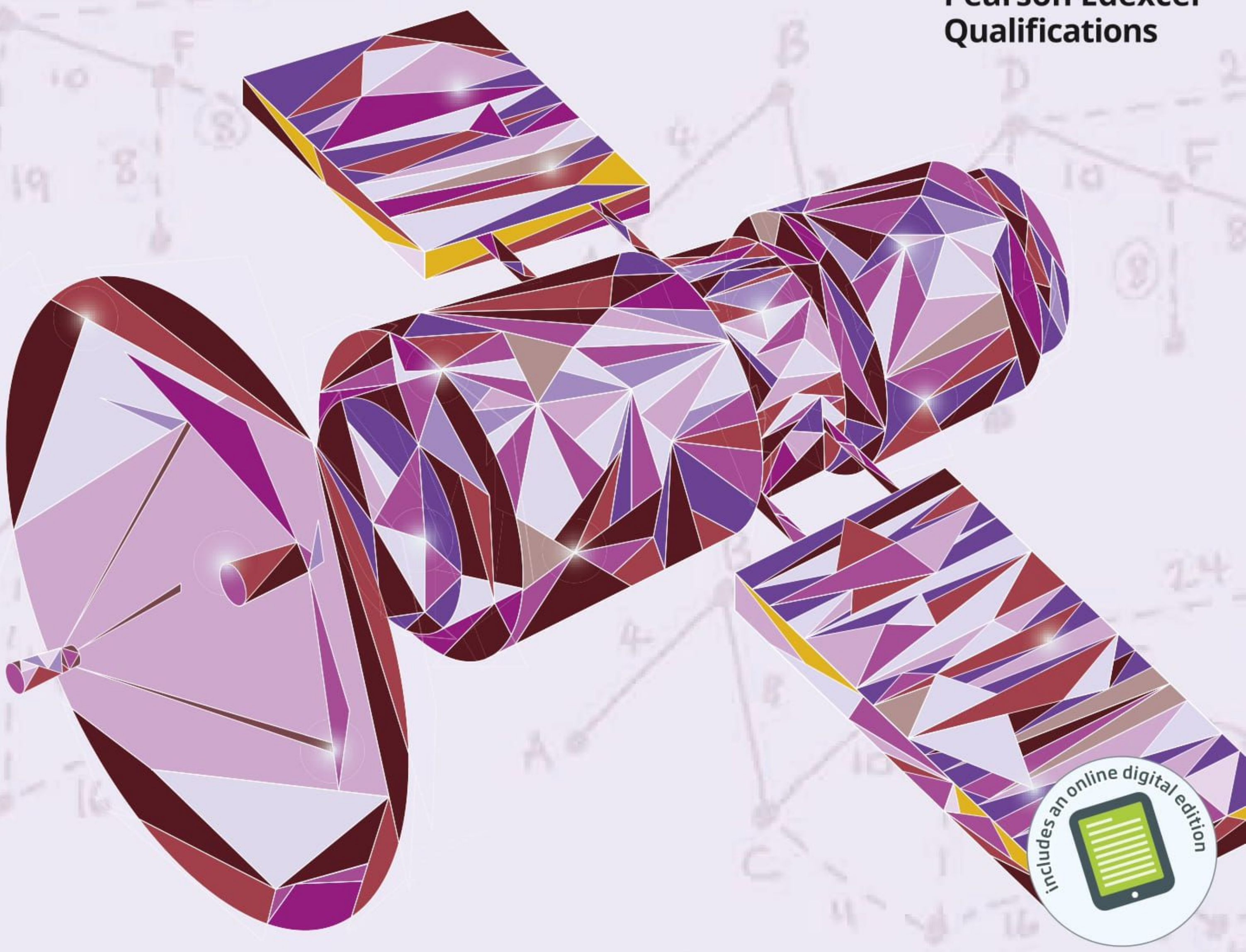
11 - 19 PROGRESSION

Endorsed for
Pearson Edexcel Qualifications



Pearson Edexcel AS and A level Further Mathematics

Decision Mathematics 1

D1

Pearson

Contents

● = A level only

Overarching themes

The following three overarching themes have been fully integrated throughout the Pearson Edexcel AS and A level Mathematics series, so they can be applied alongside your learning and practice.

