REVISE PEARSON EDEXCEL GCSE (9–1)

Mathematics

Higher

REVISION GUIDE

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Pearson

A small bit of small print

Pearson Edexcel publishes Sample Assessment Material and the Specification on its website. This is the official content and this book should be used in conjunction with it. The worked examples and questions in this book have been written to help you practise the topics in the specification, and to help you prepare for your exams. Remember that the real exam questions may not look like this, and the questions in this book will not appear in your exams.

GET THE

Your new *Revision Guide* is packed with features to help you stay ahead of the game, and on track for success in your Pearson Edexcel Higher Maths GCSE.

Examiners' report

Every year Pearson Edexcel produces reports on the most recent exams. These **examiners' reports** are jam-packed with useful advice about which questions students struggled with, where marks were dropped or picked up, and which skills students need to concentrate on. We've taken a deep dive into these reports to bring you the most relevant advice for your upcoming exams. Whenever you see this feature in the *Revision Guide*, you know that you're looking at advice based on **real students** who have sat **real exams**. To get you started, here are our **top five** examiners' tips and tricks.

TOP SECRET

Five sure-fire ways to pick up marks

1. **Write clearly** – make sure all your numbers can be read clearly, and don't write so small that the examiner can't read your working.
2. **Show all your workings** – you can pick up loads of marks for partially correct answers, but only if you have shown your method neatly.
3. **Use algebra to solve problems** – lots of problems can be solved by forming and solving a suitable equation.
4. **Don't rush** – if there are figures given in the question make sure you copy them carefully into your own working or onto your calculator, and try to be accurate when carrying out simple calculations.
5. **Answer the question** – always make sure your answer matches what is asked for in the question. Give conclusions in words or sentences, and write numerical answers on the answer line.

Problem solving

Problem solved!

You'll need to use problem-solving skills in your exam. Look out for problem-solving tips and strategies wherever you see this icon, and check out the dedicated problem-solving skills pages.

Target grades

Target grade 5

The exam-style questions in this book have been given target grades. You can use these target grades to help you track your progress. Remember that being able to answer questions at a particular target grade does not guarantee that you will achieve this grade! Your actual grade will be based on the total number of marks you get. So if you are aiming for a top grade you need to be confident with every topic, and you should attempt all the questions in your exams.

Video solutions

Worked solution video

Some of the questions in this *Revision Guide* have worked solution videos. Use your phone, tablet or webcam to scan the QR code to watch the video.

INSIDE TRACK

Hot Topic

Some topics come up year after year. We've picked 26 of the hottest topics. These pages contain key skills and knowledge that you're likely to need in your upcoming exams. If you're pushed for time you might want to revise these first.

Tricky Topic

There is some tough material in Higher GCSE. We've identified 25 of the trickiest topics. Don't worry if you find these pages difficult – they are! You might want to save these topics for days when you have a bit more time to concentrate on them.

1H	2H	3H
Date:	Date:	Date: ....
Time:	Time:	Time: ....
Location:	Location:	Location: ....

You will have to sit **three papers** for Pearson Edexcel Higher GCSE Maths. Each paper is **1 hour and 30 minutes long** and is worth **80 marks**. You can't use a calculator on Paper 1, but make sure you have one with you for Papers 2 and 3. If you know when and where your exams are taking place, write this info into your *Revision Guide* here.

The grade boundaries change every year, so you can't know exactly how many marks you need to make your target grade. But you can use this table to get a rough idea.

Target grade	Marks needed
4	20%
5	33%
6	45%
7	60%
8	70%
9	85%

In it to win it!

Many successful Higher GCSE students pick up **a few marks on lots of questions**, even if they don't get them fully correct. Pick up cheeky marks by:

- showing a bit of working
- demonstrating a method
- using mathematical language
- writing down a formula.

Whatever you do, **have a go!**

CONTENTS

1-to-1 page match with the Higher Revision Workbook ISBN 9781292210889

KEY

= Hot Topic

= Tricky Topic

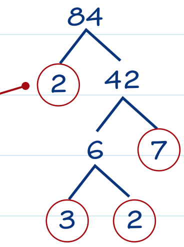
Factors and primes

The **factors** of a number are any numbers that divide into it exactly. A **prime number** has exactly two factors. The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 and so on.

Prime factors

If a number is a factor of another number **and** it is a prime number then it is called a **prime factor**. You use a factor tree to find prime factors.

Remember to circle the prime factors as you go along. The order doesn't matter.



$84 = 2 \times 2 \times 3 \times 7$

$= 2^2 \times 3 \times 7$

Remember to put in the multiplication signs.
This is called a **product** of **prime factors**.

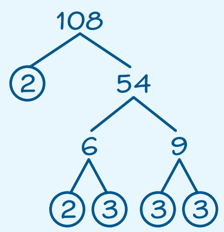
The highest common factor (HCF) of two numbers is the **highest number** that is a **factor** of both numbers.

The lowest common multiple (LCM) of two numbers is the **lowest number** that is a **multiple** of both numbers.

Worked example

Target grade 4

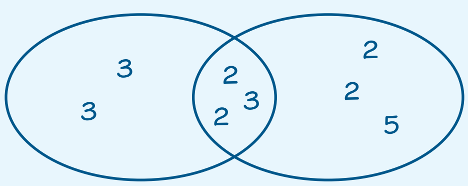
(a) Express 108 as a product of powers of its prime factors. **(3 marks)**



$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

(b) $240 = 2^4 \times 3 \times 5$
Find, as a product of powers of its prime factors,
(i) the highest common factor (HCF) of 108 and 240 **(1 mark)**

Factors of 108 Factors of 240



$\text{HCF} = 2 \times 2 \times 3 = 2^2 \times 3$

(ii) the lowest common multiple (LCM) of 108 and 240 **(1 mark)**

$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 2^4 \times 3^3 \times 5$

You can use numbers given in index form directly:
HCF: Choose the **lowest** power of each prime.
LCM: Choose the **highest** power of each prime.
For example, $\text{HCF} = 2^0 \times 3^2 \times 5^0 \times 7^1 = 3^2 \times 7$

Examiners' report

If you have to write a number as a **product** of prime factors, make sure you use $\times$ signs in your final answer. Don't use $+$, and don't just write a list of prime factors.

Real students have struggled with questions like this in recent exams – **be prepared!**

You can find the HCF and LCM by writing the products of prime factors in a **Venn diagram**. Use the powers to tell you how many times each prime factor occurs. Put the **common factors** in the intersection of the two ovals.

- HCF = product of all the prime factors in the intersection
- LCM = product of all the prime factors in the Venn diagram

There is more on Venn diagrams on page 125.

Now try this

Target grade 4

1 (a) Express 980 as a product of its prime factors. **(3 marks)**

(b) Find the highest common factor (HCF) of 980 and 56. **(2 marks)**

2 $X = 2 \times 3^5 \times 7^2$ $\quad Y = 3^2 \times 5 \times 7$

(a) Find the highest common factor (HCF) of X and Y. **(2 marks)**

(b) Find the lowest common multiple (LCM) of X and Y. **(2 marks)**

Had a look ☐ Nearly there ☐ Nailed it! ☐

Indices 1

The index laws tell you how to work with **powers** of numbers.

1 Index laws

Indices include square roots, cube roots and powers.
You can use the index laws to simplify powers and roots.

$a^m \times a^n = a^{m+n}$

$4^3 \times 4^7 = 4^{3+7} = 4^{10}$

$\frac{a^m}{a^n} = a^{m-n}$

$12^8 \div 12^3 = 12^{8-3} = 12^5$

$(a^m)^n = a^{mn}$

$(7^3)^5 = 7^{3 \times 5} = 7^{15}$

2 Cube root

The cube root of a positive number is positive.

