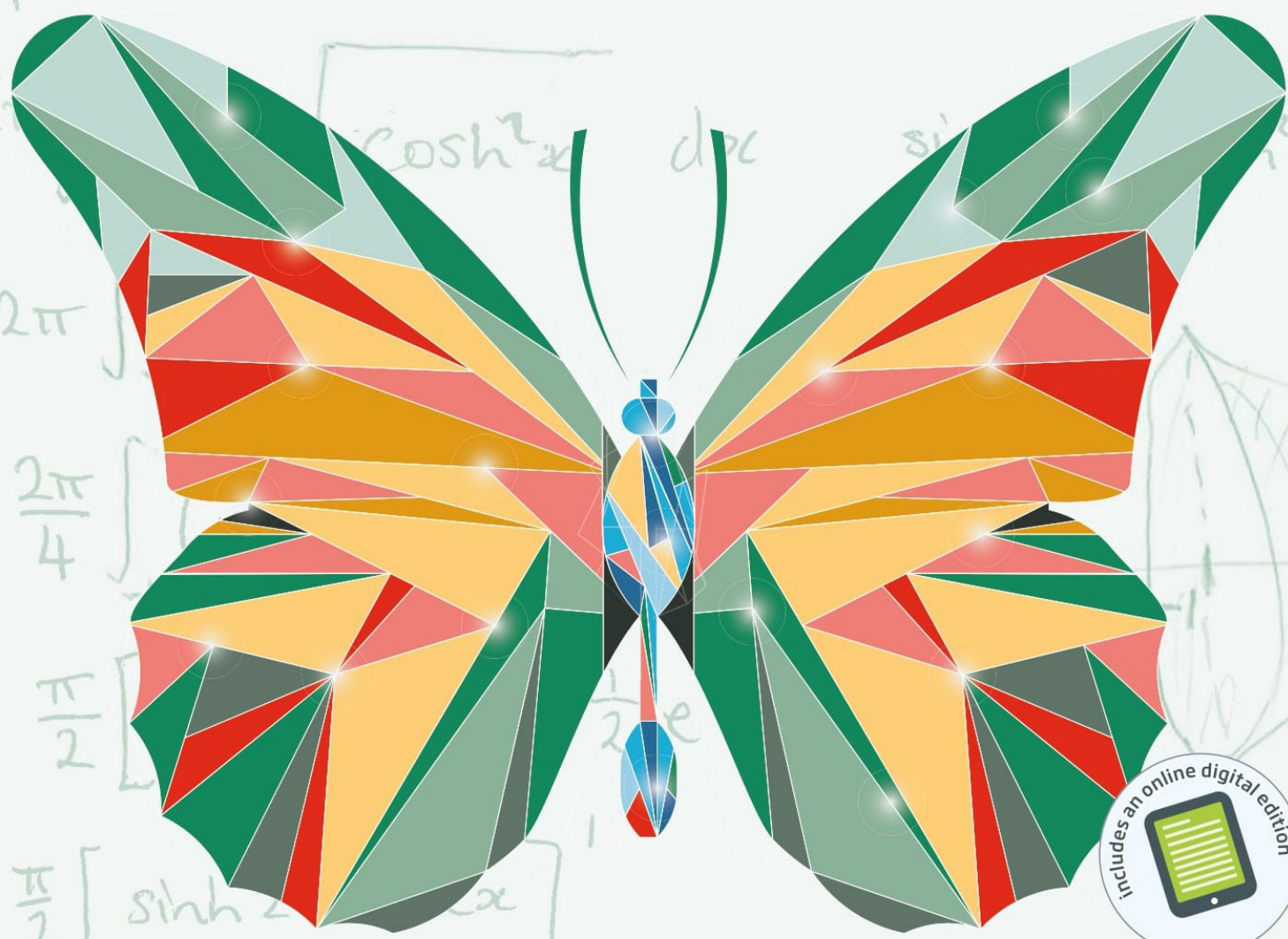




Pearson Edexcel AS and A level Further Mathematics

Core Pure Mathematics

Book 1/AS

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Overarching themes

The following three overarching themes have been fully integrated throughout the Pearson Edexcel AS and A level Mathematics series, so they can be applied alongside your learning and practice.