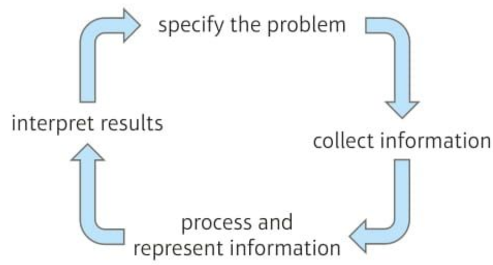
1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols
- Dedicated sections on mathematical proof explain key principles and strategies
- Opportunities to critique arguments and justify methods

2. Mathematical problem solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Structured and unstructured questions to build confidence
- Challenge boxes provide extra stretch

The Mathematical Problem-solving cycle



3. Mathematical modelling

- Dedicated modelling sections in relevant topics provide plenty of practice where you need it
- Examples and exercises include qualitative questions that allow you to interpret answers in the context of the model
- Dedicated chapter in Statistics & Mechanics Year 1/AS explains the principles of modelling in mechanics

Finding your way around the book

Access an online digital edition using the code at the front of the book.

Each chapter starts with a list of objectives

The real world applications of the maths you are about to learn are highlighted at the start of the chapter with links to relevant questions in the chapter

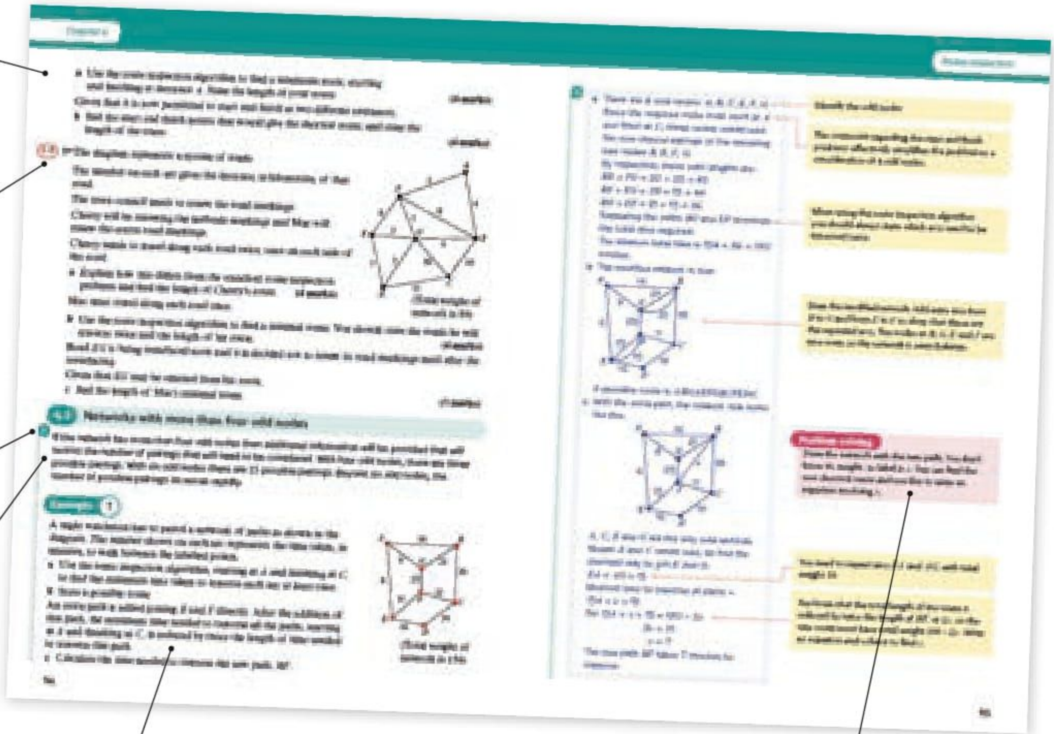
The *Prior knowledge check* helps make sure you are ready to start the chapter

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Exercises are packed with exam-style questions to ensure you are ready for the exams

A level content is clearly flagged

Each section begins with explanation and key learning points



Exam-style questions are flagged with Ⓔ

Problem-solving questions are flagged with Ⓟ

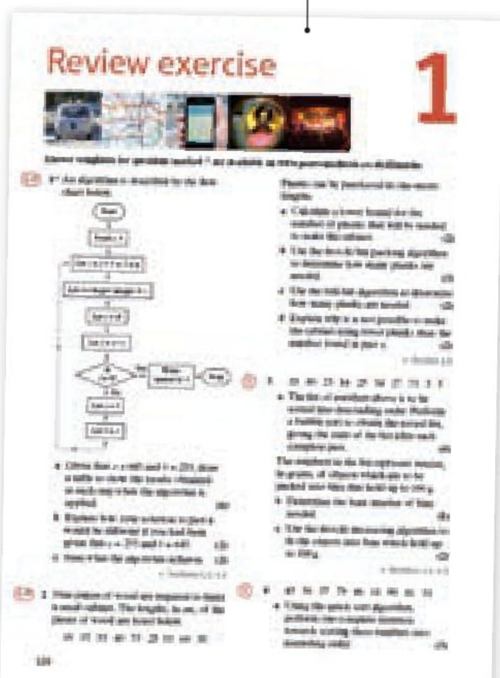
Step-by-step worked examples focus on the key types of questions you'll need to tackle

