?

Scale: 1 cm = 50 m



Rotation

A **rotation** is a type of transformation. You rotate a shape by turning it about a point called the **centre of rotation**.

To describe a rotation you need to give:

- the centre of rotation
- the angle of rotation
- the direction of rotation.

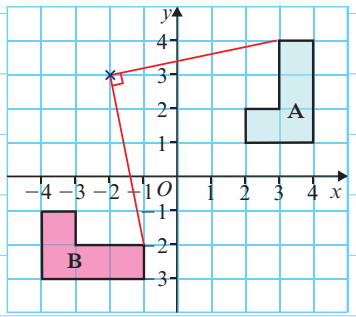
The centre of rotation may be the origin (0, 0) or another coordinate point.

For a reminder about coordinates turn to page 45.

The angle of rotation is given in degrees.

The direction of rotation will be clockwise or anticlockwise.

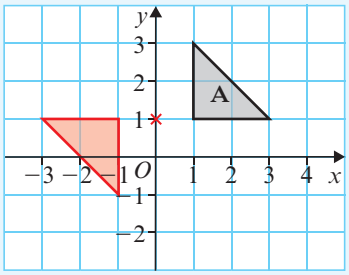
Rotated shapes are **congruent**.



A to B: Rotation 90° clockwise about the point (−2, 3)

Worked example

On the grid, rotate shape A through 180° about point (0, 1).



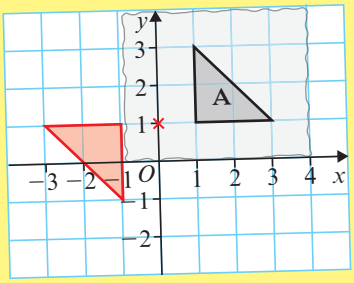
Everything in red is part of the answer.

Mark the centre of rotation (0, 1) with ×.

Trace the shape.

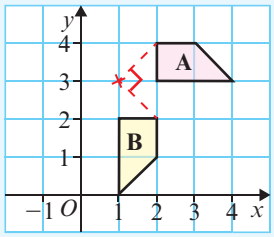
Put your pencil on the cross.

Turn the tracing paper 180° to rotate the shape.



Worked example

Describe fully the single transformation that maps shape A onto shape B.

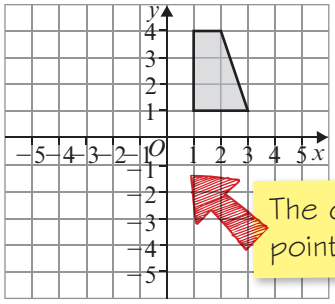


The transformation that maps shape A onto B is a rotation 90° clockwise about the point (1, 3)

To find the centre of rotation, trace the object shape then rotate the tracing paper, holding a point fixed with your pencil. Repeat for different points until your tracing covers the image.

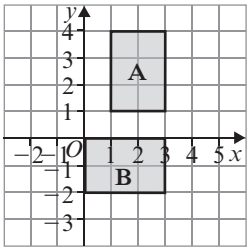
Now try this

1 Rotate this shape 180° about the origin.



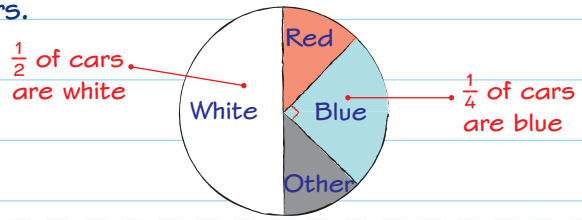
The origin is the point (0, 0).

2 Describe fully the single transformation that maps shape A onto shape B.



Pie charts

A **pie chart** is a circle divided into slices called sectors.
The whole circle represents a set of data.
Each sector represents a fraction of the data.
This pie chart shows the colours of cars in a car park.



Worked example

Kami asked 24 students about their favourite drinks. The table shows her results.
Draw a pie chart to show this information.

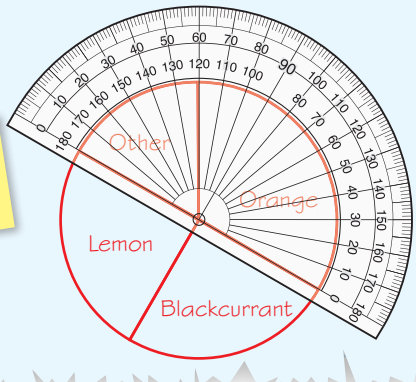
Drink	Number of students	Angle
Orange	8	8 × 15° = 120°
Blackcurrant	6	6 × 15° = 90°
Lemon	6	6 × 15° = 90°
Other	4	4 × 15° = 60°

Total number of students = 8 + 6 + 6 + 4 = 24

Angle for 1 student = 360° ÷ 24 = 15°

Check: 120° + 90° + 90° + 60° = 360° ✓

Everything in red is part of the answer.



You need a sharp pencil, compasses and a protractor to draw a pie chart.

1. Add a new column to the table and label it 'Angle'.
2. Work out how many degrees will represent 1 student.
   There are 360° in a circle and 24 students so the number of degrees for 1 student is 360° ÷ 24 = 15°.
3. Multiply the number of students by the number of degrees for 1 student to give you the angle for each sector.
4. Check that all your angles add up to 360°.
5. Draw a circle. Draw a vertical line and use a protractor to measure the first angle (120°) from this line. Draw the rest of the angles, taking care not to overlap the angles. It helps to turn your page so that you measure from the last angle you drew.
6. Label each sector of your pie chart.

For a reminder about measuring and drawing angles turn to page 67.

Now try this

The table shows information about the members of a chess club.

Members	Frequency	Angle
Boys	15	
Girls	10	
Adults	11	

Angle for 1 person = 360° ÷ number of people

(a) How many people are members of the chess club?

(b) Work out the number of degrees that represents one person.

(c) Complete the angle column in the table.

(d) Draw a pie chart to show this information.