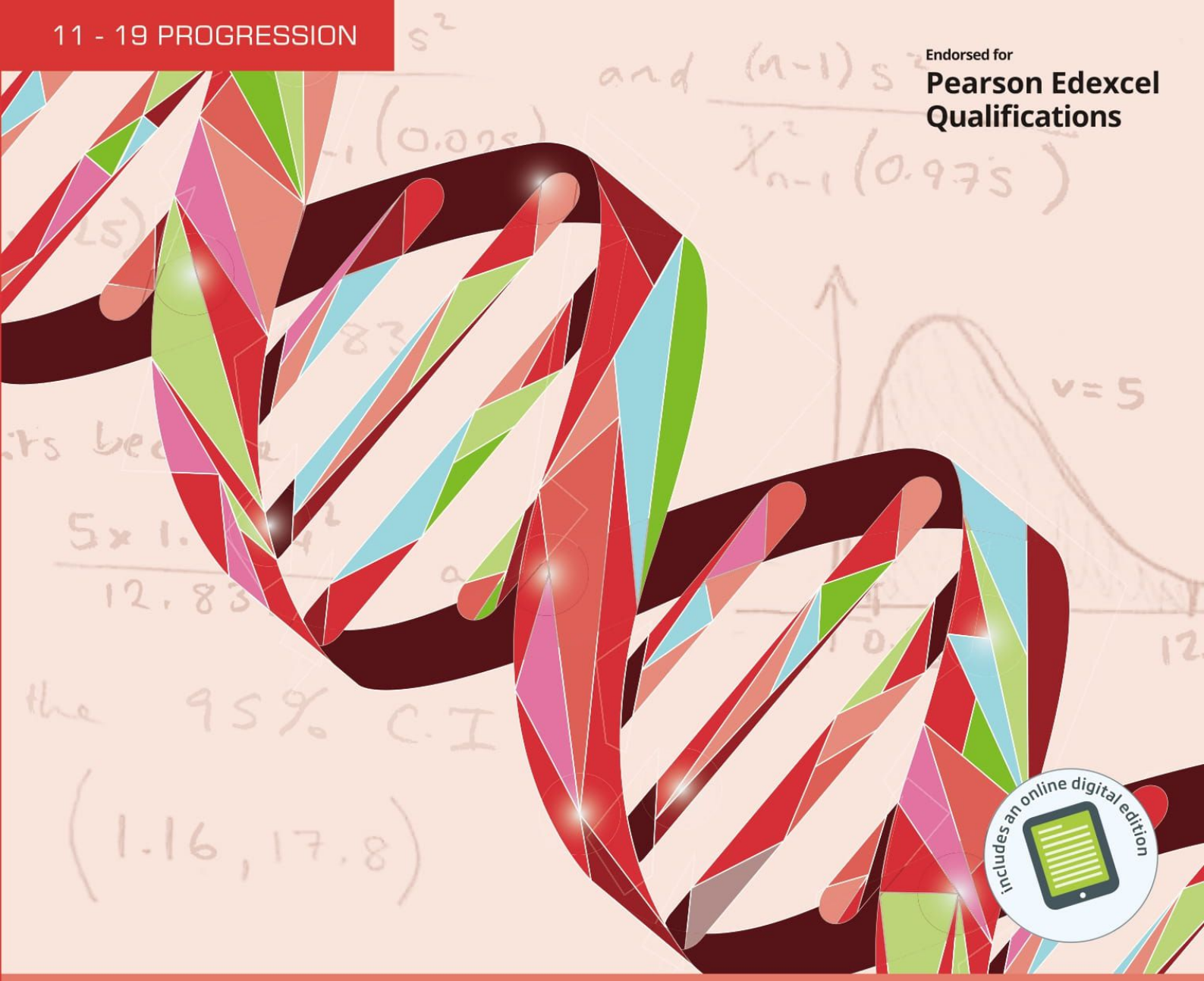
11 - 19 PROGRESSION

Endorsed for
Pearson Edexcel Qualifications



Pearson Edexcel AS and A level Further Mathematics

Further Statistics 1

FS1

Pearson

Contents

● = A level only

Overarching themes iv
Extra online content vi

1 **Discrete random variables** 1
1.1 Expected value of a discrete random variable 2
1.2 Variance of a discrete random variable 5
1.3 Expected value and variance of a function of X 7
1.4 Solving problems involving random variables 11
Mixed exercise 1 14

2 **Poisson distributions** 19
2.1 The Poisson distribution 20
2.2 Modelling with the Poisson distribution 23
2.3 Adding Poisson distributions 27
2.4 Mean and variance of a Poisson distribution 30
2.5 Mean and variance of the binomial distribution 32
2.6 Using the Poisson distribution to approximate the binomial distribution 34
Mixed exercise 2 38

● 3 **Geometric and negative binomial distributions** 43
● **3.1** The geometric distribution 44
● **3.2** Mean and variance of a geometric distribution 47
● **3.3** The negative binomial distribution 49
● **3.4** Mean and variance of the negative binomial distribution 52
Mixed exercise 3 55

4 **Hypothesis testing** 58
4.1 Testing for the mean of a Poisson distribution 59
4.2 Finding critical regions for a Poisson distribution 62
● **4.3** Hypothesis testing for the parameter p of a geometric distribution 66
● **4.4** Finding critical regions for a geometric distribution 69
Mixed exercise 4 72

● 5 **Central limit theorem** 76
● **5.1** The central limit theorem 77
● **5.2** Applying the central limit theorem to other distributions 80
Mixed exercise 5 82

Review exercise 1 85

6 **Chi-squared tests** 91
6.1 Goodness of fit 92
6.2 Degrees of freedom and the chi-squared family of distributions 96
6.3 Testing a hypothesis 99
6.4 Testing the goodness of fit with discrete data 103
6.5 Using contingency tables 113
● **6.6** Apply goodness-of-fit tests to geometric distributions 119
Mixed exercise 6 122

● 7 **Probability generating functions** 128
● **7.1** Probability generating functions 129
● **7.2** Probability generating functions of standard distributions 132
● **7.3** Mean and variance of a distribution 135
● **7.4** Sums of independent random variables 139
Mixed exercise 7 143

● 8 **Quality of tests** 146
● **8.1** Type I and Type II errors 147
● **8.2** Finding Type I and Type II errors using the normal distribution 153
● **8.3** Calculate the size and power of a test 157
● **8.4** The power function 162
Mixed exercise 8 167

Review exercise 2 173

Exam-style practice: AS 180

● **Exam-style practice: A level** 182

Appendix 185

Answers 193

Index 213

Overarching themes

The following three overarching themes have been fully integrated throughout the Pearson Edexcel AS and A level Mathematics series, so they can be applied alongside your learning and practice.

