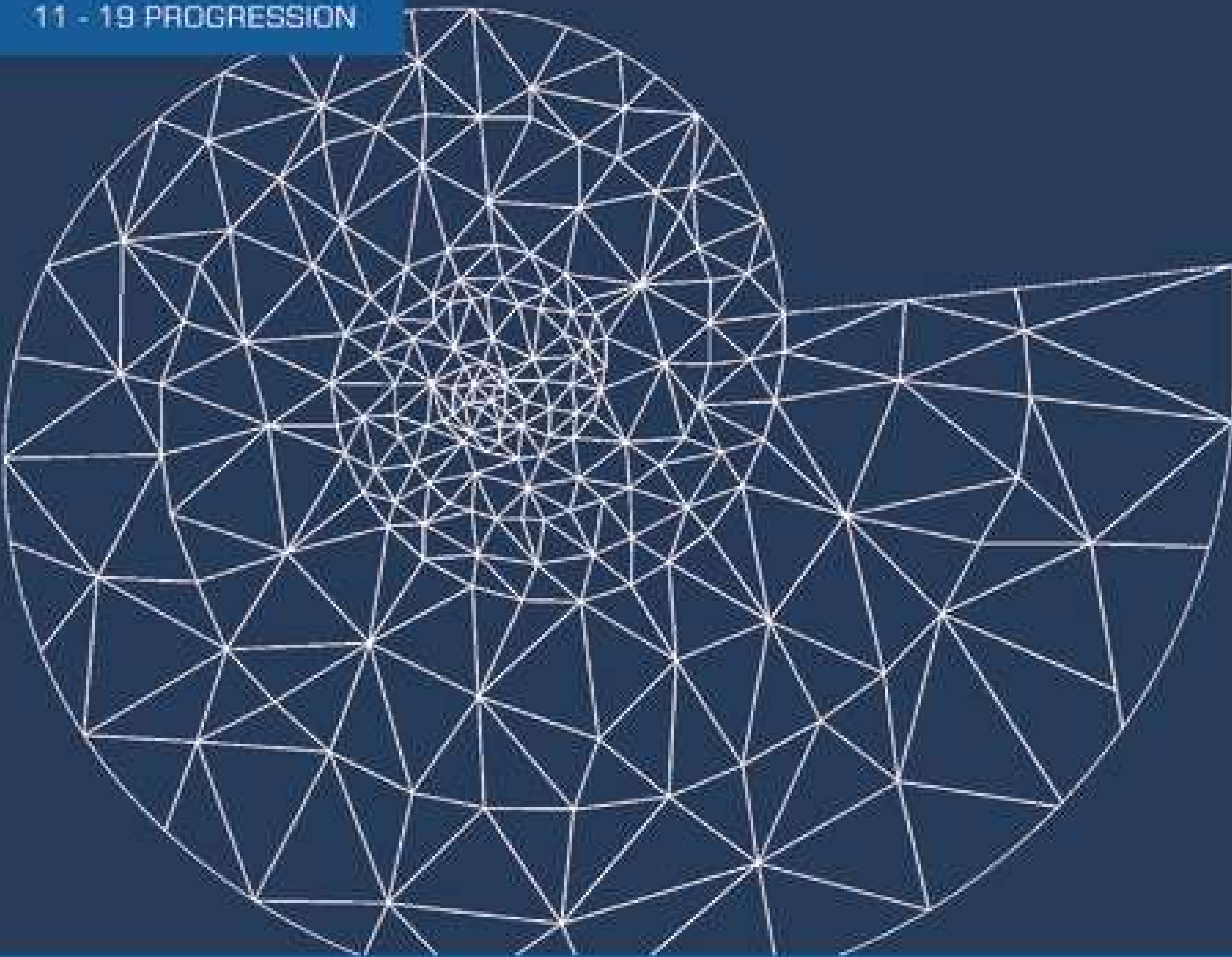
11 - 19 PROGRESSION



Pearson Edexcel A level Mathematics

Pure Mathematics

Year 2

Practice Book

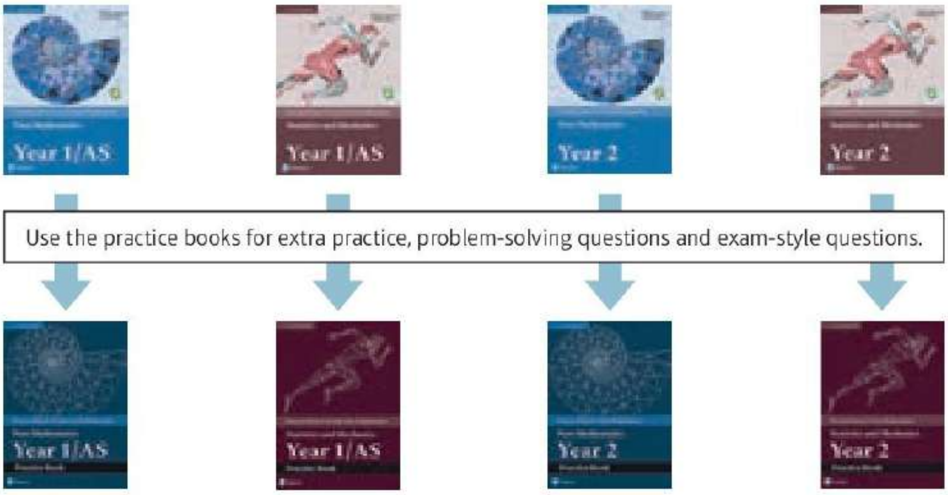
Pearson

Contents

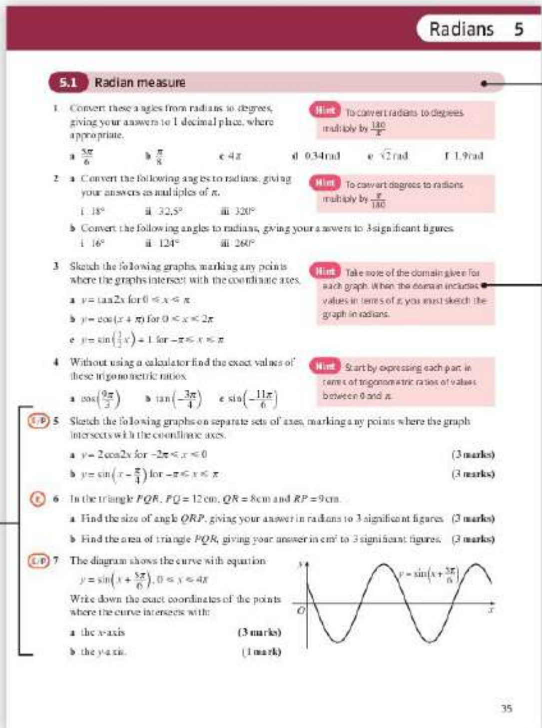
How to use this book

The Pure Mathematics Year 2 Practice Book is designed to be used alongside your Pearson Edexcel AS and A level Mathematics Pure Year 2 textbook. It provides additional practice, including problem-solving and exam-style questions, to help make sure you are ready for your exam.

- The chapters and exercises in this practice book match the chapters and sections in your textbook, so you can easily locate additional practice for any section in the textbook.
- Each chapter finishes with two sets of problem-solving practice questions at three different difficulty levels.
- An Exam question bank at the end of the book provides mixed exam-style questions to help you practise selecting the correct mathematical skills and techniques.



Finding your way around the book



One-to-one match between exercises in this practice book and sections in your textbook.

Hints in each exercise remind you of the key skills, formulae or techniques for that section. If you need more help, look at the corresponding section of your textbook.

Use the exam-style questions in every exercise to check that you are working at exam standard.