$4 \times 4 \times 4 = 64$

$4^3 = 64$

$\sqrt[3]{64} = 4$

The cube root of a negative number is negative.

$-4 \times -4 \times -4 = -64$

$(-4)^3 = -64$

$\sqrt[3]{-64} = -4$

3 Powers of 0 and 1

Anything raised to the power 0 is equal to 1.

$6^0 = 1 \quad 1^0 = 1 \quad 7223^0 = 1 \quad (-5)^0 = 1$

Anything raised to the power 1 is equal to itself.

$8^1 = 8 \quad 499^1 = 499 \quad (-3)^1 = -3$

Indices checklist

- The base numbers have to be the same. ✓
- If there's no index, the number has the power 1 ✓
- Be careful with negatives: $(-3)^2 = 9$ ✓

Worked example

Target grade 5

(a) Write $6 \times 6 \times 6 \times 6 \times 6$ as a single power of 6. **(1 mark)**

$6 \times 6 \times 6 \times 6 \times 6 = 6^5$

(b) Simplify $\frac{3^8 \times 3}{3^4}$ fully, leaving your answer in index form. **(2 marks)**

$\frac{3^8 \times 3}{3^4} = \frac{3^9}{3^4} = 3^5$

3 is the same as 3^1. For part (b), use the rule $a^m \times a^n = a^{m+n}$ to simplify the numerator; then use $\frac{a^m}{a^n} = a^{m-n}$ to simplify the fraction. Remember to write down both steps of your working and give your answer as a power.

Learn it!

You need to learn the square numbers up to 15^2 and the cubes of 1, 2, 3, 4, 5 and 10. You might need to spot square roots such as $\sqrt{169} = 13$ on your non-calculator paper.

For 1(b), start by working out $\frac{9625}{7 \times 11}$

Now try this

Target grade 5

1 (a) Write $7^3 \times 7^5$ as a single power of 7. **(1 mark)**

(b) $9625 = 5^n \times 7 \times 11$
Find the value of n. **(2 marks)**

2 $(\sqrt[3]{-27})^k = 9$
Write down the value of k. **(2 marks)**

3 (a) Simplify, leaving your answers in index form

(i) $\frac{2^9}{2^5}$ (ii) $(7^2)^6$ (iii) $5^2 \times 5^0$ **(3 marks)**

(b) $\frac{3^n}{3^2 \times 3^5} = 3^4$
Find the value of n. **(2 marks)**

Indices 2

You can use these index laws to deal with powers that are **fractions** or **negative numbers**.

1 Negative powers

$$a^{-n} = \frac{1}{a^n}$$

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Be careful!

A **negative** power can still have a **positive** answer.

2 Reciprocals

$$a^{-1} = \frac{1}{a}$$

This means that a^{-1} is the **reciprocal** of a. You can find the reciprocal of a fraction by turning it upside down.

$\left(\frac{5}{9}\right)^{-1} = \frac{9}{5}$

3 Powers of fractions

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$\left(\frac{3}{10}\right)^2 = \frac{3^2}{10^2} = \frac{9}{100}$

4 Combining rules

You can apply the rules one at a time.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$

5 Fractional powers

You can use fractional powers to represent roots.

$a^{\frac{1}{2}} = \sqrt{a}$ $\quad 49^{\frac{1}{2}} = 7$

$a^{\frac{1}{3}} = \sqrt[3]{a}$ $\quad 27^{\frac{1}{3}} = 3$

$a^{\frac{1}{4}} = \sqrt[4]{a}$ $\quad 16^{\frac{1}{4}} = 2$

Check it!

A whole number raised to a power less than 1 gets smaller.

6 More complicated indices

You can use the index laws to work out more complicated fractional powers.

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$$

Do these calculations **one step at a time.**

$27^{-\frac{2}{3}} = (27^{\frac{1}{3}})^{-2}$

$= (\sqrt[3]{27})^{-2}$

$= 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Worked example — Target grade 8

Find the value of n when $3^n = 9^{-\frac{3}{2}}$
Show each step of your working clearly. **(3 marks)**

$9^{-\frac{3}{2}} = (3^2)^{-\frac{3}{2}}$

$= 3^{2 \times -\frac{3}{2}} = 3^{-3}$

So $3^n = 3^{-3}$ and $n = -3$

Problem solved! 3^n is not the same as $3n$. You can't divide by 3 to get n on its own.
You need to make the base on the right-hand side the same as the base on the left-hand side.

1. Write 9 as a power of 3. Remember to use brackets.
2. Use $(a^n)^m = a^{nm}$ to write the right-hand side as a single power of 3.
3. Compare both sides and write down the value of n.

Now try this

Target grade 7

1 You are given that $x = 7^h$ and $y = 7^k$
Write each of the following as a single power of 7:

(a) $\frac{x}{y}$ **(1 mark)**

(b) x^2 **(1 mark)**

(c) xy^2 **(2 marks)**

Target grade 8

2 Given that $81^{-\frac{3}{4}} = 3^n$, find the value of n. **(3 marks)**

3 Write $\sqrt{\frac{49}{7^3}}$ as a single power of 7.
Show every step of your working clearly. **(3 marks)**

Start by writing 49 as a power of 7.

Calculator skills 1

These calculator keys are really useful:

x^2	Square a number.	(−)	Enter a negative number.
x^3	Cube a number.	$\sqrt{\square}$	Find the square root of a number.
x^{-1}	Find the reciprocal of a number.	$\sqrt[3]{\square}$	Find the cube root of a number. You might need to press the shift key first.
Ans	Use your previous answer in a calculation.	S⇔D	Change the answer from a fraction or surd to a decimal. Not all calculators have this key.

Rounding rules

1. To **round** a number, you look at the next digit on the right.
   5 or more → round up less than 5 → round down
2. Decimals can be rounded to a given number of **decimal places** (d.p.).
   6.475 = 6.48 correct to 2 d.p.
3. To write a number correct to **3 significant figures** (3 s.f.), look at the fourth significant figure.
   0.003 07<u>9</u> = 0.003 08 to 3 s.f.
4. Leading zeros in decimals are not counted as significant.
5. Remember that the rule for significant figures still applies to **whole numbers**.
   27 = 30 to 1 s.f.

Worked example

Target grade 4

(a) Work out the value of $\dfrac{\sqrt{8.3}}{12.5 - 7.3}$

Give your answer as a decimal.
Write down all the figures on your calculator display. **(2 marks)**

$$\frac{\sqrt{8.3}}{12.5 - 7.3} = \frac{2.880\,97}{5.2} = 0.554\,033\,088$$

(b) Find the reciprocal of 12.5. Give your answer as a decimal. **(1 mark)**

$1 \div 12.5 = 0.08$

Examiners' report

If you have to work out a calculation like this in your exam, you should work out the numerator (top) and denominator (bottom) **separately**, and **write them both down**. Then divide to work out the final answer. Read the question carefully. You have to write down **all the figures** from your calculator display.

Real students have struggled with questions like this in recent exams – **be prepared!**

Read the question carefully. You have to give the answer as a **decimal**, so you might need to use the S⇔D button on your calculator.

Now try this

Target grade 4

(a) Work out the value of $\dfrac{6.1 + 7.5}{1.8^2}$

Give your answer as a decimal.

Write down all the figures on your calculator display. **(2 marks)**

(b) Give your answer to part (a) correct to 3 significant figures. **(1 mark)**

Make sure you write down three significant figures even if the last digit is a zero.

Fractions

You need to be able to work with fractions and mixed numbers confidently **without a calculator**.

