

1.1 Laws of indices

$$1 \quad a \quad 49^{\frac{1}{2}} = \sqrt{49}$$

$$= \pm 7$$

$$b \quad \left(\frac{1}{81}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{81}}$$

$$= \frac{\sqrt{1}}{\sqrt{81}}$$

$$= \pm \frac{1}{9}$$

$$c \quad \left(\frac{25}{36}\right)^{\frac{1}{2}} = \sqrt{\frac{25}{36}}$$

$$= \frac{\sqrt{25}}{\sqrt{36}}$$

$$= \pm \frac{5}{6}$$

$$d \quad 125^{\frac{1}{3}} = \sqrt[3]{125}$$

$$= 5$$

$$e \quad \left(\frac{1}{8}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{1}{8}}$$

$$= \frac{\sqrt[3]{1}}{\sqrt[3]{8}}$$

$$= \frac{1}{2}$$

$$f \quad \left(-\frac{27}{64}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{-27}{64}}$$

$$= \frac{\sqrt[3]{-27}}{\sqrt[3]{64}}$$

$$= -\frac{3}{4}$$

$$2 \quad a \quad 25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{25}}$$

$$= \pm \frac{1}{5}$$

$$b \quad \left(\frac{1}{16}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{1}{16}\right)^{\frac{1}{2}}}$$

$$= 16^{\frac{1}{2}}$$

$$= \sqrt{16}$$

$$= \pm 4$$

$$c \quad \left(\frac{49}{100}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{49}{100}\right)^{\frac{1}{2}}}$$

$$= \left(\frac{100}{49}\right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{100}{49}}$$

$$= \pm \frac{10}{7}$$

$$d \quad 8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}}$$

$$= \frac{1}{\sqrt[3]{8}}$$

$$= \frac{1}{2}$$

$$e \quad \left(\frac{1}{64}\right)^{-\frac{1}{3}} = \frac{1}{\left(\frac{1}{64}\right)^{\frac{1}{3}}}$$

$$= 64^{\frac{1}{3}}$$

$$= 4$$

$$f \quad \left(-\frac{343}{1000}\right)^{-\frac{1}{3}} = \frac{1}{\left(-\frac{343}{1000}\right)^{\frac{1}{3}}}$$

$$= \left(-\frac{1000}{343}\right)^{\frac{1}{3}}$$

$$= \sqrt[3]{-\frac{1000}{343}}$$

$$= -\frac{10}{7}$$

$$3 \quad a \quad 25^{\frac{3}{2}} = (\sqrt{25})^3$$

$$= (\pm 5)^3$$

$$= \pm 125$$

$$b \quad 8^{\frac{2}{3}} = (\sqrt[3]{8})^2$$

$$= 2^2$$

$$= 4$$

$$c \quad 81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}}$$

$$= \frac{1}{(\sqrt[4]{81})^3}$$

$$= \frac{1}{(\pm 3)^3}$$

$$= \pm \frac{1}{27}$$

$$d \quad \left(\frac{1}{36}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{1}{36}}\right)^3$$

$$= \frac{(\sqrt{1})^3}{(\sqrt{36})^3}$$

$$= \frac{(\pm 1)^3}{(\pm 6)^3}$$

$$= \pm \frac{1}{216}$$

$$e \quad \left(-\frac{8}{27}\right)^{\frac{4}{3}} = \left(\sqrt[3]{\frac{-8}{27}}\right)^4$$

$$= \frac{(\sqrt[3]{-8})^4}{(\sqrt[3]{27})^4}$$

$$= \frac{(-2)^4}{3^4}$$

$$= -\frac{16}{81}$$

$$f \quad \left(\frac{1}{64}\right)^{-\frac{2}{3}} = \frac{1}{\left(\frac{1}{64}\right)^{\frac{2}{3}}}$$

$$= 64^{\frac{2}{3}}$$

$$= (\sqrt[3]{64})^2$$

$$= 16$$

1.1 Laws of indices (Continued)

$$4 \quad a \quad 32 = 2 \times 2 \times 2 \times 2 \times 2 \\ = 2^5 \\ n = 5$$

$$b \quad \frac{1}{8} = \frac{1}{2 \times 2 \times 2} \\ = \frac{1}{2^3} \\ = 2^{-3} \\ n = -3$$

$$c \quad (\sqrt{5})^3 = (5^{\frac{1}{2}})^3 \\ = 5^{\frac{1}{2} \times 3} \\ = 5^{\frac{3}{2}} \\ n = \frac{3}{2}$$

$$5 \quad a \quad 9^{-\frac{1}{2}} = (3^2)^{-\frac{1}{2}} \\ = 3^{2 \times -\frac{1}{2}} \\ = 3^{-1}$$

$$b \quad 27^{\frac{1}{5}} = (3^3)^{\frac{1}{5}} \\ = 3^{3 \times \frac{1}{5}} \\ = 3^{\frac{3}{5}}$$

