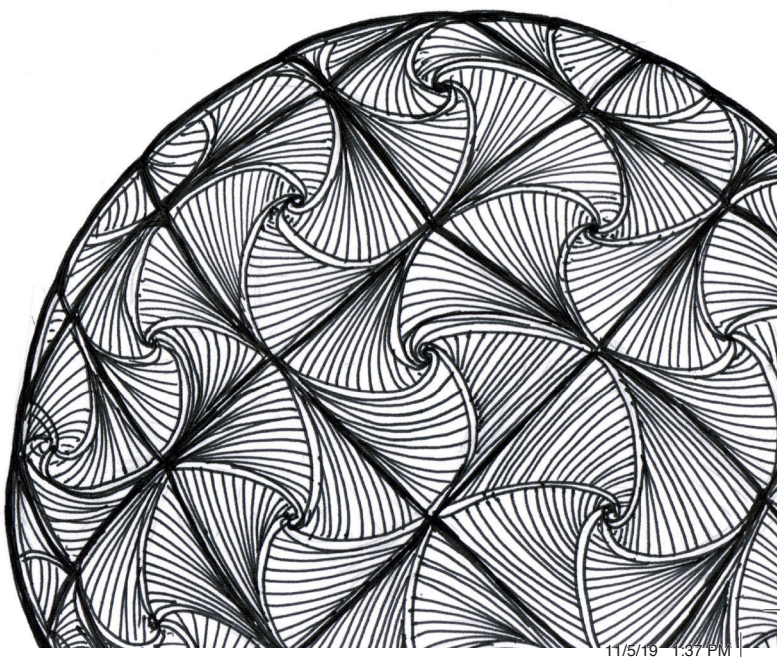
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Mathematics

GRADES 7–9 Revision & Practice

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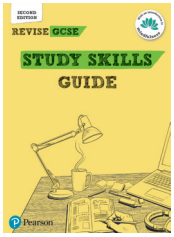
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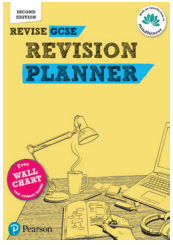
Series consultant: Harry Smith
Author: Harry Smith

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Use this quick quiz to check that you are confident with the core skills and knowledge you need for the Pearson Edexcel GCSE (9–1) Mathematics Higher exam.

Geometry and Measures

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86 Answers and worked solutions

Check your understanding with complete worked solutions to all the Exam practice questions.

A small bit of small print

Pearson Edexcel publishes Sample Assessment Material and the Specification on its website. This is the official content and this book should be used in conjunction with it. The questions in the *Exam practice* sections have been written to help you revise topics and practise answering exam questions. Remember – the real exam questions may not look like this.

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Welcome to Nail it!

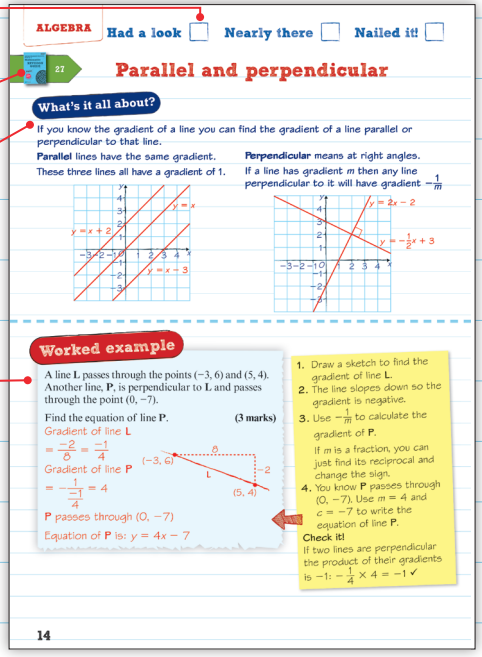
This book provides revision and practice to help you nail down a top grade in your Pearson Edexcel GCSE (9–1) Mathematics exam. Designed for students aiming for a grade 7, 8 or 9, it's packed with exam tips, support for tricky topics, and exam-style practice questions to make sure you are ready to tackle the toughest questions and achieve top marks.

Track your progress by ticking these boxes.

For more help, check out these pages in the Revise Pearson Edexcel GCSE (9–1) Mathematics Higher Revision Guide.

Revise the key facts for this topic.

Worked exam-style questions show you exactly how to tackle those trickier questions and how to set out your answer.


ALGEBRA Had a look ☐ Nearly there ☐ Nailed it! ☐

27 Parallel and perpendicular

What's it all about?

If you know the gradient of a line you can find the gradient of a line parallel or perpendicular to that line.

Parallel lines have the same gradient.

These three lines all have a gradient of 1.

Perpendicular means at right angles.

If a line has gradient m then any line perpendicular to it will have gradient $-\frac{1}{m}$

Worked example

A line **L** passes through the points (−3, 6) and (5, 4). Another line, **P**, is perpendicular to **L** and passes through the point (0, −7).