1 Adding or subtracting fractions

Add or subtract the whole numbers
↓
Write the fractions as fractions with the same denominator
↓
Add or subtract the fractions
↓
If you have an improper fraction then convert to a mixed number and add

$2\frac{2}{3} + 1\frac{1}{2}$
$= 3 + \frac{2}{3} + \frac{1}{2}$
$= 3 + \frac{4}{6} + \frac{3}{6}$
$= 3 + \frac{7}{6}$
$= 3 + 1\frac{1}{6}$
$= 4\frac{1}{6}$

3 Dividing fractions

Convert any mixed numbers to improper fractions
↓
Turn the second fraction 'upside down' and change ÷ to ×
↓
Multiply the numerators and multiply the denominators, cancelling where possible
↓
Convert any improper fractions to mixed numbers

$6\frac{1}{4} \div 1\frac{7}{8}$
$= \frac{\frac{25}{4}}{\frac{15}{8}}$
$= \frac{25}{4} \times \frac{8}{15}$
$= \frac{{}^{5}\cancel{25} \times \cancel{8}^{2}}{{}_{1}\cancel{4} \times \cancel{15}_{3}}$
$= \frac{10}{3}$
$= 3\frac{1}{3}$

2 Multiplying fractions

Convert any mixed numbers to improper fractions
↓
Multiply the numerators and multiply the denominators, cancelling where possible
↓
Convert any improper fractions to mixed numbers

$3\frac{1}{4} \times 2\frac{2}{3}$
$= \frac{13}{4} \times \frac{8}{3}$
$= \frac{13 \times \cancel{8}^{2}}{{}_{1}\cancel{4} \times 3}$
$= \frac{26}{3}$
$= 8\frac{2}{3}$

Worked example

Target grade 5

Work out $7\frac{1}{3} - 2\frac{3}{4}$ **(3 marks)**

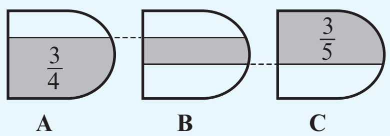
$7\frac{1}{3} - 2\frac{3}{4} = \frac{22}{3} - \frac{11}{4}$
$= \frac{88}{12} - \frac{33}{12}$
$= \frac{55}{12}$
$= 4\frac{7}{12}$

Remember you need to be able to do this **without** a calculator.

Worked example

Target grade 5

The diagram shows three identical shapes.
$\frac{3}{4}$ of shape A is shaded and $\frac{3}{5}$ of shape C is shaded.



What fraction of shape B is shaded? **(3 marks)**

$1 - \frac{1}{4} - \frac{2}{5} = \frac{20}{20} - \frac{5}{20} - \frac{8}{20} = \frac{7}{20}$

Problem solved!

White area on A $= 1 - \frac{3}{4} = \frac{1}{4}$
White area on C $= 1 - \frac{3}{5} = \frac{2}{5}$
So shaded area on B $= 1 - \frac{1}{4} - \frac{2}{5}$

Examiners' report

On the non-calculator paper, students often lose marks on basic arithmetic. Learn your times tables, and check your working!

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade 4

1 Work out

(a) $\frac{7}{10} - \frac{1}{4}$ **(2 marks)**

(b) $3\frac{4}{9} + 1\frac{5}{6}$ **(3 marks)**

(c) $\frac{3}{4} \div \frac{5}{12}$ **(2 marks)**

Target grade 5

(d) $1\frac{7}{8} \times 2\frac{2}{3}$ **(3 marks)**

Target grade 4

2 Three girls shared a full bottle of cola.
Karen drank $\frac{1}{4}$ of the bottle.
Rita drank $\frac{3}{10}$ of the bottle.
Megan drank the rest.
(a) Work out the fraction of the bottle of cola that Megan drank. **(3 marks)**
Rita drank 36 cl of cola.
(b) How much cola was in the full bottle? **(2 marks)**

Worked solution video

Had a look ☐ Nearly there ☐ Nailed it! ☐

Decimals

Terminating decimals can be written exactly. You can write a terminating decimal as a fraction with denominator 10, 100, 1000, and so on.

$0.24 = \frac{24}{100} = \frac{6}{25}$

Recurring decimals have one digit or group of digits repeating forever. You can use dots to show the recurring digit or group of digits.

$\frac{2}{3} = 0.6666... = 0.\dot{6}$

The dot tells you that the 6 repeats forever.

$\frac{346}{555} = 0.623\,4234... = 0.6\dot{2}3\dot{4}$

These dots tell you that the group of digits 234 repeats forever.

Recurring or terminating?

To check whether a fraction produces a recurring decimal or a terminating decimal, write it in its simplest form and find the prime factors of its **denominator**.

Prime factors only 2 and 5 → Terminating decimal

Prime factors other than 2 or 5 → Recurring decimal

Worked example

Target grade 5

(a) Show that $\frac{7}{50}$ can be written as a terminating decimal. **(1 mark)**

$\frac{7}{50} = \frac{14}{100} = 0.14$

(b) Show that $\frac{11}{24}$ **cannot** be written as a terminating decimal. **(2 marks)**

$\frac{11}{24} = \frac{11}{2^3 \times 3}$

Denominator contains a factor other than 2 or 5 so decimal is recurring.

Problem solved! You could also write the denominator as a product of prime factors: $\frac{7}{50} = \frac{7}{2 \times 5^2}$

The only factors are 2 and 5 so $\frac{7}{50}$ produces a terminating decimal.

Worked example

Target grade 5

(a) Show that $\frac{2}{9}$ is equivalent to 0.222… **(1 mark)**

$$9\overline{)2\,.\,{}^{2}0\,{}^{2}0\,{}^{2}0...} \quad = 0.222...$$

(b) Hence, or otherwise, write 0.7222… as a fraction. **(3 marks)**

$0.7222... = 0.222... + 0.5$

$= \frac{2}{9} + \frac{1}{2}$

$= \frac{4}{18} + \frac{9}{18} = \frac{13}{18}$

You could also use long division for part (a).

Fractions and decimals

To convert a fraction into a decimal, divide the numerator by the denominator.

$\frac{2}{5} = 2 \div 5 = 0.4$

It's useful to remember these common fraction-to-decimal conversions:

Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{3}{4}$
Decimal	0.01	0.05	0.1	0.5	0.2	0.25	0.75

Now try this

Target grade 4

1 Write these in order of size, smallest first:
0.6 $\frac{1}{2}$ $\frac{9}{20}$ 0.55 **(1 mark)**

2 Give evidence to show that $\frac{1}{250}$ can be written as a terminating decimal. **(2 marks)**

Target grade 5

3 Show that $\frac{1}{140}$ **cannot** be written as a terminating decimal. **(2 marks)**

4 Write $\frac{5}{11}$ as a recurring decimal. **(2 marks)**

Estimation

You can estimate the answer to a calculation by rounding each number to **1 significant figure**, and then doing the calculation. You can use this method to check your answers, or to estimate calculations on your **non-calculator paper**. Here are two examples:

1 $4.32 \times 18.09 \approx 4 \times 20 = 80$

The answer is approximately equal to 80.

2 $327^2 \approx 300^2 = 3^2 \times 100^2 = 90\,000$

The answer is approximately equal to 90 000.