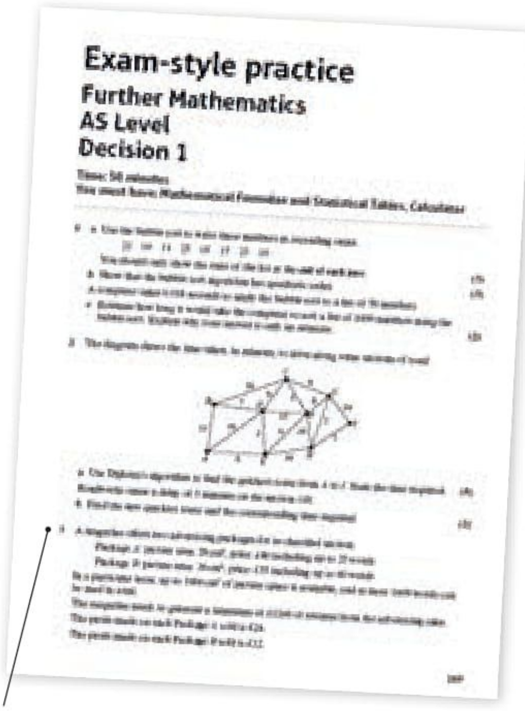
Challenge boxes give you a chance to tackle some more difficult questions

Each chapter ends with a *Mixed exercise* and a *Summary of key points*

Problem-solving boxes provide hints, tips and strategies, and *Watch out* boxes highlight areas where students often lose marks in their exams

Every few chapters a *Review exercise* helps you consolidate your learning with lots of exam-style questions





AS and A level practice papers at the back of the book help you prepare for the real thing.

Extra online content

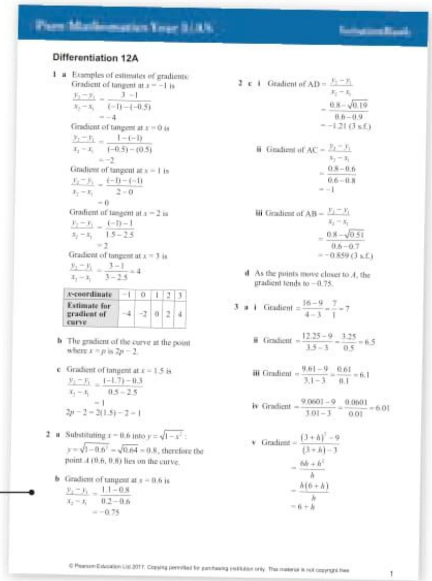
Whenever you see an *Online* box, it means that there is extra online content available to support you.

SolutionBank

SolutionBank provides a full worked solution for every question in the book.

Online Full worked solutions are available in SolutionBank.

Download all the solutions as a PDF or quickly find the solution you need online



Use of technology

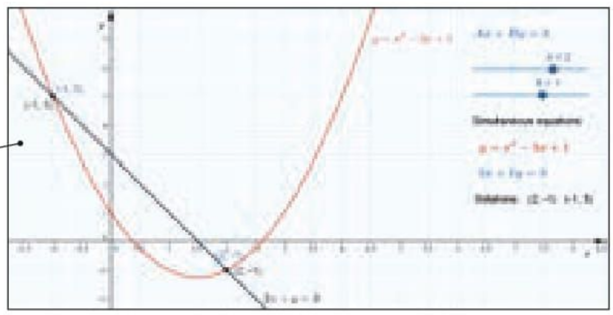
Explore topics in more detail, visualise problems and consolidate your understanding using pre-made GeoGebra activities.

Online Find the point of intersection graphically using technology.