Find the equation of line **P**. (3 marks)

Gradient of line **L** $= \frac{-2}{8} = \frac{-1}{4}$

Gradient of line **P** $= -\frac{1}{\frac{-1}{4}} = 4$

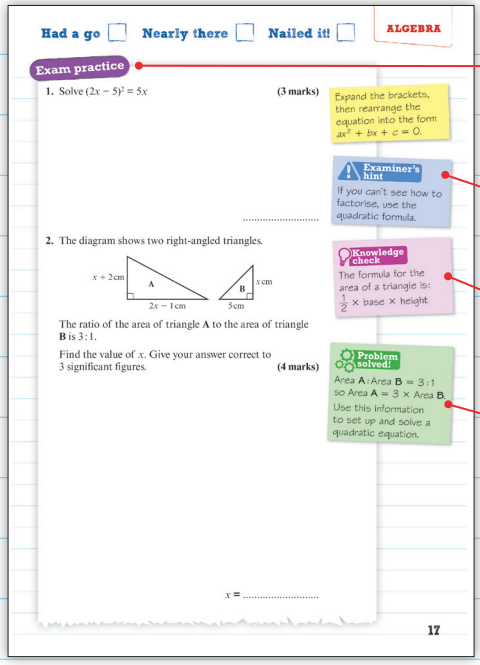
P passes through (0, −7)

Equation of **P** is: $y = 4x - 7$

1. Draw a sketch to find the gradient of line **L**.
2. The line slopes down so the gradient is negative.
3. Use $-\frac{1}{m}$ to calculate the gradient of **P**.
   If m is a fraction, you can just find its reciprocal and change the sign.
4. You know **P** passes through (0, −7). Use $m = 4$ and $c = -7$ to write the equation of line **P**.

Check it!
If two lines are perpendicular the product of their gradients is -1: $-\frac{1}{4} \times 4 = -1$ ✓

14


Had a go ☐ Nearly there ☐ Nailed it! ☐ ALGEBRA

Exam practice

1. Solve $(2x - 5)^2 = 5x$ (3 marks)

Expand the brackets, then rearrange the equation into the form $ax^2 + bx + c = 0$.

Examiner's hint
If you can't see how to factorise, use the quadratic formula.

2. The diagram shows two right-angled triangles.

The ratio of the area of triangle **A** to the area of triangle **B** is 3 : 1.

Find the value of x. Give your answer correct to 3 significant figures. (4 marks)

Knowledge check
The formula for the area of a triangle is: $\frac{1}{2} \times$ base $\times$ height

Problem solved!
Area **A** : Area **B** = 3 : 1 so Area **A** = 3 × Area **B**.
Use this information to set up and solve a quadratic equation.

$x =$ .....

17

Check that you are on track for a top grade with these exam-style questions. There are fully worked solutions at the back of the book.

Top tips for exam success.

Reminders of key information and core skills – you need to be confident with these to help you achieve that top grade.

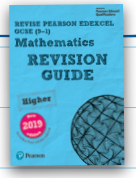
Super strategies to help you master problem-solving questions.

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Knowledge check

If you're aiming for a top grade, you need to be confident with core skills and knowledge, such as solving linear equations or working with fractions, decimals and percentages. Take this quick quiz to find out which skills you might need to brush up on before tackling the topics in this book. Tick the correct answer for each question. The answers are on page 86.

Revise core skills



Use the **Revise Pearson Edexcel GCSE (9–1) Mathematics Revision Guide** if you need to revise any of the core skills covered in this quiz. The green arrow next to each question tells you which page to look at for more help.

1

1. Express 600 as a product of its prime factors.

- ☐ **A** $2^2 \times 5^2 \times 6$
- ☐ **B** $2^3 \times 3^2 \times 5^3$
- ☐ **C** $2^3 \times 3 \times 5^2$
- ☐ **D** $2 \times 3 \times 10^2$

2

2. Write $\frac{7^{10} \times 7^3}{7}$ as a single power of 7.

- ☐ **A** 7^{29}
- ☐ **B** 7^{12}
- ☐ **C** 7^{23}
- ☐ **D** 7^{-13}

4

3. Round 0.0473508 to 3 significant figures.

- ☐ **A** 0.047
- ☐ **B** 0.0473
- ☐ **C** 0.0474
- ☐ **D** 0.05

5

4. Work out $1\frac{7}{8} \times 3\frac{2}{5}$

- ☐ **A** $6\frac{3}{8}$
- ☐ **B** $4\frac{9}{13}$
- ☐ **C** $3\frac{7}{20}$
- ☐ **D** $5\frac{1}{8}$

8

5. Write 736 000 in standard form.

- ☐ **A** 7.36×10^3
- ☐ **B** 736×10^3
- ☐ **C** 7.36×10^5
- ☐ **D** 7.36×10^{-6}

10

6. The length of a carrot is 20 cm, rounded to the nearest cm. What is the upper bound for this length?

- ☐ **A** 20.5 cm
- ☐ **B** 20.49 cm
- ☐ **C** 25 cm
- ☐ **D** 19.5 cm

13

7. This combination lock uses two letters from A to Z and two digits from 0 to 9.

C	*X*	0	5

Work out the total number of possible combinations.

