11 - 19 PROGRESSION

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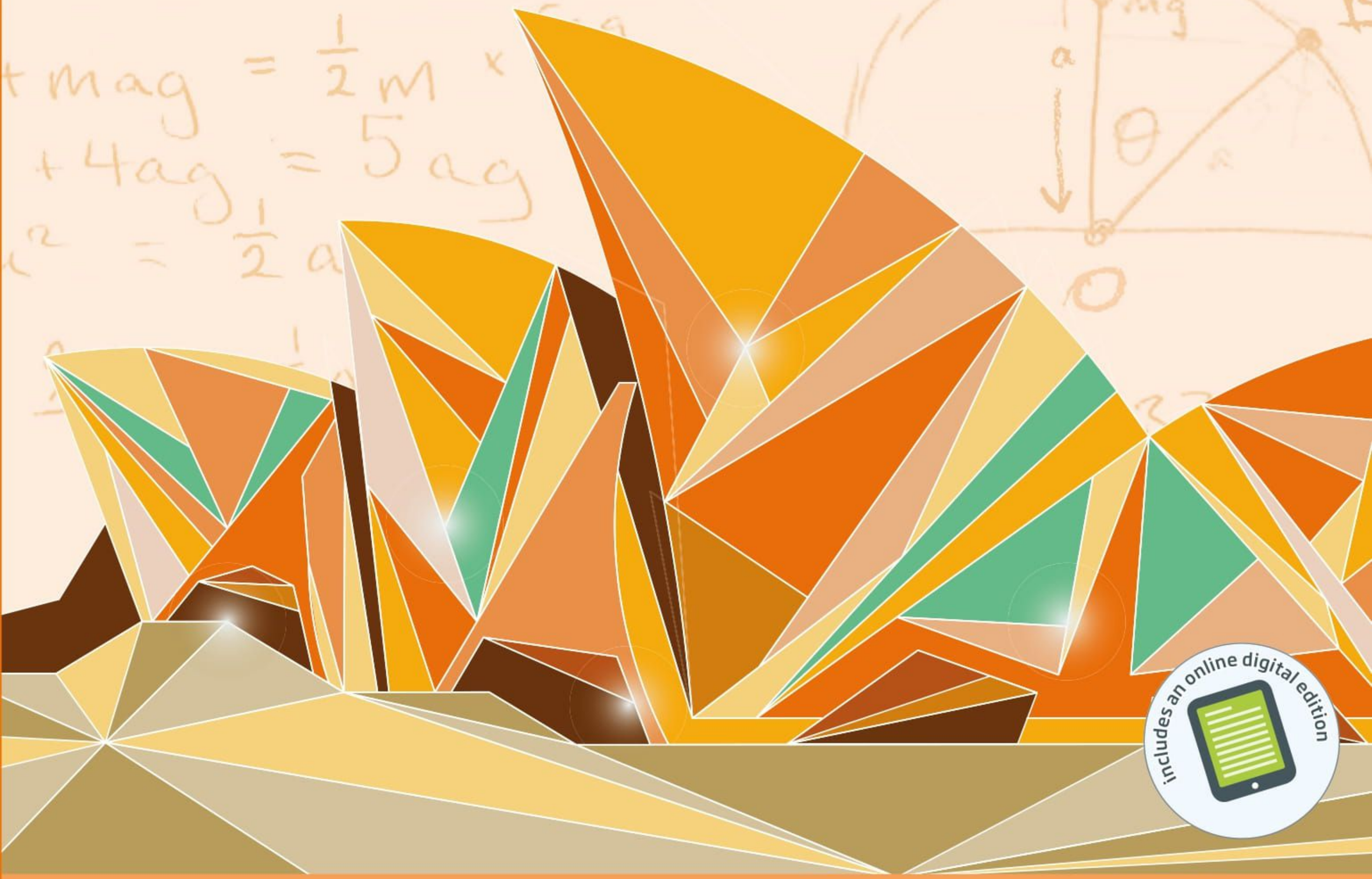
Pearson Edexcel
Qualifications



Pearson Edexcel AS and A level Further Mathematics

Further Mechanics 2

FM2

Pearson

Contents

● = A level only

Overarching themes

The following three overarching themes have been fully integrated throughout the Pearson Edexcel AS and A level Mathematics series, so they can be applied alongside your learning and practice.