GeoGebra

GeoGebra-powered interactives

Interact with the maths you are learning using GeoGebra's easy-to-use tools



Access all the extra online content for free at:

www.pearsonschools.co.uk/d1maths

You can also access the extra online content by scanning this QR code:

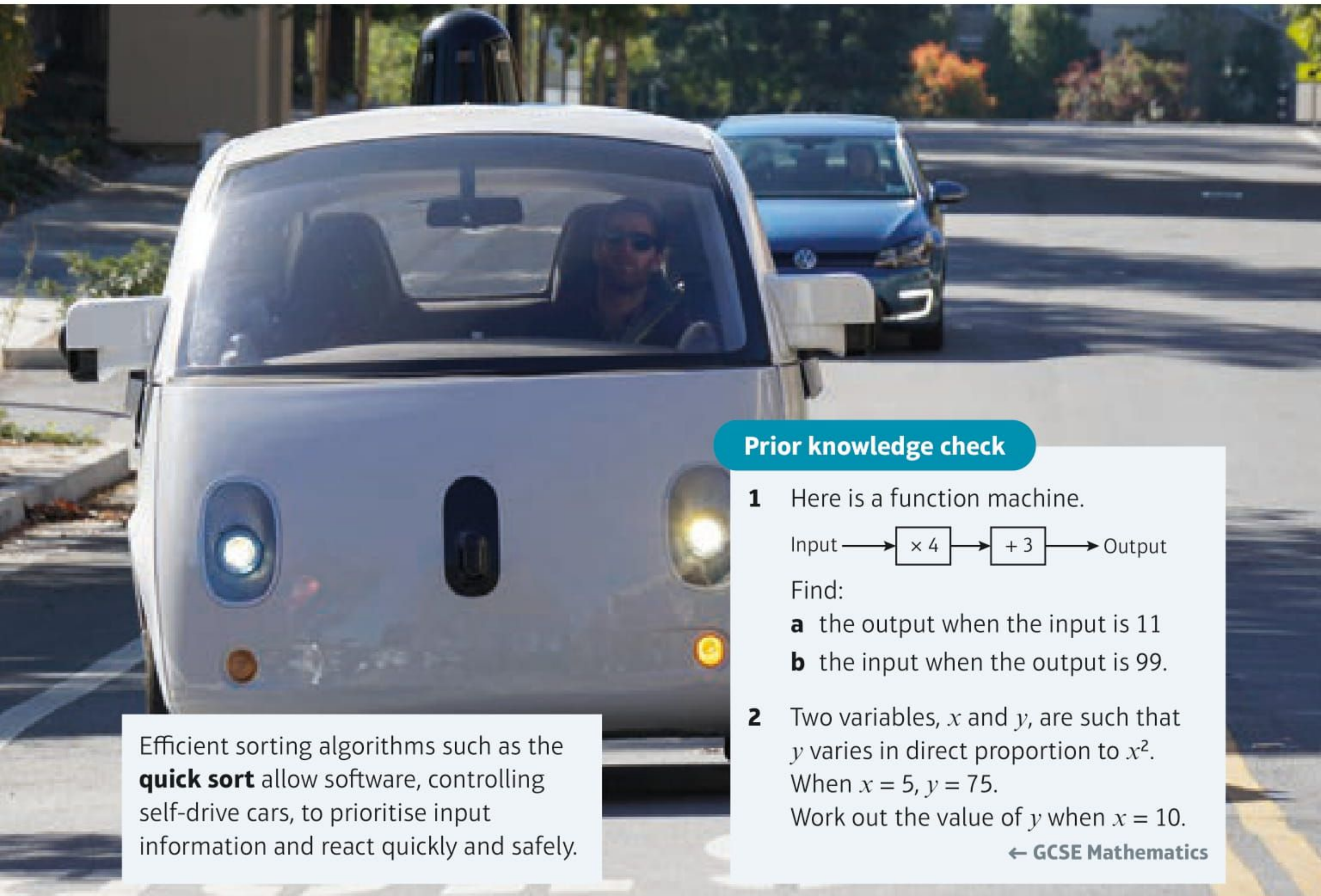
Algorithms

1

Objectives

After completing this chapter you should be able to:

- Use and understand an algorithm given in words → **pages 2–5**
- Understand how flow charts can be used to describe algorithms → **pages 6–10**
- Carry out a bubble sort → **pages 10–13**
- Carry out a quick sort → **pages 13–16**
- Carry out the three bin-packing algorithms and understand their strengths and weaknesses → **pages 16–21**
- Determine the order of an algorithm → **pages 21–24**



Efficient sorting algorithms such as the **quick sort** allow software, controlling self-drive cars, to prioritise input information and react quickly and safely.

Prior knowledge check

1 Here is a function machine.

Input → $\times 4$ → $+3$ → Output

Find:

a the output when the input is 11

b the input when the output is 99.

2 Two variables, x and y, are such that y varies in direct proportion to x^2.
When $x = 5$, $y = 75$.
Work out the value of y when $x = 10$.

← GCSE Mathematics

1.1 Using and understanding algorithms

- **An algorithm is a finite sequence of step-by-step instructions carried out to solve a problem.**

Algorithms can be given in words or in flow charts.