- ☐ **A** 67 600
- ☐ **B** 72
- ☐ **C** 6760
- ☐ **D** 260

16

8. Simplify $(x^2y)^3$

- ☐ **A** x^2y^3
- ☐ **B** x^5y^3
- ☐ **C** xy^6
- ☐ **D** x^6y^3

17

9. Expand and simplify $3(4a + b) - 2(a - 2b)$

- ☐ **A** $10a - b$
- ☐ **B** $10a + 7b$
- ☐ **C** $9a + 7b$
- ☐ **D** $5a + b$

17

10. Expand and simplify $(2x + 3)^2$

- ☐ **A** $4x^2 + 9$
- ☐ **B** $2x^2 + 6x + 9$
- ☐ **C** $4x^2 + 12x + 9$
- ☐ **D** $4x^2 + 6x + 9$

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18 **11.** Factorise $x^2 - 4x - 12$

- ☐ **A** $4x(x - 3)$
- ☐ **B** $(x - 4)(x + 3)$
- ☐ **C** $(x - 6)(x + 2)$
- ☐ **D** $(x - 4)^2$

19 **12.** Solve $6x + 2 = 8x - 1$

- ☐ **A** $x = 0.75$
- ☐ **B** $x = 2$
- ☐ **C** $x = -1$
- ☐ **D** $x = 1.5$

21 **13.** $P = 5Q^2 - 2QR$
Find the value of P when $Q = 4$ and $R = -3$

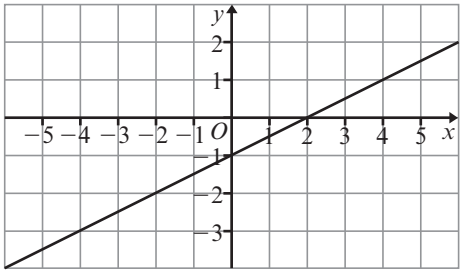
- ☐ **A** 76
- ☐ **B** 104
- ☐ **C** 112
- ☐ **D** 56

22 **14.** Find an expression for the nth term of this sequence.

8 11 14 17 20

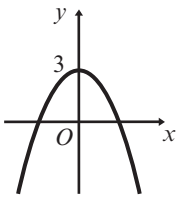
- ☐ **A** $3n + 5$
- ☐ **B** $5n + 3$
- ☐ **C** $8n + 3$
- ☐ **D** $20 - 3n$

25 **15.** Find the equation of this straight line.



- ☐ **A** $y = 2x - 1$
- ☐ **B** $y = \frac{1}{2}x - 1$
- ☐ **C** $y = -x + 2$
- ☐ **D** $y = 2x + 1$

28 **16.** Match this graph to the correct equation.



- ☐ **A** $y = 3 - x^2$
- ☐ **B** $y = x^2 - 3$
- ☐ **C** $y = x^2 + 3$
- ☐ **D** $y = x^3$

31 **17.** Solve the equation $x^2 - 10x + 16 = 0$

- ☐ **A** $x = 1.6$
- ☐ **B** $x = -4$
- ☐ **C** $x = 8$ or $x = 2$
- ☐ **D** $x = -8$ or $x = -2$

34 **18.** Solve the simultaneous equations
$5x + 6y = 5$
$x - 2y = 9$

- ☐ **A** $x = -3, y = 1.5$
- ☐ **B** $x = 4, y = -2.5$
- ☐ **C** $x = 2.5, y = 5$
- ☐ **D** $x = -3, y = -1.5$

37 **19.** Solve the inequality $3x + 1 < x - 5$

- ☐ **A** $x > 3$
- ☐ **B** $x > -3$
- ☐ **C** $x < -2$
- ☐ **D** $x < -3$

41 **20.** $M = 6R - 20$
Rearrange the formula to make R the subject.

- ☐ **A** $R = \frac{M + 20}{6}$
- ☐ **B** $R = 6M + 20$
- ☐ **C** $R = \frac{M}{6} + 20$
- ☐ **D** $R = \frac{1}{6}(M - 20)$

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50 **21.** $f(x) = (2x - 3)^2$
Find $f(-1)$.

☐ **A** −25 ☐ **B** 1
☐ **C** −11 ☐ **D** 25

59 **22.** Work out 15% of £900

☐ **A** £145 ☐ **B** £1035
☐ **C** £135 ☐ **D** £180

60 **23.** The ratio of juice to water in a drink is 3 : 2
The total amount of drink is 600 m*l*.
How much juice is in the drink?

☐ **A** 200 m*l* ☐ **B** 360 m*l*
☐ **C** 400 m*l* ☐ **D** 450 m*l*

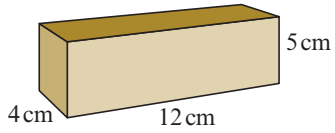
63 **24.** In a sale, prices are reduced by 20%.
The sale price of a phone is £144.
Work out its original price.

☐ **A** £180 ☐ **B** £172.80
☐ **C** £192 ☐ **D** £115.20

65 **25.** A cyclist travels 84 km at an average speed of 15 km/h.
Work out the total time taken.

☐ **A** 4.5 hours ☐ **B** 0.18 hours
☐ **C** 1260 hours ☐ **D** 5.6 hours

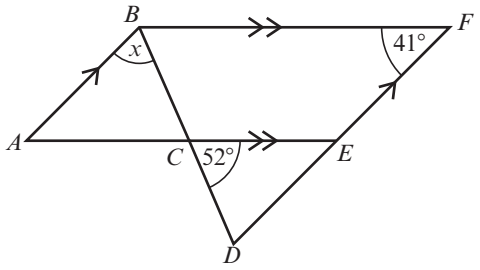
66 **26.** The diagram shows a solid brass cuboid.