$$c \quad 81^{\frac{4}{3}} = (3^4)^{\frac{4}{3}} \\ = 3^{4 \times \frac{4}{3}} \\ = 3^{\frac{16}{3}}$$

$$6 \quad 5^a \times 125^{\frac{2}{3}} = 25^{\frac{5}{2}} \\ 5^a \times (5^3)^{\frac{2}{3}} = (5^2)^{\frac{5}{2}} \\ 5^a \times 5^{3 \times \frac{2}{3}} = 5^{2 \times \frac{5}{2}} \\ 5^a \times 5^2 = 5^5 \\ a + 2 = 5 \\ a = 3$$

$$7 \quad 16^{\frac{3}{2}} \times 2^x = 8^{\frac{1}{5}} \\ (2^4)^{\frac{3}{2}} \times 2^x = (2^3)^{\frac{1}{5}} \\ 2^{4 \times \frac{3}{2}} \times 2^x = 2^{3 \times \frac{1}{5}} \\ 2^6 \times 2^x = 2^{\frac{3}{5}} \\ 6 + x = \frac{3}{5} \\ x = -5\frac{2}{5}$$

$$8 \quad 9^{\frac{3}{2}} \times 3^{2t-1} \times \frac{1}{9^2} = 81 \\ (3^2)^{\frac{3}{2}} \times 3^{2t-1} \times (3^2)^{-2} = 3^4 \\ 3^{2 \times \frac{3}{2}} \times 3^{2t-1} \times 3^{2 \times (-2)} = 3^4 \\ 3^3 \times 3^{2t-1} \times 3^{-4} = 3^4 \\ 3 + 2t - 1 - 4 = 4 \\ 2t = 6 \\ t = 3$$

$$9 \quad \frac{25^{\frac{3x+1}{2}}}{125^{1.5x}} = 5^{10-x} \\ \frac{(5^2)^{\frac{3x+1}{2}}}{(5^3)^{1.5x}} = 5^{10-x} \\ \frac{5^{2 \times \frac{3x+1}{2}}}{5^{3 \times 1.5x}} = 5^{10-x} \\ \frac{5^{3x+1}}{5^{4.5x}} = 5^{10-x}$$

$$3x + 1 - 4.5x = 10 - x \\ -9 = 0.5x \\ x = -18$$

$$10 \quad 8^{\frac{2}{3}} \times \frac{1}{4^2} \times 2^{3y-4} \times 32^{\frac{3}{5}} = 64 \\ (2^3)^{\frac{2}{3}} \times (2^2)^{-2} \times 2^{3y-4} \times (2^5)^{\frac{3}{5}} = 2^6 \\ 2^{3 \times \frac{2}{3}} \times 2^{2 \times (-2)} \times 2^{3y-4} \times 2^{5 \times \frac{3}{5}} = 2^6 \\ 2^2 \times 2^{-4} \times 2^{3y-4} \times 2^3 = 2^6 \\ 2 + (-4) + (3y - 4) + 3 = 6 \\ 3y = 9 \\ y = 3$$

$$11 \quad -27^{-\frac{2}{3}} \times 9^3 \times (-3)^{2x} = -243 \\ ((-3)^3)^{-\frac{2}{3}} \times ((-3)^2)^3 \times (-3)^{2x} = (-3)^5 \\ (-3)^{3 \times -\frac{2}{3}} \times (-3)^{2 \times 3} \times (-3)^{2x} = (-3)^5 \\ (-3)^{-2} \times (-3)^6 \times (-3)^{2x} = (-3)^5 \\ -2 + 6 + 2x = 5 \\ 2x = 1 \\ x = \frac{1}{2}$$

1.2 Surds

$$1 \quad a \quad \sqrt{20} = \sqrt{4 \times 5} \\ = \sqrt{4} \times \sqrt{5} \\ = 2\sqrt{5}$$

$$b \quad \sqrt{18} = \sqrt{9 \times 2} \\ = \sqrt{9} \times \sqrt{2} \\ = 3\sqrt{2}$$

$$c \quad \sqrt{44} = \sqrt{4 \times 11} \\ = \sqrt{4} \times \sqrt{11} \\ = 2\sqrt{11}$$

$$d \quad \sqrt{63} = \sqrt{9 \times 7} \\ = \sqrt{9} \times \sqrt{7} \\ = 3\sqrt{7}$$

$$e \quad \sqrt{98} = \sqrt{49 \times 2} \\ = \sqrt{49} \times \sqrt{2} \\ = 7\sqrt{2}$$

