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REVISE PEARSON EDEXCEL GCSE (9–1)

Mathematics

Foundation

REVISION GUIDE

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GET THE

Your new *Revision Guide* is packed with features to help you stay ahead of the game, and on track for success in your Pearson Edexcel Foundation Maths GCSE.

Examiners' report

Every year Pearson Edexcel produces reports on the most recent exams. These **examiners' reports** are jam-packed with useful advice about which questions students struggled with, where marks were dropped or picked up, and which skills students need to concentrate on. We've taken a deep dive into these reports to bring you the most relevant advice for your upcoming exams. Whenever you see this feature in the *Revision Guide*, you know that you're looking at advice based on **real students** who have sat **real exams**. To get you started, here are our **top five** examiners' tips and tricks.



Five sure-fire ways to pick up marks

1. **Write clearly** – make sure all your numbers can be read clearly, and don't write so small that the examiner can't read your working.
2. **Show all your workings** – you can pick up loads of marks for partially correct answers, but only if you have shown your method neatly.
3. **Use your calculator** – every year students lose marks on calculator papers by using mental or written methods when they could just use their calculator!
4. **Don't rush** – if there are figures given in the question make sure you copy them carefully into your own working or onto your calculator.
5. **Answer the question** – always make sure your answer matches what is asked for in the question. Give conclusions in words or sentences, and write numerical answers on the answer line.

Problem solving



You'll need to use problem-solving skills in your exam. Look out for problem-solving tips and strategies wherever you see this icon, and check out the dedicated problem-solving skills pages at the end of each main section.

Target grades



The exam-style questions in this book have been given target grades. You can use these target grades to help you track your progress. Remember that being able to answer questions at a particular target grade does not guarantee that you will achieve this grade! Your actual grade will be based on the total number of marks you get. So if you are aiming for a top grade you need to be confident with every topic, and you should attempt all the questions in your exams.

Video solutions



Some of the questions in this *Revision Guide* have worked solution videos. Use your phone, tablet or webcam to scan the QR code to watch the video.

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INSIDE TRACK

Hot Topic

Some topics come up year after year. We've picked 25 of the hottest topics. These pages contain key skills and knowledge that you're likely to need in your upcoming exams. If you're pushed for time you might want to revise these first.

Tricky Topic

There is some tough material in Foundation GCSE. We've identified 25 of the trickiest topics. Don't worry if you find these pages difficult – they are! You might want to save these topics for days when you have a bit more time to concentrate on them.

1F	2F	3F
		
Date:	Date:	Date: .....
Time:	Time:	Time: .....
Location:	Location:	Location: .....

You will have to sit **three papers** for Pearson Edexcel Foundation GCSE Maths. Each paper is **1 hour and 30 minutes long** and is worth **80 marks**. You can't use a calculator on Paper 1, but make sure you have one with you for Papers 2 and 3. If you know when and where your exams are taking place, write this information into your Revision Guide here.

The grade boundaries change every year, so you can't know exactly how many marks you need to make your target grade. But you can use this table to get a rough idea.

Target grade	Marks needed
1	15%
2	30%
3	45%
4	60%
5	75%

In it to win it!

Many successful Foundation GCSE students pick up **a few marks** on **lots of questions**, even if they don't get them fully correct. Pick up cheeky marks by:

- showing a bit of working
- demonstrating a method
- using mathematical language
- writing down a formula.

Whatever you do, **have a go!**

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KEY

= Hot Topic

= Tricky Topic

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Place value

The value of each digit in a number depends on its position. Digits that are further to the left are worth more. You can use a place value diagram to help you read and write numbers.

Worked example

Target grade 1

Write these amounts in order, smallest first. **(1 mark)**

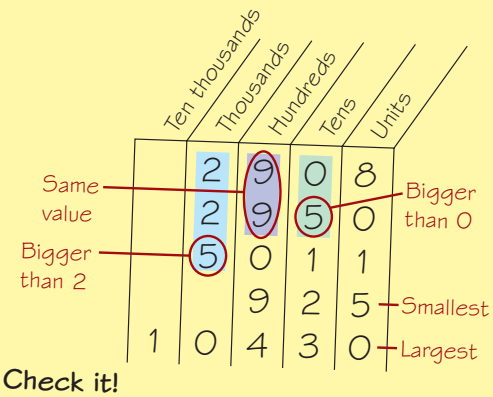
£2908 £2950 £5011 £925 £10 430

£925

£925 £10 430

£925 £5011 £10 430

£925 £2908 £2950 £5011 £10 430



Check it!

Have you included all the amounts in your final answer? ✓

Working with money

- Do all your calculations in the same units, either £ or p.
- Write either £ or p in your answer, but not both.
- 100p = £1
- Amounts in pounds need 2 decimal places. Write 280p as £2.80.

Worked example

Target grade 1

A music website sells songs and albums.
Songs cost 79p each.
Albums cost £6.99 each.
Aaron has a £25 gift card.
He buys 2 albums and spends the rest on songs.
How many songs can he afford? **(2 marks)**

$2 \times 6.99 = 13.98$

$25 - 13.98 = 11.02$

$11.02 \div 0.79 = 13.949...$

Aaron can buy 13 songs.

Examiners' report

You can get credit for attempting the correct calculation, so make sure you show all your working. Write neatly, and copy any numbers from the question carefully.
Make sure you read the question carefully – your final answer needs to be a **number of songs**, not an amount of money.
Choose whether you want to work in pounds or pence, and remember to round down because Aaron can only buy a **whole number** of songs.

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade 1

1 Write these amounts in order, smallest first:
£1974 £974 £1749 £1497 £1947 **(1 mark)**

Profit is money you make, and loss is money you lose. You need to add the individual profits, then subtract the individual losses.

2 Mark buys and sells used cars. The table shows information about five cars that Mark bought and sold.

Car	A	B	C	D	E
bought	£2200	£7800	£4200	£.....	£6500
sold	£2900	£9000	£.....	£11 200	£5750
profit or loss	£700 profit	£.....	£1500 loss	£500 profit	£..... .....

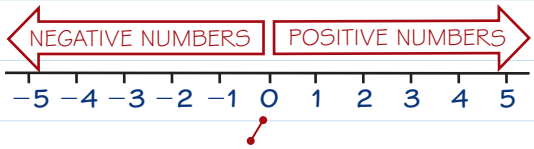
(a) Complete the table. **(2 marks)**
(b) Work out Mark's total profit or loss for these five cars. **(2 marks)**

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Had a look ☐ Nearly there ☐ Nailed it! ☐

Negative numbers

Numbers less than than 0 are called **negative** numbers.

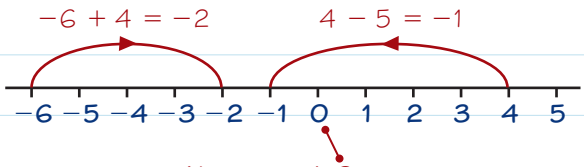


0 is neither positive nor negative.

You can use a number line to write numbers in order of size. The numbers get bigger as you move to the right.

Number lines

You can use number lines to help when adding and subtracting.



Always mark 0 on your number line.

Adding and subtracting

To add or subtract a negative number, change the double signs first.

+ − → −

12 + −3 = 12 − 3 = 9

− − → +

5 − −9 = 5 + 9 = 14

Golden rule

When you **add** a negative number the answer is **lower**.
When you **subtract** a negative number the answer is **higher**.