The density of brass is 8.6 g/cm^3.
Work out the mass of the cuboid.

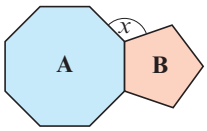
☐ **A** 498 g ☐ **B** 27.9 g
☐ **C** 2064 g ☐ **D** 14.3 kg

73 **27.** Work out the size of the angle marked x.



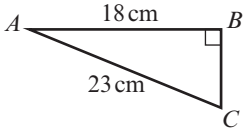
☐ **A** 52° ☐ **B** 9°
☐ **C** 87° ☐ **D** 93°

75 **28.** **A** is a regular octagon and **B** is a regular pentagon.
Work out the size of the angle marked x.



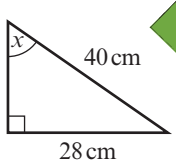
☐ **A** 117° ☐ **B** 135°
☐ **C** 108° ☐ **D** 243°

76 **29.** Work out the length of BC in this right-angled triangle, correct to 1 decimal place.



☐ **A** 5.0 cm ☐ **B** 14.3 cm
☐ **C** 29.2 cm ☐ **D** 205.0 cm

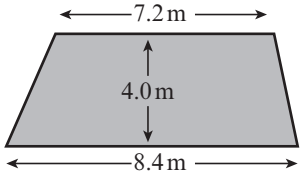
77 **30.** Work out the size of the angle marked x in this right-angled triangle, to the nearest degree.



☐ **A** 55° ☐ **B** 35°
☐ **C** 46° ☐ **D** 44°

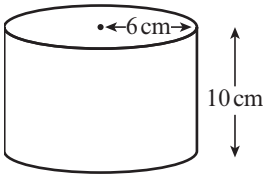
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80 **31.** Work out the area of this trapezium.



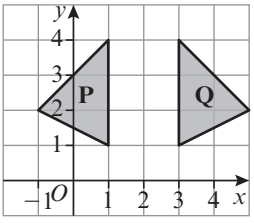
☐ **A** 33.6 m² ☐ **B** 62.4 m²
☐ **C** 31.2 m² ☐ **D** 37.2 m²

83 **32.** Work out the volume of this cylinder, to the nearest cm³.



☐ **A** 377 cm³ ☐ **B** 1131 cm³
☐ **C** 360 cm³ ☐ **D** 1885 cm³

88 **33.** Describe fully the single transformation that maps triangle **P** onto triangle **Q**.



☐ **A** Reflection
☐ **B** Rotation
☐ **C** Rotation 180° about (2, 2.5)
☐ **D** Reflection in the line $x = 2$

91 **34.** The bearing of point P from point Q is 208°.
Find the bearing of point Q from point P.

☐ **A** 018° ☐ **B** 28°
☐ **C** 028° ☐ **D** 388°

106 **35.** $\mathbf{p} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$
Work out the vector $\mathbf{p} - 2\mathbf{q}$

☐ **A** $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ ☐ **B** $\begin{pmatrix} -1 \\ 8 \end{pmatrix}$
☐ **C** $\begin{pmatrix} -4 \\ 8 \end{pmatrix}$ ☐ **D** $\begin{pmatrix} -8 \\ 11 \end{pmatrix}$

111 **36.** The table shows the number of goals scored by a team in 40 matches

Goals	0	1	2	3	4
Frequency	11	15	8	5	1

Work out the mean number of goals scored per match.

☐ **A** 2 goals ☐ **B** 8 goals
☐ **C** 1.25 goals ☐ **D** 2.5 goals

123 **37.** The table shows the probability of each score on a biased dice.

Score	1	2	3	4	5	6
Probability	0.1	x	x	x	x	$2x$

Ravi rolls the dice. Work out the probability that it lands on 6

☐ **A** 0.2 ☐ **B** 0.3
☐ **C** 0.36 ☐ **D** 1.2

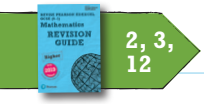
123 **38.** The two-way table shows the number of boys and girls in a class who are left- and right-handed.

	Boys	Girls
Right-handed	11	12
Left-handed	3	4

A girl is picked at random. Work out the probability that she is left-handed.

☐ **A** $\frac{1}{4}$ ☐ **B** $\frac{1}{3}$
☐ **C** $\frac{2}{15}$ ☐ **D** $\frac{7}{30}$

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Indices and surds

What's it all about?

Learn the index laws for dealing with powers of numbers or letters.

1 $a^m \times a^n = a^{m+n}$

$4^3 \times 4^7 = 4^{3+7} = 4^{10}$

2 $\frac{a^m}{a^n} = a^{m-n}$

$12^8 \div 12^3 = 12^{8-3} = 12^5$

3 $(a^m)^n = a^{mn}$

$(7^3)^5 = 7^{3\times 5} = 7^{15}$

4 $a^{-n} = \frac{1}{a^n}$

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

5 $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\left(\frac{3}{10}\right)^2 = \frac{3^2}{10^2} = \frac{9}{100}$

6 $a^{\frac{1}{n}} = \sqrt[n]{a}$

$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$

Surds

You can use surds to write exact values using square roots.