1.2 Surds (Continued)

$$\begin{aligned} \text{f } \sqrt{700} &= \sqrt{100 \times 7} \\ &= \sqrt{100} \times \sqrt{7} \\ &= 10\sqrt{7} \end{aligned}$$

$$\begin{aligned} 2 \quad \sqrt{180} &= \sqrt{36 \times 5} \\ &= \sqrt{36} \times \sqrt{5} \\ &= 6\sqrt{5} \end{aligned}$$

$$\begin{aligned} 3 \quad \text{a } \frac{\sqrt{18}}{3} &= \frac{\sqrt{9 \times 2}}{3} \\ &= \frac{\sqrt{9} \times \sqrt{2}}{3} \\ &= \frac{3\sqrt{2}}{3} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{\sqrt{75}}{5} &= \frac{\sqrt{25 \times 3}}{5} \\ &= \frac{\sqrt{25} \times \sqrt{3}}{5} \\ &= \frac{5\sqrt{3}}{5} \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{c } \frac{\sqrt{48}}{2} &= \frac{\sqrt{16 \times 3}}{2} \\ &= \frac{\sqrt{16} \times \sqrt{3}}{2} \\ &= \frac{4\sqrt{3}}{2} \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} 4 \quad \text{a } \sqrt{12} + \sqrt{75} &= \sqrt{4 \times 3} + \sqrt{25 \times 3} \\ &= \sqrt{4} \times \sqrt{3} + \sqrt{25} \times \sqrt{3} \\ &= 2\sqrt{3} + 5\sqrt{3} \\ &= 7\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b } \sqrt{192} - \sqrt{48} &= \sqrt{64 \times 3} - \sqrt{16 \times 3} \\ &= \sqrt{64} \times \sqrt{3} - \sqrt{16} \times \sqrt{3} \\ &= 8\sqrt{3} - 4\sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{c } \sqrt{500} + \sqrt{20} - \sqrt{45} &= \sqrt{100 \times 5} + \sqrt{4 \times 5} - \sqrt{9 \times 5} \\ &= \sqrt{100} \times \sqrt{5} + \sqrt{4} \times \sqrt{5} - \sqrt{9} \times \sqrt{5} \\ &= 10\sqrt{5} + 2\sqrt{5} - 3\sqrt{5} \\ &= 9\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{d } \sqrt{50} - \sqrt{18} + 2\sqrt{32} &= \sqrt{25 \times 2} - \sqrt{9 \times 2} + 2\sqrt{16 \times 2} \\ &= \sqrt{25} \times \sqrt{2} - \sqrt{9} \times \sqrt{2} + 2 \times \sqrt{16} \times \sqrt{2} \\ &= 5\sqrt{2} - 3\sqrt{2} + 8\sqrt{2} \\ &= 10\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{e } 2\sqrt{75} - 2\sqrt{12} + \sqrt{147} &= 2\sqrt{25 \times 3} - 2\sqrt{4 \times 3} + \sqrt{49 \times 3} \\ &= 2 \times \sqrt{25} \times \sqrt{3} - 2 \times \sqrt{4} \times \sqrt{3} + \sqrt{49} \times \sqrt{3} \\ &= 10\sqrt{3} - 4\sqrt{3} + 7\sqrt{3} \\ &= 13\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{f } 3\sqrt{20} + 2\sqrt{125} - 4\sqrt{180} &= 3 \times \sqrt{4 \times 5} + 2 \times \sqrt{25 \times 5} - 4 \times \sqrt{36 \times 5} \\ &= 3 \times \sqrt{4} \times \sqrt{5} + 2 \times \sqrt{25} \times \sqrt{5} - 4 \times \sqrt{36} \times \sqrt{5} \\ &= 6\sqrt{5} + 10\sqrt{5} - 24\sqrt{5} \\ &= -8\sqrt{5} \end{aligned}$$

$$\begin{aligned} 5 \quad \text{a } \sqrt{5}(2 - \sqrt{3}) &= \sqrt{5} \times 2 - \sqrt{5} \times \sqrt{3} \\ &= 2\sqrt{5} - \sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{b } \sqrt{3}(4 - \sqrt{5}) &= \sqrt{3} \times 4 - \sqrt{3} \times \sqrt{5} \\ &= 4\sqrt{3} - \sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{c } \sqrt{2}(7 + \sqrt{2}) &= \sqrt{2} \times 7 + \sqrt{2} \times \sqrt{2} \\ &= 7\sqrt{2} + 2 \end{aligned}$$