Multiplying and dividing

When multiplying and dividing, use these rules to decide whether the answer will be positive or negative.

1. If numbers have the **same** sign then the answer is **positive**.
   −3 × −7 = 21
2. If numbers have **different** signs then the answer is **negative**.
   +80 ÷ −10 = −8

Problem solved! One strategy is to try a few pairs and add up the totals. You could also work out what each total should be by adding together all 6 cards then dividing by 3:

−4 + −3 + 0 + 2 + −6 + −1 = −12

−12 ÷ 3 = −4

Check it!

Make sure you have used each card exactly once. ✓

Worked example

Target grade 1

Here are six number cards.

−4	−3	0	2	−6	−1

Sort the cards into three pairs so that each pair has the same total. **(2 marks)**

−4 and 0 Total = −4

−3 and −1 Total = −4

−6 and 2 Total = −4

Now try this

Target grade 1

1 (a) Work out −2 + −4
(b) Work out −1 − −6
(2 marks)

2 (a) Work out −4 × −12
(b) Work out 30 ÷ −5
(2 marks)

Worked solution video

3 Here are six number cards.

−10	−3	1	2	−9	−5

Sort the cards into three pairs so that each pair has the same total. **(2 marks)**

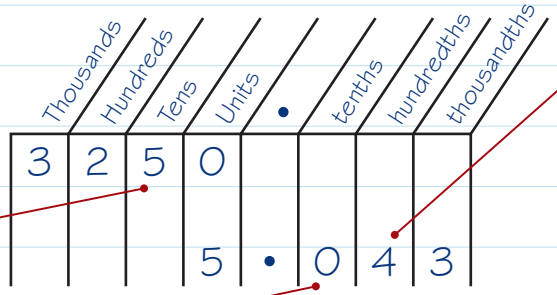
Try a few pairs and add up the totals.

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Rounding numbers

To **round** a number, you look at the next digit to the right on a place value diagram.

5 or more → round up, less than 5 → round down



To round to the nearest 100, you look at the digit in the tens column. It is a 5, so round up. 3250 rounded to the nearest 100 is 3300.

To round to 1 decimal place (1 d.p.), you look at the digit in the second decimal place. It is a 4, so round down. 5.043 rounded to 1 d.p. is 5.0 You **need** to write the 0 to show that you have rounded to 1 d.p.

To round to the nearest whole number look at the digit in the tenths column. It is a 0, so round down. 5.043 rounded to the nearest whole number is 5.

Significant figures

You always start counting **significant figures** from the first non-zero on the left.

27.05 rounded to 1 s.f. is 30 — The first non-zero digit is 2. The next digit is 7, so round up to give an answer of 30.

27.05 rounded to 2 s.f. is 27 — Look for the two digits furthest to the left, which are 2 and 7. The next digit is 0, so round down to give an answer of 27.

27.05 rounded to 3 s.f. is 27.1 — The first three significant figures are 2, 7 and 0. The next digit is 5 so round up to 27.1.

Numbers less than 1

When rounding numbers less than 1 to a given number of significant figures, remember **not** to count zero digits that are on the left.

0.0085 rounded to 1 s.f. is 0.009 — Look for the digit which is furthest to the left and which is **not** a zero. This digit is 8. The next digit is 5 so round up to give an answer of 0.009.

Worked example

Target grade 3

Round 6.9083 correct to
(a) 1 significant figure
(b) 2 significant figures
(c) 3 significant figures. **(3 marks)**

(a) 7 (1 s.f.)
(b) 6.9 (2 s.f.)
(c) 6.91 (3 s.f.)

The width of the cooker is given to the nearest whole unit. So it might be **inaccurate** by up to half a unit in either direction. Show Supraj is wrong by giving an example of a width larger than 76 cm that would round to 76 cm.

There is more about rounding errors on page 32.

Now try this

Target grade 1

1 Round the number 3756
(a) to the nearest 100 **(1 mark)**
(b) to the nearest 10 **(1 mark)**

Target grade 2

2 Write these numbers correct to 1 significant figure:
(a) 48 **(1 mark)**
(b) 3025 **(1 mark)**
(c) 0.00939 **(1 mark)**
(d) 6.5 **(1 mark)**

Worked solution video

Target grade 3

3 Round 0.179554 to 3 significant figures. **(1 mark)**

Target grade 4

4 Supraj is installing a kitchen. He orders a cooker that is 76 cm wide, to the nearest cm. He says: 'If I leave a gap exactly 76 cm wide the cooker will definitely fit perfectly.' Explain why Supraj is wrong. **(1 mark)**

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Had a look ☐ Nearly there ☐ Nailed it! ☐

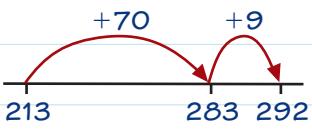
Adding and subtracting

You need to be able to add and subtract numbers without a calculator.

Mental methods

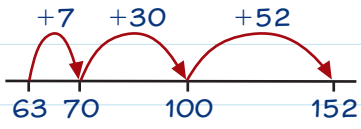
Try these methods for adding and subtracting quickly in your head.

213 + 79



Add the tens first then the units.
213 + 79 = 292

152 − 63



Count on in steps from 63 up to 152. Add up the steps to work out the difference between 152 and 63.
7 + 30 + 52 = 89
152 − 63 = 89

Worked example

Target grade 1

Work out 285 + 56 + 1091

(2 marks)

```
  285
   56
+ 1091
 1432
  2 1
```

1. **Always** add the units column first: 5 + 6 + 1 = 12. Write down the 2 and carry the **1** over to the tens column.
2. Add the tens column: 8 + 5 + 9 + **1** = 23. Make sure you include any numbers you carried over. Write down the 3 and carry the **2** over to the hundreds column.
3. Add the hundreds column: 2 + 0 + **2** = 4. Write down 4.
4. There is only one digit in the thousands column. Write this in your answer.

Worked example

Target grade 1

Work out 418 − 62

(2 marks)

```
 3 1
 4 18
 − 62
 356
```

1. **Always** subtract the units column first: 8 − 2 = 6. Remember it is (top number) − (bottom number).
2. Look at the tens column. 1 is smaller than 6 so you have to exchange 1 hundred for 10 tens. Change 4 hundreds into 3 hundreds and 10 tens.
3. Now you can subtract the tens column: 11 − 6 = 5.
4. There is nothing to subtract in the hundreds column so write 3 in your answer.

Now try this

Target grade 1

1 Work out
(a) 503 + 1126 + 85 **(2 marks)**
(b) 745 + 283 **(2 marks)**

2 (a) Work out 627 − 251 **(2 marks)**
(b) How much is 831 − 659? **(2 marks)**

3 Joe buys a magazine costing £4.45 and two birthday cards costing £1.99 each. He pays with a £10 note. How much change will he receive? **(3 marks)**

You can work in pence so you don't have to use decimal numbers. Work out 445 + 199 + 199, then subtract the result from 1000. Remember to give units with your answer.

Scan this QR code to watch a video of this question being solved.

Worked solution video

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Multiplying and dividing

You need to be able to multiply and divide numbers without a calculator.

For a reminder about multiplying and dividing by 10, 100 and 1000 have a look at page 61.

Mental methods

Try these methods for multiplying and dividing quickly in your head.

37×8

$30 \times 8 = 240$

$7 \times 8 = 56$

$37 \times 8 = 296$

Split 37 into 30 and 7. Then multiply each by 8. Add the answers to get the total.