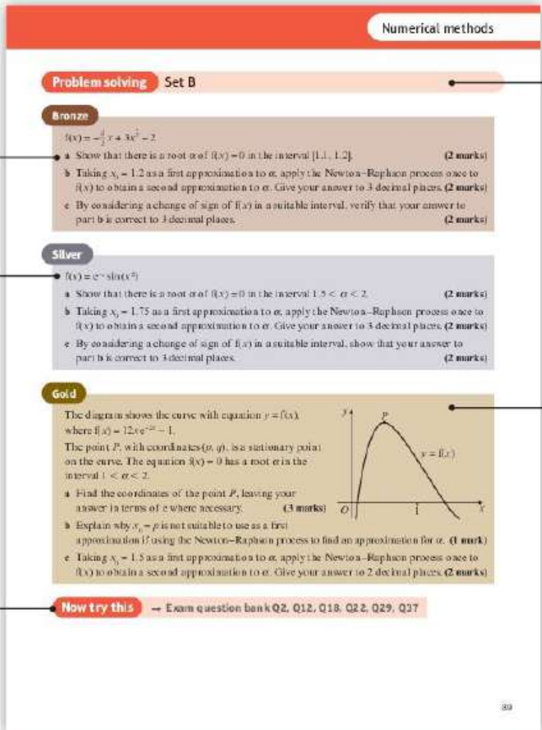
Exam-style questions are flagged with Ⓔ and have marks allocated to them.

Problem-solving questions are flagged with Ⓟ

Bronze questions might have more steps to lead you through the technique, or require a more straightforward application of the skills from that chapter.

Silver questions are more challenging, and provide less scaffolding. If you're struggling with the Silver question, try the Bronze question first.

You can find more exam-style questions on this chapter in the Exam question bank.



Numerical methods

Problem solving Set B

Bronze

$f(x) = -\frac{4}{3}x + 3x^{\frac{2}{3}} - 2$

a Show that there is a root α of $f(x) = 0$ in the interval [1.1, 1.2]. **(2 marks)**

b Taking $x_1 = 1.2$ as a first approximation to α, apply the Newton–Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places. **(2 marks)**

c By considering a change of sign of $f(x)$ in a suitable interval, verify that your answer to part b is correct to 3 decimal places. **(2 marks)**

Silver

$f(x) = e^{-x}\sin(x^2)$

a Show that there is a root α of $f(x) = 0$ in the interval $1.5 < \alpha < 2$. **(2 marks)**

b Taking $x_1 = 1.75$ as a first approximation to α, apply the Newton–Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places. **(2 marks)**

c By considering a change of sign of $f(x)$ in a suitable interval, show that your answer to part b is correct to 3 decimal places. **(2 marks)**

Gold

The diagram shows the curve with equation $y = f(x)$, where $f(x) = 12xe^{-2x} - 1$.

The point P, with coordinates (p, q), is a stationary point on the curve. The equation $f(x) = 0$ has a root α in the interval $1 < \alpha < 2$.

a Find the coordinates of the point P, leaving your answer in terms of e where necessary. **(3 marks)**

b Explain why $x_1 = p$ is not suitable to use as a first approximation if using the Newton–Raphson process to find an approximation for α. **(1 mark)**

c Taking $x_1 = 1.5$ as a first approximation to α, apply the Newton–Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places. **(2 marks)**

Now try this → Exam question bank Q2, Q12, Q18, Q22, Q29, Q37

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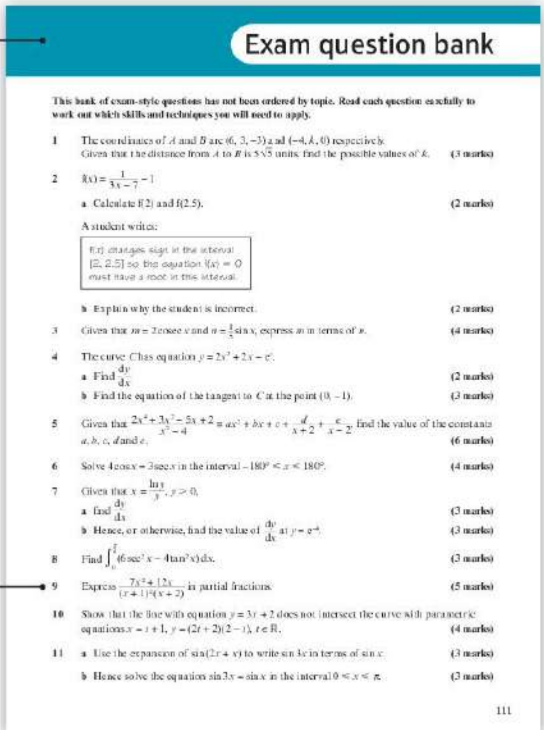
Each chapter ends with two sets of exam-style problem-solving questions which draw on material from throughout the chapter and from earlier chapters.