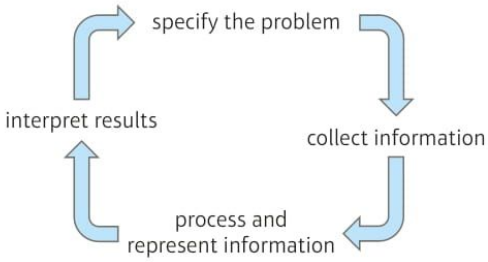
1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols
- Dedicated sections on mathematical proof explain key principles and strategies
- Opportunities to critique arguments and justify methods

2. Mathematical problem solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Structured and unstructured questions to build confidence
- Challenge boxes provide extra stretch

The Mathematical Problem-solving cycle

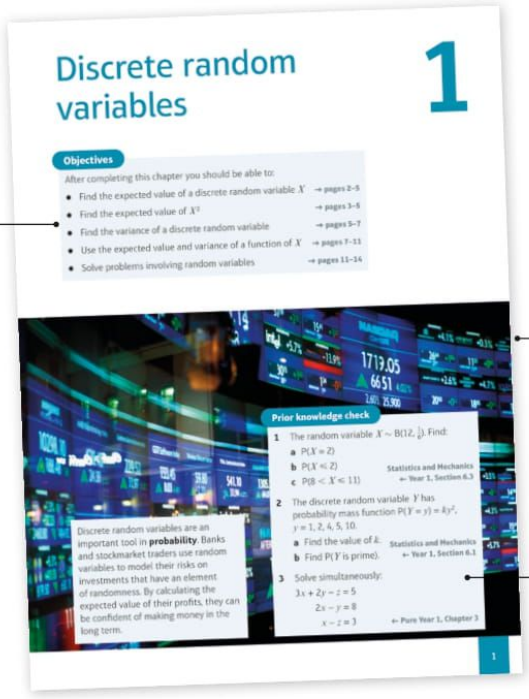


3. Mathematical modelling

- Dedicated modelling sections in relevant topics provide plenty of practice where you need it
- Examples and exercises include qualitative questions that allow you to interpret answers in the context of the model
- Dedicated chapter in Statistics & Mechanics Year 1/AS explains the principles of modelling in mechanics

Finding your way around the book

Access an online digital edition using the code at the front of the book.



Each chapter starts with a list of objectives

The real world applications of the maths you are about to learn are highlighted at the start of the chapter with links to relevant questions in the chapter

The *Prior knowledge check* helps make sure you are ready to start the chapter

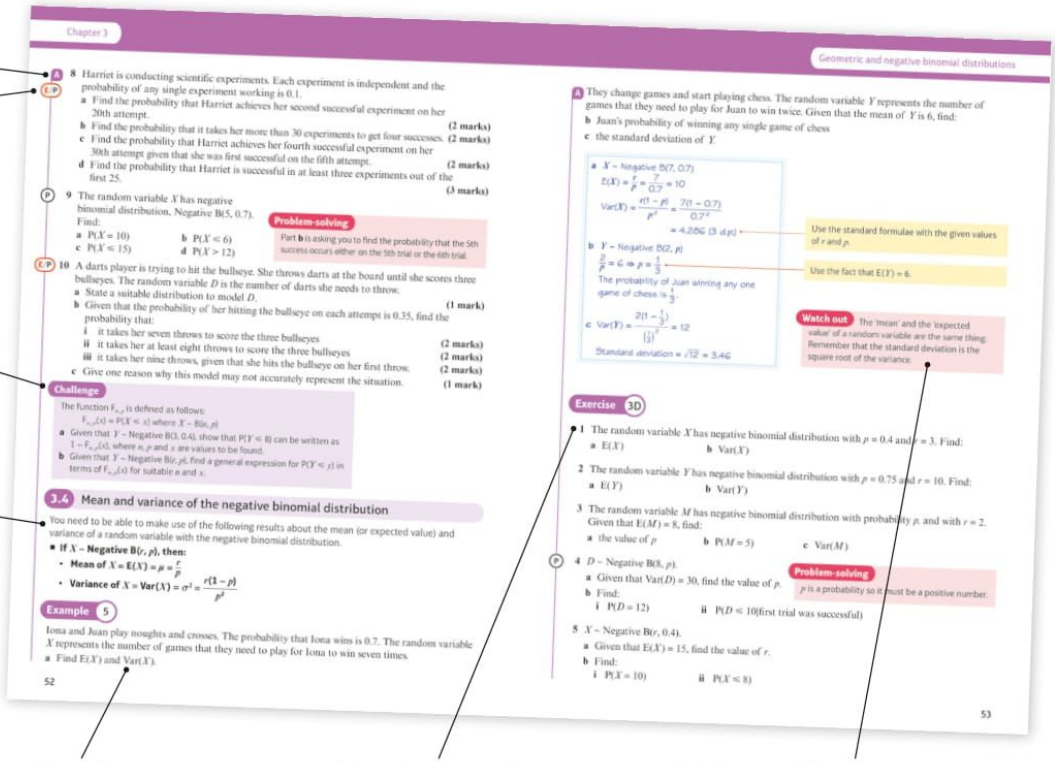
A level content is clearly flagged

Exercises are packed with exam-style questions to ensure you are ready for the exams

Challenge boxes give you a chance to tackle some more difficult questions

Each section begins with explanation and key learning points

Each chapter ends with a *Mixed exercise* and a *Summary of key points*



Chapter 3

Geometric and negative binomial distributions

8 Harriet is conducting scientific experiments. Each experiment is independent and the probability of any single experiment working is 0.1.
a Find the probability that Harriet achieves her second successful experiment on her 20th attempt. (2 marks)
b Find the probability that it takes her more than 30 experiments to get four successes. (2 marks)
c Find the probability that Harriet achieves her fourth successful experiment on her 30th attempt given that she was first successful on the fifth attempt. (2 marks)
d Find the probability that Harriet is successful in at least three experiments out of the first 25. (3 marks)

9 The random variable X has negative binomial distribution, Negative B(5, 0.7). Find:
a $P(X = 10)$ b $P(X \leqslant 6)$
c $P(X \leqslant 15)$ d $P(X > 12)$

Problem-solving: Part b is asking you to find the probability that the 5th success occurs either on the 5th trial or the 6th trial.