You need to be able to understand and use an algorithm given in words.

You have been using algorithms since you started school. Some examples of mathematical algorithms that you will be familiar with are:

- how to add several two-digit numbers
- how to multiply two two-digit numbers
- how to add, subtract, multiply or divide fractions.

It can be quite challenging to write a sequence of instructions for someone else to follow accurately.

Here are some more examples:

At the end of the street turn right and go straight over the crossroads, take the third left after the school, then ...

Affix base (*B*) to leg (*A*) using screw (*F*) and ...

Dice two large onions.
Slice 100 g mushrooms.
Grate 100 g cheese.

Example 1

The 'happy' algorithm is:

- write down any integer
- square its digits and find the sum of the squares
- continue with this number
- repeat until either the answer is 1 (in which case the number is 'happy') or until you get trapped in a cycle (in which case the number is 'unhappy')

Show that:

a 70 is happy **b** 4 is unhappy

Watch out You will need to be able to understand, describe and implement specific algorithms in your exam. You do not need to learn any of the algorithms in this section.

a $7^2 + 0^2 = 49$
$4^2 + 9^2 = 97$
$9^2 + 7^2 = 130$
$1^2 + 3^2 + 0^2 = 10$
$1^2 + 0^2 = 1$
so 70 is happy

b $4^2 = 16$
$1^2 + 6^2 = 37$
$3^2 + 7^2 = 58$
$5^2 + 8^2 = 89$
$8^2 + 9^2 = 145$
$1^2 + 4^2 + 5^2 = 42$
$4^2 + 2^2 = 20$
$2^2 + 0^2 = 4$
$4^2 = 16$
so 4 is unhappy

As soon as the sum of the squares matches a previous result, all of the steps in-between will be repeated, creating a cycle.

Example 2

a Implement this algorithm.

1 Let $n = 1$, $A = 1$, $B = 1$.

2 Print A and B.

3 Let $C = A + B$.

4 Print C.

5 Let $n = n + 1$, $A = B$, $B = C$.

6 If $n < 5$ go to 3.

7 If $n = 5$ stop.

These are not equations.
They are instructions that mean:
- replace n by $n + 1$ (add 1 to n)
- A takes B's current value
- B takes C's current value

b Describe the numbers that are generated by this algorithm.

a Use a trace table.

A **trace table** is used to record the values of each variable as the algorithm is run.

Instruction step	n	A	B	C	Print
1	1	1	1		
2					1, 1
3				2	
4					2
5	2	1	2		
6	Go to step 3				
3				3	
4					3
5	3	2	3		
6	Go to step 3				
3				5	
4					5
5	4	3	5		
6	Go to step 3				
3				8	
4					8
5	5	5	8		
6	Continue to step 7				
7	Stop				

You may be asked to complete a printed trace table in your exam. Just obey each instruction, in order.

b This algorithm produces the first few numbers in the Fibonnacci sequence.

You may be asked what the algorithm does.

Example 3

This algorithm multiplies the two numbers A and B.

1 Make a table with two columns.
Write A in the top row of the left-hand column and B in the top row of the right-hand column.

2 In the next row of the table write:
- in the left-hand column, the number that is half of A, ignoring remainders
- in the right-hand column, the number that is double B

3 Repeat step 2 until you reach the row which has a 1 in the left-hand column.

4 Delete all rows where the number in the left-hand column is even.

5 Find the sum of the non-deleted numbers in the right-hand column.
This is the product AB.

This famous algorithm is sometimes called 'the Russian peasant's algorithm' or 'the Egyptian multiplication algorithm'.

Implement this algorithm when:

a $A = 29$ and $B = 34$ **b** $A = 66$ and $B = 56$.

a

A	B
29	34
~~14~~	~~68~~
7	136
3	272
1	544
Total	986

So 29 × 34 = 986

Each time the number in the left-hand column is halved and the number in the right-hand column is doubled.

Step 4 means that rows where the number in the left-hand column is even must be deleted before summing the right-hand column.

b

A	B
~~66~~	~~56~~
33	112
~~16~~	~~224~~
~~8~~	~~448~~
~~4~~	~~896~~
~~2~~	~~1792~~
1	3584
Total	3696

So 66 × 56 = 3696

Each deleted row has an even number in its left-hand column.

Exercise 1A

1 Use the algorithm in Example 3 to evaluate:

a 244×125 b 125×244 c 256×123

2 The box below describes an algorithm.

> 1 Write the input numbers in the form $\frac{a}{b}$ and $\frac{c}{d}$
> 2 Let $e = ad$.
> 3 Let $f = bc$.
> 4 Print 'Answer is $\frac{e}{f}$'

a Implement this algorithm with the input numbers $2\frac{1}{4}$ and $1\frac{1}{3}$

b What does this algorithm do?

