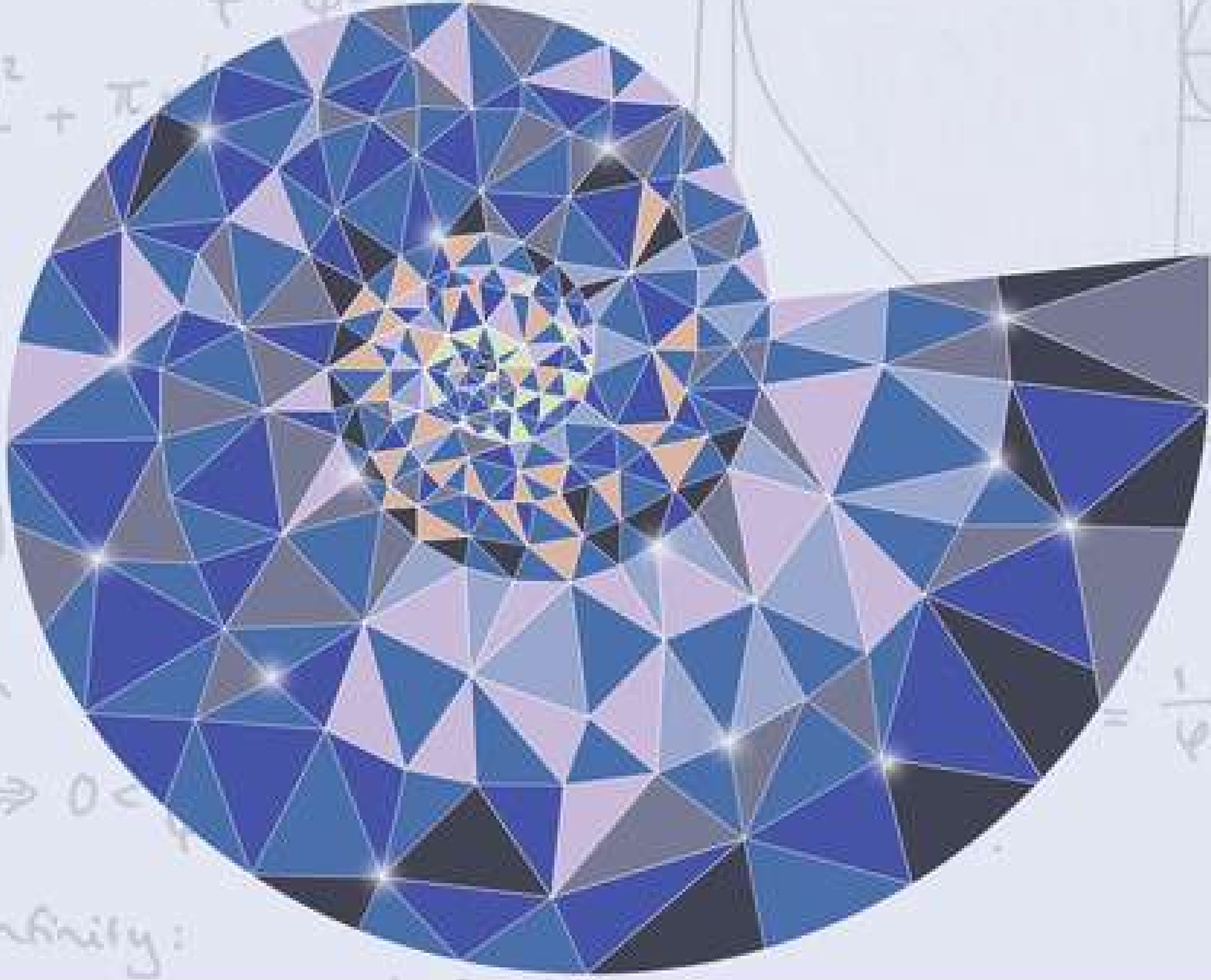
11 - 19 PROGRESSION



Edexcel A level Mathematics

Pure Mathematics

Year 2

Pearson

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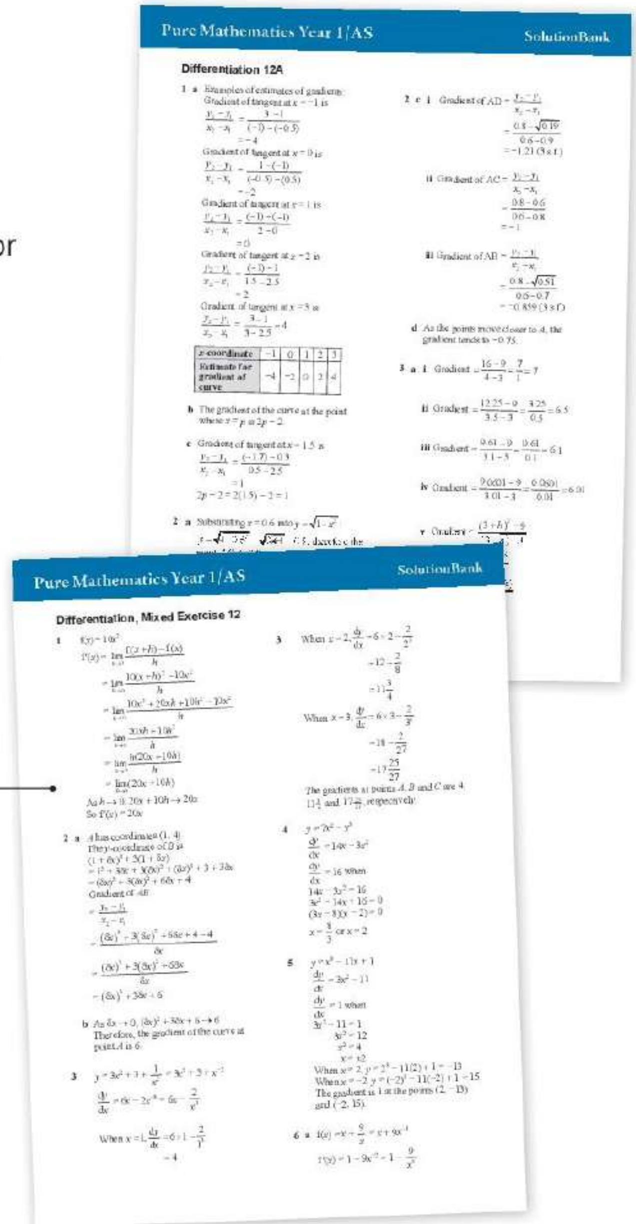
Extra online content

Whenever you see an *Online* box, it means that there is extra online content available to support you.

SolutionBank

SolutionBank provides a full worked solution for every question in the book.

Online Full worked solutions are available in SolutionBank.



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Online Find the point of intersection graphically using technology.

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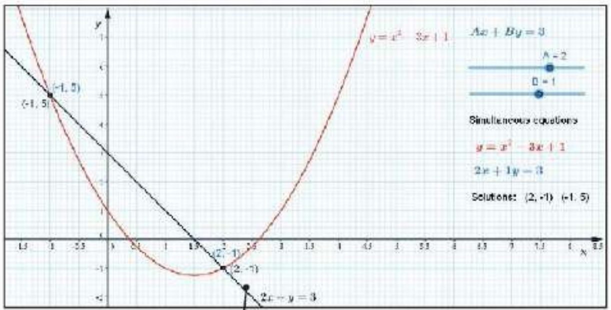