$$\begin{aligned} \text{d } (4 + \sqrt{2})(5 - \sqrt{3}) &= 4(5 - \sqrt{3}) + \sqrt{2}(5 - \sqrt{3}) \\ &= 20 - 4\sqrt{3} + 5\sqrt{2} - \sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{e } (4 + \sqrt{5})(2 - \sqrt{3}) &= 4(2 - \sqrt{3}) + \sqrt{5}(2 - \sqrt{3}) \\ &= 8 - 4\sqrt{3} + 2\sqrt{5} - \sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{f } (2 + \sqrt{3})(4 + \sqrt{3}) &= 2(4 + \sqrt{3}) + \sqrt{3}(4 + \sqrt{3}) \\ &= 8 + 2\sqrt{3} + 4\sqrt{3} + \sqrt{9} \\ &= 11 + 6\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{g } (2 - \sqrt{2})(3 - \sqrt{2}) &= 2(3 - \sqrt{2}) - \sqrt{2}(3 - \sqrt{2}) \\ &= 6 - 2\sqrt{2} - 3\sqrt{2} + \sqrt{4} \\ &= 8 - 5\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{h } (3 - \sqrt{7})(2 + \sqrt{7}) &= 3(2 + \sqrt{7}) - \sqrt{7}(2 + \sqrt{7}) \\ &= 6 + 3\sqrt{7} - 2\sqrt{7} - \sqrt{49} \\ &= -1 + \sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{i } (4 - \sqrt{5})^2 &= (4 - \sqrt{5})(4 - \sqrt{5}) \\ &= 4(4 - \sqrt{5}) - \sqrt{5}(4 - \sqrt{5}) \\ &= 16 - 4\sqrt{5} - 4\sqrt{5} + \sqrt{25} \\ &= 21 - 8\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{j } (\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) &= \sqrt{2}\sqrt{2} - \sqrt{2}\sqrt{3} + \sqrt{2}\sqrt{3} - \sqrt{3}\sqrt{3} \\ &= \sqrt{4} - \sqrt{9} \\ &= 2 - 3 \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{k } (\sqrt{3} + \sqrt{5})^2 &= (\sqrt{3} + \sqrt{5})(\sqrt{3} + \sqrt{5}) \\ &= \sqrt{3}\sqrt{3} + \sqrt{3}\sqrt{5} + \sqrt{3}\sqrt{5} + \sqrt{5}\sqrt{5} \\ &= 3 + 2\sqrt{15} + 5 \\ &= 8 + 2\sqrt{15} \end{aligned}$$

1.2 Surds (Continued)

$$\begin{aligned}
 1 \quad (\sqrt{5} - \sqrt{2}) &= (\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2}) \\
 &= \sqrt{5}\sqrt{5} - \sqrt{5}\sqrt{2} - \sqrt{2}\sqrt{5} + \sqrt{2}\sqrt{2} \\
 &= 5 - 2\sqrt{10} + 2 \\
 &= 7 - 2\sqrt{10}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad a \quad (3\sqrt{8})^2 &= 3\sqrt{8} \times 3\sqrt{8} \\
 &= 9 \times \sqrt{64} \\
 &= 9 \times 8 \\
 &= 72
 \end{aligned}$$

$$\begin{aligned}
 b \quad (\sqrt{5}) + (\sqrt{5})^2 + (\sqrt{5})^3 + (\sqrt{5})^4 &= \sqrt{5} + \sqrt{25} + \sqrt{125} + \sqrt{625} \\
 &= \sqrt{5} + 5 + \sqrt{25 \times 5} + 25 \\
 &= \sqrt{5} + 5 + 5\sqrt{5} + 25 \\
 &= 6\sqrt{5} + 30
 \end{aligned}$$

7 Sara has missed some terms out when expanding the brackets.

$$\begin{aligned}
 (3 + \sqrt{5})^2 &= (3 + \sqrt{5})(3 + \sqrt{5}) \\
 &= 9 + 3\sqrt{5} + 3\sqrt{5} + \sqrt{25} \\
 &= 9 + 6\sqrt{5} + 5 \\
 &= 14 + 6\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad \text{LHS} &= (\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) \\
 &= \sqrt{7}\sqrt{7} + \sqrt{7}\sqrt{5} - \sqrt{5}\sqrt{7} - \sqrt{5}\sqrt{5} \\
 &= \sqrt{49} - \sqrt{25} \\
 &= 7 - 5 \\
 &= 2 = \text{RHS}
 \end{aligned}$$

1.3 Rationalising denominators

$$\begin{aligned}
 1 \quad a \quad \frac{1}{\sqrt{3}} &= \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\
 &= \frac{\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 b \quad \frac{1}{\sqrt{7}} &= \frac{1 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} \\
 &= \frac{\sqrt{7}}{7}
 \end{aligned}$$

$$\begin{aligned}
 c \quad \frac{3}{\sqrt{2}} &= \frac{3 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
 &= \frac{3\sqrt{2}}{2}
 \end{aligned}$$

$$\begin{aligned}
 d \quad \frac{2}{\sqrt{2}} &= \frac{2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
 &= \frac{2\sqrt{2}}{2} \\
 &= \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 e \quad \frac{5}{\sqrt{5}} &= \frac{5 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\
 &= \frac{5\sqrt{5}}{5} \\
 &= \sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 f \quad \frac{3}{\sqrt{6}} &= \frac{3 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} \\
 &= \frac{3\sqrt{6}}{6} \\
 &= \frac{\sqrt{6}}{2}
 \end{aligned}$$

$$\begin{aligned}
 g \quad \frac{\sqrt{2}}{\sqrt{3}} &= \frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\
 &= \frac{\sqrt{6}}{3}
 \end{aligned}$$

$$\begin{aligned}
 h \quad \frac{\sqrt{2}}{\sqrt{5}} &= \frac{\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\
 &= \frac{\sqrt{10}}{5}
 \end{aligned}$$