1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols
- Dedicated sections on mathematical proof explain key principles and strategies
- Opportunities to critique arguments and justify methods

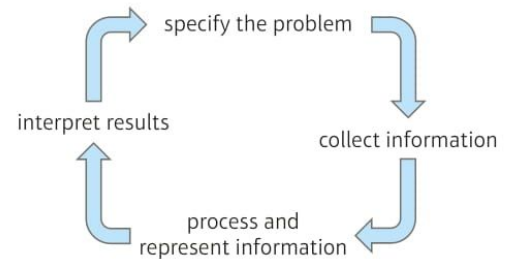
2. Mathematical problem solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Structured and unstructured questions to build confidence
- Challenge boxes provide extra stretch

3. Mathematical modelling

- Dedicated modelling sections in relevant topics provide plenty of practice where you need it
- Examples and exercises include qualitative questions that allow you to interpret answers in the context of the model
- Dedicated chapter in Statistics & Mechanics Year 1/AS explains the principles of modelling in mechanics

The Mathematical Problem-solving cycle



Finding your way around the book

Access an online digital edition using the code at the front of the book.



Each chapter starts with a list of objectives



The real world applications of the maths you are about to learn are highlighted at the start of the chapter with links to relevant questions in the chapter

The *Prior knowledge check* helps make sure you are ready to start the chapter

Step-by-step worked examples focus on the key types of questions you'll need to tackle

Problem-solving boxes provide hints, tips and strategies, and Watch out boxes highlight areas where students often lose marks in their exams

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Exam-style questions are flagged with **E**

Problem-solving questions are flagged with **P**

Exercises are packed with exam-style questions to ensure you are ready for the exams

Challenge boxes give you a chance to tackle some more difficult questions

Each section begins with explanation and key learning points

Each chapter ends with a Mixed exercise and a Summary of key points

Every few chapters a Review exercise helps you consolidate your learning with lots of exam-style questions

Review exercise 2

1 $A = \begin{pmatrix} 3 & 2 \\ 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ where p and q are integers. Determine whether or not the following products exist. Where the product exists, evaluate the product in terms of p and q . Where the product does not exist, give a reason.

- AB (1)
- BA (1)
- BAC (1)
- CBA (1)

2 $M = \begin{pmatrix} 0 & 3 \\ -1 & 2 \end{pmatrix}$, $N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Find the values of the constants, a and b , such that $MF + aM + bH = O$. (3)

3 $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$. Find an expression for A^{-1} in terms of a, b, c and d so that $A^{-1} - (a + d)A = A$, where I is the 2×2 identity matrix. (3)

4 A matrix A is given as $A = \begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}$. Given that $A^n = \begin{pmatrix} 3^n & 2^n \\ 0 & -1^n \end{pmatrix}$, find the values of a and b . (3)

5 $A = \begin{pmatrix} 2 & 3 \\ p & -1 \end{pmatrix}$, where p is a real constant. Given that A is singular, find the value of p . (2)

6 $A = \begin{pmatrix} 4 & -1 \\ -6 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 \\ 3 & p \end{pmatrix}$. Find $(AB)^{-1}$ in terms of p . (3)

7 $A = \begin{pmatrix} k & 1 & -2 \\ 0 & -1 & k \\ 0 & 0 & 0 \end{pmatrix}$, where k is a real constant. Given that A is non-singular, find A^{-1} in terms of k . (4)

8 $A = \begin{pmatrix} 2p & p & 2 \\ 3 & 0 & 0 \\ -1 & 1 & -1 \end{pmatrix}$ where p is a real constant. Given that A is non-singular, find A^{-1} in terms of p . (4)