Gold questions involve tricky problem-solving elements, or might require you to think more creatively. If you can answer the Gold questions then you can be confident that you are ready to tackle the hardest exam questions.

One challenge of the exam is that you aren't usually told which techniques or strategies you need to apply to a particular question. The questions in the Exam question bank are not ordered by topic, so you need to choose the appropriate mathematical skills.

There are a lot more questions in the Exam question bank than there will be on your exam paper. Don't try and tackle them all at once, but make sure you try some of the trickier questions from the end of the question bank.



Exam question bank

This bank of exam-style questions has not been ordered by topic. Read each question carefully to work out which skills and techniques you will need to apply.

1 The coordinates of A and B are $(6, 3, -3)$ and $(-4, k, 0)$ respectively. Given that the distance from A to B is $5\sqrt{5}$ units, find the possible values of k. **(3 marks)**

2 $f(x) = \frac{1}{3x-7} - 1$

a Calculate f(2) and f(2.5). **(2 marks)**

A student writes:

> f(x) changes sign in the interval [2, 2.5] so the equation f(x) = 0 must have a root in this interval.

b Explain why the student is incorrect. **(2 marks)**

3 Given that $m = 2\operatorname{cosec} x$ and $n = \frac{1}{2}\sin x$, express m in terms of n. **(4 marks)**

4 The curve C has equation $y = 2x^2 + 2x - e^x$.

a Find $\frac{dy}{dx}$ **(2 marks)**

b Find the equation of the tangent to C at the point $(0, -1)$. **(3 marks)**

5 Given that $\frac{2x^4 + 3x^2 - 5x + 2}{x^2 - 4} \equiv ax^2 + bx + c + \frac{d}{x+2} + \frac{e}{x-2}$, find the value of the constants a, b, c, d and e. **(6 marks)**

6 Solve $4\cos x = 3\sec x$ in the interval $-180° < x < 180°$. **(4 marks)**

7 Given that $x = \frac{\ln y}{y}$, $y > 0$,

a find $\frac{dy}{dx}$ **(3 marks)**

b Hence, or otherwise, find the value of $\frac{dy}{dx}$ at $y = e^{-4}$. **(3 marks)**

8 Find $\int_0^{\frac{\pi}{4}} (6\sec^2 x - 4\tan^2 x)\,dx$. **(3 marks)**

9 Express $\frac{7x^2 + 12x}{(x+1)^2(x+2)}$ in partial fractions. **(5 marks)**

10 Show that the line with equation $y = 3x + 2$ does not intersect the curve with parametric equations $x = t + 1$, $y = (2t + 2)(2 - t)$, $t \in \mathbb{R}$. **(4 marks)**

11 a Use the expansion of $\sin(2x + x)$ to write $\sin 3x$ in terms of $\sin x$. **(3 marks)**

b Hence solve the equation $\sin 3x = \sin x$ in the interval $0 \leq x \leq \pi$. **(3 marks)**

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1.1 Proof by contradiction

1 Write down the negation of each statement.

a There are infinitely many prime numbers.

b If n^2 is even, then n must be even.

c If pq is odd, then at least one of p and q is odd.

Hint A statement that asserts the falsehood of another statement is called the **negation** of the statement.

2 Prove by contradiction that there is no greatest even number.

Hint You prove a statement by contradiction by assuming it is **not** true. Then use logical steps to show your assumption leads to something impossible. You can then conclude that your assumption was incorrect, and the original statement was true.

3 Prove by contradiction that if a and b are integers, then $a^2 - 4b - 7 \neq 0$.

Hint Start by assuming that there exist integers a and b such that $a^2 - 4b - 7 = 0$. Rearrange and use the fact that any odd number can be written in the form $2n + 1$, where n is an integer.

