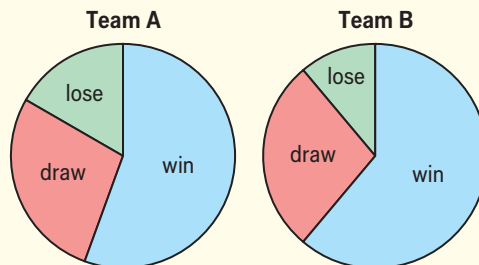


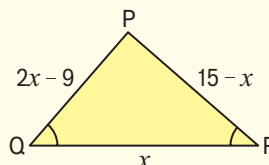
Mixed exercises A

Mixed problem-solving practice A

- Helen is going to choose a main course and a dessert from a menu. She can choose from 7 main courses and 5 desserts. Helen says, 'To work out the number of different ways of choosing a main course and a dessert you add 7 and 5.' Is Helen correct? You must give a reason for your answer.
- 20 teams play in a competition. Each team plays the other teams exactly once. Work out the total number of games played.
- Buses to Cardiff leave a bus station every 30 minutes. Buses to Bristol leave the same bus station every 18 minutes. A bus to Cardiff and a bus to Bristol both leave the bus station at 7 am. When will buses to Cardiff and to Bristol next leave the bus station at the same time?
- Can you use the pie charts to determine which team won the greatest number of matches? If so, which team was it? Explain your answer.



- PQR is a triangle. Angle PQR = angle PRQ. The length of side PQ is $(2x - 9)$ cm. The length of side PR is $(15 - x)$ cm. The length of side QR is x cm. Work out the perimeter of the triangle. Give your answer in centimetres.



- Here are the first five terms of an arithmetic sequence.
2 9 16 23 30
 - Find an expression, in terms of n , for the n th term of this sequence. Emma says, '200 is in the sequence.'
 - Is Emma right? You must explain your answer.

- The table shows information about the length of time, t , in minutes, of the phone calls Lynne had in a week.
 - Work out an estimate for the mean length of time of the phone calls. Lynne says, 'The mean may not be the best average to use to represent this information.'
 - Do you agree with Lynne? You must justify your answer.

Time, t (minutes)	Frequency
$0 < t \leq 10$	12
$10 < t \leq 20$	15
$20 < t \leq 30$	5
$30 < t \leq 40$	0
$40 < t \leq 60$	8





- 8 Mercury is approximately 5.8×10^7 km from the Sun.
Saturn is approximately 1.427×10^9 km from the Sun.
Jo says, 'Saturn is over a hundred times further away from the Sun than Mercury is.'
Is Jo right? You must show how you get your answer.



- 9 Harry went to a football match in Madrid.
His ticket cost €130.
The exchange rate was £1 = €1.15
- a Work out the cost of his ticket in pounds.
Harry bought a football shirt in Madrid.
The shirt cost €73.40
In London, the same type of shirt cost £60.
The exchange rate was £1 = €1.15
- b Compare the cost of the shirt in Madrid with the cost of the shirt in London.

- 10 Karen works out $4\frac{2}{3} + 3\frac{1}{5}$
Karen writes

$$4\frac{2}{3} + 3\frac{1}{5} = 7\frac{2}{15} + \frac{1}{15} = 7\frac{3}{15} = 7\frac{1}{5}$$

Karen is incorrect. What is Karen's mistake?



- 11 Monty makes 350 sandwiches.
He makes only four types: ham, cheese, tuna, egg.

$\frac{4}{7}$ of the sandwiches are ham.

24% of the sandwiches are cheese.

The ratio of the number of tuna sandwiches to the number of egg sandwiches is 4 : 7.

Work out the number of tuna sandwiches Monty makes.



- 12 In a sale, normal prices are reduced by 30%.
A television has a sale price of £546.
By how much money is the normal price of the television reduced?

- 13 Prove algebraically that the recurring decimal $0.3\dot{1}\dot{5}$ can be written as the fraction $\frac{52}{165}$

- 14 Here are the first five terms of a sequence.

1 8 19 34 53

Find an expression, in terms of n , for the n th term of this sequence.



Exam practice

- 15 A force of 80 newtons acts on an area of 30 cm^2 .
The force is increased by 10 newtons.
The area is increased by 10 cm^2 .
Jay says, 'The pressure decreases by less than 20%.'
Is Jay correct? You must show how you get your answer.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

(3 marks)

Adapted from IMA1/2H, June 2018, Q6

Exam feedback

ResultsPlus

Most students who achieved a **Grade 4** or above answered a similar question well.