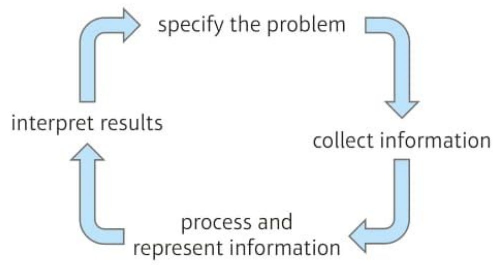
1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols
- Dedicated sections on mathematical proof explain key principles and strategies
- Opportunities to critique arguments and justify methods

2. Mathematical problem solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Structured and unstructured questions to build confidence
- Challenge boxes provide extra stretch

The Mathematical Problem-solving cycle

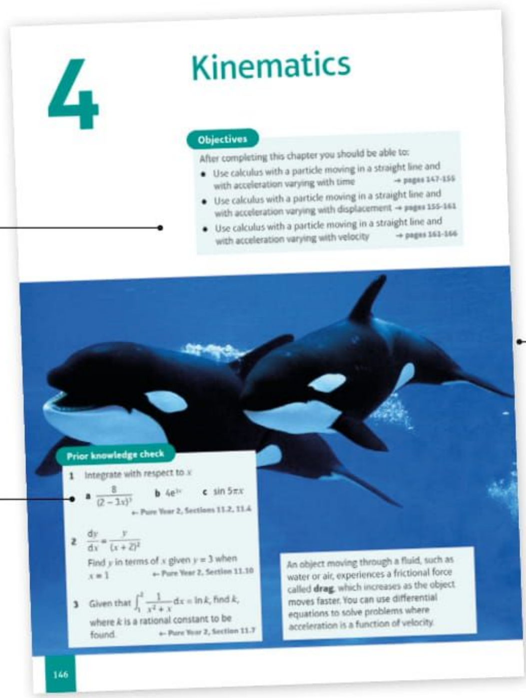


3. Mathematical modelling

- Dedicated modelling sections in relevant topics provide plenty of practice where you need it
- Examples and exercises include qualitative questions that allow you to interpret answers in the context of the model
- Dedicated chapter in Statistics & Mechanics Year 1/AS explains the principles of modelling in mechanics

Finding your way around the book

Access an online digital edition using the code at the front of the book.



Each chapter starts with a list of objectives

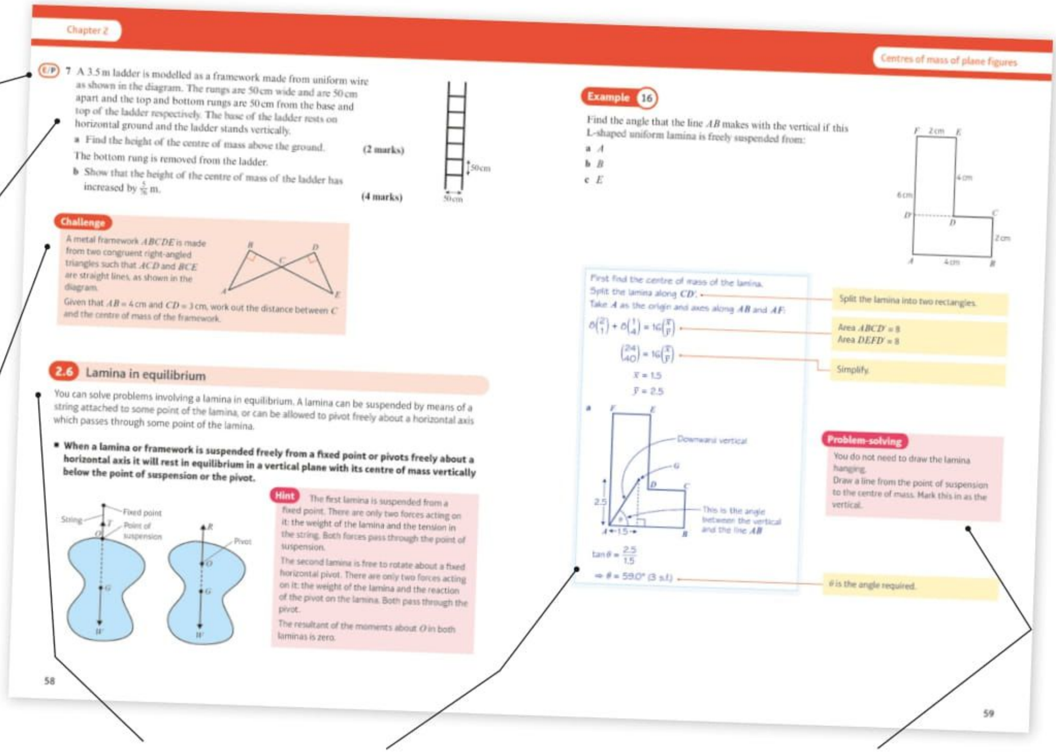
The *Prior knowledge check* helps make sure you are ready to start the chapter