$54 \div 6$

$6 \times \square = 54$

The answer is 9.

Try to find a multiplication fact using 6 with 54 as the answer.

Worked example

Target grade 1

Work out

(a) 49×3 **(2 marks)**

$$\begin{array}{r} 49 \\ \times\ 3 \\ \hline 147 \\ {\scriptstyle 2} \end{array}$$

(b) 36×24 **(3 marks)**

$$\begin{array}{r} 36 \\ \times\ 24 \\ \hline 144 \\ {\scriptstyle 2} \\ 720 \\ {\scriptstyle 1} \\ \hline 864 \end{array}$$

Always multiply from right to left.

1. $9 \times 3 = 27$. Write down 7 and carry over **2** (2 tens).
2. $4 \times 3 = 12$. Add on the carry-over. $12 + \mathbf{2} = 14$. Write down 14.

Examiners' report

Remember that this is a **non-calculator** question. You need to show all your working.

1. Work out 36×4. (Answer = 144)
2. Work out 36×20. Write down **0** and then work out 36×2. (Answer = 72**0**)
3. Add the answers. $(144 + 720 = 864)$

Multiplying and dividing are **much easier** if you are confident with your **times tables** up to 10×10.

Real students have struggled with questions like this in recent exams – **be prepared!**

Worked example

Target grade 1

Work out $288 \div 9$ **(2 marks)**

$$\begin{array}{r} 32 \\ 9\overline{)288} \\ -27 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$$

You can use long division.

1. Does 9 divide into 2? No.
2. Does 9 divide into 28? Yes. $9 \times 3 = 27$ so 9 divides into 28 three times with remainder 1.
3. Does 9 divide into 18? Yes. $9 \times 2 = 18$ so 9 divides into 18 two times with no remainder.

Using short division the calculation would look like this:

$$\begin{array}{r} 3\ 2 \\ 9\overline{)28^{1}8} \end{array}$$

Now try this

Target grade 1

1 (a) Work out 72×100 **(1 mark)**

(b) Work out 256×9 **(1 mark)**

(c) Work out 29×78 **(2 marks)**

Worked solution video

Target grade 1

2 (a) Work out $468 \div 3$ **(2 marks)**

(b) Work out $1032 \div 8$ **(2 marks)**

Target grade 2

3 There are 18 chocolate coins in a bag.
Paula buys 6 of these bags.
Paula has 7 grandchildren. She wants to give each of her grandchildren 15 coins.
Has she bought enough coins? **(3 marks)**

You need to answer 'yes' or 'no' to the question, **and** show your working to **justify** your answer.

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Had a look ☐ Nearly there ☐ Nailed it! ☐

Decimals and place value

You can use a place value diagram to help you understand and compare decimal numbers.

Remember that decimal numbers with more digits are not necessarily bigger. Try writing extra 0s so that all the numbers have the same number of decimal places.

Tens	Units	.	tenths	hundredths	thousandths
	0	.	7	5	8
	0	.	7	6	0
	0	.	7	9	0
	0	.	8	0	0

The value of the 5 in this number is 5 hundredths.

0.76 is the same as 0.760
0.76 is bigger than 0.758
6 hundredths is bigger than 5 hundredths.

0.79 is smaller than 0.8 because the digit in the tenths place is smaller.

Worked example

Target grade 1

Write these numbers in order, smallest first:
0.43 0.425 0.4 0.48 0.459
(2 marks)

~~0.43~~ ~~0.425~~ ~~0.4~~ ~~0.48~~ ~~0.459~~
0.4 0.425
0.4 0.425 0.43
0.4 0.425 0.43 0.459
0.4 0.425 0.43 0.459 0.48

All these numbers have the same tenths digit. You need to look at the hundredths digit first.
0.4 is the same as 0.40 so this is the smallest number.
0.425 has the next smallest hundredths digit so this is the next number.

Check it!
Cross out each number to make sure you include them all in your final answer. ✓

Worked example

Target grade 2

Using the information that
$58 \times 71 = 4118$
write down the value of
(a) 58×0.71 **(1 mark)**
41.18
(b) 5800×7.1 **(1 mark)**
41180

(a) 71 has been divided by 100 and 58 hasn't been changed. So the answer needs to be divided by 100. $4118 \div 100 = 41.18$

(b) 58 has been multiplied by 100 and 71 has been divided by 10.

×100 → ÷10 → is the same as → ×10 →

The answer needs to be multiplied by 10:
$4118 \times 10 = 41180$

To revise multiplying and dividing by 10, 100 and 1000 see page 61.

Now try this

Target grade 1

1 Write down the place value of the 8 in these numbers:
(a) 2.84 **(1 mark)** (b) 0.3086 **(1 mark)**

2 Write these numbers in order of size:
0.517 0.508 0.58 0.571 0.51 **(2 marks)**

You could start with the smallest number or the largest.

Target grade 2

3 Using the information that
$672 \times 13 = 8736$
write down the value of
(a) 0.672×13 **(1 mark)**
(b) $8736 \div 67.2$ **(1 mark)**

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Operations on decimals

1 Adding and subtracting

To add or subtract decimal numbers:

1. Line up digits with the same place value.
2. Line up the decimal points.
3. Write a decimal point in your answer.

See page 4 for a reminder about adding and subtracting.

Write in 0s so that both numbers have the same number of decimal places.

Worked example Target grade 1

(a) 0.75 + 1.6 **(1 mark)**

$$\begin{array}{r} 0.75 \\ +\ 1.60 \\ \hline 2.35 \\ \hline {\scriptstyle 1} \end{array}$$

(b) 3.5 − 0.21 **(1 mark)**

$$\begin{array}{r} 3.^{4}5^{1}0 \\ -\ 0.21 \\ \hline 3.29 \end{array}$$

2 Multiplying

To multiply decimal numbers:

1. Ignore the decimal points and just multiply the numbers.
2. Count the number of decimal places in the calculation.
3. Put this number of decimal places in the answer.

(a) You can use estimation to check that the decimal point is in the correct place.

$8.69 \times 12 \approx 9 \times 12 = 108$ $\quad 108 \approx 104$ ✓

(b) 8.5 × 0.04 has 3 decimal places in total

So $8.5 \times 0.04 = 0.340 = 0.34$

Write a 0 before the decimal point and simplify your answer.

Worked example Target grade 1 / Target grade 2

(a) 8.69 × 12 **(2 marks)**

$$\begin{array}{r} 869 \\ \times\ 12 \\ \hline 1738 \\ +\ 8690 \\ \hline 10428 \\ {\scriptstyle 1\ 1} \end{array}$$

$8.69 \times 12 = 104.28$

(b) 8.5 × 0.04 **(2 marks)**

$$\begin{array}{r} 85 \\ \times\ 4 \\ \hline 340 \\ {\scriptstyle 2} \end{array}$$

$8.5 \times 0.04 = 0.34$

3 Dividing

To divide by a decimal number:

1. Multiply both numbers by 10, 100 or 1000 to make the second number a whole number.
2. Divide by the whole number.

Multiply 40.6 and 1.4 by 10.

If you multiply both numbers in a division by the same amount, the answer stays the same.