Exam-style practice

Further Mathematics AS Level

Paper 1: Core Pure Mathematics

Time: 1 hour 40 minutes
You must have: Mathematical Formulae and Statistical Tables, Calculator

- With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations:
 $l_1: r = (-3i + 5k) + z(3i - k)$
 $l_2: r = (10i - j + 15k) + z(4i - 2j + 4k)$
 Show that lines l_1 and l_2 do not meet. (4)
- $M = \begin{pmatrix} 2 & k & 3 \\ 3 & -1 & 2 \end{pmatrix}$ where k is an integer.
 a Find $\det M$, giving your answer in terms of k . (3)
 Three planes A , B and C are defined by the following Cartesian equations:
 $A: 2x + ky + 3z = 1$
 $B: x - 3y + z = -2$
 $C: 3x - y + 2z = 3$
 b Given that the planes do not meet at a single point, determine whether the three equations form a consistent system, and give a geometric interpretation of your answer. You must show sufficient working to justify your conclusions. (5)
- The cubic equation $2x^3 - 3x^2 - 7x - 1 = 0$ has roots α, β and γ . Without solving the equation, find the cubic equation whose roots are $(2\alpha - 1), (2\beta - 1)$ and $(2\gamma - 1)$, giving your answer in the form $px^3 + qx^2 + rx + s = 0$ where p, q, r and s are integers to be found. (6)
- Prove by induction that for all positive integers n ,
 $\sum_{k=1}^n k^2 = \frac{1}{6}n(n+1)(2n+1)$ (6)
 b Use the standard results for $\sum_{k=1}^n k$ and $\sum_{k=1}^n k^2$ to show that for all positive integers n ,
 $\sum_{k=1}^n 2k^2 + 1 = \frac{1}{3}n(n+1)(n+2)$ (4)
 c Hence show that $n = 1$ is the only value of n that satisfies
 $\sum_{k=1}^n 2k^2 + 1 = 4 \sum_{k=1}^n k^2$ (5)

A full AS level practice paper at the back of the book helps you prepare for the real thing.

Extra online content

Whenever you see an *Online* box, it means that there is extra online content available to support you.



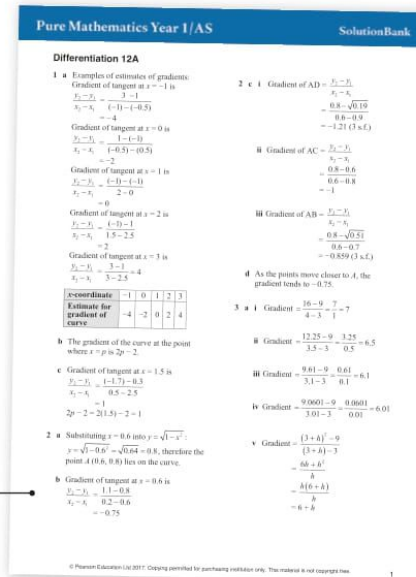
SolutionBank

SolutionBank provides a full worked solution for every question in the book.

Online Full worked solutions are available in SolutionBank.



Download all the solutions as a PDF or quickly find the solution you need online



Use of technology

Explore topics in more detail, visualise problems and consolidate your understanding using pre-made GeoGebra activities.

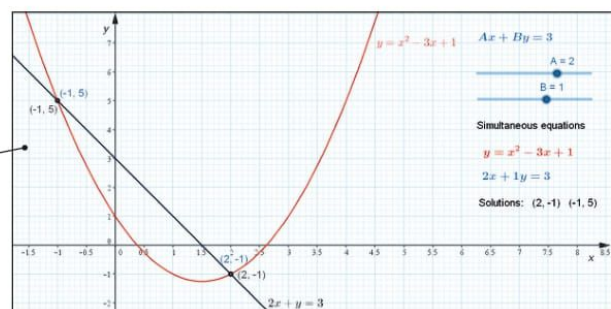
Online Find the point of intersection graphically using technology.