10 A darts player is trying to hit the bullseye. She throws darts at the board until she scores three bullseyes. The random variable D is the number of darts she needs to throw.
a State a suitable distribution to model D. (1 mark)
b Given that the probability of her hitting the bullseye on each attempt is 0.35, find the probability that:
i it takes her seven throws to score the three bullseyes (2 marks)
ii it takes her at least eight throws to score the three bullseyes (2 marks)
iii it takes her nine throws, given that she hits the bullseye on her first throw. (2 marks)
c Give one reason why this model may not accurately represent the situation. (1 mark)

Challenge

The function $F_{n,p}(x)$ is defined as follows:
$F_{n,p}(x) = P(X \leqslant x)$ where $X \sim B(n, p)$
a Given that $Y \sim$ Negative B(3, 0.4), show that $P(Y \leqslant 8)$ can be written as $1 - F_{n,p}(x)$, where n, p and x are values to be found.
b Given that $Y \sim$ Negative B(r, p), find a general expression for $P(Y \leqslant y)$ in terms of $F_{n,p}(x)$ for suitable n and x.

3.4 Mean and variance of the negative binomial distribution

You need to be able to make use of the following results about the mean (or expected value) and variance of a random variable with the negative binomial distribution.

- **If $X \sim$ Negative B(r, p), then:**
- **Mean of $X = E(X) = \mu = \frac{r}{p}$**
- **Variance of $X = \text{Var}(X) = \sigma^2 = \frac{r(1-p)}{p^2}$**

Example 5

Iona and Juan play noughts and crosses. The probability that Iona wins is 0.7. The random variable X represents the number of games that they need to play for Iona to win seven times.
a Find $E(X)$ and $\text{Var}(X)$.

52

A They change games and start playing chess. The random variable Y represents the number of games that they need to play for Juan to win twice. Given that the mean of Y is 6, find:
b Juan's probability of winning any single game of chess
c the standard deviation of Y.

a $X \sim$ Negative B(7, 0.7)
$E(X) = \frac{r}{p} = \frac{7}{0.7} = 10$
$\text{Var}(X) = \frac{r(1-p)}{p^2} = \frac{7(1-0.7)}{0.7^2}$
$= 4.2$ (3 d.p.)

Use the standard formulae with the given values of r and p.

b $Y \sim$ Negative B(2, p)
$\frac{2}{p} = 6 \Rightarrow p = \frac{1}{3}$
The probability of Juan winning any one game of chess is $\frac{1}{3}$.

Use the fact that $E(Y) = 6$.

c $\text{Var}(Y) = \frac{2\left(1-\frac{1}{3}\right)}{\left(\frac{1}{3}\right)^2} = 12$
Standard deviation $= \sqrt{12} = 3.46$

Watch out: The 'mean' and the 'expected value' of a random variable are the same thing. Remember that the standard deviation is the square root of the variance.

Exercise 3D

1 The random variable X has negative binomial distribution with $p = 0.4$ and $r = 3$. Find:
a $E(X)$ b $\text{Var}(X)$

2 The random variable Y has negative binomial distribution with $p = 0.75$ and $r = 10$. Find:
a $E(Y)$ b $\text{Var}(Y)$

3 The random variable M has negative binomial distribution with probability p, and with $r = 2$. Given that $E(M) = 8$, find:
a the value of p b $P(M = 5)$ c $\text{Var}(M)$

4 $D \sim$ Negative B(8, p).
a Given that $\text{Var}(D) = 30$, find the value of p.
b Find:
i $P(D = 12)$ ii $P(D \leqslant 10 | \text{first trial was successful})$

Problem-solving: p is a probability so it must be a positive number.

5 $X \sim$ Negative B(r, 0.4).
a Given that $E(X) = 15$, find the value of r.
b Find:
i $P(X = 10)$ ii $P(X \leqslant 8)$

53

Step-by-step worked examples focus on the key types of questions you'll need to tackle

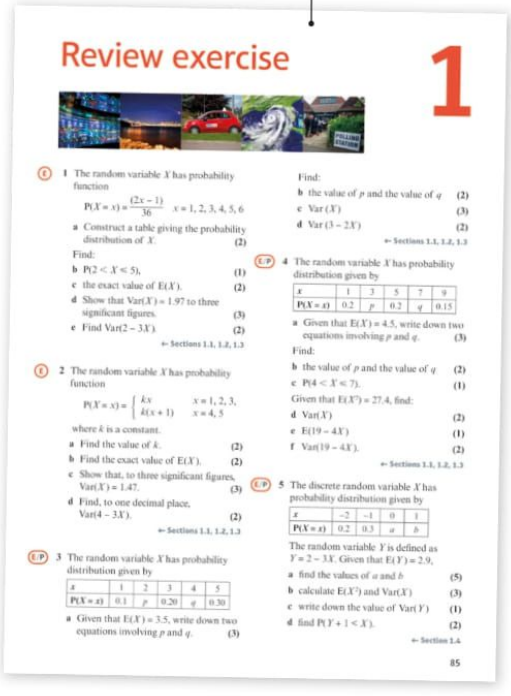
Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Problem-solving boxes provide hints, tips and strategies, and *Watch out* boxes highlight areas where students often lose marks in their exams

Exam-style questions are flagged with (E)

Problem-solving questions are flagged with (P)

Every few chapters a *Review exercise* helps you consolidate your learning with lots of exam-style questions