Worked example Target grade 2

(a) 55.8 ÷ 3 **(1 mark)**

$$3\overline{)5^{2}5.^{1}8}\ \ 18.6$$

(b) 40.6 ÷ 1.4 **(2 marks)**

$40.6 \div 1.4 = 406 \div 14$

$$\begin{array}{r} 29 \\ 14\overline{)406} \\ -28 \\ \hline 126 \\ -126 \\ \hline 0 \end{array}$$

$40.6 \div 1.4 = 29$

Now try this

Target grade 1

1 (a) Work out 12.5 + 7.93 **(2 marks)**
(b) Work out 8.14 + 3 + 0.772 **(2 marks)**

2 (a) Work out 16.5 − 9.72 **(2 marks)**
(b) Work out 5.76 × 34 **(2 marks)**

Target grade 2

3 (a) A kitchen stool costs £39.90.
Work out the cost of 6 kitchen stools. **(2 marks)**
(b) Tom buys 12 pencils for £11.28.
Work out the cost of 1 pencil. **(2 marks)**

Worked solution video

Make sure you line up the decimal points.

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Had a look ☐ Nearly there ☐ Nailed it! ☐

Squares, cubes and roots

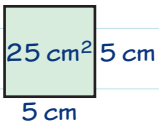
Squares and square roots

When a whole number is multiplied by itself the answer is a **square number**. You can write square numbers using index notation.

Multiplication	Index notation	Square number
2×2	2^2	4
5×5	5^2	25
9×9	9^2	81
13×13	13^2	169

Square numbers are the areas of squares with whole number side lengths.

There is more about area on page 80.



Square roots are the opposite of squares. You use the symbol $\sqrt{\ }$ to represent a square root.

$\sqrt{4} = 2$ $\sqrt{25} = 5$ $\sqrt{81} = 9$

You need to be able to **remember** the square numbers up to 15×15 and the corresponding square roots.

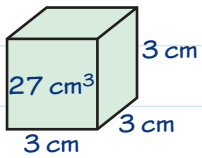
Cubes and cube roots

When a whole number is multiplied by itself then multiplied by itself again, the answer is a **cube number**. You can write cube numbers using index notation.

Multiplication	Index notation	Cube number
$2 \times 2 \times 2$	2^3	8
$3 \times 3 \times 3$	3^3	27
$5 \times 5 \times 5$	5^3	125
$10 \times 10 \times 10$	10^3	1000

Cube numbers are the volumes of cubes with whole number side lengths.

There is more about volume on page 83.



Cube roots are the opposite of cubes. You use the symbol $\sqrt[3]{\ }$ to represent a cube root.

$\sqrt[3]{8} = 2$ $\sqrt[3]{27} = 3$ $\sqrt[3]{1000} = 10$

You need to be able to **remember** the cubes of 2, 3, 4, 5 and 10, and the corresponding cube roots.

Worked example

Target grade 3

Pavel says
'A square number cannot be a prime number.'
Is Pavel correct? Explain your answer. (2 marks)

Yes because a prime number has exactly two factors: 1 and itself. The square number 1 has only one factor (1) so it is not prime. Every other square number has at least three factors (1, itself and its square root) and so is not prime.

Problem solved! If a question says 'explain' you must answer the question and give a **reason**. Start your answer with: 'Yes because...' or 'No because...'. If you're not sure you could **try some numbers** to see what is going on:

Factors of 9 are 1, 3, 9, so 9 is **not prime**.

Factors of 49 are 1, 7, 49, so 49 is **not prime**.

Revise factors and prime numbers on page 11.

Now try this

Target grade 1

1 Work out
(a) 6^2 **(1 mark)**
(b) $\sqrt[3]{125}$ **(1 mark)**
(c) 4^3 **(1 mark)**
(d) $\sqrt{121}$ **(1 mark)**

You should know these without using your calculator.

Target grade 3

2 Amy says
'The product of two square numbers is always an even number.'
Give an example to show that Amy is wrong. **(1 mark)**

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Indices

Indices include square roots, cube roots and powers. You can use the **index laws** to simplify powers.

1 $a^m \times a^n = a^{m+n}$

$4^3 \times 4^7 = 4^{3+7} = 4^{10}$

2 $\frac{a^m}{a^n} = a^{m-n}$

$12^8 \div 12^3 = 12^{8-3} = 12^5$

3 $(a^m)^n = a^{mn}$

$(7^3)^5 = 7^{3\times 5} = 7^{15}$

4 Negative powers

$$a^{-n} = \frac{1}{a^n}$$

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Be careful!
A **negative** power can still have a **positive** answer.

5 Reciprocals

$$a^{-1} = \frac{1}{a}$$

This means that a^{-1} is the **reciprocal** of a.
You can find the reciprocal of a fraction by turning it upside down.

$\left(\frac{5}{9}\right)^{-1} = \frac{9}{5}$

6 Powers of fractions

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$\left(\frac{3}{10}\right)^2 = \frac{3^2}{10^2} = \frac{9}{100}$

7 Combining rules

You can apply the rules one at a time.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$

Worked example

Target grade 4

(a) Write $6 \times 6 \times 6 \times 6 \times 6$ as a single power of 6. **(1 mark)**

$6 \times 6 \times 6 \times 6 \times 6 = 6^5$

(b) Simplify $\frac{3^7 \times 3}{3^4}$ fully, leaving your answer in index form. **(2 marks)**

$\frac{3^7 \times 3}{3^4} = \frac{3^8}{3^4} = 3^4$

Powers of 0 and 1

Anything raised to the power 0 is equal to 1.

$6^0 = 1 \quad 1^0 = 1 \quad 7223^0 = 1 \quad (-5)^0 = 1$

Anything raised to the power 1 is equal to itself.

$8^1 = 8 \quad 499^1 = 499 \quad (-3)^1 = -3$

3 is the same as 3^1. For part (b), use the rule $a^m \times a^n = a^{m+n}$ to simplify the numerator, then use $\frac{a^m}{a^n} = a^{m-n}$ to simplify the fraction. Remember to write down both steps of your working and give your answer as a power.

Now try this

Target grade 4

1 Simplify, leaving your answers as a power of 7.

(a) $7^2 \times 7^8$ **(1 mark)**

(b) $7^9 \div 7^2$ **(1 mark)**

(c) $\frac{7^4 \times 7^5}{7^6}$ **(2 marks)**

(d) $(7^5)^3$ **(1 mark)**

2 (a) Write $\frac{5 \times 5^8}{5^4}$ as a single power of 5. **(2 marks)**

(b) Work out $\frac{4 \times 4^{10}}{4^5 \times 4^3}$ **(2 marks)**

Target grade 5

3 Write down the value of

(a) 4^0 **(1 mark)**

(b) 4^{-1} **(1 mark)**

(c) 4^{-2} **(1 mark)**

(d) $\left(\frac{2}{5}\right)^2$ **(1 mark)**

(e) $\left(\frac{3}{4}\right)^{-3}$ **(1 mark)**

Worked solution video

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Had a look ☐ Nearly there ☐ Nailed it! ☐

Hot Topic

Estimation

You can estimate the answer to a calculation by rounding each number to **1 significant figure**, and then doing the calculation. You can use this method to check your answers, or to estimate calculations on your **non-calculator paper**. Here are two examples:

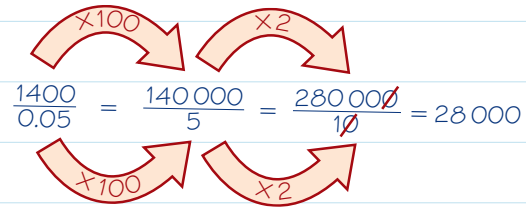
1 $4.32 \times 18.09 \approx 4 \times 20 = 80$
The answer is approximately equal to 80.