$\approx$ means 'is approximately equal to'

Decimal division trick

You might have to divide by a decimal on your non-calculator paper. If you multiply both numbers in a division by the same amount the answer stays the same.

$$\frac{1400}{0.05} \overset{\times 100}{=} \frac{140\,000}{5} \overset{\times 2}{=} \frac{280\,00\cancel{0}}{1\cancel{0}} = 28\,000$$

Worked example

Target grade 4

Work out an estimate for

(a) $\dfrac{4.31 \times 278}{0.487}$ **(2 marks)**

$\dfrac{4.31 \times 278}{0.487} \approx \dfrac{4 \times 300}{0.5} = \dfrac{1200}{0.5} = 2400$

(b) 37.4^3 **(2 marks)**

$37.4^3 \approx 40^3 = 4^3 \times 10^3 = 64 \times 1000 = 64\,000$

Round all the numbers to **1 significant figure**. Then **write out** the calculation with the rounded values before calculating your estimate.

You can use the laws of indices to work out 40^3 without a calculator.
$(ab)^n = a^n \times b^n$
so $40^3 = (4 \times 10)^3 = 4^3 \times 10^3$

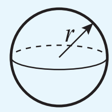
Problem solved! On your non-calculator paper start by writing $\pi = 3.142$ then round to 1 s.f. to make your estimate.

Worked example

Target grade 5

A spherical ball-bearing has a radius of 2.35 cm.

Surface area of sphere $= 4\pi r^2$



(a) Work out an estimate for its surface area in square centimetres. **(2 marks)**

$4\pi r^2 = 4 \times 3.142 \times 2.35^2$
$\approx 4 \times 3 \times 2^2 = 48\text{ cm}^2$

(b) Is your answer to part (a) an overestimate or an underestimate? Give a reason for your answer. **(1 mark)**

$3 < 3.142$ and $2 < 2.35$
so the answer is an underestimate.

Examiners' report

You have rounded **both values down** so your answer will be an underestimate. The question says 'give a reason' so show working and write a conclusion **in words**.

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade 4

1 Showing your rounding, work out an estimate for

$\dfrac{82 \times 285}{64 \times 35}$ **(2 marks)**

Worked solution video

Scan this QR code to watch a video of this question being solved.

Target grade 5

2 A scientist models a raindrop as a sphere with radius 3.2 mm.
Volume of a sphere $= \frac{4}{3}\pi r^3$

(a) Work out an estimate for the volume of the raindrop. **(2 marks)**

(b) Is your answer to part (a) an overestimate or an underestimate?
Give a reason for your answer. **(1 mark)**

Standard form

Numbers in standard form have two parts.

$$7.3 \times 10^{-6}$$

This part is a number greater than or equal to 1 and less than 10.

This part is a power of 10.

You can use standard form to write very large or very small numbers.

$$920\,000 = 9.2 \times 10^5$$

Numbers greater than 10 have a positive power of 10.

$$0.007\,03 = 7.03 \times 10^{-3}$$

Numbers less than 1 have a negative power of 10.

Counting decimal places

You can count decimal places to convert between numbers in standard form and ordinary numbers.

3 jumps

$7900 = 7.9 \times 10^3$

7900 > 10
So the power is positive

4 jumps

$0.00035 = 3.5 \times 10^{-4}$

0.000 35 < 1
So the power is negative

Be careful!
Don't just count zeros to work out the power.

Worked example

Target grade 5

Work out the value of $(8.3 \times 10^6) - (4.1 \times 10^5)$
Give your answer in standard form. **(2 marks)**

$$\begin{array}{r} \overset{7}{\cancel{8}}\,\overset{12}{\cancel{3}}\overset{1}{0}0\,000 \\ -\quad 410\,000 \\ \hline 7\,890\,000 \end{array}$$

$7\,890\,000 = 7.89 \times 10^6$

Examiners' report

You need to be able to work with numbers in standard form on your **non-calculator paper**. To add or subtract, write the numbers as ordinary numbers first, then write your final answer in standard form.

Real students have struggled with questions like this in recent exams – **be prepared!**

Non-calculator multiplying

1

Rearrange so powers of 10 are together

Multiply the number parts

Add the powers

Rewrite your answer in standard form if necessary

$(3 \times 10^3) \times (5 \times 10^6)$
$= (3 \times 5) \times (10^3 \times 10^6)$
$= 15 \times 10^9$

$a^m \times a^n = a^{m+n}$

$= 1.5 \times 10^1 \times 10^9$
$= 1.5 \times 10^{10}$

Non-calculator dividing

2

Rearrange so powers of 10 are together

Divide the number parts

Subtract the powers

Rewrite your answer in standard form if necessary

$(1.2 \times 10^8) \div (2 \times 10^4)$
$= (1.2 \div 2) \times (10^8 \div 10^4)$
$= 0.6 \times 10^4$

$a^m \div a^n = a^{m-n}$

$= 6 \times 10^{-1} \times 10^4$
$= 6 \times 10^3$

Using a calculator

You can enter numbers in standard form using the $\times 10^x$ key.

To enter 3.7×10^{-6} press

3 . 7 $\times 10^x$ (–) 6

If you are using a calculator with numbers in standard form it is a good idea to put brackets around each number.

Now try this

Target grade 5

The mass of one *E. coli* bacterium is 6×10^{-16} grams. Find the total mass of 3×10^6 bacteria. **(2 marks)**

Have a go at this question **without a calculator** first. Then use your calculator to check your answer.

Recurring decimals

You can use algebra to convert a recurring decimal into a fraction. Here is the strategy:

Write the recurring decimal as n. → Multiply by 10, 100 or 1000. → Subtract to remove the recurring part. → Divide by 9, 99 or 999 to write as a fraction.

If you need to do this in your exam you must show **all your working**. For a reminder about recurring decimals have a look at page 6.

Worked example

Target grade 7

Prove that the recurring decimal $0.\dot{2}\dot{4}$ has the value $\frac{8}{33}$ **(2 marks)**

Let $n = 0.242\,424\,24\ldots$

$100n = 24.242\,424\,24\ldots$

$-\ n = 0.242\,424\,24\ldots$

$99n = 24$

$n = \frac{24}{99} = \frac{8}{33}$

Some calculators will convert recurring decimals into fractions for you. But the question says 'Prove that…' so you must write down all the steps shown here.

1. Write the recurring decimal equal to n, and write out some of its digits.
2. Multiply both sides by 100 as there are 2 recurring digits.
3. Subtract n to remove the recurring part.
4. Divide both sides by 99 to write n as a fraction.
5. Simplify the fraction.

Multiply by…

- ✓ 10 if 1 digit recurs.
- ✓ 100 if 2 digits recur.
- ✓ 1000 if 3 digits recur.

Problem solved! In this recurring decimal the digit 4 does not recur. Follow the same steps to write n as a fraction. After you divide by 99, multiply the top and bottom of your fraction by 10 to convert the decimal in the numerator into an integer.

Worked example

Target grade 8

Show that $0.4\dot{7}\dot{3}$ can be written as the fraction $\frac{469}{990}$ **(2 marks)**

Let $n = 0.473\,737\,37\ldots$

$100n = 47.373\,737\,37\ldots$

$-\ n = 0.473\,737\,37\ldots$

$99n = 46.9$

$n = \frac{46.9}{99}$

$n = \frac{469}{990}$

Worked solution video

Now try this

Target grade 7

1 Work out the recurring decimal $0.\dot{5}\dot{4}$ as a fraction in its simplest form. **(2 marks)**

Target grade 8

2 Prove that the recurring decimal $0.0\dot{1}\dot{8}$ has the value $\frac{1}{55}$ **(2 marks)**

3 Show that $0.\dot{3}5\dot{1}$ can be written as the fraction $\frac{13}{37}$ **(2 marks)**

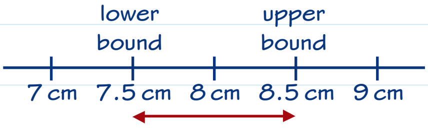
$0.\dot{3}5\dot{1} = 0.351351351\ldots$

There are 3 recurring digits so you need to write $0.\dot{3}5\dot{1}$ as n, then multiply by 1000. You will get a fraction with denominator 999 which you can simplify.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Upper and lower bounds

Upper and lower bounds are a measure of accuracy. For example, the width of a postcard is given as 8 cm to the nearest cm.