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Complex numbers

1

Objectives

After completing this chapter you should be able to:

- Understand and use the definitions of imaginary and complex numbers → page 2
- Add and subtract complex numbers → pages 2–3
- Multiply complex numbers → pages 5–6
- Understand the definition of a complex conjugate → pages 6–8
- Divide complex numbers → pages 7–8
- Solve quadratic equations that have complex roots → pages 8–10
- Solve cubic or quartic equations that have complex roots → pages 10–14

Prior knowledge check

- 1 Simplify each of the following:
a $\sqrt{50}$ **b** $\sqrt{108}$ **c** $\sqrt{180}$
← Pure Year 1, Chapter 1
- 2 In each case, determine the number of distinct real roots of the equation $f(x) = 0$.
a $f(x) = 3x^2 + 8x + 10$
b $f(x) = 2x^2 - 9x + 7$
c $f(x) = 4x^2 + 12x + 9$
← Pure Year 1, Chapter 2
- 3 Find the solutions of $x^2 - 8x + 6 = 0$, giving your answers in the form $a \pm \sqrt{b}$ where a and b are integers.
← Pure Year 1, Chapter 2
- 4 Write $\frac{7}{4 - \sqrt{3}}$ in the form $p + q\sqrt{3}$ where p and q are rational numbers.
← Pure Year 1, Chapter 1

Complex numbers contain a real and an imaginary part. Engineers and physicists often describe quantities with two components using a single complex number. This allows them to model complicated situations such as air flow over a cyclist.

1.1 Imaginary and complex numbers

The quadratic equation $ax^2 + bx + c = 0$ has solutions given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If the expression under the square root is negative, there are no real solutions.

You can find solutions to the equation in all cases by extending the number system to include $\sqrt{-1}$. Since there is no real number that squares to produce -1 , the number $\sqrt{-1}$ is called an **imaginary number**, and is represented using the letter **i**. Sums of real and imaginary numbers, for example $3 + 2i$, are known as **complex numbers**.

- $i = \sqrt{-1}$
- An imaginary number is a number of the form bi , where $b \in \mathbb{R}$.
- A complex number is written in the form $a + bi$, where $a, b \in \mathbb{R}$.

Links For the equation $ax^2 + bx + c = 0$, the **discriminant** is $b^2 - 4ac$.

- If $b^2 - 4ac > 0$, there are two distinct real roots.
- If $b^2 - 4ac = 0$, there are two equal real roots.
- If $b^2 - 4ac < 0$, there are no real roots.

← Pure Year 1, Section 2.5

Notation The set of all complex numbers is written as $\mathbb{C}$.

For the complex number $z = a + bi$:

- $\text{Re}(z) = a$ is the real part
- $\text{Im}(z) = b$ is the imaginary part

Example 1

Write each of the following in terms of i .

a $\sqrt{-36}$ b $\sqrt{-28}$

a $\sqrt{-36} = \sqrt{36 \times (-1)} = \sqrt{36} \sqrt{-1} = 6i$

b $\sqrt{-28} = \sqrt{28 \times (-1)} = \sqrt{28} \sqrt{-1}$
 $= \sqrt{4} \sqrt{7} \sqrt{-1} = (2\sqrt{7})i$

You can use the rules of surds to manipulate imaginary numbers.

Watch out An alternative way of writing $(2\sqrt{7})i$ is $2i\sqrt{7}$. Avoid writing $2\sqrt{7}i$ as this can easily be confused with $2\sqrt{7i}$.

In a complex number, the real part and the imaginary part cannot be combined to form a single term.

- Complex numbers can be added or subtracted by adding or subtracting their real parts and adding or subtracting their imaginary parts.
- You can multiply a real number by a complex number by multiplying out the brackets in the usual way.