$$\begin{aligned}
 i \quad \frac{\sqrt{3}}{\sqrt{7}} &= \frac{\sqrt{3} \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} \\
 &= \frac{\sqrt{21}}{7}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad a \quad \frac{1}{5 + \sqrt{3}} &= \frac{1 \times (5 - \sqrt{3})}{(5 + \sqrt{3})(5 - \sqrt{3})} \\
 &= \frac{5 - \sqrt{3}}{25 - 5\sqrt{3} + 5\sqrt{3} - 3} \\
 &= \frac{5 - \sqrt{3}}{22}
 \end{aligned}$$

$$\begin{aligned}
 b \quad \frac{1}{3 - \sqrt{2}} &= \frac{1 \times (3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} \\
 &= \frac{3 + \sqrt{2}}{9 + 3\sqrt{2} - 3\sqrt{2} - 2} \\
 &= \frac{3 + \sqrt{2}}{7}
 \end{aligned}$$

$$\begin{aligned}
 c \quad \frac{2}{1 - \sqrt{5}} &= \frac{2 \times (1 + \sqrt{5})}{(1 - \sqrt{5})(1 + \sqrt{5})} \\
 &= \frac{2 + 2\sqrt{5}}{1 + \sqrt{5} - \sqrt{5} - 5} \\
 &= \frac{2 + 2\sqrt{5}}{-4} \\
 &= \frac{1 + \sqrt{5}}{-2}
 \end{aligned}$$

$$\begin{aligned}
 d \quad \frac{2}{1 + \sqrt{2}} &= \frac{2 \times (1 - \sqrt{2})}{(1 + \sqrt{2})(1 - \sqrt{2})} \\
 &= \frac{2 - 2\sqrt{2}}{1 - \sqrt{2} + \sqrt{2} - 2} \\
 &= \frac{2 - 2\sqrt{2}}{-1} \\
 &= -2 + 2\sqrt{2}
 \end{aligned}$$

1.3 Rationalising denominators (Continued)

$$\begin{aligned} \text{e} \quad \frac{3}{2-\sqrt{3}} &= \frac{3 \times (2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} \\ &= \frac{6+3\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}-3} \\ &= \frac{6+3\sqrt{3}}{1} \\ &= 6+3\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{f} \quad \frac{4}{5+\sqrt{7}} &= \frac{4 \times (5-\sqrt{7})}{(5+\sqrt{7})(5-\sqrt{7})} \\ &= \frac{20-4\sqrt{7}}{25-5\sqrt{7}+5\sqrt{7}-7} \\ &= \frac{20-4\sqrt{7}}{18} \\ &= \frac{10-2\sqrt{7}}{9} \end{aligned}$$

$$\begin{aligned} \text{g} \quad \frac{\sqrt{2}}{1+\sqrt{3}} &= \frac{\sqrt{2} \times (1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} \\ &= \frac{\sqrt{2}-\sqrt{6}}{1-\sqrt{3}+\sqrt{3}-3} \\ &= \frac{\sqrt{2}-\sqrt{6}}{-2} \end{aligned}$$

$$\begin{aligned} \text{h} \quad \frac{\sqrt{3}}{1-\sqrt{3}} &= \frac{\sqrt{3} \times (1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} \\ &= \frac{\sqrt{3}+3}{1+\sqrt{3}-\sqrt{3}-3} \\ &= \frac{3+\sqrt{3}}{-2} \end{aligned}$$

$$\begin{aligned} \text{i} \quad \frac{\sqrt{3}}{2-\sqrt{2}} &= \frac{\sqrt{3} \times (2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} \\ &= \frac{2\sqrt{3}+\sqrt{6}}{4+2\sqrt{2}-2\sqrt{2}-2} \\ &= \frac{2\sqrt{3}+\sqrt{6}}{2} \end{aligned}$$

$$\begin{aligned} \text{j} \quad \frac{\sqrt{5}}{3+\sqrt{3}} &= \frac{\sqrt{5} \times (3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})} \\ &= \frac{3\sqrt{5}-\sqrt{15}}{9-3\sqrt{3}+3\sqrt{3}-3} \\ &= \frac{3\sqrt{5}-\sqrt{15}}{6} \end{aligned}$$

$$\begin{aligned} \text{k} \quad \frac{\sqrt{5}}{4-\sqrt{5}} &= \frac{\sqrt{5} \times (4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})} \\ &= \frac{4\sqrt{5}+5}{16+4\sqrt{5}-4\sqrt{5}-5} \\ &= \frac{4\sqrt{5}+5}{11} \end{aligned}$$

$$\begin{aligned} \text{l} \quad \frac{\sqrt{7}}{7-\sqrt{7}} &= \frac{\sqrt{7} \times (7+\sqrt{7})}{(7-\sqrt{7})(7+\sqrt{7})} \\ &= \frac{7\sqrt{7}+7}{49+7\sqrt{7}-7\sqrt{7}-7} \\ &= \frac{7+7\sqrt{7}}{42} \\ &= \frac{1+\sqrt{7}}{6} \end{aligned}$$