The actual width of the postcard could be anything between 7.5 cm and 8.5 cm.

7.5 cm is called the **lower bound**.

8.5 cm is called the **upper bound**.

Using upper and lower bounds in calculations

To find the overall upper and lower bounds of the answer to a calculation use these rules.

	+	−	×	÷
Overall upper bound	UB + UB	UB − LB	UB × UB	UB ÷ LB
Overall lower bound	LB + LB	LB − UB	LB × LB	LB ÷ UB

Overall lower bound of $a + b$ = lower bound of a + lower bound of b.

Worked example

Target grade 7

A roll of ribbon is 150 cm long, correct to 2 significant figures.
A 21-cm piece of ribbon is cut off the roll, correct to the nearest cm.
Calculate the lower bound, in cm, for the amount of ribbon remaining on the roll. **(3 marks)**

	Lower bound	Upper bound
Length of ribbon	145 cm	155 cm
Length of piece cut off	20.5 cm	21.5 cm

Lower bound of remaining length
= 145 − 21.5
= 123.5 cm

If you're answering questions about upper and lower bounds, it's a good idea to write out the upper bound and lower bound for **all values** given in the question before you start. To work out the **lower bound** for $a - b$ you need to use the **lower bound** for a and the **upper bound** for b.

Different values might be given to **different degrees of accuracy**. The length of the roll is correct to 2 significant figures, or to the nearest **10 cm**. The length of the piece cut off is correct to the nearest **cm**. Be really careful when you're working out your upper and lower bounds.

Now try this

Target grade 8

1 The area of a rectangle is 320 cm^2.
The length of the rectangle is 22 cm.
Both values are correct to 2 significant figures.
Calculate the lower bound for the width of the rectangle. Show your working clearly. **(3 marks)**

2 Correct to 2 decimal places, the volume of a solid cube is 3.37 m^3. Calculate the upper bound for the surface area of the cube. **(4 marks)**

For a cube with edges of length x, the volume is x^3 and the surface area is $6x^2$

Accuracy and error

When a question involves **upper and lower bounds**, you might need to give your answer to an **appropriate** degree of accuracy. You can use this strategy:

Calculate the **lower bound** for the answer. → Calculate the **upper bound** for the answer. → Choose the most accurate value that **both bounds** would round to.

If you're not confident with upper and lower bounds, revise them now on page 10.

Worked example

Target grade 9

A cylinder has a volume of 115 cm^3, to the nearest cm^3. Its radius is 2.3 cm, correct to 1 decimal place.
Find the height of the cylinder to an appropriate degree of accuracy. You must explain why your answer is to this degree of accuracy. **(4 marks)**

	Upper bound	Lower bound
Volume	115.5 cm^3	114.5 cm^3
Radius	2.35 cm	2.25 cm

UB for height $= \dfrac{115.5}{\pi \times 2.25^2} = 7.26218\ldots$

LB for height $= \dfrac{114.5}{\pi \times 2.35^2} = 6.59963\ldots$

Height = 7 cm (1 s.f.)

UB and LB both round to 7 cm to 1 s.f.

Problem solved!

Choose an answer that **both** bounds will round to. Check 2 significant figures:

UB = 7.3 cm (2 s.f.) and LB = 6.6 cm (2 s.f.) ✗

Check 1 significant figure:

UB = 7 cm (1 s.f.) and LB = 7 cm (1 s.f.) ✓

Examiners' report

When you are told that values in the question have been rounded, there is a good chance that you will have to work with bounds – you can score some marks on these questions just by writing down the upper and lower bounds of the rounded values.

Real students have struggled with questions like this in recent exams – **be prepared!**

Error intervals

If you need to write an error interval, you should use **inequalities** to show the lower and upper bounds for a rounded value.

If $x = 4.7$ to 1 decimal place, then the error interval for x is $4.65 \leqslant x < 4.75$

If $y = 840$ to 2 significant figures, then the error interval for y is $835 \leqslant y < 845$

Use 'less than or equal to' for the lower bound.

Use 'less than' for the upper bound.

Worked example

Target grade 4

Aron rounds a number, n, to 2 decimal places. The result is 8.64. Using inequalities, write down the error interval for n. **(2 marks)**

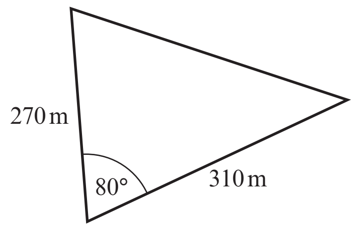
$8.635 \leqslant n < 8.645$

Now try this

Target grade 9

1 A go-karting course is 425 m long, correct to the nearest metre.
Nisha completes the course in 1 minute 25 seconds, to the nearest second.
Calculate her average speed in m/s to an appropriate degree of accuracy.
You must explain why your answer is to this degree of accuracy. **(4 marks)**

2 The diagram shows a farmer's field.

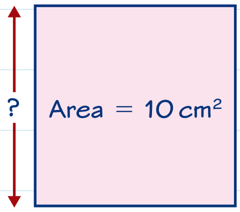


Worked solution video

The lengths of the sides of the field have been measured to the nearest 10 m, and the angle given is exact. The farmer wants to plant grass in the field.
1 bag of seed covers 500 m^2, to the nearest 100 m^2.
The farmer has 90 bags of grass seed.
Does the farmer definitely have enough bags?
Show all your working. **(4 marks)**

Surds 1

You can give exact answers to calculations by leaving some numbers as square roots.



This square has a side length of $\sqrt{10}$ cm.

You can't write $\sqrt{10}$ exactly as a decimal number. It is called a **surd**.

Rules for simplifying square roots

These are the most important rules to remember when dealing with surds:

1. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ $\quad \sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$
2. $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$ $\quad \sqrt{\dfrac{3}{25}} = \dfrac{\sqrt{3}}{\sqrt{25}} = \dfrac{\sqrt{3}}{5}$

You need to remember these rules for your exam.

Worked example

Target grade 7

Show that $\sqrt{45} = 3\sqrt{5}$

Show each stage of your working clearly. **(2 marks)**

$$\sqrt{45} = \sqrt{9 \times 5}$$
$$= \sqrt{9} \times \sqrt{5}$$
$$= 3\sqrt{5}$$

This question says 'Show that…' so you can't use your calculator. You need to show each step of your working clearly:

1. Look for a factor of 45 which is a square number: $45 = 9 \times 5$
2. Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ to split the square root into two square roots.
3. Write $\sqrt{9}$ as a whole number.

Rationalising the denominator of a fraction means making the denominator a whole number.

You can do this by multiplying the top **and** bottom of the fraction by the surd part in the denominator.

$$\frac{5}{3\sqrt{2}} \xrightarrow{\times\sqrt{2}} \frac{5\sqrt{2}}{6}$$

The surd part of the denominator is $\sqrt{2}$

Remember that $\sqrt{2} \times \sqrt{2} = 2$

So $3\sqrt{2} \times \sqrt{2} = 3 \times 2 = 6$

Good form

Most surd questions ask you to write a number or answer in a certain **form**.

This means you need to find **integers** for all the letters in the expression.

$6\sqrt{3}$ is in the form $k\sqrt{3}$

$k = 6$

The integers can be positive or negative.