Review exercise 1

(E) 1 The random variable X has probability function
$P(X = x) = \frac{(2x-1)}{36}$ $x = 1, 2, 3, 4, 5, 6$
a Construct a table giving the probability distribution of X. (2)
Find:
b $P(2 < X \leqslant 5)$, (1)
c the exact value of $E(X)$. (2)
d Show that $\text{Var}(X) = 1.97$ to three significant figures. (3)
e Find $\text{Var}(2 - 3X)$. (2)
← Sections 1.1, 1.2, 1.3

(E) 2 The random variable X has probability function
$P(X = x) = \begin{cases} kx & x = 1, 2, 3, \\ k(x+1) & x = 4, 5 \end{cases}$
where k is a constant.
a Find the value of k. (2)
b Find the exact value of $E(X)$. (2)
c Show that, to three significant figures, $\text{Var}(X) = 1.47$. (3)
d Find, to one decimal place, $\text{Var}(4 - 3X)$. (2)
← Sections 1.1, 1.2, 1.3

(E/P) 3 The random variable X has probability distribution given by

x	1	2	3	4	5
$P(X = x)$	0.1	p	0.20	q	0.30

a Given that $E(X) = 3.5$, write down two equations involving p and q. (3)
Find:
b the value of p and the value of q (2)
c $\text{Var}(X)$ (3)
d $\text{Var}(3 - 2X)$ (2)
← Sections 1.1, 1.2, 1.3

(E/P) 4 The random variable X has probability distribution given by

x	1	3	5	7	9
$P(X = x)$	0.2	p	0.2	q	0.15

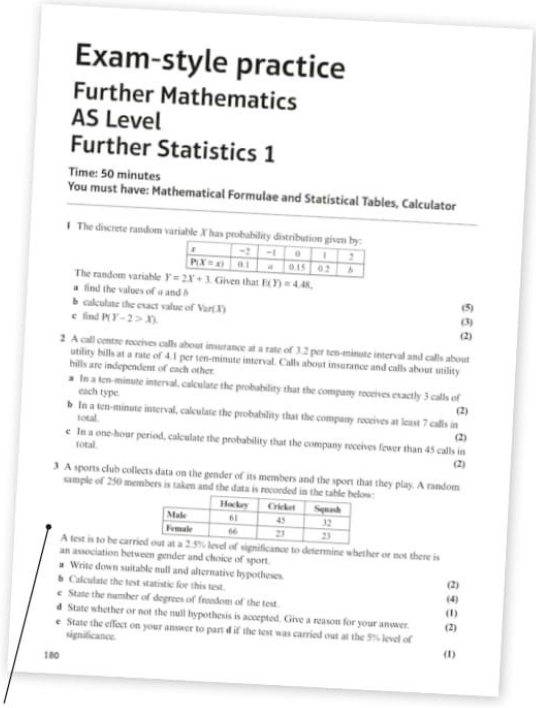
a Given that $E(X) = 4.5$, write down two equations involving p and q. (3)
Find:
b the value of p and the value of q (2)
c $P(4 < X < 7)$. (1)
Given that $E(X^2) = 27.4$, find:
d $\text{Var}(X)$ (2)
e $E(19 - 4X)$ (1)
f $\text{Var}(19 - 4X)$. (2)
← Sections 1.1, 1.2, 1.3

(E/P) 5 The discrete random variable X has probability distribution given by

x	−2	−1	0	1
$P(X = x)$	0.2	0.3	a	b

The random variable Y is defined as $Y = 2 - 3X$. Given that $E(Y) = 2.9$,
a find the values of a and b (5)
b calculate $E(X^2)$ and $\text{Var}(X)$ (3)
c write down the value of $\text{Var}(Y)$ (1)
d find $P(Y + 1 < X)$. (2)
← Section 1.4

85



Exam-style practice
Further Mathematics
AS Level
Further Statistics 1

Time: 50 minutes
You must have: Mathematical Formulae and Statistical Tables, Calculator

1 The discrete random variable X has probability distribution given by:

x	−2	−1	0	1	2
$P(X = x)$	0.1	a	0.15	0.2	b

The random variable $Y = 2X + 3$. Given that $E(Y) = 4.48$,
a find the values of a and b (5)
b calculate the exact value of $\text{Var}(X)$ (3)
c find $P(Y - 2 > X)$. (2)

2 A call centre receives calls about insurance at a rate of 3.2 per ten-minute interval and calls about utility bills at a rate of 4.1 per ten-minute interval. Calls about insurance and calls about utility bills are independent of each other.
a In a ten-minute interval, calculate the probability that the company receives exactly 3 calls of each type. (2)
b In a ten-minute interval, calculate the probability that the company receives at least 7 calls in total. (2)
c In a one-hour period, calculate the probability that the company receives fewer than 45 calls in total. (2)

3 A sports club collects data on the gender of its members and the sport that they play. A random sample of 250 members is taken and the data is recorded in the table below:

	Hockey	Cricket	Squash
Male	61	42	32
Female	66	23	23

A test is to be carried out at a 2.5% level of significance to determine whether or not there is an association between gender and choice of sport.
a Write down suitable null and alternative hypotheses. (2)
b Calculate the test statistic for this test. (4)
c State the number of degrees of freedom of the test. (1)
d State whether or not the null hypothesis is accepted. Give a reason for your answer. (2)
e State the effect on your answer to part d if the test was carried out at the 5% level of significance. (1)

180

AS and A level practice papers at the back of the book help you prepare for the real thing.

Extra online content

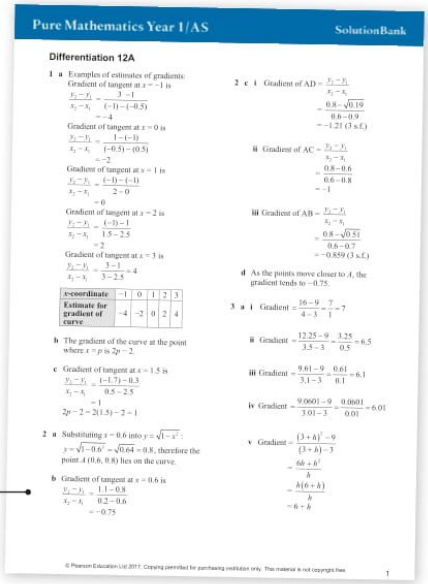
Whenever you see an *Online* box, it means that there is extra online content available to support you.