Example 2

Simplify each of the following, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

a $(2 + 5i) + (7 + 3i)$ b $(2 - 5i) - (5 - 11i)$ c $2(5 - 8i)$ d $\frac{10 + 6i}{2}$

a $(2 + 5i) + (7 + 3i) = (2 + 7) + (5 + 3)i$
 $= 9 + 8i$

b $(2 - 5i) - (5 - 11i) = (2 - 5) + (-5 - (-11))i$
 $= -3 + 6i$

Add the real parts and add the imaginary parts.

Subtract the real parts and subtract the imaginary parts.

$$\text{c } 2(5 - 8i) = (2 \times 5) - (2 \times 8)i = 10 - 16i$$

$$\text{d } \frac{10 + 6i}{2} = \frac{10}{2} + \frac{6i}{2} = 5 + 3i$$

$2(5 - 8i)$ can also be written as $(5 - 8i) + (5 - 8i)$.

First separate into real and imaginary parts.

Exercise 1A

Do not use your calculator in this exercise.

1 Write each of the following in the form bi where b is a real number.

a $\sqrt{-9}$

b $\sqrt{-49}$

c $\sqrt{-121}$

d $\sqrt{-10\,000}$

e $\sqrt{-225}$

f $\sqrt{-5}$

g $\sqrt{-12}$

h $\sqrt{-45}$

i $\sqrt{-200}$

j $\sqrt{-147}$

2 Simplify, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

a $(5 + 2i) + (8 + 9i)$

b $(4 + 10i) + (1 - 8i)$

c $(7 + 6i) + (-3 - 5i)$

d $(\frac{1}{2} + \frac{1}{3}i) + (\frac{5}{2} + \frac{5}{3}i)$

e $(20 + 12i) - (11 + 3i)$

f $(2 - i) - (-5 + 3i)$

g $(-4 - 6i) - (-8 - 8i)$

h $(3\sqrt{2} + i) - (\sqrt{2} - i)$

i $(-2 - 7i) + (1 + 3i) - (-12 + i)$

j $(18 + 5i) - (15 - 2i) - (3 + 7i)$

3 Simplify, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

a $2(7 + 2i)$

b $3(8 - 4i)$

c $2(3 + i) + 3(2 + i)$

d $5(4 + 3i) - 4(-1 + 2i)$

e $\frac{6 - 4i}{2}$

f $\frac{15 + 25i}{5}$

g $\frac{9 + 11i}{3}$

h $\frac{-8 + 3i}{4} - \frac{7 - 2i}{2}$

P 4 Write in the form $a + bi$, where a and b are simplified surds.

a $\frac{4 - 2i}{\sqrt{2}}$

b $\frac{2 - 6i}{1 + \sqrt{3}}$

5 Given that $z = 7 - 6i$ and $w = 7 + 6i$, find, in the form $a + bi$, where $a, b \in \mathbb{R}$:

a $z - w$

b $w + z$

Notation

Complex numbers are often represented by the letter z or the letter w .

E 6 Given that $z_1 = a + 9i$, $z_2 = -3 + bi$ and $z_2 - z_1 = 7 + 2i$, find a and b where $a, b \in \mathbb{R}$. **(2 marks)**

P 7 Given that $z_1 = 4 + i$ and $z_2 = 7 - 3i$, find, in the form $a + bi$, where $a, b \in \mathbb{R}$:

a $z_1 - z_2$

b $4z_2$

c $2z_1 + 5z_2$

P 8 Given that $z = a + bi$ and $w = a - bi$, $a, b \in \mathbb{R}$, show that:

a $z + w$ is always real

b $z - w$ is always imaginary

You can use complex numbers to find solutions to any quadratic equation with real coefficients.

- If $b^2 - 4ac < 0$ then the quadratic equation $ax^2 + bx + c = 0$ has two distinct complex roots, neither of which are real.