The real world applications of the maths you are about to learn are highlighted at the start of the chapter with links to relevant questions in the chapter

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Exercises are packed with exam-style questions to ensure you are ready for the exams

Challenge boxes give you a chance to tackle some more difficult questions



Exam-style questions are flagged with (E)

Problem-solving questions are flagged with (P)

A level content is clearly flagged with (A)

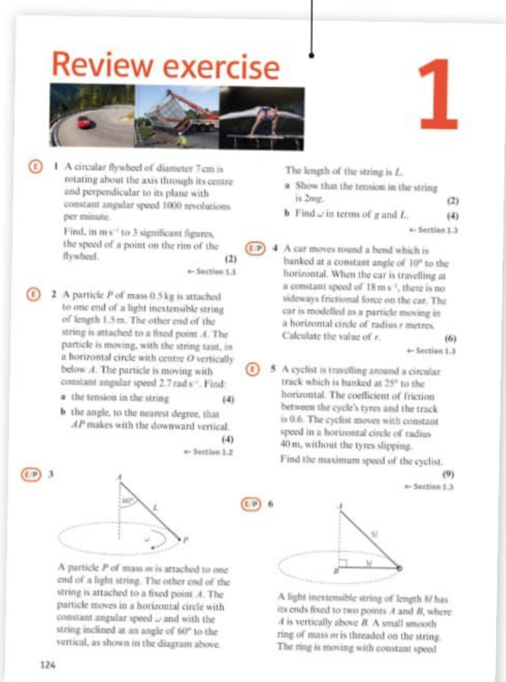
Each section begins with explanation and key learning points

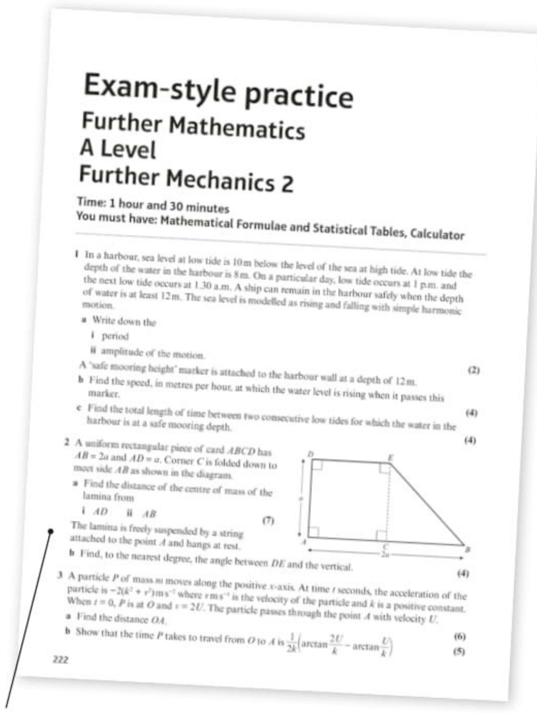
Step-by-step worked examples focus on the key types of questions you'll need to tackle

Each chapter ends with a *Mixed exercise* and a *Summary of key points*

Problem-solving boxes provide hints, tips and strategies, and *Watch out* boxes highlight areas where students often lose marks in their exams

Every few chapters a *Review exercise* helps you consolidate your learning with lots of exam-style questions





AS and A level practice papers at the back of the book help you prepare for the real thing.