SolutionBank

SolutionBank provides a full worked solution for every question in the book.

Online Full worked solutions are available in SolutionBank.

Download all the solutions as a PDF or quickly find the solution you need online


Pure Mathematics Year 1/AS SolutionBank

Differentiation 12A

1 a Examples of estimates of gradients:
Gradient of tangent at $x = -1$ is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{(-1)-(-0.5)} = -4$
Gradient of tangent at $x = 0$ is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-1)}{(-0.5)-(0.5)} = -2$
Gradient of tangent at $x = 1$ is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1)-(-1)}{2-0} = 0$
Gradient of tangent at $x = 2$ is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1)-1}{1.5-2.5} = 2$
Gradient of tangent at $x = 3$ is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{3-2.5} = 4$

x-coordinate	−1	0	1	2	3
Estimate for gradient of curve	−4	−2	0	2	4

b The gradient of the curve at the point where $x = p$ is $2p - 2$.

c Gradient of tangent at $x = 1.5$ is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1.7)-0.3}{0.5-2.5} = 1$
$2p - 2 = 2(1.5) - 2 = 1$

2 a Substituting $x = 0.6$ into $y = \sqrt{1 - x^2}$:
$y = \sqrt{1 - 0.6^2} = \sqrt{0.64} = 0.8$, therefore the point $A\,(0.6, 0.8)$ lies on the curve.

b Gradient of tangent at $x = 0.6$ is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1.1-0.8}{0.2-0.6} = -0.75$

2 c i Gradient of AD $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.8-\sqrt{0.19}}{0.6-0.9} = -1.21$ (3 s.f.)

ii Gradient of AC $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.8-0.6}{0.6-0.8} = -1$

iii Gradient of AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.8-\sqrt{0.51}}{0.6-0.7} = -0.859$ (3 s.f.)

d As the points move closer to A, the gradient tends to -0.75.

3 a i Gradient $= \frac{16-9}{4-3} = \frac{7}{1} = 7$

ii Gradient $= \frac{12.25-9}{3.5-3} = \frac{3.25}{0.5} = 6.5$

iii Gradient $= \frac{9.61-9}{3.1-3} = \frac{0.61}{0.1} = 6.1$

iv Gradient $= \frac{9.0601-9}{3.01-3} = \frac{0.0601}{0.01} = 6.01$

v Gradient $= \frac{(3+h)^2-9}{(3+h)-3} = \frac{6h+h^2}{h} = \frac{h(6+h)}{h} = 6+h$

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Use of technology

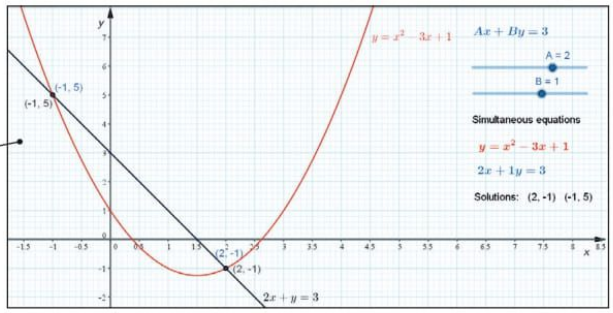
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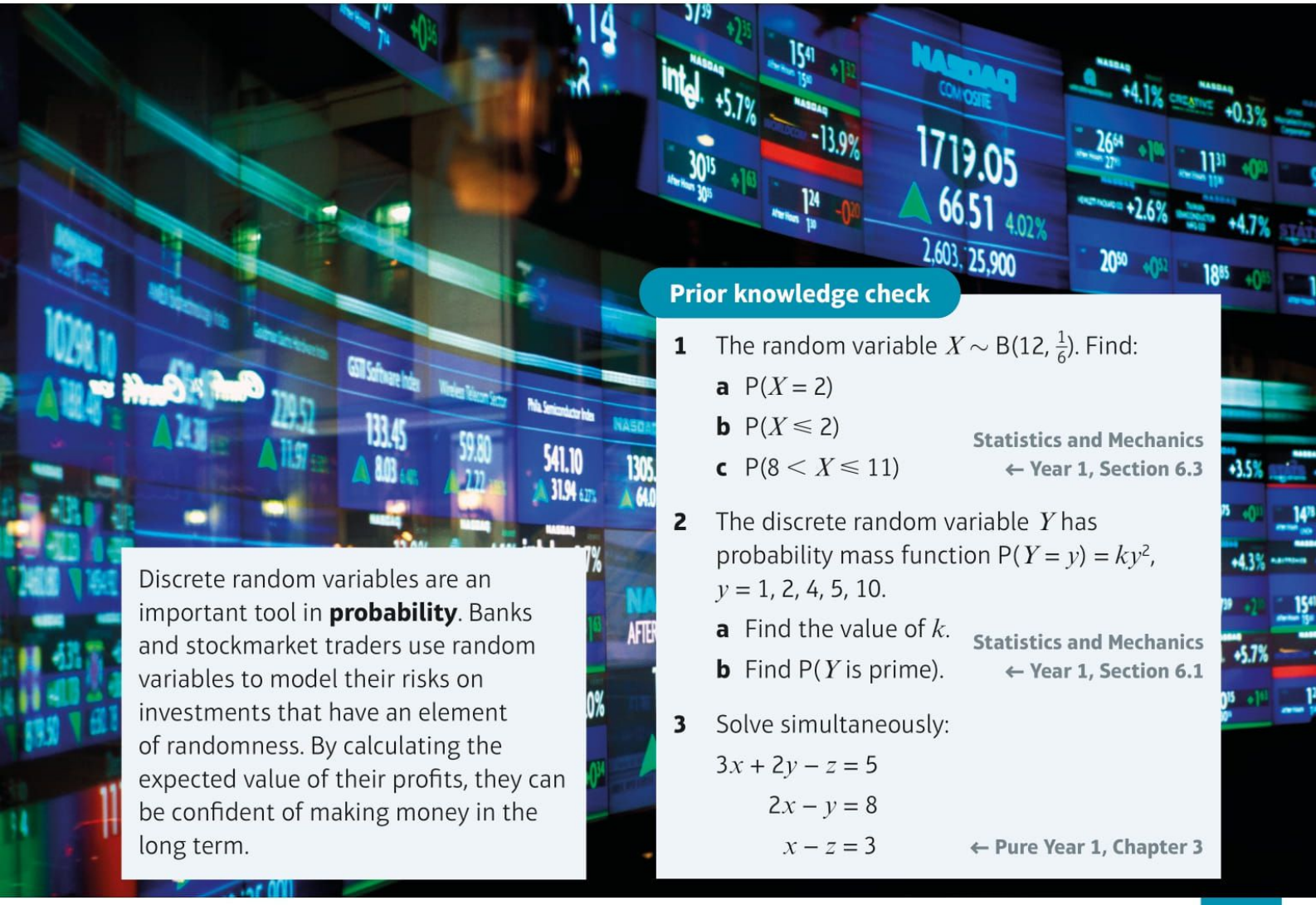
Discrete random variables