Exam practice

- 16** On Saturday, some adults and some children were in a cinema.
The ratio of the number of adults to the number of children was 5 : 2.
Each person had a seat in screen 1 or a seat in screen 2.
 $\frac{1}{4}$ of the children had seats in screen 1.
108 children had seats in screen 2.
There are only two screens in the cinema. There are exactly 800 seats in the cinema.
On this Saturday, were there people on more than 60% of the seats?
You must show how you get your answer.

(5 marks)

Adapted from IMA1/2H, June 2017, Q2

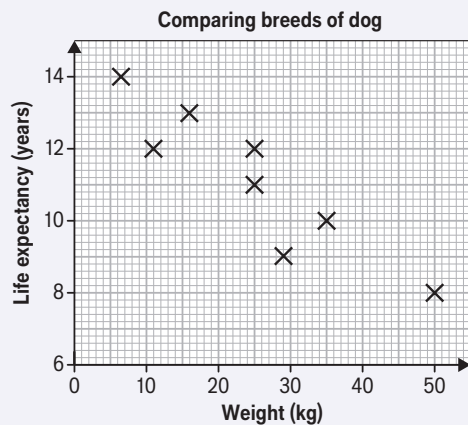
Exam feedback

ResultsPlus

Most students who achieved a **Grade 6** or above answered a similar question well.

Exam practice

- 17** The scatter graph shows the mean weight and life expectancy for eight different breeds of dog.
One of the breeds of dog has a weight of 16 kg.
- Write down the life expectancy of this dog. (1 mark)
 - Write down the type of correlation for the scatter graph. (1 mark)
- A vet says, 'Heavier dogs have a lower life expectancy.'
- Does the scatter graph support what the vet says?
Give a reason for your answer. (1 mark)
 - Another breed of dog has an average weight of 20 kg.
Estimate the life expectancy of this breed of dog. (2 marks)



(2 marks)

Adapted from IMA1/1H, May 2017, Q1

Exam feedback

ResultsPlus

Most students who achieved a **Grade 3** or above answered a similar question well.



Exam practice

- 18** $T = \sqrt{\frac{x}{y}}$
 $x = 7.2 \times 10^{-6}$
 $y = 1.8 \times 10^{-4}$
- Work out the value of T . (2 marks)
 x is increased by 5%.
 y is increased by 10%.
Kieran says, 'The value of T will increase because both x and y are increased.'
 - Kieran is wrong. Explain why. (2 marks)

Adapted from IMA1/3H, June 2018, Q9

Exam feedback

ResultsPlus

Q18a: Most students who achieved a **Grade 7** or above answered a similar question well.



Exam practice

19 Solve

$$\frac{2x-3}{2} - \frac{5x+2}{9} = \frac{1-x}{6}$$

(4 marks)

Adapted from IMA1/2H, June 2017, Q11

Exam feedback

ResultsPlus

Most students who achieved a **Grade 8** or above answered a similar question well.

Exam practice

20 a Factorise $p^2 - q^2$

(1 mark)

b Hence, or otherwise, simplify fully $(x^2 + 9)^2 - (x^2 - 3)^2$

(3 marks)

Adapted from IMA1/1H, June 2018, Q15

Exam feedback

ResultsPlus

Q20a: Most students who achieved a **Grade 8** or above answered a similar question well.

Q20b: Most students who achieved a **Grade 9** answered a similar question well.



Exam practice

21 $8^{\frac{2}{5}} \times 2^x = 4^{\frac{1}{4}}$

Work out the exact value of x .

(3 marks)

Adapted from IMA1/2H, June 2017, Q18

Exam feedback

ResultsPlus

Most students who achieved a **Grade 9** answered a similar question well.



Exam practice

22 Kate rationalised the denominator of $\frac{3}{\sqrt{20}}$

Here is Kate's answer.

$$\begin{aligned} \frac{3}{\sqrt{20}} &= \frac{3\sqrt{20}}{\sqrt{20} \times \sqrt{20}} \\ &= \frac{3 \times 5\sqrt{2}}{20} \\ &= \frac{3\sqrt{2}}{4} \end{aligned}$$

Kate's answer is wrong.

Find Kate's mistake.

(1 mark)

Adapted from IMA1/2H, June 2018, Q20b

Exam feedback

ResultsPlus

Most students who achieved a **Grade 8** or above answered a similar question well.