Extra online content

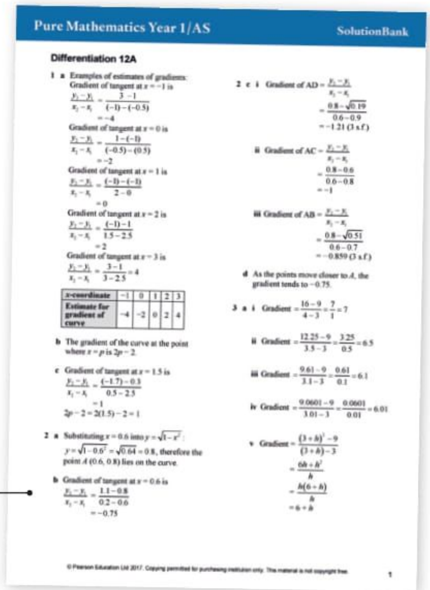
Whenever you see an *Online* box, it means that there is extra online content available to support you.

SolutionBank

SolutionBank provides a full worked solution for every question in the book.

Online Full worked solutions are available in SolutionBank.

Download all the solutions as a PDF or quickly find the solution you need online


Pure Mathematics Year 1/AS — SolutionBank

Differentiation 12A

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Use of technology

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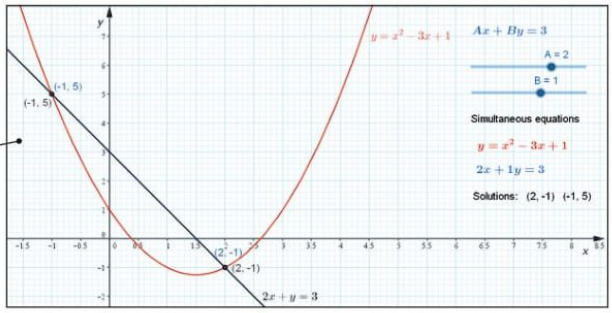

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Circular motion

1

Objectives

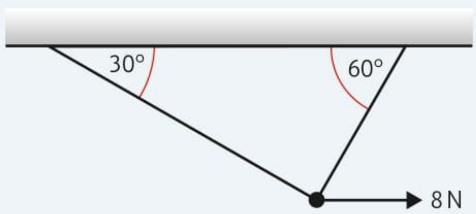
After completing this chapter you should be able to:

- Understand and calculate angular speed of an object moving in a circle → pages 2–4
- Understand and calculate angular acceleration of an object moving on a circular path → pages 5–10
- Solve problems with objects moving in horizontal circles → pages 11–18
- Solve problems with objects moving in vertical circles → pages 19–25
- Solve problems when objects do not stay on a circular path → pages 26–30

A car travelling around a bend can be modelled as a particle on a circular path. Police use models such as this to determine likely speeds of cars following accidents. → Exercise 1C, Q18

Prior knowledge check

1 A smooth ring is threaded on a light inextensible string. The ends of the string are attached to a horizontal ceiling, and make angles of 30° and 60° with the ceiling respectively. The ring is held in equilibrium by a horizontal force of magnitude 8 N.



Find

a the tension in the string **b** the mass of the ring.

← Statistics and Mechanics 2, Section 7.1

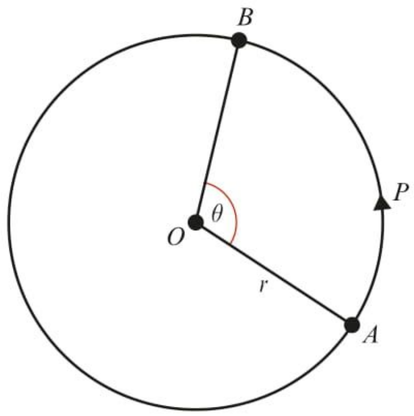
2 A box of mass 4 kg is projected with speed 10 m s^{-1} up the line of greatest slope of a rough plane, which is inclined at an angle of 20° to the horizontal. The coefficient of friction between the box and the plane is 0.15. Find:

a the distance travelled by the box before it comes to instantaneous rest

b the work done against friction as the box reaches instantaneous rest. ← Further Mechanics 1, Section 2.3