✓ $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ $\quad \sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

✓ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\quad \sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{\sqrt{25}} = \frac{\sqrt{3}}{5}$

Worked example

Show that $(3-\sqrt{8})^2$ can be written in the form $p + q\sqrt{2}$, where p and q are integers. **(3 marks)**

$$(3-\sqrt{8})(3-\sqrt{8}) = 9 - 3\sqrt{8} - 3\sqrt{8} + \sqrt{8}\sqrt{8}$$
$$= 17 - 6\sqrt{8}$$
$$= 17 - 6 \times 2\sqrt{2}$$
$$= 17 - 12\sqrt{2}$$

Knowledge check

Use FOIL to multiply out brackets:

First: 3×3

Outside: $3 \times -\sqrt{8}$

Inside: $-\sqrt{8} \times 3$

Last: $\sqrt{8} \times \sqrt{8} = 8$

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Exam practice

1. (a) Work out the value of $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$ **(2 marks)**

Work out the powers one step at a time. Remember $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

.....

(b) $2^x = \frac{1}{2}$ $2^y = 4\sqrt{2}$ $2^z = \frac{1}{\sqrt{2}}$

Work out the value of $x + y + z$ **(2 marks)**

Problem solved!

Write each expression as a power of 2 to find the values of x, y and z.

.....

2. Show that $\frac{20 + \sqrt{12}}{\sqrt{3} + 1}$ can be written in the form $a\sqrt{3} - b$, where a and b are integers. **(3 marks)**

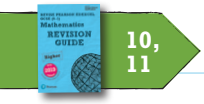
You need to **rationalise the denominator** in this expression. You can do this by multiplying the top and the bottom of the fraction by $\sqrt{3} - 1$

Examiner's hint

Check that you have written your final answer in the form asked for in the question.

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10, 11

Accuracy and bounds

What's it all about?

When amounts are rounded, the actual amounts could be greater or less than the recorded values. You can show the range of possible actual values using an **error interval**. Here are two examples:

1. If $x = 4.7$ to 1 decimal place, then the error interval for x is $4.65 \leqslant x < 4.75$

2. If $y = 840$ to 2 significant figures, then the error interval for y is $835 \leqslant y < 845$

This value is called the **lower bound** for y

This value is called the **upper bound** for y

To give an answer to an **appropriate degree of accuracy**:

Calculate the **lower bound** for the answer. → Calculate the **upper bound** for the answer. → Choose the most accurate value that **both bounds** would round to.

Worked example

A cylinder has a volume of 115 cm^3, to the nearest cm^3. Its radius is 2.3 cm, correct to 1 decimal place.

Find the height of the cylinder to an appropriate degree of accuracy. You must explain why your answer is to this degree of accuracy. **(4 marks)**

	Upper bound	Lower bound
Volume	115.5 cm^3	114.5 cm^3
Radius	2.35 cm	2.25 cm

UB for height $= \dfrac{115.5}{\pi \times 2.25^2} = 7.26218\ldots$

LB for height $= \dfrac{114.5}{\pi \times 2.35^2} = 6.59963\ldots$

Height = 7 cm (1 s.f.)

UB and LB both round to 7 cm to 1 s.f.

Problem solved!

height $= \dfrac{\text{volume}}{\pi \times \text{radius}^2}$

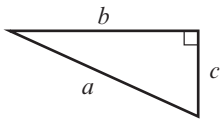
To find the **upper** bound for the height, use the upper bound for the volume and the lower bound for the radius.

To find the **lower** bound for the height, use the lower bound for the volume and the upper bound for the height.

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Exam practice

1.



$a = 20\,\text{m}$ to the nearest metre.
$b = 17\,\text{m}$ to the nearest metre.

Calculate the lower bound for c. **(4 marks)**

.....m

Examiner's hint

If you are using bounds in a question, it's a good idea to write the upper and lower bounds for each value before you start.

Knowledge check

Use Pythagoras' theorem: $a^2 = b^2 + c^2$

2. Poppy is riding her bike to school. She travels a distance of 0.85 km, at an average speed of 7.1 m/s.
Both figures are correct to 2 significant figures.

By considering bounds, work out the time taken for Poppy to complete her journey. Give your answer to a suitable degree of accuracy.

You must show your working and give a reason for your final answer. **(5 marks)**

Knowledge check

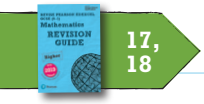
Learn the formula triangle for speed, distance and time:



Make sure your units are consistent before using the formula.

You need to work out the upper and lower bounds for the time, then choose the most accurate value that **both** of these bounds would round to.

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Brackets and factorising

What's it all about?

Expanding triple brackets

Follow these two steps to expand the product of **three factors**:

1. Choose two of the factors. Expand and simplify them, then write brackets around the result.
2. Multiply every term in this expression by **each** term in the third factor. Then simplify by collecting like terms.

Factorising $ax^2 + bx + c$

If the coefficient of x^2 is more than 1, find two numbers that add up to b and multiply to make ac. For example:

$8x^2 + 22x + 15$

$ac = 120 = 12 \times 10$, and $12 + 10 = 22$

so $8x^2 + 22x + 15$

$= 8x^2 + 10x + 12x + 15$

$= 2x(4x + 5) + 3(4x + 5)$

$= (4x + 5)(2x + 3)$

Difference of two squares

You can factorise expressions that are written as

(something)^2 − (something else)^2

Use this rule:

$a^2 - b^2 = (a + b)(a - b)$

$x^2 - 36 = x^2 - (6)^2$

$= (x + 6)(x - 6)$

36 is a square number.