$$\begin{aligned} 3 \quad \text{a} \quad \frac{1}{\sqrt{2}-\sqrt{3}} &= \frac{1 \times (\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} \\ &= \frac{\sqrt{2}+\sqrt{3}}{2+\sqrt{6}-\sqrt{6}-3} \\ &= \frac{\sqrt{2}+\sqrt{3}}{-1} \\ &= -\sqrt{2}-\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b} \quad \frac{5}{\sqrt{3}+\sqrt{5}} &= \frac{5 \times (\sqrt{3}-\sqrt{5})}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})} \\ &= \frac{5\sqrt{3}-5\sqrt{5}}{3-\sqrt{15}+\sqrt{15}-5} \\ &= \frac{5\sqrt{3}-5\sqrt{5}}{-2} \\ &= \frac{5}{2}(-\sqrt{3}+\sqrt{5}) \end{aligned}$$

$$\begin{aligned} \text{c} \quad \frac{2\sqrt{3}}{\sqrt{3}-\sqrt{7}} &= \frac{2\sqrt{3} \times (\sqrt{3}+\sqrt{7})}{(\sqrt{3}-\sqrt{7})(\sqrt{3}+\sqrt{7})} \\ &= \frac{2 \times \sqrt{9}+2\sqrt{21}}{3+\sqrt{21}-\sqrt{21}-7} \\ &= \frac{6+2\sqrt{21}}{-4} \\ &= \frac{3+\sqrt{21}}{-2} \end{aligned}$$

$$\begin{aligned} \text{d} \quad \frac{2+\sqrt{3}}{\sqrt{3}+\sqrt{5}} &= \frac{(2+\sqrt{3})(\sqrt{3}-\sqrt{5})}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})} \\ &= \frac{2\sqrt{3}-2\sqrt{5}+3-\sqrt{15}}{3-\sqrt{15}+\sqrt{15}-5} \\ &= \frac{2\sqrt{3}-2\sqrt{5}+3-\sqrt{15}}{-2} \end{aligned}$$

$$\begin{aligned} \text{e} \quad \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} &= \frac{(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} \\ &= \frac{2+\sqrt{6}+\sqrt{6}+3}{2+\sqrt{6}-\sqrt{6}-3} \\ &= \frac{5+2\sqrt{6}}{-1} \\ &= -5-2\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{f} \quad \frac{\sqrt{7}-\sqrt{11}}{\sqrt{11}+\sqrt{7}} &= \frac{(\sqrt{7}-\sqrt{11})(\sqrt{11}-\sqrt{7})}{(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7})} \\ &= \frac{\sqrt{77}-7-11+\sqrt{77}}{11-\sqrt{77}+\sqrt{77}-7} \\ &= \frac{2\sqrt{77}-18}{4} \\ &= \frac{\sqrt{77}-9}{2} \end{aligned}$$

$$\begin{aligned} 4 \quad \text{a} \quad \frac{1}{(2+\sqrt{3})^2} &= \frac{1}{(2+\sqrt{3})(2+\sqrt{3})} \\ &= \frac{1}{4+2\sqrt{3}+2\sqrt{3}+3} \\ &= \frac{1}{7+4\sqrt{3}} \\ &= \frac{1 \times (7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})} \\ &= \frac{7-4\sqrt{3}}{49-28\sqrt{3}+28\sqrt{3}-48} \\ &= \frac{7-4\sqrt{3}}{1} \\ &= 7-4\sqrt{3} \end{aligned}$$

1.3 Rationalising denominators (Continued)

$$\begin{aligned}
 \text{b } \frac{1}{(2-\sqrt{5})^2} &= \frac{1}{(2-\sqrt{5})(2-\sqrt{5})} \\
 &= \frac{1}{4-2\sqrt{5}-2\sqrt{5}+5} \\
 &= \frac{1}{9-4\sqrt{5}} \\
 &= \frac{1 \times (9+4\sqrt{5})}{(9-4\sqrt{5})(9+4\sqrt{5})} \\
 &= \frac{9+4\sqrt{5}}{81+36\sqrt{5}-36\sqrt{5}-80} \\
 &= \frac{9+4\sqrt{5}}{1} \\
 &= 9+4\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \frac{3}{(5-\sqrt{3})^2} &= \frac{3}{(5-\sqrt{3})(5-\sqrt{3})} \\
 &= \frac{3}{25-5\sqrt{3}-5\sqrt{3}+3} \\
 &= \frac{3}{28-10\sqrt{3}} \\
 &= \frac{3 \times (28+10\sqrt{3})}{(28-10\sqrt{3})(28+10\sqrt{3})} \\
 &= \frac{84+30\sqrt{3}}{784+280\sqrt{3}-280\sqrt{3}-300} \\
 &= \frac{84+30\sqrt{3}}{484} \\
 &= \frac{42+15\sqrt{3}}{242}
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \frac{4}{(6+\sqrt{2})^2} &= \frac{4}{(6+\sqrt{2})(6+\sqrt{2})} \\
 &= \frac{4}{36+6\sqrt{2}+6\sqrt{2}+2} \\
 &= \frac{4}{38+12\sqrt{2}} \\
 &= \frac{2}{19+6\sqrt{2}} \\
 &= \frac{2 \times (19-6\sqrt{2})}{(19+6\sqrt{2})(19-6\sqrt{2})} \\
 &= \frac{38-12\sqrt{2}}{361-114\sqrt{2}+114\sqrt{2}-72} \\
 &= \frac{38-12\sqrt{2}}{289}
 \end{aligned}$$