1.1 Angular speed

When an object is moving in a straight line, the speed, usually measured in m s^{-1} or km h^{-1}, describes the rate at which distance is changing. For an object moving on a circular path, you can use the same method for measuring speed, but it is often simpler to measure the speed by considering the rate at which the radius is turning.



As the particle P moves from point A to point B on the circumference of a circle of radius r m, the radius of the circle turns through an angle θ radians.

The distance moved by P is $r\theta$ m, so if P is moving at v m s^{-1}

we know that $v = \frac{\mathrm{d}}{\mathrm{d}t}(r\theta) = r\frac{\mathrm{d}\theta}{\mathrm{d}t} = r \times \dot{\theta}$

Notation

$\dot{\theta}$ is the rate at which the radius is turning about O.

It is called the **angular speed of the particle** about O.

The angular speed of a particle is usually denoted by ω, and measured in rad s^{-1}.

- **If a particle is moving around a circle of radius r m with linear speed v m s^{-1} and angular speed ω rad s^{-1} then $v = r\omega$.**

Example 1

A particle moves in a circle of radius 4 m with speed 2 m s^{-1}. Calculate the angular speed.

Using $v = r\omega$, $2 = 4\omega$, so $\omega = 0.5$ rad s^{-1}

Example 2

Express an angular speed of 200 revolutions per minute in radians per second.

Each complete revolution is 2π radians, so 200 revolutions is 400π radians per minute.
Therefore the angular speed is
$\frac{400\pi}{60} = 20.9$ rad s^{-1} (3 s.f.)

Watch out Sometimes an angular speed is described in terms of the number of revolutions completed in a given time.

Example 3

A particle moves round a circle in 10 seconds at a constant speed of $15\,\text{m}\,\text{s}^{-1}$. Calculate the angular speed of the particle and the radius of the circle.

The particle rotates through an angle of 2π radians in 10 seconds, so $\omega = \frac{2\pi}{10} = 0.628\,\text{rad}\,\text{s}^{-1}$ (3 s.f.)

Using $v = r\omega$, $r = \frac{v}{\omega} = \frac{15}{0.628} = 23.9\,\text{m}$ (3 s.f.)

Exercise 1A

1 Express:

a an angular speed of 5 revolutions per minute in $\text{rad}\,\text{s}^{-1}$

b an angular speed of 120 revolutions per minute in $\text{rad}\,\text{s}^{-1}$

c an angular speed of $4\,\text{rad}\,\text{s}^{-1}$ in revolutions per minute

d an angular speed of $3\,\text{rad}\,\text{s}^{-1}$ in revolutions per hour.

2 Find the speed in $\text{m}\,\text{s}^{-1}$ of a particle moving on a circular path of radius 20 m at:

a $4\,\text{rad}\,\text{s}^{-1}$

b $40\,\text{rev}\,\text{min}^{-1}$

3 A particle moves on a circular path of radius 25 cm at a constant speed of $2\,\text{m}\,\text{s}^{-1}$. Find the angular speed of the particle:

a in $\text{rad}\,\text{s}^{-1}$

b in $\text{rev}\,\text{min}^{-1}$

4 Find the speed in $\text{m}\,\text{s}^{-1}$ of a particle moving on a circular path of radius 80 cm at:

a $2.5\,\text{rad}\,\text{s}^{-1}$

b $25\,\text{rev}\,\text{min}^{-1}$

5 An athlete is running round a circular track of radius 50 m at $7\,\text{m}\,\text{s}^{-1}$.

a How long does it take the athlete to complete one circuit of the track?

b Find the angular speed of the athlete in $\text{rad}\,\text{s}^{-1}$.