$36 = 6^2$ so $a = x$ and $b = 6$

Worked example

Show that

$(x + 5)^2(3x - 2) = 3x^3 + 28x^2 + 55x - 50$

for all values of x. **(3 marks)**

$(x + 5)(x + 5)(3x - 2) = (x^2 + 10x + 25)(3x - 2)$

$= 3x^3 + 30x^2 + 75x - 2x^2 - 20x - 50$

$= 3x^3 + 28x^2 + 55x - 50$

Knowledge check

$(x + 5)^2 = (x + 5)(x + 5)$

$= x^2 + 10x + 25$

Examiner's hint

In a 'show that' question you must show each step of your working clearly.

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Exam practice

1. Factorise $5x^2 + 13x + 6$ **(2 marks)**

.....

> $ac = 30$ and $b = 13$, so look for two numbers that add up to 13 and multiply to make 30. Use these numbers to split the x term into two terms.

2. Show that $(2x + 1)(x - 5)(4x - 3)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are positive integers. **(3 marks)**

.....

> **Problem solved!**
>
> Use the x^3 term and the number term to check your answer:
>
> $2x \times x \times 4x = 8x^3$ so $a = 8$
>
> $1 \times -5 \times -3 = 15$ so $d = 15$

3. (a) Factorise $x^2 - y^2$ **(2 marks)**

.....

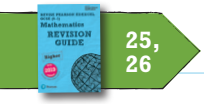
(b) Hence, or otherwise, simplify fully $(p^2 + q^2)^2 - (p^2 - q^2)^2$ **(3 marks)**

.....

> **Examiner's hint**
>
> If a question says 'Hence, or otherwise' you can save time by using your earlier answer. In this case, set $x = p^2 + q^2$ and $y = p^2 - q^2$ and use the answer to part (a).

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Straight-line graphs

What's it all about?

You need to be able to find the equations of straight lines confidently.

1 Given one point and the gradient:

- Substitute the gradient for m in $y = mx + c$
- Substitute the x- and y-values given into the equation
- Solve the equation to find c
- Write out the equation

2 Given two points:

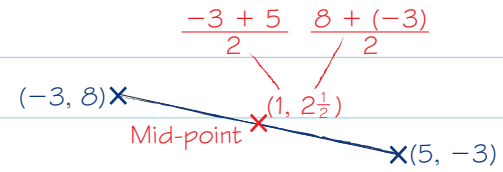
- Draw a sketch showing the two points
- Work out the gradient of the line using a triangle
- Use method 1 (on the left) and one of the points given to find the equation

Mid-points

A **line segment** is a short section of a straight line.

You can find the **mid-point** of a line segment if you know the coordinates of the ends.

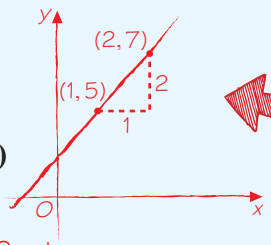
Coordinates of mid-point = (average of x-coordinates, average of y-coordinates)



Worked example

A line passes through the points with coordinates (1, 5) and (2, 7).

Find the equation of the line. **(3 marks)**



Gradient, $m = \frac{2}{1} = 2$

Equation of line: $y = mx + c \rightarrow y = 2x + c$

For point (1, 5), $x = 1$, $y = 5$

Substitute these values into the equation:

$5 = 2 + c \rightarrow c = 3$

The equation is $y = 2x + 3$

Examiner's hint

Drawing a sketch will help you check that your answer makes sense.

Knowledge check

The equation of a straight line is $y = mx + c$, where m is the gradient and $(0, c)$ is the y-intercept.

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Exam practice

1. The line **L** has gradient 3 and passes through the point (2, 1).

Find the equation of the line. **(2 marks)**

Examiner's hint

You can find the *y*-intercept by 'counting back'. The gradient is 3, so for each unit across, the line goes 3 units up. The line passes through (2, 1) and $1 - (2 \times 3) = -5$, so it will pass through $(0, -5)$.

2. A straight line passes through the points (2, 7) and (8, −1).

Find the equation of the line. Give your answer in the form $ax + by = c$, where a, b and c are integers. **(3 marks)**

Find the equation in the form $y = mx + c$, then multiply through by a suitable number and rearrange.

3. A straight line passes through the points with coordinates (−5, 2), (1, k) and (3, $2k$).

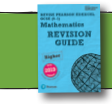
Find the equation of the line. **(4 marks)**

Problem solved!

The gradient between each pair of points must be the same. Use this fact to find the value of k, then find the equation.

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Had a look ☐ Nearly there ☐ Nailed it! ☐



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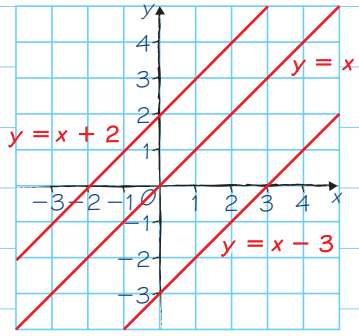
Parallel and perpendicular

What's it all about?