3 The box below describes an algorithm.

> 1 Let $A = 1$, $n = 1$.
> 2 Print A.
> 3 Let $A = A + 2n + 1$.
> 4 Let $n = n + 1$.
> 5 If $n \leqslant 10$, go to **2**.
> 6 Stop.

a Implement the algorithm.

b Describe the numbers produced by the algorithm.

(P) 4 The box below describes an algorithm.

> 1 Input A, r.
> 2 Let $C = \frac{A}{r}$ to 3 decimal places.
> 3 If $|r - C| \leqslant 10^{-2}$ go to **7**.
> 4 Let $s = \frac{1}{2}(r + C)$ to 3 decimal places.
> 5 Let $r = s$.
> 6 Go to **2**.
> 7 Print r.
> 8 Stop.

Hint This algorithm requires you to use the modulus function. If $x \neq y$, $|x - y|$ is the positive difference between x and y. For example $|3.2 - 7| = 3.8$.

a Use a trace table to implement the algorithm above when:

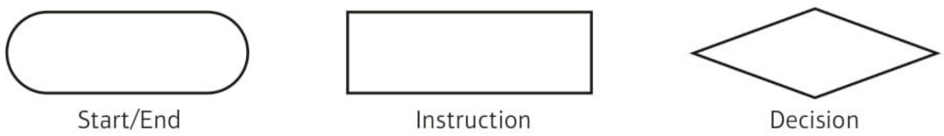
i $A = 253$ and $r = 12$ ii $A = 79$ and $r = 10$ iii $A = 4275$ and $r = 50$

b What does the algorithm produce?

1.2 Flow charts

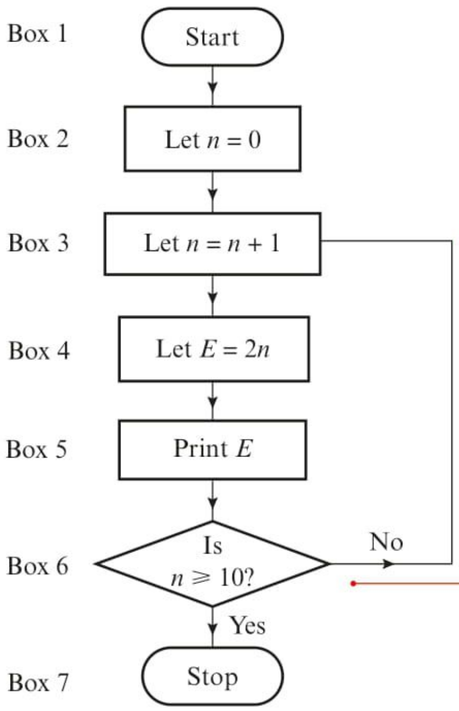
You need to be able to implement an algorithm given as a flow chart.

- **In a flow chart, the shape of each box tells you about its function.**



The boxes in a flow chart are linked by arrowed lines. As with an algorithm written in words, you need to follow each step in order.

Example 4



n is acting as a counter. It ensures that the algorithm stops after this loop has been completed 10 times.

A decision box will contain a question to which the answer is either ‘yes’ or ‘no’.

a Implement this algorithm using a trace table.

b Alter box 4 to read ‘Let $E = 3n$’ and implement the algorithm again.
How does this alter the algorithm?

Example 12

Nine boxes of fixed cross-section have heights, in metres, as follows.

0.3 0.7 0.8 0.8 1.0 1.1 1.1 1.2 1.5

They are to be packed into bins with the same fixed cross-section and height 2 m.
Determine the lower bound for the number of bins needed.

$0.3 + 0.7 + 0.8 + 0.8 + 1.0 + 1.1 + 1.1 + 1.2 + 1.5 = 8.5$ m

$\frac{8.5}{2} = 4.25$ bins

So a minimum of 5 bins will be needed.

Sum the heights and divide by the bin size. You must always round **up** to determine the lower bound.

Watch out In practice, it may not be possible to pack these boxes into 5 bins. All that the lower bound is telling us, is that **at least** five bins will be needed.

With small amounts of data it is often possible to 'spot' an optimal answer.

The algorithms you will learn in this chapter will not necessarily find an optimal solution, but can be implemented quickly.