4 Prove by contradiction that there is no smallest positive rational number.

Hint A **rational number** can be written in the form $\frac{a}{b}$, where a and b are integers. An **irrational number** cannot be written in the form $\frac{a}{b}$, where a and b are integers. The set of rational numbers can be written as $\mathbb{Q}$.

E/P 5 Prove by contradiction that if n^3 is odd, then n must be odd. **(3 marks)**

E/P 6 Prove by contradiction that there are no non-zero integer solutions to the equation $x^2 - y^2 = 1$. **(4 marks)**

E/P 7 Prove that $\sqrt{5}$ is irrational. **(6 marks)**

E/P 8 Prove by contradiction that the difference between any rational number and any irrational number is irrational. **(6 marks)**

1.2 Algebraic fractions

1 Simplify:

a $(x - 3) \times \frac{1}{x^2 - 9}$ **b** $\frac{x^2 - 1}{3} \times \frac{1}{x^2 + 2x + 1}$

c $\frac{x^2 + 5x}{y - 2} \times \frac{y^2 - 2y}{x^2}$

Hint To multiply algebraic fractions, first factorise the numerators and denominators where possible. Then cancel any common factors, and multiply the numerators and multiply the denominators.

2 Simplify:

a $\dfrac{x}{x+2} \div \dfrac{x^3}{x^2+x-2}$ b $\dfrac{9x^2-16}{5x-10} \div \dfrac{3x-4}{10}$

c $\dfrac{2x^2+3xy-2y^2}{5xy} \div \dfrac{x^2+2xy}{10x^2}$

Hint To divide two algebraic fractions, multiply the first fraction by the reciprocal of the second fraction.

3 Express as a single fraction in its simplest form:

a $\dfrac{5}{x+3} + \dfrac{2}{x-1}$ b $\dfrac{4}{2(x-3)} + \dfrac{5}{3(x+1)}$

c $\dfrac{2x}{x-5} - \dfrac{3x}{x+5}$ d $\dfrac{x+1}{2x-3} - \dfrac{2x}{x+2}$

Hint To add or subtract two fractions, find a common denominator.

4 Express as a single fraction in its simplest form:

a $\dfrac{5x}{4x^2-9} + \dfrac{3}{2x-3}$ b $\dfrac{2}{x^2+x-12} + \dfrac{1}{x^2-5x+6}$

c $\dfrac{5}{4x^2+4x+1} - \dfrac{3}{4x^2-1}$ d $\dfrac{x-1}{x^2+3x+2} - \dfrac{x-2}{x^2-2x-3}$

Hint You may need to factorise the denominators to find the lowest common multiple.

E/P 5 Simplify $\dfrac{x^2+4x+4}{y^2-6y+9} \div \dfrac{x^2-4}{y^2-9}$ **(4 marks)**

E/P 6 Simplify $\dfrac{x^2-2x-15}{2x^2-12} \times \dfrac{x^3-6x^2}{x^2-5x-24}$ **(4 marks)**

E/P 7 Express $\dfrac{2x^2-5x}{25x^2-1} + \dfrac{3x}{5x-1}$ as a fraction in its simplest form. **(4 marks)**

E/P 8 Express $\dfrac{3x-2}{2x^2-5x-3} - \dfrac{5}{2x+1}$ as a fraction in its simplest form. **(4 marks)**

E/P 9 $f(x) = x + \dfrac{5}{x+3} + \dfrac{40}{x^2-2x-15}, x \in \mathbb{R}, x \neq -3, x \neq 5$

a Show that $f(x) = \dfrac{x^3-2x^2-10x+15}{(x+3)(x-5)}$ **(4 marks)**

b Hence show that $f(x)$ can be further simplified to give $f(x) = \dfrac{x^2-5x+5}{x-5}$ **(4 marks)**

1.3 Partial fractions

1 Express in partial fractions:

a $\dfrac{8x-1}{(x+1)(x-2)}$

b $\dfrac{2x+13}{(2x-1)(x+3)}$

c $\dfrac{7-11x}{(3x-1)(2x+1)}$

Hint For part **a**, write $\dfrac{8x-1}{(x+1)(x-2)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x-2}$

Then find A and B by rearranging and substituting suitable values of x.