$4 - 9\sqrt{2}$ is in the form $p + q\sqrt{2}$

$p = 4$ and $q = -9$

You can check your answer by writing down the integer value for each letter.

Now try this

Target grade 8

1 Write $\sqrt{32} + \sqrt{98}$ in the form $p\sqrt{2}$ where p is an integer. Show each stage of your working clearly. **(2 marks)**

Find factors of 32 and 98 which are square numbers.

2 Show that $\dfrac{35}{\sqrt{7}} = 5\sqrt{7}$ **(2 marks)**

Rationalise the denominator by multiplying top and bottom by $\sqrt{7}$

Target grade 9

3 x is an integer such that

$$\frac{\sqrt{x} \times \sqrt{18}}{\sqrt{3}} = 8\sqrt{3}$$

Find the value of x. **(4 marks)**

Counting strategies

You might need to find strategies for counting the total number of possible **combinations**. One way of finding combinations is to make a systematic list. Here are all the possible three-digit numbers that can be made from the number cards shown on the right.

4 5 6

456 465 546 564 645 654 There are six possibilities.

Start by writing out the numbers that begin with 4.

Then write the numbers beginning with 5.

Finally, write the numbers that begin with 6.

Counting with calculation

Sometimes it's impossible to write down every possible combination. You can count combinations by **multiplying** the number of choices for each option. Here is a four-character password for a website:

p7m$

First character must be a letter: 26 choices.

Second character must be a digit from 0 to 9: 10 choices.

Third character must be a letter: 26 choices.

Fourth character must be one of: $, ·, _ or # 4 choices.

The total number of possible passwords is $26 \times 10 \times 26 \times 4 = 27\,040$.

Worked example

Target grade 8

The lock on this briefcase has three dials. The first dial can be any letter and the last two dials can be any digit from 0 to 9. Here is one possible combination. M 7 0

(a) How many different ways are there of setting the code? **(2 marks)**

$26 \times 10 \times 10 = 2600$

A different suitcase has four dials. The first two dials can be any letter from A to E, and the last two dials can be any **even** digit greater than 0. Here is one possible combination. E B 4 2

(b) How many different ways are there of setting this code? **(2 marks)**

$5 \times 5 \times 4 \times 4 = 400$

Work out how many choices there are for each dial.

Dial 1: Letter = 26 choices

Dial 2: Digit from 0 to 9 = 10 choices

Dial 3: Digit from 0 to 9 = 10 choices

Multiply the number of choices to work out the total number of possible combinations.

Problem solved! The number of choices for each dial has **changed** here, so be really careful.

Dials 1 and 2: Letter from A to E = 5 choices

Dials 3 and 4: Digits 2, 4, 6 or 8 = 4 choices

It's important to **show your strategy** as well as your answer, so write out the calculation you use.

Now try this

Target grade 8

In a lottery competition players select 5 numbers between 1 and 50, and 2 additional numbers between 1 and 11. Players are allowed to select the same number more than once.
Work out how many different ways there are of selecting these 7 numbers. Give your answer in standard form, correct to 3 significant figures. **(2 marks)**

In most real-life lotteries, players cannot select the same number more than once. This would change the calculation.

Problem-solving practice 1

Throughout your Higher GCSE exam you will need to **problem-solve**, **reason**, **interpret** and **communicate** mathematically. If you come across a tricky or unfamiliar question in your exam you can try some of these strategies:

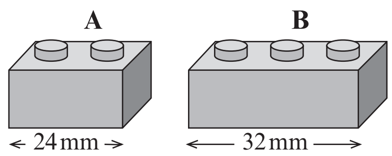
- Sketch a diagram to see what is going on.
- Try the problem with smaller or easier numbers.
- Plan your strategy before you start.
- Write down any formulae you might be able to use.
- Use x or n to represent an unknown value.

AO2

AO3

Now try this

1 The diagram shows two types of plastic building block.



Worked solution video

Block A is 24 mm long.
Block B is 32 mm long.
Jeremy joins some type A blocks together to make a straight row.
He then joins some type B blocks together to make a straight row of the same length.
Write down the shortest possible length of this row. **(4 marks)**

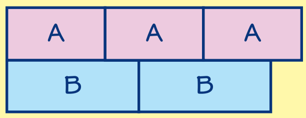
Factors and primes page 1

Target grade 4

You have to use a whole number of building blocks in each row, so the length of each row has to be a multiple of the length of one block. The answer will be the lowest common multiple of 24 and 32. You can't get all the marks just by writing down the answer. You need to show how you found the answer clearly and neatly.

TOP TIP

If you're not sure how to start, draw a sketch. This might help you see that the lengths are multiples of 24 and 32.



2 Susan has 2 dogs.
Each dog is fed $\frac{3}{8}$ kg of dog food each day.
Susan buys dog food in bags.
Each bag weighs 14 kg.
For how many days can Susan feed the 2 dogs from 1 bag of dog food?
You must show **all** your working. **(5 marks)**

Fractions page 5

Target grade 4

There are lots of steps in this question so make sure you keep track of your working and write it down clearly.

TOP TIP

Write words with each calculation to explain what you are doing.

3 Prove that the recurring decimal $0.9\dot{2}\dot{8}$ can be written as $\frac{919}{990}$ **(3 marks)**

Worked solution video

Recurring decimals page 9

Target grade 8

Start by writing $x = 0.928\,282\,8\ldots$ If you work out $1000x$ and $10x$ you can keep all your working in whole numbers.

TOP TIP

If a question says 'Show that…' or 'Prove that…' you have to show every step of your working clearly and neatly.

Problem-solving practice 2

Now try this

4 (a) $a = 4 \times 10^{2n}$ where n is an integer. Find, in standard form, an expression for $\sqrt{a}$ **(2 marks)**

(b) $b = 8 \times 10^{3m}$ where m is an integer. Find, in standard form, an expression for $b^{\frac{4}{3}}$ **(3 marks)**

Target grade 8

Standard form page 8
Indices 2 page 3

Remember that $(xy)^n = x^n y^n$

Be careful in part (b): the first part of a number in standard form must be greater than or equal to 1 and less than 10.

TOP TIP

Questions on indices can involve unknowns, so your calculator won't be able to help you.
Make sure you know the laws of indices.

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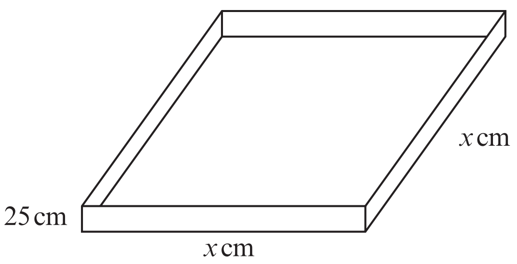
5 The diagram shows a wooden planting box in the shape of a cuboid.



The volume of the box is 810 000 cm^3 correct to 2 significant figures.

The depth of the box is 25 cm, to the nearest cm.

The box has a square base with sides of length x cm.

Find the lower bound for x. Give your answer correct to 3 significant figures.

(4 marks)

Target grade 8

Upper and lower bounds page 10
Volumes of 3-D shapes page 85

Complete this table showing the upper and lower bounds for each measurement before you start:

	25 cm	810 000 cm^3
Upper	25.5 cm	
Lower		805 000 cm^3

You are **dividing** the volume by the depth to work out x^2. Choose the values you use carefully to make the answer as **small** as possible.

TOP TIP

When answering questions about upper and lower bounds, it's a good idea to write out the upper and lower bounds for all the values before you start.