6 A disc of radius 12 cm rotates at a constant angular speed, completing one revolution every 10 seconds. Find:

a the angular speed of the disc in $\text{rad}\,\text{s}^{-1}$

b the speed of a particle on the outer rim of the disc in $\text{m}\,\text{s}^{-1}$

c the speed of a particle at a point 8 cm from the centre of the disc in $\text{m}\,\text{s}^{-1}$.

7 A cyclist completes two circuits of a circular track in 45 seconds. Calculate:

a his angular speed in rad s^{-1}

b the radius of the track given that his speed is 40 km h^{-1}.

8 Anish and Bethany are on a fairground roundabout. Anish is 3 m from the centre and Bethany is 5 m from the centre. If the roundabout completes 10 revolutions per minute, calculate the speeds with which Anish and Bethany are moving.

9 A model train completes one circuit of a circular track of radius 1.5 m in 26 seconds. Calculate:

a the angular speed of the train in rad s^{-1}

b the linear speed of the train in m s^{-1}.

10 A train is moving at 150 km h^{-1} round a circular bend of radius 750 m. Calculate the angular speed of the train in rad s^{-1}.

(P) 11 The hour hand on a clock has radius 10 cm, and the minute hand has radius 15 cm. Calculate:

a the angular speed of the end of each hand

b the linear speed of the end of each hand.

12 The drum of a washing machine has diameter 50 cm. The drum spins at 1200 rev min^{-1}. Find the linear speed of a point on the drum.

13 A gramophone record rotates at 45 rev min^{-1}. Find:

a the angular speed of the record in rad s^{-1}

b the distance from the centre of a point moving at 12 cm s^{-1}.

(P) 14 The Earth completes one orbit of the sun in a year. Taking the orbit to be a circle of radius 1.5×10^{11} m, and a year to be 365 days, calculate the speed at which the Earth is moving.

(P) 15 A bead moves around a hoop of radius r m with angular velocity 1 rad s^{-1}. The bead moves at a speed greater than 5 m s^{-1}. Find the range of possible values for r.

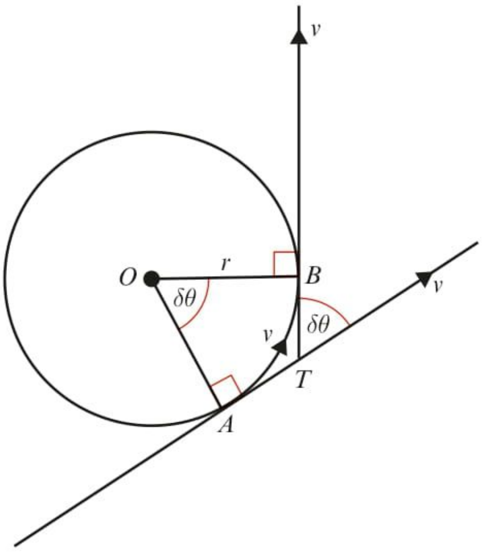
Challenge

Two separate circular turntables, with different radii, are both mounted horizontally on a common vertical axis which acts as the centre of rotation for both. The smaller turntable, of radius 18 cm, is uppermost and rotates clockwise. The larger turntable has radius 20 cm and rotates anticlockwise. Both turntables have constant angular velocities, with magnitudes in the same ratio as their radii.

A blue dot is placed at a point on the circumference of the smaller turntable, and a red dot likewise on the larger one. Starting from the instant that the two dots are at their closest possible distance apart, it is known that 10 seconds later these dots are at their maximum distance apart for the first time. Find the exact angular velocity of the larger turntable.

1.2 Acceleration of an object moving on a horizontal circular path

When an object moves round a horizontal circular path at constant speed, the direction of the motion is changing. If the direction is changing, then, although the speed is constant, the velocity is not constant. If the velocity is changing then the object must have an acceleration.



Suppose that the object is moving on a circular path of radius r at constant speed v.

Let the time taken to move from A to B be δt, and the angle AOB be $\delta\theta$.

At A, the velocity is v along the tangent AT. At B, the velocity is v along the tangent TB.

The velocity at B can be resolved into components:

$v\cos\delta\theta$ parallel to AT and

$v\sin\delta\theta$ perpendicular to AT.