1

Objectives

After completing this chapter you should be able to:

- Find the expected value of a discrete random variable X → pages 2–5
- Find the expected value of X^2 → pages 3–5
- Find the variance of a discrete random variable → pages 5–7
- Use the expected value and variance of a function of X → pages 7–11
- Solve problems involving random variables → pages 11–14



Discrete random variables are an important tool in **probability**. Banks and stockmarket traders use random variables to model their risks on investments that have an element of randomness. By calculating the expected value of their profits, they can be confident of making money in the long term.

Prior knowledge check

1 The random variable $X \sim B(12, \frac{1}{6})$. Find:

a $P(X = 2)$

b $P(X \leqslant 2)$

c $P(8 < X \leqslant 11)$

Statistics and Mechanics ← Year 1, Section 6.3

2 The discrete random variable Y has probability mass function $P(Y = y) = ky^2$, $y = 1, 2, 4, 5, 10$.

a Find the value of k.

b Find $P(Y$ is prime$)$.

Statistics and Mechanics ← Year 1, Section 6.1

3 Solve simultaneously:

$$3x + 2y - z = 5$$
$$2x - y = 8$$
$$x - z = 3$$

← Pure Year 1, Chapter 3

1.1 Expected value of a discrete random variable

Recall that a **random variable** is a variable whose value depends on a random event. The random variable is **discrete** if it can only take certain numerical values.

Links The probabilities of any discrete random variable add up to 1. For a discrete random variable, X, you write $\sum P(X = x) = 1$.
← Statistics and Mechanics Year 1, Chapter 6

If you take a set of observations from a discrete random variable, you can find the mean of those observations. As the number of observations increases, this value will get closer and closer to the **expected value** of the discrete random variable.

Watch out The expected value is a theoretical quantity, and gives information about the probability distribution of a random variable.

- **The expected value of the discrete random variable X is denoted $E(X)$ and defined as $E(X) = \sum xP(X = x)$.**

Notation The expected value is sometimes referred to as the **mean**, and is denoted by μ.

Example 1

A fair six-sided dice is rolled. The number on the uppermost face is modelled by the random variable X.

a Write down the probability distribution of X.

b Use the probability distribution of X to calculate $E(X)$.

a

x	1	2	3	4	5	6
$P(x = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Since the dice is fair, each side is equally likely to end facing up, so the probability of any face ending up as the uppermost is $\frac{1}{6}$

b The expected value of X is:

$$E(X) = \sum xP(X = x) = \frac{1}{6} + \frac{2}{6} + \ldots + \frac{6}{6}$$

$$= \frac{21}{6} = \frac{7}{2} = 3.5$$

Substitute values from the probability distribution into the formula then simplify.

If you know the probability distribution of X then you can calculate the expected value. Notice that in Example 1 the expected value is 3.5, but $P(X = 3.5) = 0$. The expected value of a random variable does not have to be a value that the random variable can actually take. Instead this tells us that in the long run, we would expect the average of all rolls to get close to 3.5.

Example 2

The random variable X has a probability distribution as shown in the table.

x	1	2	3	4	5
$p(x)$	0.1	p	0.3	q	0.2

a Given that $E(X) = 3$, write down two equations involving p and q.

b Find the value of p and the value of q.

a $p + q + 0.1 + 0.3 + 0.2 = 1$

$p + q = 1 - 0.6$

$p + q = 0.4$ (1)

$(1 \times 0.1) + 2p + (3 \times 0.3) + 4q + (5 \times 0.2) = 3$

$2p + 4q = 3 - (0.1 + 0.9 + 1)$

$2p + 4q = 1$ (2)

b $2p + 4q = 1$

$2p + 2q = 0.8$

so $2q = 0.2$

$q = 0.1$

$p = 0.4 - q$

$= 0.4 - 0.1$

$= 0.3$

Problem-solving

Remember that the probabilities must add up to 1. You will often have to use $\sum P(X = x) = 1$ when solving problems involving discrete random variables.

$E(X) = \sum xP(X = x)$

From (2)

Multiply (1) by 2.

Subtract bottom line from top line.

Substitute 0.1 for q.

If X is a discrete random variable, then X^2 is also a discrete random variable. You can use this rule to determine the expected value of X^2.

- $\mathbf{E(X^2) = \sum x^2 P(X = x)}$

Links Any function of a random variable is also a random variable. → Section 1.3

Example 3

A discrete random variable X has the following probability distribution

x	1	2	3	4
$P(X = x)$	$\frac{12}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

a Write down the probability distribution for X^2.

b Find $E(X^2)$.

a The probability distribution for X^2 is

x	1	2	3	4
x^2	1	4	9	16
$P(X^2 = x^2)$	$\frac{12}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

b $E(X^2) = \sum x^2 P(X^2 = x^2)$

$= 1 \times \frac{12}{25} + 4 \times \frac{6}{25} + 9 \times \frac{4}{25} + 16 \times \frac{3}{25}$

$= \frac{120}{25}$

$= 4.8$

X can take values 1, 2, 3, 4, so X^2 can take values 1^2, 2^2, 3^2, 4^2.