Interact with the maths you are learning using GeoGebra's easy-to-use tools

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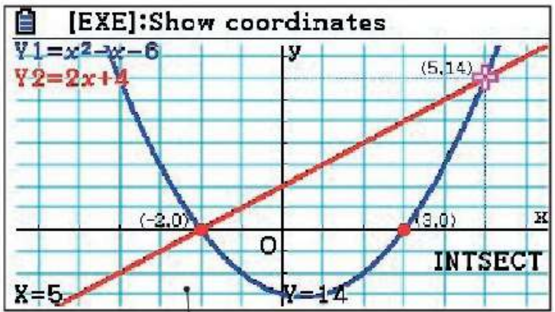
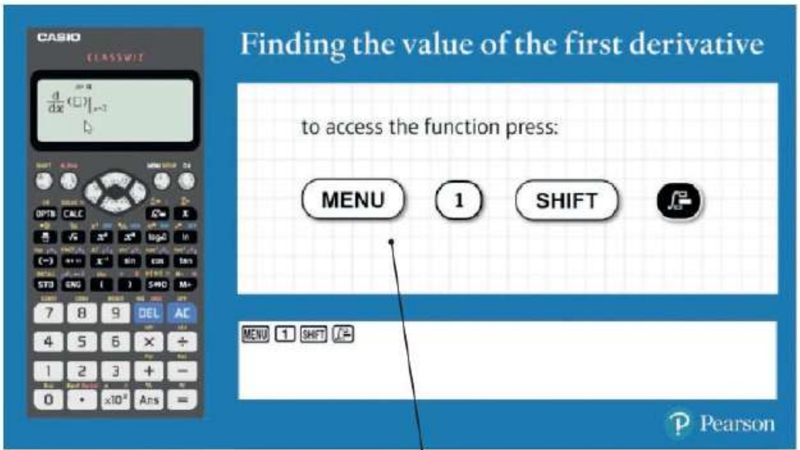


Explore the maths you are learning and gain confidence in using a graphic calculator

Calculator tutorials

Our helpful video tutorials will guide you through how to use your calculator in the exams. They cover both Casio's scientific and colour graphic calculators.



Step-by-step guide with audio instructions on exactly which buttons to press and what should appear on your calculator's screen

Online Work out each coefficient quickly using the ${}^nC_r$ and power functions on your calculator.

Overarching themes

The following three overarching themes have been fully integrated throughout the Pearson Edexcel AS and A level Mathematics series, so they can be applied alongside your learning and practice.

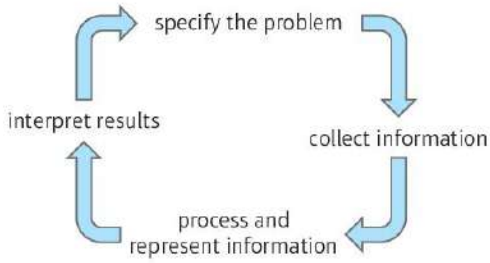
1. Mathematical argument, language and proof

- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols
- Dedicated sections on mathematical proof explain key principles and strategies
- Opportunities to critique arguments and justify methods

2. Mathematical problem solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Structured and unstructured questions to build confidence
- Challenge boxes provide extra stretch

The Mathematical Problem-solving cycle

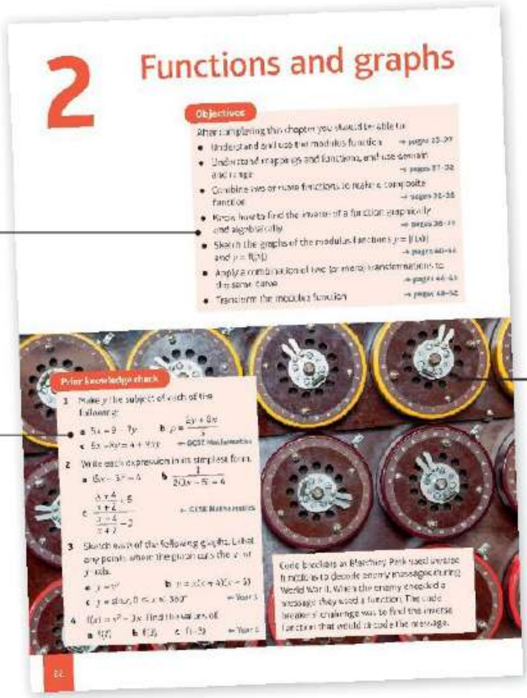


3. Mathematical modelling

- Dedicated modelling sections in relevant topics provide plenty of practice where you need it
- Examples and exercises include qualitative questions that allow you to interpret answers in the context of the model
- Dedicated chapter in Statistics & Mechanics Year 1/AS explains the principles of modelling in mechanics

Finding your way around the book

Access an online digital edition using the code at the front of the book.