---

6 (a) Nisha writes down a four digit whole number. All the digits are odd, and no digit is repeated. How many different possible numbers could Nisha have written? **(2 marks)**

(b) In a popular word game, there are 1310 allowable three-letter words. A monkey types 3 letters at random. Calculate the probability that it types a word that is allowable in the game. **(3 marks)**

Target grade 8

Counting strategies page 13
Probability page 123

There are five possible choices for the first digit (1, 3, 5, 7 or 9). Once the first digit has been chosen, there are only four possible choices for the second digit, and so on.

TOP TIP

Remember to write down any calculations you carry out so you can show your strategy.

Algebraic expressions

You need to be able to work with algebraic expressions confidently. For a reminder about using the index laws with **numbers** have a look at pages 2 and 3.

1 You can use the **index laws** to simplify algebraic expressions.

$a^m \times a^n = a^{m+n}$

$x^4 \times x^3 = x^{4+3} = x^7$

$\frac{a^m}{a^n} = a^{m-n}$

$m^8 \div m^2 = m^{8-2} = m^6$

$(a^m)^n = a^{mn}$

$(n^2)^4 = n^{2\times 4} = n^8$

2 You can square or cube a whole expression.

$(4x^3y)^2 = (4)^2 \times (x^3)^2 \times (y)^2$

$= 16x^6y^2$

$16 = (4)^2$

$(x^3)^2 = x^{3\times 2} = x^6$

You need to square everything inside the brackets.

Remember that if a letter appears on its own then it has the power 1.

3 Algebraic expressions may also contain negative and fractional indices.

$a^{-m} = \frac{1}{a^m}$

$(c^2)^{-3} = c^{2\times -3} = c^{-6} = \frac{1}{c^6}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$(8p^3)^{\frac{1}{3}} = (8)^{\frac{1}{3}} \times (p^3)^{\frac{1}{3}}$

$= \sqrt[3]{8} \times p^{3\times\frac{1}{3}}$

$= 2p$

One at a time

When you are **multiplying** expressions:

1. Multiply any number parts first.
2. Add the powers of each letter to work out the new power.

$6p^2q \times 3p^3q^2 = 18p^5q^3$

$6 \times 3 = 18$

$p^2 \times p^3 = p^{2+3} = p^5$

$q \times q^2 = q^{1+2} = q^3$

When you are **dividing** expressions:

1. Divide any number parts first.
2. Subtract the powers of each letter to work out the new power.

$\frac{12a^5b^3}{3a^2b^2} = 4a^3b$

$12 \div 3 = 4$

$b^3 \div b^2 = b^{3-2} = b$

$a^5 \div a^2 = a^{5-2} = a^3$

Worked example

Target grade 4

Simplify fully

(a) $m \times m \times m \times m$ **(1 mark)**

m^4

(b) $(x^3)^3$ **(1 mark)**

x^9

Target grade 5

(c) $\frac{4y^2 \times 3y^7}{6y}$ **(2 marks)**

$\frac{4y^2 \times 3y^7}{6y} = \frac{12y^9}{6y} = 2y^8$

(a) $m = m^1$, so $m \times m \times m \times m$
$= m^{1+1+1+1}$

(b) Use $(a^m)^n = a^{mn}$

(c) Start by simplifying the top part of the fraction. Do the number part first then the powers. Use $a^m \times a^n = a^{m+n}$

Next divide the expressions. Divide the number part, then divide the indices using $\frac{a^m}{a^n} = a^{m-n}$

Now try this

Worked solution video

Target grade 4

1 Simplify $(h^2)^6$ **(1 mark)**

2 Simplify fully

(a) $(2a^5b)^4$ **(2 marks)**

Target grade 5

(b) $5x^4y^2 \times 3x^3y^7$ **(2 marks)**

(c) $18d^8g^{10} \div 6d^2g^5$ **(2 marks)**

Target grade 7

3 (a) Simplify $(16p^{10})^{\frac{1}{2}}$ **(2 marks)**

(b) Simplify $(64x^9y^2)^{-\frac{1}{3}}$ **(2 marks)**

Apply the power outside the brackets to everything inside the brackets.

Expanding brackets

Expanding or multiplying out brackets is a key algebra skill.

You have to multiply the expression outside the bracket by everything inside the bracket.

$4n \times n^2 = 4n^3$

$$4n(n^2 + 2) = 4n^3 + 8n$$

$4n \times 2 = 8n$

'Expand and simplify' means multiply out and then collect like terms.

Golden rule

When you expand, you need to be careful with negative signs in front of the bracket.

Negative signs belong to the term to their right.

$-2 \times x$ $\quad -2 \times -y$

$$x - 2(x - y) = x - 2x + 2y$$
$$= -x + 2y$$

Multiply out the brackets first and then collect like terms if possible.

You can use the **grid method** to expand two brackets.

$$(x + 7)(x - 5) = x^2 - 5x + 7x - 35$$
$$= x^2 + 2x - 35$$

Remember to collect like terms if possible.

	x	-5
x	x^2	$-5x$
7	$7x$	-35

The negative sign belongs to the 5. You need to write it in your grid.

Or

You can use the acronym FOIL to expand two brackets.

$2a^2$ $\quad -b^2$

$$(2a + b)(a - b) = 2a^2 - 2ab + ab - b^2$$
$$= 2a^2 - ab - b^2$$

ab

$-2ab$

First terms
Outer terms
Inner terms
Last terms

Some people remember this as a 'smiley face'.

Worked example

Target grade 6

Be careful with the negative signs:
$-4 \times -4 = 16 \qquad 3p \times -4 = -12p$

(a) Expand and simplify $(3p - 4)^2$ **(2 marks)**

$$(3p - 4)^2 = (3p - 4)(3p - 4)$$
$$= 9p^2 - 12p - 12p + 16$$
$$= 9p^2 - 24p + 16$$

	$3p$	-4
$3p$	$9p^2$	$-12p$
-4	$-12p$	16

(b) Expand and simplify $x(x - 2)(x + 3)$ **(2 marks)**

$$x(x - 2)(x + 3) = x(x^2 + 3x - 2x - 6)$$
$$= x(x^2 + x - 6)$$
$$= x^3 + x^2 - 6x$$

Examiners' report

If you have to multiply three factors, don't try to do it all in one step. Expand $(x - 2)(x + 3)$ and write brackets around the whole expansion. Then multiply every term inside the brackets by x.

Check it!
Try an easy value, like $x = 5$
$x(x - 2)(x + 3) = 5 \times 3 \times 8 = 120$
$x^3 + x^2 - 6x = 125 + 25 - 30 = 120$ ✓

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Worked solution video

Target grade 4

1 Expand and simplify

(a) $(x + 5)(x - 1)$ **(2 marks)**

Target grade 5

(b) $(p - 6)^2$ **(2 marks)**

Target grade 6

2 Expand and simplify

(a) $x(x + 3)(x + 9)$ **(2 marks)**

Target grade 7

(b) $(n + 5)(n + 3)^2$ **(3 marks)**

$(n + 5)(n + 3)^2 = n(n + 3)^2 + 5(n + 3)^2$

Hot Topic

Factorising

Factorising is the opposite of expanding brackets:

Expanding brackets

$$2x(x + 3) = 2x^2 + 6x$$

Factorising

You need to look for the **largest factor** you can take out of every term in the expression.

$10a^2 + 5ab = 5(2a^2 + ab)$

This expression has only been **partly factorised**.

$10a^2 + 5ab = 5a(2a + b)$

This expression has been **completely factorised**.

Factorising $x^2 + bx + c$

You need to write the expression with **two brackets**.