2 $327^2 \approx 300^2 = 3^2 \times 100^2 = 90\,000$
The answer is approximately equal to 90 000.

≈ means 'is approximately equal to'

Decimal division trick

You might have to divide by a decimal on your non-calculator paper. If you multiply both numbers in a division by the same amount the answer stays the same.



Worked example

Target grade 4

Work out an estimate for

(a) $\dfrac{4.31 \times 278}{0.487}$ **(2 marks)**

$\dfrac{4.31 \times 278}{0.487} \approx \dfrac{4 \times 300}{0.5} = \dfrac{1200}{0.5} = 2400$

(b) 37.4^3 **(2 marks)**

$37.4^3 \approx 40^3 = 4^3 \times 10^3 = 64 \times 1000 = 64\,000$

Round all the numbers to **1 significant figure**. Then **write out** the calculation with the rounded values before calculating your estimate.

You can use the laws of indices to work out 40^3 without a calculator.
$(ab)^n = a^n \times b^n$
so $40^3 = (4 \times 10)^3 = 4^3 \times 10^3$

Problem solved! On your non-calculator paper you can use $\pi = 3.142$ then round to 1 s.f. to make your estimate.

Examiners' report

You have rounded **both values down** so your answer will be an underestimate. The question says 'give a reason' so show working and write a conclusion in **words**.

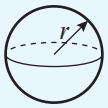
Real students have struggled with questions like this in recent exams – **be prepared!**

Worked example

Target grade 5

A spherical ball-bearing has a radius of 2.35 cm.

Surface area of sphere $= 4\pi r^2$



(a) Work out an estimate for its surface area in square centimetres. **(2 marks)**

$4\pi r^2 = 4 \times 3.142 \times 2.35^2$
$\approx 4 \times 3 \times 2^2 = 48\ \text{cm}^2$

(b) Is your answer to part (a) an overestimate or an underestimate? Give a reason for your answer. **(1 mark)**

$3 < 3.142$ and $2 < 2.35$
so the answer is an underestimate.

Now try this

Target grade 4

1 Showing your rounding, work out an estimate for

$\dfrac{82 \times 285}{64 \times 35}$ **(2 marks)**

Worked solution video

Target grade 5

2 A scientist models a raindrop as a sphere with radius 3.2 mm.

Volume of a sphere $= \frac{4}{3}\pi r^3$

(a) Work out an estimate for the volume of the raindrop. **(2 marks)**

(b) Is your answer to part (a) an overestimate or an underestimate? Give a reason for your answer. **(1 mark)**

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Factors, multiples and primes

Factors and multiples

The **factors** of a number are any whole numbers that divide into it exactly.

1 and the number itself are both factors of any number.
The factors of 12 are 1, 2, 3, 4, 6 and 12.

Factors come in pairs. Each pair is a multiplication with the number as its answer.
The factor pairs of 12 are 1×12, 2×6 and 3×4.

A common factor is a number that is a factor of two or more numbers.
2 is a common factor of 6 and 12.

The **multiples** of a number are all the numbers in its times table.
The multiples of 7 are 7, 14, 21, 28, 35, ...

A common multiple is a number that is a multiple of two or more numbers.
12 is a common multiple of 6 and 4.

Primes

A **prime number** has exactly two factors. It can only be divided by 1 and by itself.

The first ten prime numbers are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
1 is not a prime number. It has only 1 factor.

Factor trees

You can use a factor tree to find prime factors.

1. Choose a factor pair of the number.
2. Circle the prime factors as you go along.
3. Continue until every branch ends with a prime number.
4. At the end write down **all** the circled numbers, putting in multiplication signs.

Worked example

Target grade 1

Here is a list of numbers:
16 8 3 17 6 20 12
From this list write down

(a) a prime number **(1 mark)**

17

(b) a multiple of 5 **(1 mark)**

20

(c) two factors of 24 which have a sum of 15. **(2 marks)**

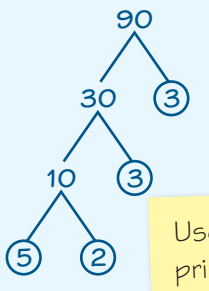
12 and 3

(a) 3 is also a prime number.
(c) 3, 6, 8 and 12 are all factors of 24. Only 12 and 3 have a sum of 15.

Worked example

Target grade 4

Write 90 as the product of prime factors.
Give your answer in index form. **(3 marks)**



$90 = 5 \times 2 \times 3 \times 3$
$= 5 \times 2 \times 3^2$

Use a factor tree to find all the prime factors. 3 appears twice in the factor tree, so you have to write 3^2.

Check it!
$5 \times 2 \times 3^2 = 10 \times 9 = 90$ ✓

Now try this

Target grade 1

1 Here is a list of numbers:
2 8 15 18 21 24 37 44
From the list, write down

(a) the number that is a multiple of 7 **(1 mark)**
(b) the number that is a factor of 45 **(1 mark)**
(c) the number that is a multiple of 6 and a factor of 48 **(1 mark)**
(d) two prime numbers. **(2 marks)**

Target grade 4

2 Write 280 as the product of its prime factors.
Give your answer in index form.
(3 marks)

Make sure you can recognise all the prime numbers below 50.

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Had a look ☐ Nearly there ☐ Nailed it! ☐

HCF and LCM

The **highest common factor (HCF)** of two numbers is the highest number that is a factor of both numbers.

The **lowest common multiple (LCM)** of two numbers is the lowest number that is a multiple of both numbers.

Examiners' report

If you have to write a number as a **product** of prime factors, make sure you use × signs in your final answer. Don't use +, and don't just write a list of prime factors.

Real students have struggled with questions like this in recent exams – **be prepared!**

You can find the HCF and LCM by writing the products of prime factors in a **Venn diagram**. Use the powers to tell you how many times each prime factor occurs. Put the **common factors** in the intersection of the two ovals.

- HCF = product of all the prime factors in the intersection
- LCM = product of all the prime factors in the Venn diagram

There is more on Venn diagrams on page 132.

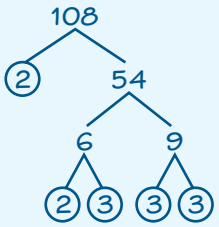
You can use **index notation** (powers) to simplify your answers.

For a reminder about this look at page 9.

Worked example

Target grade 4

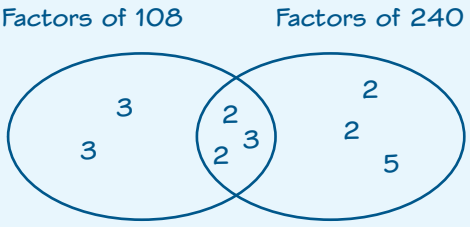
(a) Express 108 as a product of powers of its prime factors. **(3 marks)**



$108 = 2 \times 2 \times 3 \times 3 \times 3$
$= 2^2 \times 3^3$

(b) $240 = 2^4 \times 3 \times 5$
Find, as a product of powers of its prime factors

(i) the highest common factor (HCF) of 108 and 240 **(1 mark)**



$HCF = 2 \times 2 \times 3 = 2^2 \times 3$

(ii) the lowest common multiple (LCM) of 108 and 240. **(1 mark)**

$LCM = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 2^4 \times 3^3 \times 5$

Worked example

Target grade 4

n is a number.
10 is the highest common factor of 20 and n.
Work out two different possible values for n. **(2 marks)**

10 and 30

You can use prime factors to answer this question.