$$\begin{aligned}
 \text{e } \frac{1}{(3-\sqrt{2})(2+\sqrt{2})} &= \frac{1}{6+3\sqrt{2}-2\sqrt{2}-2} \\
 &= \frac{1}{4+\sqrt{2}} \\
 &= \frac{1 \times (4-\sqrt{2})}{(4+\sqrt{2})(4-\sqrt{2})} \\
 &= \frac{4-\sqrt{2}}{16-4\sqrt{2}+4\sqrt{2}-2} \\
 &= \frac{4-\sqrt{2}}{14}
 \end{aligned}$$

$$\begin{aligned}
 \text{f } \frac{3}{(2+\sqrt{5})(4-\sqrt{5})} &= \frac{3}{8-2\sqrt{5}+4\sqrt{5}-5} \\
 &= \frac{3}{3+2\sqrt{5}} \\
 &= \frac{3 \times (3-2\sqrt{5})}{(3+2\sqrt{5})(3-2\sqrt{5})} \\
 &= \frac{9-6\sqrt{5}}{9-6\sqrt{5}+6\sqrt{5}-20} \\
 &= \frac{9-6\sqrt{5}}{-11} \\
 &= \frac{-9+6\sqrt{5}}{11}
 \end{aligned}$$

$$\begin{aligned}
 \text{g } \frac{\sqrt{3}}{(1+\sqrt{3})^2} &= \frac{\sqrt{3}}{(1+\sqrt{3})(1+\sqrt{3})} \\
 &= \frac{\sqrt{3}}{1+\sqrt{3}+\sqrt{3}+3} \\
 &= \frac{\sqrt{3}}{4+2\sqrt{3}} \\
 &= \frac{\sqrt{3} \times (4-2\sqrt{3})}{(4+2\sqrt{3})(4-2\sqrt{3})} \\
 &= \frac{4\sqrt{3}-6}{16-8\sqrt{3}+8\sqrt{3}-12} \\
 &= \frac{4\sqrt{3}-6}{4} \\
 &= \frac{2\sqrt{3}-3}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{h } \frac{\sqrt{5}}{(5-\sqrt{2})^2} &= \frac{\sqrt{5}}{(5-\sqrt{2})(5-\sqrt{2})} \\
 &= \frac{\sqrt{5}}{25-5\sqrt{2}-5\sqrt{2}+2} \\
 &= \frac{\sqrt{5}}{27-10\sqrt{2}} \\
 &= \frac{\sqrt{5} \times (27+10\sqrt{2})}{(27-10\sqrt{2})(27+10\sqrt{2})} \\
 &= \frac{27\sqrt{5}+10\sqrt{10}}{729+270\sqrt{2}-270\sqrt{2}-200} \\
 &= \frac{27\sqrt{5}+10\sqrt{10}}{529}
 \end{aligned}$$

$$\begin{aligned}
 \text{i } \frac{\sqrt{2}}{(5-\sqrt{2})(1+\sqrt{2})} &= \frac{\sqrt{2}}{5+5\sqrt{2}-\sqrt{2}-2} \\
 &= \frac{\sqrt{2}}{3+4\sqrt{2}} \\
 &= \frac{\sqrt{2} \times (3-4\sqrt{2})}{(3+4\sqrt{2})(3-4\sqrt{2})} \\
 &= \frac{3\sqrt{2}-8}{9-12\sqrt{2}+12\sqrt{2}-32} \\
 &= \frac{3\sqrt{2}-8}{-23} \\
 &= \frac{8-3\sqrt{2}}{23}
 \end{aligned}$$

$$\begin{aligned}
 5 \text{ a } \sqrt{75} - \sqrt{12} &= \sqrt{25 \times 3} - \sqrt{4 \times 3} \\
 &= \sqrt{25} \times \sqrt{3} - \sqrt{4} \times \sqrt{3} \\
 &= 5\sqrt{3} - 2\sqrt{3} \\
 &= 3\sqrt{3}
 \end{aligned}$$