2 Express in partial fractions:

a $\frac{7x-15}{x^2-5x}$ **b** $\frac{3(x+7)}{x^2-9}$

c $\frac{9x-1}{2x^2-9x-5}$

Hint Factorise each denominator to work out the denominators for the partial fractions.

3 Express in partial fractions:

a $\frac{6x^2-43x+50}{x(x-2)(x-5)}$

b $\frac{4x^2+11x+9}{(x-1)(x+2)(x+3)}$

c $\frac{5x^2-22x+6}{x(x-3)(2x-1)}$

Hint For part **a**, write

$\frac{6x^2-43x+50}{x(x-2)(x-5)} \equiv \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-5}$

You can also find A, B and C by multiplying across and comparing coefficients. Often a combination of substitution and comparing coefficients is the most efficient way to find the numerators.

4 Given that $\frac{1+15x-10x^2}{(x-2)(1-2x)} \equiv A + \frac{B}{x-2} + \frac{C}{1-2x}$, find the values of the constants A, B and C.

Hint Start by multiplying both sides by $(x-2)(1-2x)$ to remove the fractions.

E/P **5** Given that $\frac{4}{(x+1)(3x-2)} \equiv \frac{A}{x+1} + \frac{B}{3x-2}$, find the values of the constants A and B. **(3 marks)**

E/P **6** Express $\frac{5x-3}{(2x+3)(x+2)}$ in partial fractions. **(3 marks)**

E/P **7** Express $\frac{3}{9-y^2}$ in partial fractions. **(3 marks)**

E/P **8** Given that $\frac{33x-x^2-44}{(x-1)(x+5)(2x-3)} \equiv \frac{A}{x-1} + \frac{B}{x+5} + \frac{C}{2x-3}$, find the values of the constants A, B and C. **(4 marks)**

E/P **9** Given that $\frac{2x^2-11}{(x+1)(x-2)} \equiv A + \frac{B}{x+1} + \frac{C}{x-2}$, find the values of the constants A, B and C. **(4 marks)**

1.4 Repeated factors

1 $f(x) = \frac{5x^2-5x+2}{x^2(x-2)}$, $x \neq 0$, $x \neq 2$. Given that $f(x)$ can be expressed in the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$, find the values of the constants A, B and C.

Hint x is a repeated factor, so you need separate denominators of x and x^2 in the expanded partial fraction.

2 $g(x) = \frac{8x^2-x+3}{(x+1)^2(2x-1)}$, $x \neq -1$, $x \neq \frac{1}{2}$

Given that $g(x)$ can be expressed in the form $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-1}$, find the values of the constants A, B and C.

Hint $(x+1)$ is a repeated linear factor, so you need denominators of $(x+1)$ and $(x+1)^2$.

(E/P) 3 $\dfrac{4x-1}{(2x+1)^2} \equiv \dfrac{A}{2x+1} + \dfrac{B}{(2x+1)^2}$. Find the values of the constants A and B. **(3 marks)**

(E/P) 4 Given that $\dfrac{x^2+7x+32}{(x-2)(x+3)^2} \equiv \dfrac{A}{x-2} + \dfrac{B}{x+3} + \dfrac{C}{(x+3)^2}$, find the values of the constants A, B and C. **(4 marks)**

(E/P) 5 Express $\dfrac{6x^2-13x+15}{2x^3-3x^2}$ in the form $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{2x-3}$, where A, B and C are constants to be found. **(4 marks)**

(E/P) 6 Express $\dfrac{x^2-5x+16}{x^3-4x^2+4x}$ as a sum of partial fractions. **(6 marks)**

1.5 Algebraic division

1 Use algebraic division to show that
$$\frac{x^2-5x+7}{x-3} \equiv x-2+\frac{1}{x-3}$$

Hint This is an improper algebraic fraction because the degree of the numerator is greater than or equal to the degree of the denominator. You can divide the numerator by the denominator to obtain a polynomial and a remainder.