$20 = 2 \times 2 \times 5$

$10 = 2 \times 5$

Any number which contains the prime factors 2 and 5 exactly once will work.

Now try this

Target grade 4

1 (a) Write 132 and 110 as products of powers of their prime factors. **(4 marks)**
(b) Find the HCF of 132 and 110. **(1 mark)**
(c) Find the LCM of 132 and 110. **(1 mark)**

Worked solution video

2 Given that $X = 3^2 \times 5 \times 7^4$ and $Y = 2 \times 3^3 \times 5^2$ write down, as a product of powers of its prime factors,
(a) the HCF of X and Y **(1 mark)**
(b) the LCM of X and Y. **(1 mark)**

Expand the index notation, then draw a Venn diagram showing the prime factors of each number.

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Fractions

You need to be able to work confidently with fractions, with or without a calculator.

1 Dividing objects

You can use fractions to divide an object into parts.



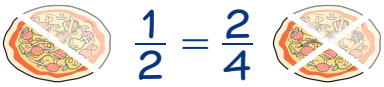
$\frac{2}{3}$ of this rectangle is shaded.

The top number is called the **numerator**.

The bottom number is called the **denominator**.

2 Equivalent fractions

Different fractions can describe the same amount.

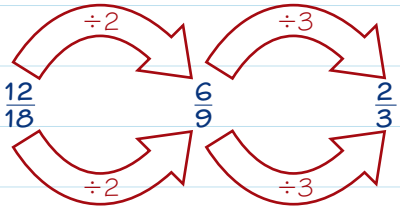


$\frac{1}{2}$ and $\frac{2}{4}$ are called equivalent fractions.

You can find equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

3 Cancelling fractions

To **cancel** or **reduce** a fraction you divide the top and bottom by the same number.



When you can't cancel the fraction any further it is in its **simplest form**.

4 Finding a fraction of an amount

Divide the amount by the denominator

Multiply by the numerator

Work out $\frac{3}{10}$ of 200 kg:

200 kg ÷ 10 = 20 kg

20 kg × 3 = 60 kg

To see how to convert between fractions and decimals look at page 56.

Worked example

Target grade 1

(a) Write $\frac{20}{80}$ as a fraction in its simplest form.

$\frac{20}{80} = \frac{2}{8} = \frac{1}{4}$ **(1 mark)**

(b) Work out $\frac{2}{5}$ of £240.

240 ÷ 5 = 48

48 × 2 = 96

$\frac{2}{5}$ of £240 is £96 **(2 marks)**

Examiners' report

It is **much easier** to find equivalent fractions if you are confident with your times tables. Write down all the steps when you are simplifying.

Real students have struggled with questions like this in recent exams – **be prepared!**

$\frac{2}{5}$ is less than 1 so the answer should be less than £240. ✓

Now try this

Target grade 2

Plan your strategy before you start. Work out how much money he makes in total, compare this with £8 and then write a conclusion.

1 David buys 80 chocolates for £20.
He sells $\frac{3}{4}$ of the chocolates for 40p each.
He then sells the remaining chocolates for 20p each.
Work out the total profit that David makes. **(4 marks)**

2 Sandeep buys some pencils to sell at the school fete.
He buys 40 pencils for £8.
He sells a quarter of the pencils for 25p each and sells half of the pencils for 30p each.
The remaining pencils are broken and are thrown away.
Does Sandeep make a profit or loss?
You must show your working. **(4 marks)**

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Had a look ☐ Nearly there ☐ Nailed it! ☐

Operations on fractions

Make sure you can add, subtract, multiply and divide fractions **without** a calculator.

1 Adding or subtracting

Write both fractions as equivalent fractions with the same denominator

↓

Add or subtract the numerators

↓

Do not change the denominator

18 is the lowest common multiple (LCM) of 9 and 6. This is the easiest common denominator to use.

For a reminder about LCMs see page 12.

Worked example Target grade 2

Work out

(a) $\frac{1}{5} + \frac{3}{10}$ **(2 marks)**

$= \frac{2}{10} + \frac{3}{10}$

$= \frac{5}{10} = \frac{1}{2}$

(b) $\frac{8}{9} - \frac{1}{6}$ **(2 marks)**

$= \frac{16}{18} - \frac{3}{18}$

$= \frac{13}{18}$

2 Multiplying

Write any whole numbers on their own as fractions with denominator 1

↓

Multiply the numerators and multiply the denominators

Worked example Target grade 2

Work out

(a) $\frac{2}{3} \times \frac{7}{10}$ **(2 marks)**

$= \frac{2 \times 7}{3 \times 10}$

$= \frac{14}{30} = \frac{7}{15}$

(b) $3 \times \frac{2}{11}$ **(2 marks)**

$= \frac{3}{1} \times \frac{2}{11}$

$= \frac{6}{11}$

3 Dividing

Write any whole numbers on their own as fractions with denominator 1

↓

Turn the second fraction 'upside down'

↓

Change ÷ to ×

↓

Multiply the numerators and multiply the denominators

Worked example Target grade 2

Work out

(a) $\frac{2}{5} \div \frac{3}{4}$ **(2 marks)**

$= \frac{2}{5} \times \frac{4}{3}$

$= \frac{8}{15}$

Change $\frac{3}{4}$ into $\frac{4}{3}$ and change ÷ to ×.

(b) $6 \div \frac{2}{3}$ **(2 marks)**

$= \frac{6}{1} \div \frac{2}{3}$

$= \frac{6}{1} \times \frac{3}{2}$

$= \frac{18}{2}$

$= 9$

Watch out!

1. You do not have to cancel your final answer unless the question asks you to 'give your answer in its simplest form'.
2. You can compare and order fractions by using equivalent fractions with the same denominator.

See page 13 for a reminder about equivalent fractions and simplest form.

Now try this

Target grade 2

Worked solution video

1 Work out

(a) $\frac{5}{9} + \frac{1}{3}$ **(2 marks)**

(b) $\frac{3}{10} \div \frac{8}{15}$ **(2 marks)**

2 Work out

(a) $\frac{7}{8} - \frac{1}{2}$ **(2 marks)** (b) $\frac{8}{11} - \frac{3}{5}$ **(2 marks)**

3 Work out

(a) $\frac{5}{9} \times \frac{3}{10}$ **(2 marks)** (b) $\frac{8}{15} \div \frac{4}{7}$ **(3 marks)**

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Mixed numbers

Mixed numbers have a whole number part and a fraction part.

$3\frac{1}{4}$ This mixed number is the same as $3 + \frac{1}{4}$

Improper fractions have a numerator larger than their denominator.

$\frac{5}{2}$, $\frac{21}{5}$ and $\frac{4}{3}$ are all improper fractions.

Converting between mixed numbers and improper fractions

To convert a mixed number into an improper fraction you...

Multiply this... $3\frac{1}{4} = \frac{3 \times 4 + 1}{4} = \frac{13}{4}$

... by this... $3 \times 4 = 12$

... add it to this. $12 + 1 = 13$

Keep the same denominator.

To convert an improper fraction into a mixed number you...

Divide this... $\frac{23}{5} = 23 \div 5 = 4\frac{3}{5}$

... by this.