■ **The first-fit algorithm works by considering items in the order they are given.**

First-fit algorithm

1 Take the items **in the order given**.

2 Place each item in the first available bin that can take it. Start from bin 1 each time.

Advantage: It is quick to implement.
Disadvantage: It is not likely to lead to a good solution.

Online See the operation of the first-fit algorithm using GeoGebra.

Example 13

Use the first-fit algorithm to pack the following items into bins of size 20. (The numbers in brackets are the size of the item.) State the number of bins used and the amount of wasted space.

A(8) B(7) C(14) D(9) E(6) F(9) G(5) H(15) I(6) J(7) K(8)

Bin 1:	A(8)	B(7)	G(5)
Bin 2:	C(14)	E(6)	
Bin 3:	D(9)	F(9)	
Bin 4:	H(15)		
Bin 5:	I(6)	J(7)	
Bin 6:	K(8)		

This used 6 bins and there are
$2 + 5 + 7 + 12 = 26$ units of waste of space.

A(8) goes into bin 1, leaving space of 12.
B(7) goes into bin 1, leaving space of 5.
C(14) goes into bin 2, leaving space of 6.
D(9) goes into bin 3, leaving space of 11.
E(6) goes into bin 2, leaving space of 0.
F(9) goes into bin 3, leaving space of 2.
G(5) goes into bin 1, leaving space of 0.
H(15) goes into bin 4, leaving space of 5.
I(6) goes into bin 5, leaving space of 14.
J(7) goes into bin 5, leaving space of 7.
K(8) goes into bin 6, leaving space of 12.

The total number of comparisons would then be:

$$1 + 2 + 3 + \ldots + (n - 4) + (n - 3) + (n - 2) + (n - 1)$$
$$= \tfrac{1}{2}(n - 1)n = \tfrac{1}{2}n^2 - \tfrac{1}{2}n$$

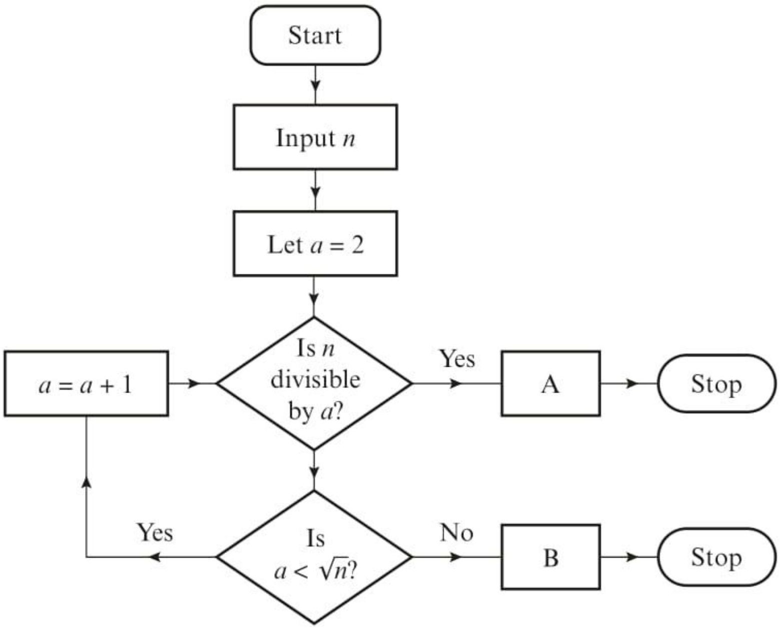
Links This is the sum of the first $(n - 1)$ natural numbers. ← **Core Pure Book 1, Section 3.1**

Since this is a quadratic expression, the bubble sort is taken to have **quadratic order**.

Watch out A different algorithm may require $50n^2 + 11n + 90$ steps to complete a problem of size n. This algorithm would also be described as having quadratic order.

Example 17

An algorithm is defined by this flow diagram, where $n > 2$ and n is an integer.



a Describe what the algorithm does.

b Suggest suitable output text for boxes A and B.

c Determine the order of the algorithm.

a The algorithm tests whether or not n is prime.

b Box A: n is not prime.
Box B: n is prime.

c Let the size of the algorithm be n.
At each step the algorithm tests whether n is divisible by a.
If n is prime, the answer at this step will never be 'yes' so the algorithm will continue until $a \geqslant \sqrt{n}$.
The maximum number of steps needed is given by the integer part of $\sqrt{n}$.
So the algorithm has order $\sqrt{n}$.

Problem-solving

The maximum number of steps will not always be needed. If n is even, then the algorithm will only require one step. In general you should consider the worst case scenario when determining the order of an algorithm.

E **6** A DIY enthusiast requires the following 14 pieces of wood as shown in the table.