Each chapter starts with a list of objectives

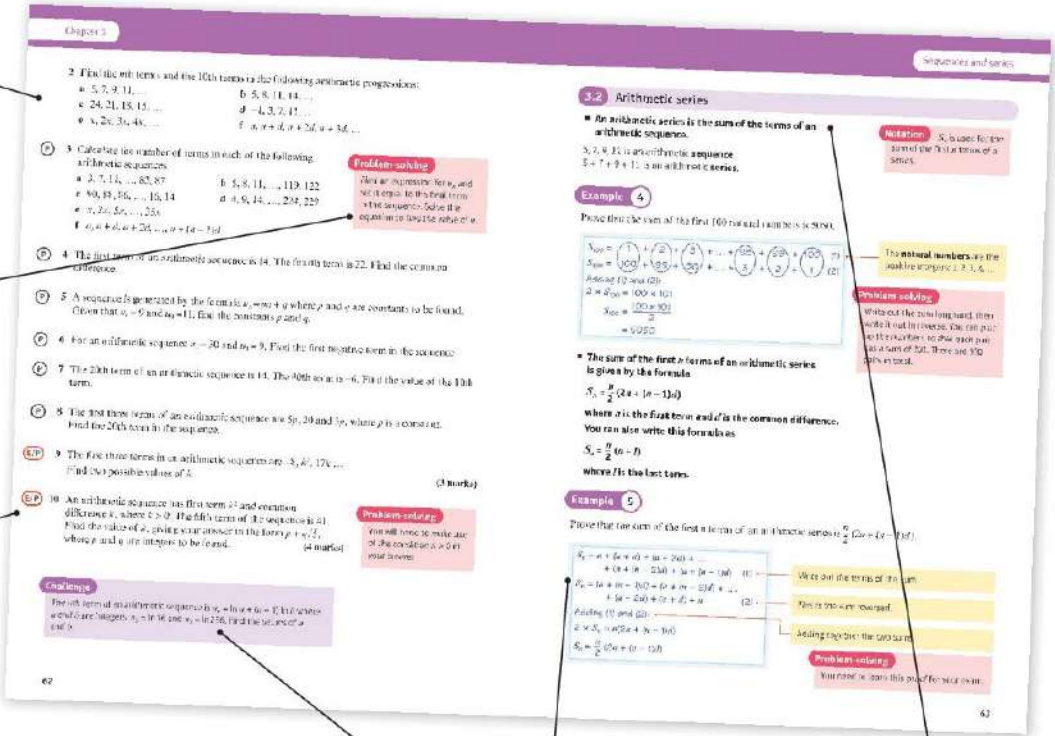
The *Prior knowledge check* helps make sure you are ready to start the chapter

The real world applications of the maths you are about to learn are highlighted at the start of the chapter with links to relevant questions in the chapter

Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Problem-solving boxes provide hints, tips and strategies, and *Watch out* boxes highlight areas where students often lose marks in their exams

Exercises are packed with exam-style questions to ensure you are ready for the exams



Exam-style questions are flagged with (E)

Problem-solving questions are flagged with (P)