We know that acceleration $= \dfrac{\text{change in velocity}}{\text{time}}$, so to find the acceleration of the object at the instant when it passes point A, we need to consider what happens to $\dfrac{v\cos\delta\theta - v}{\delta t}$ and $\dfrac{v\sin\delta\theta - 0}{\delta t}$ as $\delta t \to 0$.

These will be the components of the acceleration parallel to AT and perpendicular to AT respectively.

For a small angle $\delta\theta$ measured in radians, $\cos\delta\theta \approx 1$ and $\sin\delta\theta \approx \delta\theta$, so the acceleration parallel to AT is zero, and the acceleration perpendicular to AT is $v\dfrac{\delta\theta}{\delta t} = v\omega$.

Using $v = r\omega$, $v\omega$ can be written as $r\omega^2$ or $\dfrac{v^2}{r}$.

- **An object moving on a circular path with constant linear speed v and constant angular speed ω has acceleration $r\omega^2$ or $\dfrac{v^2}{r}$, towards the centre of the circle.**

Example 4

A particle is moving on a horizontal circular path of radius 20 cm with constant angular speed 2 rad s^{-1}. Calculate the acceleration of the particle.

Acceleration
$= 0.2 \times 2^2$
$= 0.8$ m s^{-2} towards the centre of the circle.

The radius needs to be measured in metres if the answer is to be in m s^{-2}.

Using $a = r\omega^2$.

A

Given that the rod remains on the surface of the sphere,

b show that the time taken for the particle to make one complete revolution is at least $\pi\sqrt{\frac{6r}{g}}$.

(3 marks)

c Without further calculation, state how your answer to part **b** would change if the particle was moved:

i up the rod towards the pivot

ii down the rod away from the pivot. **(2 marks)**

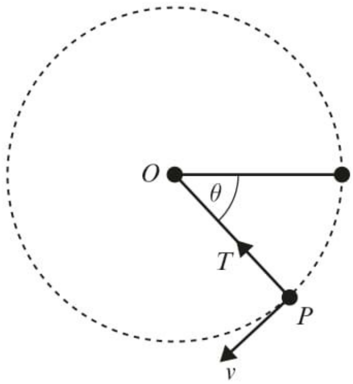
E/P **11** A rough disc rotates in a horizontal plane with constant angular velocity ω about a fixed vertical axis. A particle P of mass m lies on the disc at a distance $\frac{3}{5}a$ from the axis. The coefficient of friction between P and the disc is $\frac{3}{7}$. Given that P remains at rest relative to the disc,

a prove that $\omega^2 \leqslant \frac{5g}{7a}$ **(7 marks)**

The particle is now connected to the axis by a horizontal light elastic string of natural length $\frac{a}{2}$ and modulus of elasticity $\frac{5mg}{2}$. The disc again rotates with constant angular velocity ω about the axis and P remains at rest relative to the disc at a distance $\frac{3}{5}a$ from the axis.

b Find the range of possible values of ω^2. **(8 marks)**

12 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at a point O. The particle is held with the string taut and OP horizontal. It is then projected vertically downwards with speed u, where $u^2 = \frac{4}{3}ga$. When OP has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in the diagram. Find:



a an expression for v^2 in terms of a, g and θ

b an expression for T in terms of m, g and θ

c the value of θ when the string becomes slack to the nearest degree.

d Explain why P would not complete a vertical circle if the string were replaced by a light rod.

13 A particle P of mass 0.4 kg is attached to one end of a light inelastic string of length 1 m. The other end of the string is fixed at point O. P is hanging in equilibrium below O when it is projected horizontally with speed u m s^{-1}. When OP is horizontal it meets a small smooth peg at Q, where $OQ = 0.8$ m. Calculate the minimum value of u if P is to describe a complete circle about Q.

E/P **14** A smooth solid hemisphere is fixed with its plane face on a horizontal table and its curved surface uppermost. The plane face of the hemisphere has centre O and radius a. The point A is the highest point on the hemisphere. A particle P is placed on the hemisphere at A.