Keep the same denominator.

Write the remainder as the numerator.

Golden rule

You need to write mixed numbers as improper fractions before you do any calculations.

Worked example

Target grade 4

Work out $4\frac{1}{2} \times 3$ **(2 marks)**

$= \frac{9}{2} \times \frac{3}{1}$

$= \frac{27}{2}$

$= 13\frac{1}{2}$

$4\frac{1}{2} = \frac{4 \times 2 + 1}{2} = \frac{9}{2}$

Worked example

Target grade 4

Work out $3\frac{2}{5} - 1\frac{1}{2}$ **(3 marks)**

$= \frac{17}{5} - \frac{3}{2}$

$= \frac{34}{10} - \frac{15}{10} = \frac{19}{10}$

$= 1\frac{9}{10}$

Worked example

Target grade 4

Kumar has $2\frac{1}{2}$ pizzas. He wants to share them equally between 3 people.
What fraction of a pizza does each person receive? **(2 marks)**

$2\frac{1}{2} \div 3 = \frac{5}{2} \div \frac{3}{1}$

$= \frac{5}{2} \times \frac{1}{3}$

$= \frac{5}{6}$

Each person receives $\frac{5}{6}$ of a pizza.

Examiners' report

Working with mixed numbers in word problems is tricky. Here are some helpful tips:

- Convert mixed numbers to improper fractions before calculating.
- If you are sharing then divide.
- Use common sense to check your answers. In this question, Kumar has less than 3 whole pizzas, so each person will get less than 1 whole pizza. You can use this fact to check your answer.

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade 4

1 Work out
(a) $3\frac{1}{3} \times 1\frac{3}{4}$ **(3 marks)**
(b) $4\frac{1}{5} \div 1\frac{7}{8}$ **(3 marks)**

Write both numbers as improper fractions before you do the calculation. Remember you can't use a calculator for these questions.

2 A cup holds $\frac{1}{3}$ litre of water.
How many **full** cups can be filled from a $2\frac{1}{2}$ litre jug of water? **(3 marks)**

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Had a look ☐ Nearly there ☐ Nailed it! ☐

Hot Topic

Calculator and number skills

These calculator keys are really useful:

x^2	Square a number.	(−)	Enter a negative number.
x^3	Cube a number.	$\sqrt{\square}$	Find the square root of a number.
x^{-1}	Find the reciprocal of a number.	$\sqrt[3]{\square}$	Find the cube root of a number. You might need to press the shift key first.
Ans	Use your previous answer in a calculation.	S⇔D	Change the answer from a fraction or surd to a decimal. Not all calculators have this key.

Order of operations

You need to use the correct **priority of operations when doing a calculation.**

Brackets
Indices
Division
Multiplication
Addition
Subtraction

$(10 - 7) + 4 \times 3^2$
$= 3 + 4 \times 3^2$
$= 3 + 4 \times 9$
$= 3 + 36$
$= 39$

Reciprocals

The reciprocal of a number is 1 divided by that number. You can find it by writing the number as a fraction then turning it upside down.

$7 = \frac{7}{1} \rightarrow \frac{1}{7}$ The reciprocal of 7 is $\frac{1}{7}$

$\frac{3}{4} \rightarrow \frac{4}{3}$ The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

You can use the x^{-1} key on your calculator to find reciprocals.

Worked example

Target grade 4

(a) Work out the value of $\frac{\sqrt{8.3}}{12.5 - 7.3}$

Give your answer as a decimal.
Write down all the figures on your calculator display. **(2 marks)**

$\frac{\sqrt{8.3}}{12.5 - 7.3} = \frac{2.880\,97}{5.2} = 0.554\,033\,088$

(b) Find the reciprocal of 12.5. Give your answer as a decimal. **(1 mark)**

$1 \div 12.5 = 0.08$

Read the question carefully. You have to give the answer as a **decimal**, so you might need to use the S⇔D button on your calculator.

Examiners' report

If you have to work out a calculation like this in your exam, you should work out the numerator (top) and denominator (bottom) **separately**, and **write them both down**. Then divide to work out the final answer. Read the question carefully. You have to write down **all the figures** from your calculator display.

Check it!
Check your answer by working out the whole calculation in one go using the = key.

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Try doing these **without a calculator**. Make sure you use the correct priority of operations.

Target grade 1

1 Work out
(a) $10 - 3 \times 2$ **(1 mark)**
(b) $84 \div (6 + 6)$ **(1 mark)**
(c) $8 + 3^2$ **(1 mark)**
(d) $(9 - 4)^3$ **(1 mark)**

Target grade 2

2 Work out
(a) $\sqrt{244 + 717}$ **(1 mark)**
(b) $\sqrt[3]{3.1 - 0.356}$ **(1 mark)**

Worked solution video

Use a calculator and give your answers as decimal numbers. You might need to use the S⇔D key to convert a fraction answer to a decimal.

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Standard form 1

Numbers in standard form are written as the product of two parts.

$$7.3 \times 10^{-6}$$

This part is a number greater than or equal to 1 and less than 10

This part is an integer power of 10

You can use standard form to write very large or very small numbers.

$920\,000 = 9.2 \times 10^5$

Numbers greater than 10 have a positive power of 10

$0.007\,03 = 7.03 \times 10^{-3}$

Numbers less than 1 have a negative power of 10

Counting decimal places

You can count decimal places to convert between numbers in standard form and ordinary numbers.

3 jumps

$7\,900 = 7.9 \times 10^3$

7900 > 10 so the power is positive

4 jumps

$0.000\,35 = 3.5 \times 10^{-4}$

0.000 35 < 1 so the power is negative

Be careful!
Don't just count zeros to work out the power.

Worked example

Target grade 5

(a) Write 1 630 000 in standard form. **(1 mark)**

1.63×10^6

(b) Write 4.2×10^{-3} as an ordinary number. **(1 mark)**

0.0042

Count the number of decimal places you need to move to get a number between 1 and 10. 1 630 000 is bigger than 10 so the power will be positive.

Using a calculator

You can enter numbers in standard form using the $\times 10^x$ key.

To enter 3.7×10^{-6} press

3 . 7 $\times 10^x$ (−) 6

If you are using a calculator with numbers in standard form it is a good idea to put brackets around each number.

Your calculator might give the answer as a normal number. You need to convert it into standard form.

Worked example

Target grade 5

A and B are standard form numbers.
$A = 1.9 \times 10^4$ $B = 4.2 \times 10^5$
Calculate, giving your answers in standard form

(a) $A + B$ **(1 mark)**

$(1.9 \times 10^4) + (4.2 \times 10^5) = 439\,000$
$= 4.39 \times 10^5$

(b) $B \div 6$ **(1 mark)**

$(4.2 \times 10^5) \div 6 = 70\,000 = 7 \times 10^4$

Now try this

Target grade 5

1 (a) Write 0.000 28 in standard form. **(1 mark)**

(b) Write 3.91×10^5 as an ordinary number. **(1 mark)**

2 p and q are standard form numbers.
$p = 3.5 \times 10^6$ $q = 7.9 \times 10^5$
Calculate, giving your answers in standard form

(a) $p - q$ **(1 mark)**

(b) $5p$ **(1 mark)**

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Had a look ☐ Nearly there ☐ Nailed it! ☐

Standard form 2

This page shows **non-calculator** methods for doing calculations with standard form.