Example 3Solve the equation $z^2 + 9 = 0$.

$$\begin{aligned}
 z^2 &= -9 \\
 z &= \pm\sqrt{-9} = \pm\sqrt{9 \times -1} = \pm\sqrt{9}\sqrt{-1} = \pm 3i \\
 z &= +3i, z = -3i
 \end{aligned}$$

Note that just as $z^2 = 9$ has two roots $+3$ and -3 , $z^2 = -9$ also has two roots $+3i$ and $-3i$.

Example 4Solve the equation $z^2 + 6z + 25 = 0$.**Method 1** (Completing the square)

$$\begin{aligned}
 z^2 + 6z &= (z + 3)^2 - 9 \\
 z^2 + 6z + 25 &= (z + 3)^2 - 9 + 25 = (z + 3)^2 + 16 \\
 (z + 3)^2 + 16 &= 0 \\
 (z + 3)^2 &= -16 \\
 z + 3 &= \pm\sqrt{-16} = \pm 4i \\
 z &= -3 \pm 4i \\
 z &= -3 + 4i, \quad z = -3 - 4i
 \end{aligned}$$

Method 2 (Quadratic formula)

$$\begin{aligned}
 z &= \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 25}}{2} \\
 &= \frac{-6 \pm \sqrt{-64}}{2} \\
 z &= \frac{-6 \pm 8i}{2} = -3 \pm 4i \\
 z &= -3 + 4i, \quad z = -3 - 4i
 \end{aligned}$$

Because $(z + 3)^2 = (z + 3)(z + 3) = z^2 + 6z + 9$

$$\sqrt{-16} = \sqrt{16 \times (-1)} = \sqrt{16}\sqrt{-1} = 4i$$

You can use your calculator to find the complex roots of a quadratic equation like this one.

Using $z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\sqrt{-64} = \sqrt{64 \times (-1)} = \sqrt{64}\sqrt{-1} = 8i$$

Exercise 1B**Do not use your calculator in this exercise.**1 Solve each of the following equations. Write your answers in the form $\pm bi$.

a $z^2 + 121 = 0$

b $z^2 + 40 = 0$

c $2z^2 + 120 = 0$

d $3z^2 + 150 = 38 - z^2$

e $z^2 + 30 = -3z^2 - 66$

f $6z^2 + 1 = 2z^2$

2 Solve each of the following equations.

Write your answers in the form $a \pm bi$.

a $(z - 3)^2 - 9 = -16$

b $2(z - 7)^2 + 30 = 6$

c $16(z + 1)^2 + 11 = 2$

Hint The left-hand side of each equation is in completed square form already. Use inverse operations to find the values of z .

3 Solve each of the following equations. Write your answers in the form $a \pm bi$.

a $z^2 + 2z + 5 = 0$

b $z^2 - 2z + 10 = 0$

c $z^2 + 4z + 29 = 0$

d $z^2 + 10z + 26 = 0$

e $z^2 + 5z + 25 = 0$

f $z^2 + 3z + 5 = 0$

4 Solve each of the following equations. Write your answers in the form $a \pm bi$.

a $2z^2 + 5z + 4 = 0$

b $7z^2 - 3z + 3 = 0$

c $5z^2 - z + 3 = 0$

5 The solutions to the quadratic equation $z^2 - 8z + 21 = 0$ are z_1 and z_2 .

Find z_1 and z_2 , giving each in the form $a \pm i\sqrt{b}$.

E/P 6 The equation $z^2 + bz + 11 = 0$, where $b \in \mathbb{R}$, has distinct non-real complex roots.

Find the range of possible values of b .

(3 marks)

1.2 Multiplying complex numbers

You can multiply complex numbers using the same technique that you use for multiplying brackets in algebra. You can use the fact that $i = \sqrt{-1}$ to simplify powers of i .