You need to find two numbers which add up to 7… $5 + 2 = 7$

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

… and multiply to make 10. $5 \times 2 = 10$

When factorising $x^2 + bx + c$, use this table to help you find the two numbers:

b	*c*	Factors
positive	positive	both numbers positive
positive	negative	bigger number positive and smaller number negative
negative	negative	bigger number negative and smaller number positive
negative	positive	both numbers negative

Factorising $ax^2 + bx + c$

$2x^2 - 7x - 15 = (2x \quad)(x \quad)$

One of the brackets must contain a $2x$ term. Try pairs of numbers which have a product of -15. Check each pair by multiplying out the brackets.

$(2x + 5)(x - 3) = 2x^2 - x - 15$ ✗

$(2x - 3)(x + 5) = 2x^2 + 7x - 15$ ✗

$(2x + 3)(x - 5) = 2x^2 - 7x - 15$ ✓

Difference of two squares

You can factorise expressions that are written as

$$(\text{something})^2 - (\text{something else})^2$$

Use this rule:

$a^2 - b^2 = (a + b)(a - b)$

$x^2 - 36 = x^2 - (6)^2$

$\qquad = (x + 6)(x - 6)$

36 is a square number.

$36 = 6^2$ so $a = x$ and $b = 6$

Worked example

Factorise fully

Target grade 4

(a) $p^2 - 3p$ **(2 marks)**

$p(p - 3)$

Target grade 5

(b) $15x^2 + 5xy$ **(2 marks)**

$5x(3x + y)$

You need to look for the **largest** factor you can take out of every term.

Partly factorised: $x(15x + 5y)$ ✗

Partly factorised: $5(3x^2 + xy)$ ✗

Fully factorised: $5x(3x + y)$ ✓

Now try this

Target grade 4

1 Factorise

(a) $4a - 6$ **(1 mark)**

(b) $y^2 + 5y$ **(1 mark)**

Target grade 5

2 Factorise fully

(a) $12g + 3g^2$ **(2 marks)**

(b) $p^2 - 15p + 14$ **(2 marks)**

(c) $6x^2 - 8xy$ **(2 marks)**

3 Factorise

(a) $4ma - 24m^2a$ **(2 marks)**

(b) $p^2 - 64$ **(1 mark)**

Target grade 6

4 Factorise $3x^2 - 8x + 4$ **(2 marks)**

Worked solution video

Linear equations 1

To solve a linear equation you need to get the letter on its own on one side.
It is really important to write your working **neatly** when you are solving equations.

$$\begin{aligned} 5x + 3 &= 18 \quad (-\,3) \\ 5x &= 15 \quad (\div\,5) \\ x &= 3 \end{aligned}$$

Write down the operation you are carrying out. Remember to do the same thing to both sides of the equation.

Every line of working should have an equals sign in it.

Start a new line for each step. Do one operation at a time.

Line up the equals signs.

Letter on both sides?

To solve an equation you have to get the letter on its own on one side of the equation.

Start by collecting like terms so that all the letters are together.

$$\begin{aligned} 2 - 2x &= 26 + 4x \quad (+\,2x) \\ 2 &= 26 + 6x \quad (-\,26) \\ -24 &= 6x \quad (\div\,6) \\ -4 &= x \end{aligned}$$

You can write your answer as $-4 = x$ or as $x = -4$

Equations with brackets

Always start by multiplying out the brackets then collecting like terms.

For a reminder about multiplying out brackets have a look at page 17.

$$\begin{aligned} 19 &= 8 - 2(5 - 3y) \\ 19 &= 8 - 10 + 6y \\ 19 &= -2 + 6y \quad (+\,2) \\ 21 &= 6y \quad (\div\,6) \\ \tfrac{21}{6} &= y \\ y &= \tfrac{7}{2} \text{ or } 3\tfrac{1}{2} \text{ or } 3.5 \end{aligned}$$

Your answer can be written as a fraction or decimal.

Worked example

Target grade 4

Solve $7r + 2 = 5(r - 4)$ **(3 marks)**

$$\begin{aligned} 7r + 2 &= 5r - 20 \quad (-\,5r) \\ 2r + 2 &= -20 \quad (-\,2) \\ 2r &= -22 \quad (\div\,2) \\ r &= -11 \end{aligned}$$

Multiply out the brackets then collect all the terms in r on one side. You need to write down each step of your working clearly.

Examiners' report

Don't use a trial and improvement method to solve an equation. You probably won't find the correct answer, and you can't get any method marks.

Real students have struggled with questions like this in recent exams – **be prepared!**

Now try this

Target grade 4

1 Solve
(a) $5w - 17 = 2w + 4$ **(3 marks)**
(b) $2(x + 11) = 20$ **(3 marks)**

2 Solve
(a) $6y - 9 = 2(y - 8)$ **(3 marks)**
(b) $4m - 2(m - 3) = 7m - 14$ **(3 marks)**

Expand the brackets first.

Expand the brackets then collect all the m terms on one side of the equation.

Linear equations 2

Equations with fractions

When you have an equation with fractions, you need to get rid of any fractions before solving. You can do this by multiplying every term by the lowest common multiple (LCM) of the denominators.

$$\frac{x}{3} + \frac{x-1}{5} = 11 \qquad (\times 15)$$

The LCM of 3 and 5 is 15.

$$\frac{^5\cancel{15}x}{\cancel{3}_1} + \frac{^3\cancel{15}(x-1)}{\cancel{5}_1} = 165$$

Cancel the fractions. There is more about simplifying algebraic fractions on page 47.

$$5x + 3x - 3 = 165$$
$$8x - 3 = 165 \qquad (+\,3)$$
$$8x = 168 \qquad (\div\,8)$$
$$x = 21$$

Multiplying by an expression

You might have to multiply by an expression to get rid of the fractions.

$$\frac{20}{n-3} = -5 \qquad (\times(n-3))$$
$$20 = -5(n-3)$$

Worked example

Target grade 4

Solve $\frac{29-x}{4} = x + 5$ **(3 marks)**

$$\frac{4(29-x)}{4} = 4(x+5)$$
$$29 - x = 4(x+5)$$
$$29 - x = 4x + 20 \qquad (+\,x)$$
$$29 = 5x + 20 \qquad (-\,20)$$
$$9 = 5x \qquad (\div\,5)$$
$$\frac{9}{5} = x$$

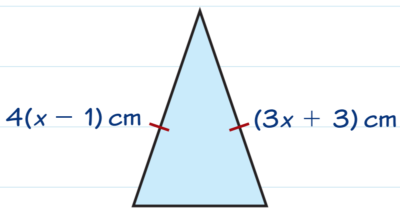
Eliminate fractions **before** you start solving the equation. You can do this by multiplying both sides of the equation by 4.
Use brackets to show that you are multiplying everything by 4.
$4(x+5)$ ✓ $4x + 5$ ✗
Multiply out the brackets, then solve the equation normally. Remember that your answer could be a fraction.

Top tip!

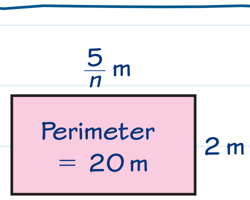
It's OK to leave the answer to an equation as an improper fraction. Don't waste time converting to mixed numbers or decimals.

Writing your own equations

You can find unknown values by writing and solving equations.



$$4(x-1) = 3x + 3$$



$$\frac{5}{n} + \frac{5}{n} + 2 + 2 = 20$$

Now try this

Target grade 4

1 Solve

(a) $\frac{25-3w}{4} = 10$ **(3 marks)**

(b) $5x - 10 = \frac{18-x}{3}$ **(3 marks)**

Target grade 6

2 Solve

(a) $\frac{2y}{3} + \frac{y-4}{2} = 5$ **(3 marks)**

(b) $\frac{3m-1}{4} - \frac{2m+4}{3} = 1.5$ **(3 marks)**