It is then given an initial horizontal speed u, where $u^2 = \frac{ag}{2}$. When OP makes an angle θ with OA, and while P remains on the hemisphere, the speed of P is v.

A

a Find an expression for v^2. **(2 marks)**

b Show that P is still on the hemisphere when $\theta = \arccos 0.9$. **(2 marks)**

c Find the value of:

i $\cos\theta$ when P leaves the hemisphere

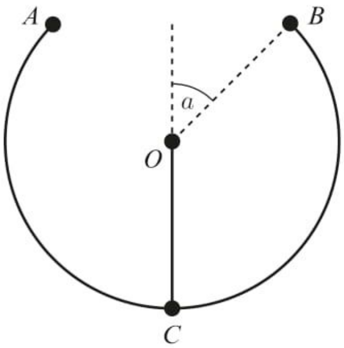
ii v when P leaves the hemisphere. **(3 marks)**

After leaving the hemisphere P strikes the table at B, find:

d the speed of P at B **(2 marks)**

e the angle, to the nearest degree, at which P strikes the table. **(3 marks)**

E/P **15** Part of a hollow spherical shell, centre O and radius r, is removed to form a bowl with a plane circular rim. The bowl is fixed with the circular rim uppermost and horizontal. The point C is the lowest point of the bowl. The point B is on the rim of the bowl and OB is at an angle α to the upward vertical as shown in the diagram. Angle α satisfies $\tan\alpha = \frac{4}{3}$. A smooth small marble of mass m is placed inside the bowl at C and given an initial horizontal speed u. The direction of motion of the marble lies in the vertical plane COB. The marble stays in contact with the bowl until it reaches B. When the marble reaches B it has speed v.



a Find an expression for v^2. **(4 marks)**

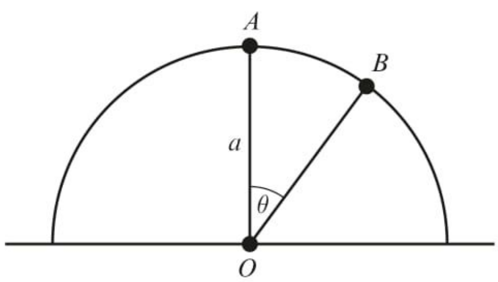
b If $u^2 = 4gr$, find the normal reaction of the bowl on the marble as the marble reaches B. **(3 marks)**

c Find the least possible value of u for the marble to reach B. **(3 marks)**

The point A is the other point of the rim of the bowl lying in the vertical plane COB.

d Find the value of u which will enable the marble to leave the bowl at B and meet it again at A. **(4 marks)**

E/P **16** A particle is at the highest point A on the outer surface of a fixed smooth hemisphere of radius a and centre O. The hemisphere is fixed to a horizontal surface with the plane face in contact with the surface. The particle is projected horizontally from A with speed u, where $u < \sqrt{ag}$. The particle leaves the sphere at the point B, where OB makes an angle θ with the upward vertical, as shown in the diagram.



a Find an expression for $\cos\theta$ in terms of u, g and a. **(3 marks)**

The particle strikes the horizontal surface with speed $\sqrt{\frac{5ag}{2}}$.

b Find the value of θ, to the nearest degree. **(4 marks)**

Pearson Edexcel AS and A level Further Mathematics
Further Mechanics 2

Series Editor: Harry Smith

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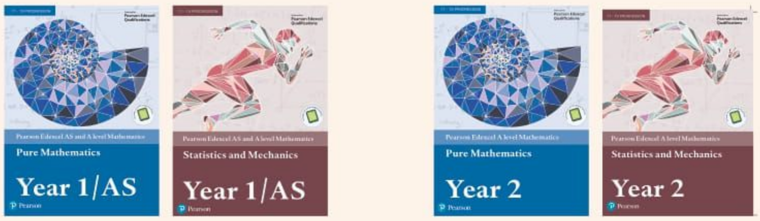

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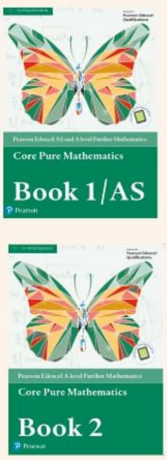

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