Note that because X takes only positive values, $P(X^2 = x^2) = P(X = x)$.

Watch out $E(X^2)$ is, in general, not equal to $(E(X))^2$. In this example $E(X) = 1.92$ and $1.92^2 \neq 4.8$.

Exercise 1A

1 For each of the following probability distributions write out the distribution of X^2 and calculate both $E(X)$ and $E(X^2)$.

a

x	2	4	6	8
$P(X = x)$	0.3	0.3	0.2	0.2

b

x	−2	−1	1	2
$P(X = x)$	0.1	0.4	0.1	0.4

Watch out Note that, for example, $P(X^2 = 4) = P(X = 2) + P(X = -2)$.

2 The score on a biased dice is modelled by a random variable X with probability distribution

x	1	2	3	4	5	6
$P(X = x)$	0.1	0.1	0.1	0.2	0.4	0.1

Find $E(X)$ and $E(X^2)$.

3 The random variable X has a probability function

$$P(X = x) = \frac{1}{x} \qquad x = 2, 3, 6$$

a Construct tables giving the probability distributions of X and X^2.

b Work out $E(X)$ and $E(X^2)$.

c State whether or not $(E(X))^2 = E(X^2)$.

4 The random variable X has a probability function given by

$$P(X = x) = \begin{cases} 2^{-x} & x = 1, 2, 3, 4 \\ 2^{-4} & x = 5 \end{cases}$$

a Construct a table giving the probability distribution of X.

b Calculate $E(X)$ and $E(X^2)$.

c State whether or not $(E(X))^2 = E(X^2)$.

E/P **5** The random variable X has the following probability distribution:

x	1	2	3	4	5
$P(X = x)$	0.1	a	b	0.2	0.1

Given that $E(X) = 2.9$, find the value of a and the value of b. **(5 marks)**

E/P **6** The random variable X has the following probability distribution:

x	−2	−1	1	2
$P(X = x)$	0.1	a	b	c

Given that $E(X) = 0.3$ and $E(X^2) = 1.9$, find a, b and c. **(7 marks)**

Problem-solving

You can use the given information to write down simultaneous equations for a, b and c which can be solved using the matrix inverse operation on your calculator.

← Core Pure Book 1, Section 6.6

E/P **7** The discrete random variable X has probability function

$$P(X = x) = \begin{cases} a(1 - x) & x = -2, -1, 0 \\ b & x = 5 \end{cases}$$

Given that $E(X) = 1.2$, find the value of a and the value of b. **(6 marks)**

E/P **8** A biased six-sided dice has a $\frac{1}{8}$ chance of landing on any of the numbers 1, 2, 3 or 4. The probabilities of landing on 5 or 6 are unknown. The outcome is modelled as a random variable, X. Given that $E(X) = 4.1$,

a find the probability distribution of X. **(5 marks)**

The dice is rolled 10 times.

b Calculate the probability that the dice lands on 6 at least 3 times. **(3 marks)**

E **9** Jorge has designed a game for his school fete. Students can pay £1 to roll a fair six-sided dice. If they score a 6 they win a prize of £5. If they score a 4 or a 5, they win a smaller prize of £P.

By modelling the amount paid out in prize money as a discrete random variable, determine the maximum value of P in order for Jorge to not make a loss on his game. **(5 marks)**

Hint The expected profit from the game is the cost of playing the game minus the expected value of the amount paid out in prize money.

Challenge

Three fair six-sided dice are rolled. The discrete random variable X is defined as the largest value of the three values shown. Find $E(X)$.

1.2 Variance of a discrete random variable

If you take a set of observations from a discrete random variable, you can find the variance of those observations. As the number of observations increases, this value will get closer and closer to the **variance** of the discrete random variable.

Notation The variance is sometimes denoted by σ^2, where σ is the standard deviation.

- **The variance of X is usually written as $\text{Var}(X)$ and is defined as $\text{Var}(X) = E((X - E(X))^2)$**

The random variable $(X - E(X))^2$ is the squared deviation from the expected value of X. It is large when X takes values that are very different to $E(X)$.

Online Explore probability distributions of a discrete random variable and compare the theoretical distribution with observed results generated from that discrete random variable using GeoGebra.

- **Sometimes it is easier to calculate the variance using the formula $\text{Var}(X) = E(X^2) - (E(X))^2$**

From the definition you can see that $\text{Var}(X) \geqslant 0$ for any random variable X. The larger $\text{Var}(X)$ the more variable X is. In other words, the more likely it is to take values very different to its expected value.

Example 4

A fair six-sided dice is rolled. The number on the uppermost face is modelled by the random variable X.

Find $\text{Var}(X)$.

Method 1

We have that $\text{E}(X) = 3.5$.

This was calculated in the first example of the previous section.

The distributions of X, X^2 and $(X - \text{E}(X))^2$ are given by

x	1	2	3	4	5	6
x^2	1	4	9	16	25	36
$(x - \text{E}(X))^2$	6.25	2.25	0.25	0.25	2.25	6.25
$\text{P}(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

So the variance is

$\text{Var}(X) = \sum(x - \text{E}(X))^2 \text{P}(X = x)$

$= 6.25 \times \frac{2}{6} + 2.25 \times \frac{2}{6} + 0.25 \times \frac{2}{6}$

Substitute values into the formula for variance.

$= (6.25 + 2.25 + 0.25) \times \frac{1}{3} = \frac{35}{12}$

Method 2

The expected value of X^2 is

$\text{E}(X^2) = \sum x^2 \text{P}(X = x) = \frac{1}{6}(1 + 4 + \ldots + 36) = \frac{91}{6}$

So using the alternative formula

$\text{Var}(X) = \text{E}(X^2) - (\text{E}(X))^2 = \frac{91}{6} - \frac{49}{4} = \frac{35}{12}$

It is usually quicker to use this method to find the variance of a random variable.

Exercise 1B

1 The random variable X has a probability distribution given by

x	−1	0	1	2	3
$\text{P}(X = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

a Find $\text{E}(X)$.

b Find $\text{Var}(X)$.