If you know the gradient of a line you can find the gradient of a line parallel or perpendicular to that line.

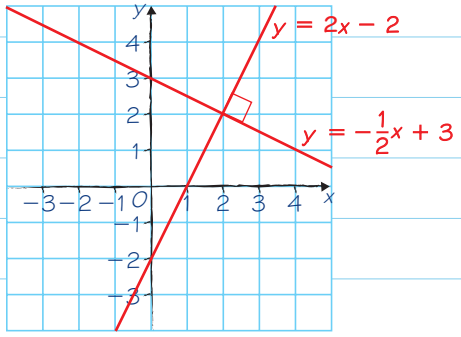
Parallel lines have the same gradient.

These three lines all have a gradient of 1.



Perpendicular means at right angles.

If a line has gradient m then any line perpendicular to it will have gradient $-\frac{1}{m}$



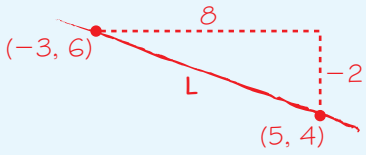
Worked example

A line **L** passes through the points (−3, 6) and (5, 4). Another line, **P**, is perpendicular to **L** and passes through the point (0, −7).

Find the equation of line **P**. **(3 marks)**

Gradient of line **L**

$= \frac{-2}{8} = \frac{-1}{4}$



Gradient of line **P**

$= -\frac{1}{\frac{-1}{4}} = 4$

P passes through (0, −7)

Equation of **P** is: $y = 4x - 7$

1. Draw a sketch to find the gradient of line **L**.
2. The line slopes down so the gradient is negative.
3. Use $-\frac{1}{m}$ to calculate the gradient of **P**.

   If m is a fraction, you can just find its reciprocal and change the sign.
4. You know **P** passes through (0, −7). Use $m = 4$ and $c = -7$ to write the equation of line **P**.

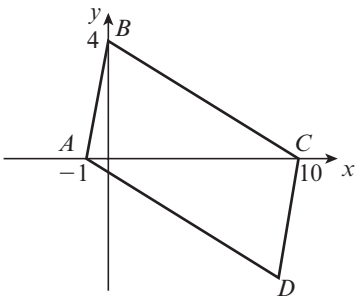
Check it!

If two lines are perpendicular the product of their gradients is −1: $-\frac{1}{4} \times 4 = -1$ ✓

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Exam practice

1. The diagram shows a parallelogram.



Find the equation of the line that passes through C and is perpendicular to AD. **(4 marks)**

Knowledge check

A parallelogram has two pairs of parallel sides, so CD is parallel to AB.

Sketch the line on the diagram, and check that the equation looks roughly correct.

.....

2. The point A has coordinates $(p, 2p)$
The point B has coordinates $(7, 4)$
The line with equation $5y - 2x = 3$ is parallel to the line segment AB.

Find the value of p. **(4 marks)**

Problem solved!

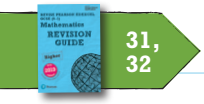
Start by writing an expression for the gradient of AB in terms of p.

Knowledge check

To find the gradient of the line with equation $5y - 2x = 3$, rearrange it into the form $y = mx + c$.

$p =$ .....

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Quadratic equations

What's it all about?

Quadratic equations can have two real solutions, one real solution or no real solutions. To solve a quadratic equation, write it in the form:

$$ax^2 + bx + c = 0$$

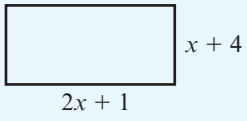
1. If you can, **factorise** the left-hand side then set each factor equal to zero and solve to find the solutions.

2. You can use the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Worked example

The diagram shows a rectangle. The measurements are given in cm.



The area of the rectangle is 60 cm.

Find the perimeter of the rectangle. **(5 marks)**

$$(2x + 1)(x + 4) = 60$$
$$2x^2 + 8x + x + 4 = 60$$
$$2x^2 + 9x - 56 = 0$$

You might need to form and solve a quadratic equation to solve a problem. Set the area of the rectangle equal to 60, then rearrange and solve the equation.

Method 1

$$(2x - 7)(x + 8) = 0$$
$$2x - 7 = 0 \text{ or } x + 8 = 0$$
$$x = \frac{7}{2} \qquad x = -8$$

Method 2

$$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 2 \times (-56)}}{2 \times 2}$$
$$= \frac{-9 \pm 23}{4}$$
$$= 3.5 \text{ or } -8$$

Problem solved!

Lengths must be positive numbers. If $x = -8$ then both dimensions would be negative, so you can ignore this solution.

length = 8, width = 7.5, so
perimeter = 2 × 8 + 2 × 7.5 = 31 cm

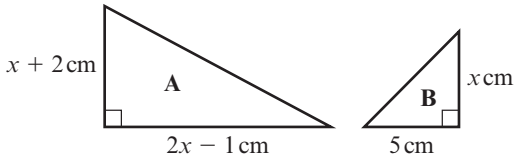
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Exam practice

1. Solve $(2x - 5)^2 = 5x$ **(3 marks)**

.....

2. The diagram shows two right-angled triangles.



The ratio of the area of triangle **A** to the area of triangle **B** is 3 : 1.

Find the value of x. Give your answer correct to 3 significant figures. **(4 marks)**

x = ...