10 Probability

10.1 Combined events

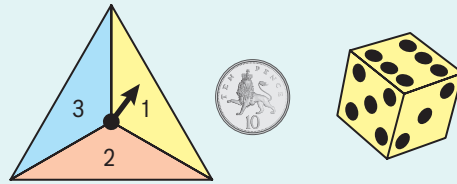
Key points

- A sample space diagram, or possibility space diagram, shows all the possible outcomes of two events.
- Probability = $\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

△ Purposeful practice 1

Write all the possible outcomes when

- a the coin is flipped
 - b the spinner is spun
 - c the coin is flipped **and** the spinner is spun
- a the coin is flipped
 - b the dice is rolled
 - c the coin is flipped **and** the dice is rolled
- a the spinner is spun
 - b the dice is rolled
 - c the spinner is spun **and** the dice is rolled



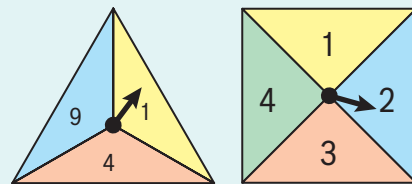
Reflect and reason

How can you use the number of possible outcomes from two separate events to work out the number of possible outcomes when both events happen together?

△ Purposeful practice 2

Fay spins each spinner once. Both spinners are fair. She adds the two numbers together to get her score.

- 1 Make a possibility space diagram for each possible score.
- 2 Find the probability that Fay's score is
 - a 8
 - b less than 8
 - c more than 8
 - d 8 or more
 - e 8 or less



Reflect and reason

Which of your answers to Q2 can you add together to make 1? Explain why.

Problem-solving practice

- Mel rolls two ordinary dice.
He adds the two scores.
What is the probability that Mel's total score is a prime number?
- When you roll two ordinary dice at the same time, what is the probability that both dice show the same score?
- Box 1 contains a £5 note, a £10 note and a £20 note.
Box 2 contains a £10 note and a £20 note.
Dan picks a note from each box at random.
What is the probability he gets a total of less than £30?
- Amy has a set of cards labelled 1 to 10.
She picks one card at random.
She also throws an ordinary dice once.
 - How many possible outcomes include 5 on the dice?
 - Work out the probability of picking an even number and rolling a 5.
- Kim and Zoe play a game.
They roll two dice and multiply the numbers to get a score.
Kim wins if the score is less than 12.
Zoe wins if the score is 12 or more.
Is this game fair?
Explain.
- Arrange these cards into two sets, so that there are 15 different possible outcomes for 'pick one card from set A and one card from set B'.



Exam practice

- Paul has a bag of stationary.
There are 40 pens in the bag.
The table shows the types of pens in the bag.

	Red	Green	Blue
Ballpoint pen	4	7	9
Felt tip pen	2	5	4
Fountain pen	1	0	8

Paul takes at random a pen from the bag.

- Write down the probability that the pen is a red ballpoint pen. **(1 mark)**
- Work out the probability the pen is a felt tip. **(2 marks)**

Adapted from IMA0/2H, June 2015, Q1a and b

10.2 Mutually exclusive events

Key points

- When events are mutually exclusive, you can add their probabilities.
For mutually exclusive events, $P(A \text{ or } B) = P(A) + P(B)$
- The probabilities of an exhaustive set of mutually exclusive events sum to 1.
- For mutually exclusive events A and not A, $P(\text{not } A) = 1 - P(A)$

△ Purposeful practice 1



For this set of counters, write down these probabilities

- | | | | |
|--------------------------------------|-------------------------------------|--------------------------------------|--------------------------------------|
| 1 $P(R)$ | 2 $P(B)$ | 3 $P(Y)$ | 4 $P(W)$ |
| 5 $P(R \text{ or } Y)$ | 6 $P(R \text{ or } W)$ | 7 $P(R \text{ or } B \text{ or } Y)$ | 8 $P(R \text{ or } B \text{ or } W)$ |
| 9 $P(B \text{ or } Y \text{ or } W)$ | 10 $P(\text{not } B)$ | 11 $P(\text{not } Y)$ | 12 $P(\text{not } W)$ |
| 13 $P(\text{not } B \text{ or } R)$ | 14 $P(\text{not } B \text{ or } W)$ | 15 $P(\text{not } W \text{ or } Y)$ | |

Reflect and reason

Use your answers to **Q1–15** to show that

$$P(R \text{ or } W) = P(R) + P(W)$$

$$P(B \text{ or } Y \text{ or } W) = P(B) + P(Y) + P(W)$$

$$P(\text{not } W) = 1 - P(W)$$

△ Purposeful practice 2

In each question, there are only blue (B), yellow (Y), red (R) and green (G) counters in a bag.

- The table shows the probabilities of getting a blue or yellow or green counter.
Work out the probability of getting a red counter.
- The table shows the probabilities of getting a blue or yellow or red counter.
Work out the probability of getting a green counter.
- The table shows the probabilities of getting a blue or red or green counter.
Work out the probability of getting a yellow counter.
- The table shows the probabilities of getting a blue or yellow or red counter.
Work out the probability of getting a green counter.