Length in metres	0.4	0.6	1	1.2	1.4	1.6
Number of pieces	3	4	3	2	1	1

The DIY store sells wood in 2 m and 2.4 m lengths. He considers buying six 2 m lengths of wood.

a Explain why he will not be able to cut all of the lengths he requires from these six 2 m lengths. **(2 marks)**

b He eventually decides to buy 2.4 m lengths. Use a first-fit decreasing bin-packing algorithm to show how he could use six 2.4 m lengths to obtain the pieces he requires. **(4 marks)**

c Obtain a solution that requires only five 2.4 m lengths. **(4 marks)**

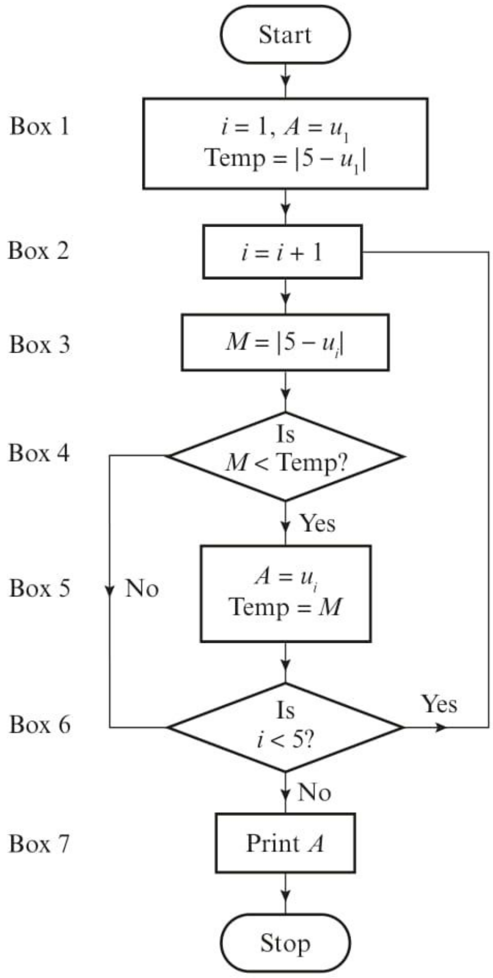
E/P **7** The algorithm described by the flow chart below is to be applied to the five pieces of data below.

$u_1 = 6.1,\ u_2 = 6.9,\ u_3 = 5.7,\ u_4 = 4.8,\ u_5 = 5.3$

Hint This question uses the modulus function. If $x \neq y$, $|x - y|$ is the positive difference between x and y, e.g. $|5 - 6.1| = 1.1$.

a Obtain the final output of the algorithm using the five values given for u_1 to u_5. **(4 marks)**

b In general, for any set of values u_1 to u_5, explain what the algorithm achieves. **(2 marks)**



c If Box 4 in the flow chart is altered to 'Is $M >$ Temp?' state what the algorithm achieves now. **(1 mark)**

E **8** A plumber is cutting lengths of PVC pipe for a bathroom installation. The lengths needed, in metres, are:

0.3 2.0 1.3 1.6 0.3 1.3 0.2 0.1 2.0 0.5

The pipe is sold in 2 m lengths.

a Carry out a bubble sort to produce a list of the lengths needed in **descending** order. Give the state of the list after each pass. **(4 marks)**

b Apply the first-fit decreasing bin-packing algorithm to your ordered list to determine the total number of 2 m lengths of pipe needed. **(3 marks)**

c Does the answer to part **b** use the minimum number of 2 m lengths? You must justify your answer. **(2 marks)**

E/P **9** Here are the names of eight students in an A level group:

Maggie, Vivien, Cath, Alana, Daisy, Beth, Kandis, Sara

a Use a quick sort to put the names in alphabetical order. Show the result of each pass and identify the pivots. **(5m arks)**

The quick sort algorithm has order $n \log n$.
A computer program can sort a list of 100 names in 0.3 seconds using a quick sort.

b Estimate the time needed for this computer program to apply a quick sort to a list of 1000 names. **(2 marks)**

Challenge

An algorithm for factorising an n-digit integer is found to have order 1.1^n. A computer uses the algorithm to factorise 8 788 751, taking 0.734 seconds.

a Estimate the time needed for the computer to factorise:
i 3 744 388 667 **ii** a number with 100 digits

Internet security is based on large, hard-to-factorise numbers.
A cryptographer wants to choose a number which will take at least one year to factorise using this algorithm.

b Determine the minimum number of digits the cryptographer should use for their number.

c Suggest a reason why the run time of this algorithm might vary widely depending on the choice of number to be factorised.

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