■ $i^2 = -1$

Example 5

Express each of the following in the form $a + bi$, where a and b are real numbers.

a $(2 + 3i)(4 + 5i)$

b $(7 - 4i)^2$

$$\begin{aligned} \text{a } (2 + 3i)(4 + 5i) &= 2(4 + 5i) + 3i(4 + 5i) \\ &= 8 + 10i + 12i + 15i^2 \\ &= 8 + 10i + 12i - 15 \\ &= (8 - 15) + (10i + 12i) \\ &= -7 + 22i \end{aligned}$$

$$\begin{aligned} \text{b } (7 - 4i)^2 &= (7 - 4i)(7 - 4i) \\ &= 7(7 - 4i) - 4i(7 - 4i) \\ &= 49 - 28i - 28i + 16i^2 \\ &= 49 - 28i - 28i - 16 \\ &= (49 - 16) + (-28i - 28i) \\ &= 33 - 56i \end{aligned}$$

Multiply the two brackets as you would with real numbers.

Use the fact that $i^2 = -1$.

Add real parts and add imaginary parts.

Multiply out the two brackets as you would with real numbers.

Use the fact that $i^2 = -1$.

Add real parts and add imaginary parts.

Example 6

Simplify: a i^3 b i^4 c $(2i)^5$

$$\begin{aligned} \text{a } i^3 &= i \times i \times i = i^2 \times i = -i \\ \text{b } i^4 &= i \times i \times i \times i = i^2 \times i^2 = (-1) \times (-1) = 1 \\ \text{c } (2i)^5 &= 2i \times 2i \times 2i \times 2i \times 2i \\ &= 32(i \times i \times i \times i \times i) = 32(i^2 \times i^2 \times i) \\ &= 32 \times (-1) \times (-1) \times i = 32i \end{aligned}$$

$i^2 = -1$

$(2i)^5 = 2^5 \times i^5$
First work out $2^5 = 32$.

Exercise 1C**Do not use your calculator in this exercise.****1** Simplify each of the following, giving your answers in the form $a + bi$.

a $(5 + i)(3 + 4i)$

b $(6 + 3i)(7 + 2i)$

c $(5 - 2i)(1 + 5i)$

d $(13 - 3i)(2 - 8i)$

e $(-3 - i)(4 + 7i)$

f $(8 + 5i)^2$

g $(2 - 9i)^2$

h $(1 + i)(2 + i)(3 + i)$

i $(3 - 2i)(5 + i)(4 - 2i)$

j $(2 + 3i)^3$

HintFor part **h**, begin by multiplying the first pair of brackets.**(P)** **2 a** Simplify $(4 + 5i)(4 - 5i)$, giving your answer in the form $a + bi$.**b** Simplify $(7 - 2i)(7 + 2i)$, giving your answer in the form $a + bi$.**c** Comment on your answers to parts **a** and **b**.**d** Prove that $(a + bi)(a - bi)$ is a real number for any real numbers a and b .**(P)** **3** Given that $(a + 3i)(1 + bi) = 25 - 39i$, find two possible pairs of values for a and b .**4** Write each of the following in its simplest form.

a i^6

b $(3i)^4$

c $i^5 + i$

d $(4i)^3 - 4i^3$

(P) **5** Express $(1 + i)^6$ in the form $a + bi$, where a and b are integers to be found.**(P)** **6** Find the value of the real part of $(3 - 2i)^4$.**(P)** **7** $f(z) = 2z^2 - z + 8$

Find: **a** $f(2i)$

b $f(3 - 6i)$

Problem-solvingYou can use the binomial theorem to expand $(a + b)^n$. ← **Pure Year 1, Section 8.3****(E/P)** **8** $f(z) = z^2 - 2z + 17$ Show that $z = 1 - 4i$ is a solution to $f(z) = 0$.**(2 marks)****9 a** Given that $i^1 = i$ and $i^2 = -1$, write i^3 and i^4 in their simplest forms.**b** Write i^5 , i^6 , i^7 and i^8 in their simplest forms.**c** Write down the value of:

i i^{100}

ii i^{253}

iii i^{301}

Challenge**a** Expand $(a + bi)^2$.**b** Hence, or otherwise, find $\sqrt{40 - 42i}$, giving your answer in the form $a - bi$, where a and b are positive integers.**Notation**The **principal square root** of a complex number, $\sqrt{z}$, has a positive real part.**1.3 Complex conjugation****■** For any complex number $z = a + bi$, the **complex conjugate of the number** is defined as $z^* = a - bi$.**Notation**Together z and z^* are called a **complex conjugate pair**.**Example 7**Given that $z = 2 - 7i$,**a** write down z^* **b** find the value of $z + z^*$ **c** find the value of zz^*

- (E/P)** 10 Given that $z = a + bi$, show that $\frac{z}{z^*} = \left(\frac{a^2 - b^2}{a^2 + b^2}\right) + \left(\frac{2ab}{a^2 + b^2}\right)i$ (4 marks)
- (E/P)** 11 The complex number z is defined by $z = \frac{3 + qi}{q - 5i}$, where $q \in \mathbb{R}$.
 Given that the real part of z is $\frac{1}{13}$,
a find the possible values of q (4 marks)
b write the possible values of z in the form $a + bi$, where a and b are real constants. (1 mark)
- (E/P)** 12 Given that $z = x + iy$, find the value of x and the value of y such that $z + 4iz^* = -3 + 18i$ where z^* is the complex conjugate of z . (5 marks)
- 13 $z = 9 + 6i$, $w = 2 - 3i$
 Express $\frac{z}{w}$ in the form $a + bi$, where a and b are real constants.
- (E/P)** 14 The complex number z is given by $z = \frac{q + 3i}{4 + qi}$ where q is an integer.
 Express z in the form $a + bi$ where a and b are rational and are given in terms of q . (4 marks)
- (E)** 15 Given that $6 - 2i$ is one of the roots of a quadratic equation with real coefficients,
a write down the other root of the equation (1 mark)
b find the quadratic equation, giving your answer in the form $z^2 + bz + c = 0$ where b and c are real constants. (2 marks)
- (E/P)** 16 Given that $z = 4 - ki$ is a root of the equation $z^2 - 2mz + 52 = 0$, where k and m are positive real constants, find the value of k and the value of m . (4 marks)
- (E/P)** 17 $h(z) = z^3 - 11z + 20$
 Given that $2 + i$ is a root of the equation $h(z) = 0$, solve $h(z) = 0$ completely. (4 marks)
- (E/P)** 18 $f(z) = z^3 + 6z + 20$
 Given that $f(1 + 3i) = 0$, solve $f(z) = 0$ completely. (4 marks)
- (E/P)** 19 $f(z) = z^3 + 3z^2 + kz + 48$, $k \in \mathbb{R}$
 Given that $f(4i) = 0$,
a find the value of k (2 marks)
b find the other two roots of the equation. (3 marks)
- (E)** 20 $f(z) = z^4 - z^3 - 16z^2 - 74z - 60$
a Write $f(z)$ in the form $(z^2 - 5z - 6)(z^2 + bz + c)$, where b and c are real constants to be found. (2 marks)
b Hence find all the solutions to $f(z) = 0$. (3 marks)
- (E/P)** 21 $g(z) = z^4 - 6z^3 + 19z^2 - 36z + 78$
 Given that $g(3 - 2i) = 0$, find all the roots of $g(z) = 0$. (4 marks)
- (E/P)** 22 $f(z) = z^4 - 2z^3 - 5z^2 + pz + 24$
 Given that $f(4) = 0$,
a find the value of p (1 mark)
b solve completely the equation $f(z) = 0$. (5 marks)

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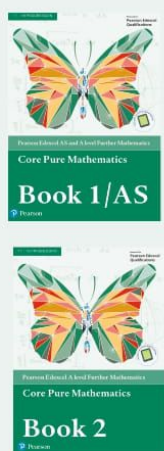


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