Expand the brackets, then rearrange the equation into the form $ax^2 + bx + c = 0$.

Examiner's hint

If you can't see how to factorise, use the quadratic formula.

Knowledge check

The formula for the area of a triangle is:

$\frac{1}{2}$ × base × height

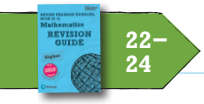
Problem solved!

Area **A** : Area **B** = 3 : 1 so Area **A** = 3 × Area **B**.

Use this information to set up and solve a quadratic equation.

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Had a look ☐ Nearly there ☐ Nailed it! ☐



Sequences

What's it all about?

In an **arithmetic** sequence the **difference** between consecutive terms is constant.

−2.5 −2.5 −2.5 −2.5

4 1.5 −1 −3.5 −6

In a **geometric** sequence the **ratio** between consecutive terms is constant.

×3 ×3 ×3 ×3

3 9 27 81 243...

Quadratic sequences

The nth term of a quadratic sequence can be written as

$$u_n = an^2 + bn + c$$

where a, b and c are numbers and $a \neq 0$.

Finding nth terms

You can find the nth term of a quadratic sequence by looking at the **second differences**. The sequence with nth term $an^2 + bn + c$ will have second differences equal to $2a$.

Worked example

The first five terms of a quadratic sequence are

3 11 25 45 71

Find an expression, in terms of n, for the nth term of this sequence. **(3 marks)**

3 11 25 45 71

+8 +14 +20 +26

+6 +6 +6

The second difference is +6, so the coefficient of n^2 will be 3.

$2a = 6$ so $a = 3$

$u_n = 3n^2 + bn + c$

n	1	2	3	4	5
u_n	3	11	25	45	71
$3n^2$	3	12	27	48	75
$u_n - 3n^2$	0	−1	−2	−3	−4

Work out $u_n - 3n^2$ for each term. This will be an arithmetic sequence.

$u_n - 3n^2 = -n + 1$

So $u_n = 3n^2 - n + 1$

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Exam practice

1. The first three terms of a geometric sequence are

$$8 \quad 2p \quad p + 12$$

where p is a positive constant.

(a) Find the value of p. **(3 marks)**

$p =$ .....

(b) Find the 5th term of the sequence. **(2 marks)**

.....

Problem solved!

The sequence is geometric so the ratios between consecutive terms are constant:

$$\frac{2p}{8} = \frac{p + 12}{2p}$$

2. Here are the first five terms of a sequence.

$$3 \quad 10 \quad 21 \quad 36 \quad 55$$

Find an expression, in terms of n, for the nth term of this sequence. **(3 marks)**

.....

This is a quadratic sequence. Find the second differences then draw a table like the one in the Worked example opposite.

3. The rule to get from one term to the next term in a sequence is:

Add 3 then multiply by k

where k is a positive constant. The second term is 19 and the fourth term is 145.

Find the first term in the sequence. **(5 marks)**

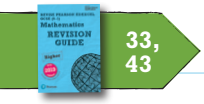
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Use the second term to find an expression for the fourth term in terms of k.

Examiner's hint

Don't assume that answers will be integers.

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Completing the square

What's it all about?

You can use **completed square form** $(x + p)^2 + q$ or $a(x + p)^2 + q$ to solve quadratic equations and find the turning points of graphs. Use this identity to help you complete the square when the coefficient of x^2 is 1:

$$x^2 + bx + c \equiv \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

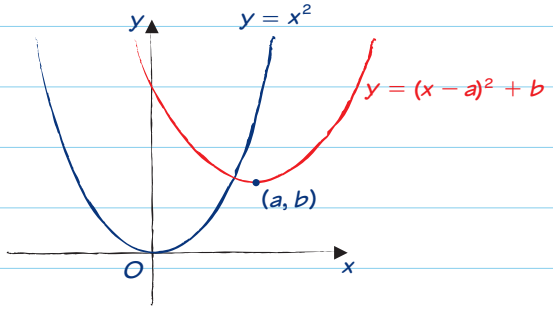
Halve the x-coefficient to find the number in the brackets. Then subtract the square of this number outside the brackets.

$-\left(\frac{b}{2}\right)^2 + c$ are number terms so you can combine them.

Turning points

The graph of $y = (x - a)^2 + b$ has a turning point at (a, b). You can think of it as a translation of the graph of $y = x^2$ by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$.

There is more about transformations of graphs on page 28.



Worked example

(a) Show that $3x^2 + 24x - 1$ can be written in the form $a(x + p)^2 + q$ where a, p and q are integers. **(3 marks)**

$$\begin{aligned} 3x^2 + 24x - 1 &= 3(x^2 + 8x) - 1 \\ &= 3((x + 4)^2 - 16) - 1 \\ &= 3(x + 4)^2 - 48 - 1 \\ &= 3(x + 4)^2 - 49 \end{aligned}$$

Problem solved!

You can complete the square in $ax^2 + bx + c$ when $a \neq 1$ by taking a factor of a out of the first two terms.

$x^2 + 8x = (x + 4)^2 - 4^2$

(b) Hence write down the coordinates of the turning point on the graph with equation $y = 3x^2 + 24x - 1$. **(1 mark)**

$(-4, -49)$

The turning point will occur when $(x + 4)^2 = 0$. This happens when $x = -4$.