1 Multiplying numbers in standard form

Rearrange so powers of 10 are together

Multiply the number parts

Add the powers

Rewrite your answer in standard form if necessary

$(3 \times 10^3) \times (5 \times 10^6)$
$= (3 \times 5) \times (10^3 \times 10^6)$
$= 15 \times 10^9$

$a^m \times a^n = a^{m+n}$

$= 1.5 \times 10^1 \times 10^9$
$= 1.5 \times 10^{10}$

2 Dividing numbers in standard form

Rearrange so powers of 10 are together

Divide the number parts

Subtract the powers

Rewrite your answer in standard form if necessary

$(1.2 \times 10^8) \div (2 \times 10^4)$
$= (1.2 \div 2) \times (10^8 \div 10^4)$
$= 0.6 \times 10^4$

$a^m \div a^n = a^{m-n}$

$= 6 \times 10^{-1} \times 10^4$
$= 6 \times 10^3$

3 Adding and subtracting numbers in standard form

Write both numbers as ordinary numbers

Add or subtract normally

Convert the answer back into standard form

$3.9 \times 10^5 + 2.5 \times 10^4$
$= 390\,000 + 25\,000$

390 000
+ 25 000
415 000
1

$= 415\,000$
$= 4.15 \times 10^5$

Worked example

Target grade 5

A and B are standard form numbers.
$A = 8.2 \times 10^6$ $B = 2 \times 10^4$
Calculate $A \times B$ giving your answer in standard form. **(2 marks)**

$(8.2 \times 10^6) \times (2 \times 10^4)$
$= (8.2 \times 2) \times (10^6 \times 10^4)$
$= 16.4 \times 10^{10}$
$= 1.64 \times 10^1 \times 10^{10}$
$= 1.64 \times 10^{11}$

Problem solved! If you're not sure which operation to use, try using **simpler numbers**:

An egg weighs 80 g and a box of eggs weighs 960 g. Calculate the number of eggs in the box.

You would divide the total weight by the weight of each egg. So for thie worked example, divide the mass of the gold bar by the mass of one atom.

Worked example

Target grade 5

A single atom of gold has a mass of 3×10^{-22} g.
A gold bar has a mass of 1.2 kg.
Calculate the number of atoms of gold in the bar. **(3 marks)**

$1.2\,\text{kg} = 1200\,\text{g} = 1.2 \times 10^3\,\text{g}$

$\frac{1.2 \times 10^3}{3 \times 10^{-22}} = \frac{1.2}{3} \times \frac{10^3}{10^{-22}}$
$= 0.4 \times 10^{25}$
$= 4 \times 10^{-1} \times 10^{25}$
$= 4 \times 10^{24}$

Now try this

Target grade 5

1 (a) Work out
$(3 \times 10^7) \times (9 \times 10^4)$
Write your answer in standard form. **(2 marks)**

Worked solution video

(b) Work out
$(6.4 \times 10^{-9}) \div (2 \times 10^{-3})$
Write your answer in standard form. **(2 marks)**

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Counting strategies

You might need to make a list of possible combinations. You need to find a **systematic** way of doing this to make sure you have found every possible combination. You could use the three number cards on the right to make **six** different three-digit numbers. Here is one way of writing them out systematically.



1. 4 5 6
2. 4 6 5

Start by writing out the numbers that begin with 4.

3. 5 4 6
4. 5 6 4

Then write the numbers beginning with 5.

5. 6 4 5
6. 6 5 4

Finally, write the numbers that begin with 6.

Worked example

Target grade 3

James is making badges to sell. He can choose three different colours:
Red (R) Green (G) Blue (B)
He can also choose three shapes:
Circle (C) Star (S) Oval (O)
How many possible combinations of badge colour and shape can James make? **(2 marks)**

RC GC BC
RS GS BS
RO GO BO
There are 9 possible combinations.

You need to use a systematic method to make sure you have written down every possible combination. Start by writing down all the possible red badges. Then write down all the possible green badges and then all the possible blue badges.

Make sure you **answer the question**. You need to show a systematic strategy **and** write the number of combinations.

Problem solved! You can save time in your exam by numbering a list of items like this. You can still clearly **show your strategy** using the numbers, and you don't have to write as much.

Read the question carefully. Each team must play the other teams only once, so be careful not to count each match more than once.

You might need to combine counting with probability.
Have a look at page 129 to revise this skill.

Worked example

Target grade 4

A school hockey league contains five teams:
Benfield (1) Gosforth (2) Kenton (3)
St Cuthbert's (4) Sacred Heart (5)
Each team must play each of the other teams once. How many matches will be played in total?
(2 marks)

1 vs 2, 1 vs 3, 1 vs 4, 1 vs 5
2 vs 3, 2 vs 4, 2 vs 5
3 vs 4, 3 vs 5
4 vs 5 There will be 10 matches in total.

Now try this

Target grade 3

1 Seth has three tickets to go to the cinema. He chooses two friends to go with him. He chooses one friend from Ali, Ben and Carl, and one friend from Yvonne and Zara.
List all the possible combinations of friends that Seth could choose.
(2 marks)

2 A local darts tournament has six clubs competing for a place in the national finals: Compton, Penn, Goldthorn, Lanesfield, Tettenhall, Wightwick.
Over a four-week period each club must play all the other clubs once each.
How many darts matches will be played in total during the tournament? **(2 marks)**

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Had a look ☐ Nearly there ☐ Nailed it! ☐

Problem-solving practice 1

About half of the questions in your Foundation GCSE exam will require you to **problem-solve**, **reason**, **interpret** or **communicate** mathematically. If you come across a tricky or unfamiliar question in your exam you can try some of these strategies:

- Sketch a diagram to see what is going on.
- Try the problem with smaller or easier numbers.
- Plan your strategy before you start.
- Write down any formulae you might be able to use.
- Use *x* or *n* to represent an unknown value.

AO2 AO3

Now try this

1 Wilfred has these coins.



Nisha has these coins.



Wilfred gives Nisha one coin. Wilfred now has twice as much money as Nisha.
What value is the coin Wilfred gives to Nisha? **(4 marks)**

Target grade 1

Place value page 1

In money questions you should decide whether you are going to work in pounds or pence. Don't mix up your units.

Remember that if Wilfred gives a coin to Nisha his amount of money is **reduced** by that value and hers is **increased** by the same amount.

TOP TIP

With a question like this it's easy to check your answer. Write down the new amounts of money Wilfred and Nisha have and check that Wilfred's total is twice Nisha's.

2 Liam is planning a trip for a group of 20 children and 5 adults.
They can go to either the theatre or the zoo.
If they go to the theatre, they will go by train.
If they go to the zoo, they will go by coach.
Liam has information about the costs.

Theatre ticket prices	Return train fares
Stalls: £22	Adults: £11.50
Circle: £15	Child: £5.75

Zoo admission	Coach hire
Adult: £18	20 Seats: £190
Child: £12	30 Seats: £240
	40 Seats: £300

What is the lowest possible total cost of the trip? You must show all your working. **(5 marks)**

Target grade 2

Calculator and number skills page 16

Work out the total cost of each trip. Remember to choose the cheapest ticket price for the theatre tickets, and write down all your working. You have to say which trip is the cheapest to complete your answer.

TOP TIP

Plan how you will lay out your answer. You need to show **what** you are working out at each stage, so write short headings to go with your workings. You could use these headings:

Cost of theatre trip
Cost of zoo trip
Conclusion