2 Find the expected value and variance of the random variable X with probability distributions given by:

a

x	1	2	3
$\text{P}(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

b

x	−1	0	1
$\text{P}(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

c

x	−2	−1	1	2
$\text{P}(X = x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

3 Given that Y is the score when a single, unbiased, eight-sided dice is rolled, find $\text{E}(Y)$ and $\text{Var}(Y)$.

(P) **8** The random variable X has the probability distribution

x	1	2	3	4	5
$\mathbf{P}(X = x)$	0.1	p	q	0.3	0.1

a Given that $\mathrm{E}(X) = 3.1$, write down two equations involving p and q.

Find:

b the value of p and the value of q **c** $\mathrm{Var}(X)$

d $\mathrm{Var}(2X - 3)$

(E) **9** The random variable X has the probability function

$$\mathrm{P}(X = x) = \begin{cases} kx & x = 1, 2 \\ k(x - 2) & x = 3, 4, 5 \end{cases}$$

where k is a constant.

a Find the value of k. **(2 marks)**

b Find the exact value of $\mathrm{E}(X)$. **(1 mark)**

c Show that, to three significant figures, $\mathrm{Var}(X) = 2.02$. **(2 marks)**

d Find, to one decimal place, $\mathrm{Var}(3 - 2X)$. **(1 mark)**

(E) **10** The random variable X has the discrete uniform distribution

$$\mathrm{P}(X = x) = \tfrac{1}{6} \qquad x = 1, 2, 3, 4, 5, 6$$

a Write down $\mathrm{E}(X)$ and show that $\mathrm{Var}(X) = \frac{35}{12}$ **(4 marks)**

b Find $\mathrm{E}(2X - 1)$. **(2 marks)**

c Find $\mathrm{Var}(3 - 2X)$. **(2 marks)**

d Find $\mathrm{E}(2^X)$. **(3 marks)**

11 The random variable X has the probability function

$$\mathrm{P}(X = x) = \frac{3x - 1}{26} \qquad x = 1, 2, 3, 4$$

a Construct a table giving the probability distribution of X.

Find:

b $\mathrm{P}(2 < X \leqslant 4)$ **c** the exact value of $\mathrm{E}(X)$.

d Show that $\mathrm{Var}(X) = 0.92$ to two significant figures. **e** Find $\mathrm{Var}(1 - 3X)$.

12 The random variable Y has mean 2 and variance 9.

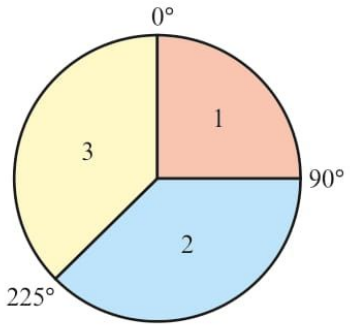
Find:

a $\mathrm{E}(3Y + 1)$ **b** $\mathrm{E}(2 - 3Y)$ **c** $\mathrm{Var}(3Y + 1)$

d $\mathrm{Var}(2 - 3Y)$ **e** $\mathrm{E}(Y^2)$ **f** $\mathrm{E}((Y - 1)(Y + 1))$

13 The random variable T has a mean of 20 and a standard deviation of 5.
The random variable S is defined as $S = 3T + 4$.
Find $\mathrm{E}(S)$ and $\mathrm{Var}(S)$.

14 A fair spinner is made from the disc in the diagram and the random variable X represents the number it lands on after being spun.



a Write down the distribution of X.
b Work out $E(X)$.
c Find $Var(X)$.
d Find $E(2X + 1)$.
e Find $Var(3X - 1)$.

15 The discrete variable X has the probability distribution

x	−1	0	1	2
$P(X = x)$	0.2	0.5	0.2	0.1

Find:

a $E(X)$ **b** $Var(X)$ **c** $E(\frac{1}{3}X + 1)$ **d** $Var(\frac{1}{3}X + 1)$

E/P **16** The discrete random variable X has a probability distribution given by

x	−1	0	1	2
$P(X = x)$	0.1	0.3	a	b

The random variable Y is defined $Y = 3X - 1$. Given that $E(Y) = 1.1$,

a find the values of a and b. **(5 marks)**
b Calculate $E(X^2)$ and $Var(X)$ using the values of a and b that you found in part **a**. **(3 marks)**
c Write down the value of $Var(Y)$. **(1 mark)**
d Find $P(Y + 2 > X)$. **(2 marks)**

E/P **17** The discrete random variable X has a probability distribution given by

x	−2	0	2	3	4
$P(X = x)$	a	b	a	b	c

The random variable Y is defined as $Y = \dfrac{2 - 3X}{5}$

You are given that $E(Y) = -0.98$ and $P(Y \geqslant -1) = 0.4$.

a Write down three simultaneous equations in a, b and c. **(4 marks)**
b Solve this system to find the values of a, b and c. **(3 marks)**
c Find $P(-2X > 10Y)$. **(2 marks)**

Challenge

A Let n be a positive integer and suppose that X is a discrete random variable with $P(X = i) = \frac{1}{n}$ for $i = 1, \ldots, n$.

Show that $E(X) = \dfrac{n + 1}{2}$ and $Var(X) = \dfrac{(n + 1)(n - 1)}{12}$

Hint You can make use of the following results:

$$\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}$$

← Core Pure Book 1, Chapter 3

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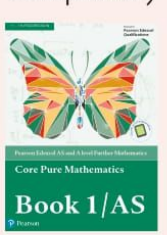


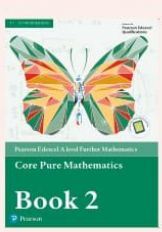
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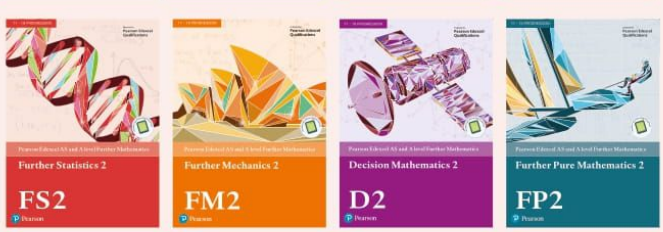
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