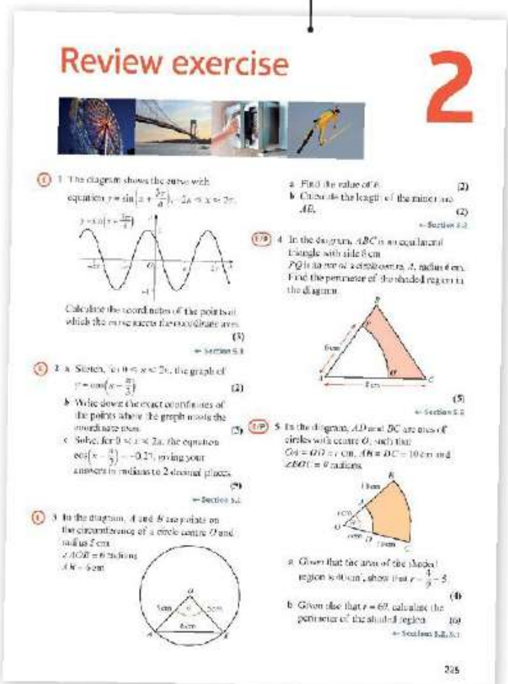
Each chapter ends with a *Mixed exercise* and a *Summary of key points*

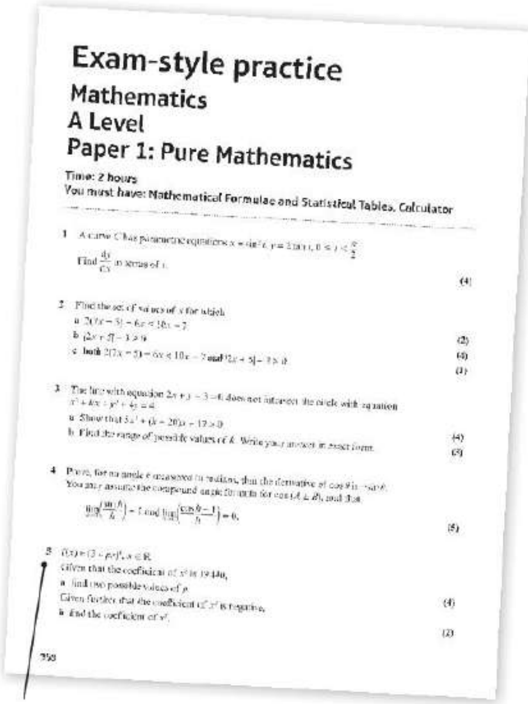
Challenge boxes give you a chance to tackle some more difficult questions

Step-by-step worked examples focus on the key types of questions you'll need to tackle

Each section begins with explanation and key learning points

Every few chapters a *Review exercise* helps you consolidate your learning with lots of exam-style questions





Two A level practice papers at the back of the book help you prepare for the real thing