2 **a** Find the remainder when x^2-5x+9 is divided by $x+1$.

b Hence, or otherwise, write $\dfrac{x^2-5x+9}{x+1}$ in the form $Ax+B+\dfrac{C}{x+1}$, where A, B and C are constants to be found.

Hint For part **b**, you can use the relationship $F(x) = Q(x) \times \text{divisor} + \text{remainder}$ to write $\dfrac{F(x)}{\text{divisor}} = Q(x) + \dfrac{\text{remainder}}{\text{divisor}}$

3 Given that $\dfrac{x^3-x^2-9}{x+3} \equiv Ax^2+Bx+C+\dfrac{D}{x+3}$, find the values of the constants A, B, C and D.

Hint You could use algebraic long division, or you could multiply both sides by $(x+3)$ and use substitution and comparing coefficients.

4 Given that
$$x^3-2x^2+5 \equiv (Ax^2+Bx+C)(x-4)+D$$
find the values of the constants A, B, C and D.

Hint You can compare coefficients when the identity is written in this form.

(E/P) 5 $\dfrac{18x^2+22x-7}{(x+2)(3x-1)} \equiv A + \dfrac{B}{x+2} + \dfrac{C}{3x-1}$. Find the values of the constants A, B and C. **(4 marks)**

(E/P) 6 Given that $\dfrac{2x^3-4x^2+5x-1}{x-3} \equiv Ax^2+Bx+C+\dfrac{D}{x-3}$, find the values of the constants A, B, C and D. **(4 marks)**

(E/P) 7 Given that $\dfrac{x^4-3x^2+5}{x^2+2} \equiv Ax^2+B+\dfrac{C}{x^2+2}$, find the values of the constants A, B and C. **(4 marks)**

(E/P) 8 Given that $\dfrac{4x^2-5x-3}{(x+1)(2x-1)} \equiv A + \dfrac{B}{x+1} + \dfrac{C}{2x-1}$, find the values of the constants A, B and C. **(4 marks)**

E/P **9** $3x^4 - 5x^3 + 6x^2 - 12x + 5 \equiv (Ax^2 + Bx + C)(x^2 + 2) + Dx + E$

Find the values of the constants A, B, C, D and E. **(5 marks)**

Problem solving Set A

Bronze

a Simplify $\dfrac{3x^2 + x - 2}{x^2 - 1}$ **(3 marks)**

b Hence, or otherwise, express $\dfrac{3x^2 + x - 2}{x^2 - 1} - \dfrac{1}{x^2 - x}$ as a single fraction in its simplest form. **(3 marks)**

Silver

Express $\dfrac{2x^2 - 3x}{2x^2 + x - 6} - \dfrac{6}{x^2 + x - 2}$ as a single fraction in its simplest form. **(7 marks)**

Gold

Express $\dfrac{2x^3 + 3x^2 - 5x - 6}{3x^2 + 5x + 2}$ in the form $Ax + B + \dfrac{C}{Dx + E}$ where A, B, C, D and E are constants to be found. **(9 marks)**

Problem solving Set B

Bronze

Express $\dfrac{39x^2 - 49x + 15}{(3x - 2)^2(1 - x)}$ as a sum of partial fractions. **(4 marks)**

Silver

Show that $\dfrac{3x^2 - 2x + 4}{x^2 - x - 6}$ can be written in the form $A + \dfrac{B}{x - 3} + \dfrac{C}{x + 2}$ and find the values of the constants A, B and C. **(5 marks)**

Gold

$$f(x) = \frac{3x^2 + 3}{2x^3 - 5x^2 - 4x + 3}$$

The graph of $y = f(x)$ has a vertical asymptote with equation $x = -1$. Use this information to express $f(x)$ as the sum of three fractions with linear denominators. **(8 marks)**

Now try this → Exam question bank Q5, Q9, Q49, Q62, Q69, Q75, Q78

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Pure Mathematics Year 2

Series Editor: Harry Smith

Practice Book

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