1.3 Rationalising denominators (Continued)

$$\begin{aligned} \text{b } \frac{4+\sqrt{5}}{\sqrt{75}-\sqrt{12}} &= \frac{4+\sqrt{5}}{3\sqrt{3}} \\ &= \frac{(4+\sqrt{5}) \times \sqrt{3}}{3\sqrt{3} \times \sqrt{3}} \\ &= \frac{4\sqrt{3}+\sqrt{15}}{9} \end{aligned}$$

$$\begin{aligned} \text{6 a } \frac{2+\sqrt{5}}{\sqrt{50}-\sqrt{8}} &= \frac{2+\sqrt{5}}{\sqrt{25 \times 2}-\sqrt{4 \times 2}} \\ &= \frac{2+\sqrt{5}}{5\sqrt{2}-2\sqrt{2}} \\ &= \frac{2+\sqrt{5}}{3\sqrt{2}} \\ &= \frac{(2+\sqrt{5}) \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}} \\ &= \frac{2\sqrt{2}+\sqrt{10}}{6} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{3-\sqrt{3}}{\sqrt{27}+\sqrt{75}} &= \frac{3-\sqrt{3}}{\sqrt{9 \times 3}+\sqrt{25 \times 3}} \\ &= \frac{3-\sqrt{3}}{3\sqrt{3}+5\sqrt{3}} \\ &= \frac{3-\sqrt{3}}{8\sqrt{3}} \\ &= \frac{(3-\sqrt{3}) \times \sqrt{3}}{8\sqrt{3} \times \sqrt{3}} \\ &= \frac{3\sqrt{3}-3}{24} \\ &= \frac{\sqrt{3}-1}{8} \end{aligned}$$

$$\begin{aligned} \text{c } \frac{1+\sqrt{2}}{\sqrt{80}+\sqrt{45}} &= \frac{1+\sqrt{2}}{4\sqrt{5}+3\sqrt{5}} \\ &= \frac{1+\sqrt{2}}{7\sqrt{5}} \\ &= \frac{(1+\sqrt{2}) \times \sqrt{5}}{7\sqrt{5} \times \sqrt{5}} \\ &= \frac{\sqrt{5}+\sqrt{10}}{35} \end{aligned}$$

$$\begin{aligned} \text{d } \frac{\sqrt{3}-4}{2\sqrt{32}-\sqrt{18}} &= \frac{\sqrt{3}-4}{8\sqrt{2}-3\sqrt{2}} \\ &= \frac{\sqrt{3}-4}{5\sqrt{2}} \\ &= \frac{(\sqrt{3}-4) \times \sqrt{2}}{5\sqrt{2} \times \sqrt{2}} \\ &= \frac{\sqrt{6}-4\sqrt{2}}{10} \end{aligned}$$

$$\begin{aligned} \text{e } \frac{2+\sqrt{5}}{1-2\sqrt{48}+\sqrt{12}} &= \frac{2+\sqrt{5}}{1-8\sqrt{3}+2\sqrt{3}} \\ &= \frac{2+\sqrt{5}}{1-6\sqrt{3}} \\ &= \frac{(2+\sqrt{5}) \times (1+6\sqrt{3})}{(1-6\sqrt{3}) \times (1+6\sqrt{3})} \\ &= \frac{2+12\sqrt{3}+\sqrt{5}+6\sqrt{15}}{1+6\sqrt{3}-6\sqrt{3}-108} \\ &= \frac{2+12\sqrt{3}+\sqrt{5}+6\sqrt{15}}{-107} \end{aligned}$$

$$\begin{aligned} \text{f } \frac{5+\sqrt{3}}{2-3\sqrt{20}+\sqrt{80}} &= \frac{5+\sqrt{3}}{2-3\sqrt{4 \times 5}+\sqrt{16 \times 5}} \\ &= \frac{5+\sqrt{3}}{2-6\sqrt{5}+4\sqrt{5}} \\ &= \frac{5+\sqrt{3}}{2-2\sqrt{5}} \\ &= \frac{(5+\sqrt{3})(2+2\sqrt{5})}{(2-2\sqrt{5})(2+2\sqrt{5})} \\ &= \frac{10+10\sqrt{5}+2\sqrt{3}+2\sqrt{15}}{4+4\sqrt{5}-4\sqrt{5}-20} \\ &= \frac{5+5\sqrt{5}+\sqrt{3}+\sqrt{15}}{-8} \end{aligned}$$

$$\begin{aligned} \text{7 } \frac{\sqrt{7}+7}{7-\sqrt{7}} &= \frac{(\sqrt{7}+7)(7+\sqrt{7})}{(7-\sqrt{7})(7+\sqrt{7})} \\ &= \frac{7\sqrt{7}+7+49+7\sqrt{7}}{49+7\sqrt{7}-7\sqrt{7}-7} \\ &= \frac{56+14\sqrt{7}}{42} \\ &= \frac{4+\sqrt{7}}{3} \end{aligned}$$

$$\begin{aligned} \text{8 } \frac{2+\sqrt{3}}{\sqrt{80}-\sqrt{20}} &= \frac{2+\sqrt{3}}{4\sqrt{5}-2\sqrt{5}} \\ &= \frac{2+\sqrt{3}}{2\sqrt{5}} \\ &= \frac{(2+\sqrt{3}) \times \sqrt{5}}{2\sqrt{5} \times \sqrt{5}} \\ &= \frac{2\sqrt{5}+\sqrt{15}}{10} \end{aligned}$$