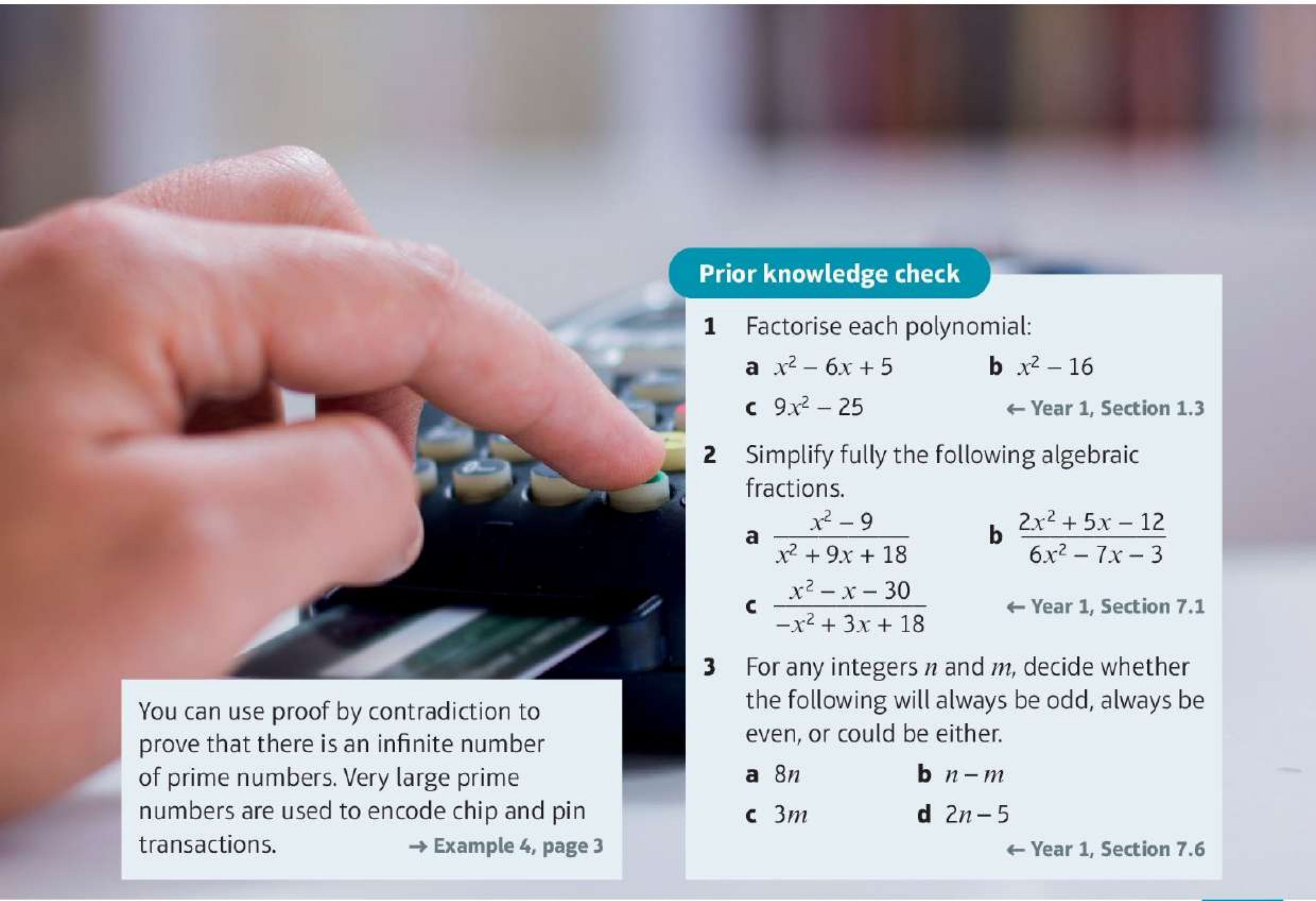
Algebraic methods

1

Objectives

After completing this chapter you should be able to:

- Use proof by contradiction to prove true statements → **pages 2–5**
- Multiply and divide two or more algebraic fractions → **pages 5–7**
- Add or subtract two or more algebraic fractions → **pages 7–8**
- Convert an expression with linear factors in the denominator into partial fractions → **pages 9–11**
- Convert an expression with repeated linear factors in the denominator into partial fractions → **pages 12–13**
- Divide algebraic expressions → **pages 14–17**
- Convert an improper fraction into partial fraction form → **pages 17–18**



You can use proof by contradiction to prove that there is an infinite number of prime numbers. Very large prime numbers are used to encode chip and pin transactions. → **Example 4, page 3**

Prior knowledge check

1 Factorise each polynomial:

a $x^2 - 6x + 5$ **b** $x^2 - 16$

c $9x^2 - 25$ ← **Year 1, Section 1.3**

2 Simplify fully the following algebraic fractions.

a $\dfrac{x^2 - 9}{x^2 + 9x + 18}$ **b** $\dfrac{2x^2 + 5x - 12}{6x^2 - 7x - 3}$

c $\dfrac{x^2 - x - 30}{-x^2 + 3x + 18}$ ← **Year 1, Section 7.1**

3 For any integers n and m, decide whether the following will always be odd, always be even, or could be either.

a $8n$ **b** $n - m$

c $3m$ **d** $2n - 5$

← **Year 1, Section 7.6**

1.1 Proof by contradiction

A **contradiction** is a disagreement between two statements, which means that both cannot be true. Proof by contradiction is a powerful technique.

- **To prove a statement by contradiction you start by assuming it is not true. You then use logical steps to show that this assumption leads to something impossible (either a contradiction of the assumption, or a contradiction of a fact you know to be true). You can conclude that your assumption was incorrect, and the original statement was true.**

Notation A statement that asserts the falsehood of another statement is called the negation of that statement.

Example 1

Prove by contradiction that there is no greatest odd integer.

Assumption: there is a greatest odd integer, n.

$n + 2$ is also an integer and $n + 2 > n$

$n + 2 = \text{odd} + \text{even} = \text{odd}$

So there exists an odd integer greater than n.

This contradicts the assumption that the greatest odd integer is n.

Therefore, there is no greatest odd integer.

Begin by assuming the original statement is false. This is the negation of the original statement.

You need to use logical steps to reach a contradiction. Show all of your working.

The existence of an odd integer greater than n contradicts your initial assumption.

Finish your proof by concluding that the original statement must be true.

Example 2

Prove by contradiction that if n^2 is even, then n must be even.

Assumption: there exists a number n such that n^2 is even but n is odd.

n is odd so write $n = 2k + 1$

$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

So n^2 is odd.

This contradicts the assumption that n^2 is even.

Therefore, if n^2 is even then n must be even.

This is the negation of the original statement.

You can write any odd number in the form $2k + 1$ where k is an integer.

All multiples of 2 are even numbers, so 1 more than a multiple of 2 is an odd number.

Finish your proof by concluding that the original statement must be true.

- **A rational number can be written as $\frac{a}{b}$, where a and b are integers.**
- **An irrational number cannot be expressed in the form $\frac{a}{b}$, where a and b are integers.**

Notation $\mathbb{Q}$ is the set of all rational numbers.

Example 3

Prove by contradiction that $\sqrt{2}$ is an irrational number.

Assumption: $\sqrt{2}$ is a rational number.

Then $\sqrt{2} = \frac{a}{b}$ for some integers, a and b.

Also assume that this fraction cannot be reduced further: there are no common factors between a and b.

So $2 = \frac{a^2}{b^2}$ or $a^2 = 2b^2$

This means that a^2 must be even, so a is also even.

If a is even, then it can be expressed in the form $a = 2n$, where n is an integer.

So $a^2 = 2b^2$ becomes $(2n)^2 = 2b^2$ which means $4n^2 = 2b^2$ or $2n^2 = b^2$.

This means that b^2 must be even, so b is also even.

If a and b are both even, they will have a common factor of 2.

This contradicts the statement that a and b have no common factors.

Therefore $\sqrt{2}$ is an irrational number.

Begin by assuming the original statement is false.

This is the definition of a rational number.

If a and b did have a common factor you could just cancel until this fraction was in its simplest form.

Square both sides and make a^2 the subject.

We proved this result in Example 2.

Again using the result from Example 2.

All even numbers are divisible by 2.

Finish your proof by concluding that the original statement must be true.

Example 4

Prove by contradiction that there are infinitely many prime numbers.

Assumption: there is a finite number of prime numbers.

List all the prime numbers that exist:

$p_1, p_2, p_3, \ldots, p_n$

Consider the number

$N = p_1 \times p_2 \times p_3 \times \ldots \times p_n + 1$

When you divide N by any of the prime numbers $p_1, p_2, p_3, \ldots, p_n$ you get a remainder of 1. So none of the prime numbers $p_1, p_2, p_3, \ldots, p_n$ is a factor of N.

So N must either be prime or have a prime factor which is not in the list of all possible prime numbers.

This is a contradiction.

Therefore, there is an infinite number of prime numbers.

Begin by assuming the original statement is false.

This is a list of all possible prime numbers.

This new number is one more than the product of the existing prime numbers.

This contradicts the assumption that the list $p_1, p_2, p_3, \ldots, p_n$ contains all the prime numbers.

Conclude your proof by stating that the original statement must be true.

Exercise 1A

(P) **1** Select the statement that is the negation of 'All multiples of three are even'.

A All multiples of three are odd.

B At least one multiple of three is odd.

C No multiples of three are even.

(P) **2** Write down the negation of each statement.

a All rich people are happy.

b There are no prime numbers between 10 million and 11 million.

c If p and q are prime numbers then $(pq + 1)$ is a prime number.

d All numbers of the form $2^n - 1$ are either prime numbers or multiples of 3.

e At least one of the above four statements is true.

(P) **3** Statement: If n^2 is odd then n is odd.

a Write down the negation of this statement.

b Prove the original statement by contradiction.

(P) **4** Prove the following statements by contradiction.

a There is no greatest even integer.

b If n^3 is even then n is even.

c If pq is even then at least one of p and q is even.

d If $p + q$ is odd then at least one of p and q is odd.

(E/P) **5** **a** Prove that if ab is an irrational number then at least one of a and b is an irrational number. **(3 marks)**

b Prove that if $a + b$ is an irrational number then at least one of a and b is an irrational number. **(3 marks)**

c A student makes the following statement:

If $a + b$ is a rational number then at least one of a and b is a rational number.

Show by means of a counterexample that this statement is not true. **(1 mark)**

(P) **6** Use proof by contradiction to show that there exist no integers a and b for which $21a + 14b = 1$.

Hint Assume the opposite is true, and then divide both sides by the highest common factor of 21 and 14.

(E/P) **7** **a** Prove by contradiction that if n^2 is a multiple of 3, n is a multiple of 3. **(3 marks)**

b Hence prove by contradiction that $\sqrt{3}$ is an irrational number. **(3 marks)**

Hint Consider numbers in the form $3k + 1$ and $3k + 2$.

(P) **8** Use proof by contradiction to prove the statement:
'There are no integer solutions to the equation
$x^2 - y^2 = 2$'

Hint You can assume that x and y are positive, since $(-x)^2 = x^2$.

(E/P) **9** Prove by contradiction that $\sqrt[3]{2}$ is irrational. **(5 marks)**

(E/P) **10** This student has attempted to use proof by contradiction to show that there is no least positive rational number:

> **Assumption:** There is a least positive rational number.
> Let this least positive rational number be n.
> As n is rational, $n = \frac{a}{b}$ where a and b are integers.
> $n - 1 = \frac{a}{b} - 1 = \frac{a-b}{b}$
> Since a and b are integers, $\frac{a-b}{b}$ is a rational number that is less than n.
> This contradicts the statement that n is the least positive rational number.
> Therefore, there is no least positive rational number.

Problem-solving You might have to analyse student working like this in your exam. The question says, 'the error', so there should only be one error in the proof.

a Identify the error in the student's proof. **(1 mark)**

b Prove by contradiction that there is no least positive rational number. **(5 marks)**

1.2 Algebraic fractions

Algebraic fractions work in the same way as numeric fractions. You can simplify them by cancelling common factors and finding common denominators.

- **To multiply fractions, cancel any common factors, then multiply the numerators and multiply the denominators.**

Example 5

Simplify the following products:

a $\frac{3}{5} \times \frac{5}{9}$ **b** $\frac{a}{b} \times \frac{c}{a}$ **c** $\frac{x+1}{2} \times \frac{3}{x^2-1}$

a $\frac{^1\cancel{3}}{_1\cancel{5}} \times \frac{\cancel{5}^1}{\cancel{9}_3} = \frac{1 \times 1}{1 \times 3} = \frac{1}{3}$ — Cancel any common factors and multiply numerators and denominators.

b $\frac{^1\cancel{a}}{b} \times \frac{c}{\cancel{a}_1} = \frac{1 \times c}{b \times 1} = \frac{c}{b}$ — Cancel any common factors and multiply numerators and denominators.

c $\frac{x+1}{2} \times \frac{3}{x^2-1} = \frac{x+1}{2} \times \frac{3}{(x+1)(x-1)}$ — Factorise $(x^2 - 1)$.

$= \frac{^1\cancel{x+1}}{2} \times \frac{3}{_1\cancel{(x+1)}(x-1)}$ — Cancel any common factors and multiply numerators and denominators.

$= \frac{3}{2(x-1)}$

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Pure Mathematics Year 2

Series Editor: Harry Smith

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