Colour	B	Y	R	G
Probability	0.1	0.2		0.3

Colour	B	Y	R	G
Probability	0.1	0.25	0.6	

Colour	B	Y	R	G
Probability	$\frac{3}{8}$		$\frac{1}{4}$	$\frac{1}{4}$

Colour	B	Y	R	G
Probability	15%	15%	25%	

Reflect and reason

For **Q2** Karl writes,

$$0.1 + 0.25 + 0.6 = 0.95, \text{ so } P(\text{green}) = 0.5$$

Explain what Karl has done wrong.

Problem-solving practice

- The probability that a train is late is 0.03.
What is the probability that the train is not late?
- The weather forecast says the probability of rain is $<10\%$.
What is the probability that it does not rain?
- A counter is picked at random from a bag.
The table shows the probabilities of getting a blue or yellow counter.
The probability of getting a red counter is the same as the probability of getting a green counter.
Work out the probability of getting a green counter.

Colour	blue	yellow	red	green
Probability	0.15	0.25		

- The table shows the probabilities of getting a blue or yellow counter.
The probability of getting a red counter is twice the probability of getting a green counter.
Work out the probability of getting a red counter.

Colour	blue	yellow	red	green
Probability	0.3	0.4		

- The probability of picking a black counter from a bag of counters is $\frac{1}{12}$.
Alex says there are 6 counters in the bag.
Explain why there cannot be only 6 counters in the bag.
- The table shows the probabilities of getting different colours of counters.

Colour	pink	black	white	green
Probability	0.3	0.15	0.45	0.1

There are 12 pink counters in the bag.

- Which colour counter is half as likely as pink?
- Work out the numbers of black, white and green counters in the bag.

Exam practice

- There are some cubes in a bag.
The cubes are red or blue or green or white.
Sam is going to take a cube at random from the bag.
The table shows each of the probabilities that the cube will be red or will be white.

Colour	red	blue	green	white
Probability	0.4			0.45

There are 8 red cubes in the bag.

The probability that the cube Sam takes will be blue is twice the probability that the cube will be green.

Work out the number of green cubes in the bag.

(4 marks)

Adapted from IMA1/3H, June 2018, Q6a

Exam feedback

Most students who achieved a **Grade 5** or above answered a similar question well.

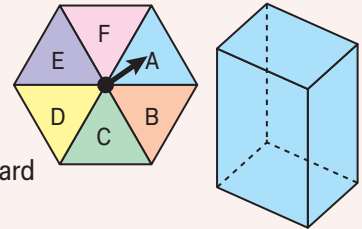
ResultsPlus

Problem-solving practice

- 1 A train company advertises, 'The probability that one of our trains is late is only 2%.' The company runs 1400 trains each week. Work out an estimate for the number of these trains that are likely to be late each week.
- 2 In a probability experiment, Shan picks a ball from a bag, records its colour, and then replaces it in the bag. She does this 50 times. Here are her results. There are 20 balls in the bag. Calculate an estimate for the number of each colour.

Colour	Red	Blue	Green	White
Frequency	15	20	5	10

- 3 a Which of these probabilities need to be estimated from a probability experiment? The probability that
- a drawing pin lands point up when you drop it
 - a spinner with 6 equal sections, A–F, lands on a vowel
 - a light bulb lasts more than 1 year in normal use
 - a card picked at random from a normal pack is a picture card
 - more than one egg breaks when you drop a box of 6 eggs
 - a cuboid-shaped matchbox lands on one of its smallest faces when dropped
- b Which of the probabilities in a can be calculated as a theoretical probability?



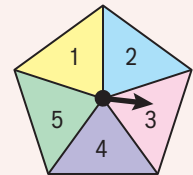
- 4 The table shows the probabilities that a biased dice lands on 1, 2, 3, 4 and 5.

Score	1	2	3	4	5	6
Probability	0.21	0.13	0.15	0.24	0.11	

This dice is rolled 120 times. Work out an estimate for the number of times it will land on 6.

- 5 The table shows the results of spinning this five-sided spinner 80 times.

Score	1	2	3	4	5
Frequency	18	14	19	13	16



- Is the spinner likely to be fair? Show your working to explain.
- Explain how you could improve the experiment to test whether the spinner is fair.

Exam practice

- 1 When a piece of buttered toast is dropped, it can land butter side up or butter side down. Kay, Jay and Min each dropped a piece of buttered toast a number of times. Their results are shown in the table.

	Kay	Jay	Min
Butter side up	7	15	36
Butter side down	12	13	54

Dane is going to drop a piece of buttered toast once. Whose results will give the best estimate for the probability that the toast will land butter side down? Give a reason for your answer.

(1 mark)

Adapted from IMA1/3H, November 2017, Q8

Exam feedback

ResultsPlus

In a similar question, students identified the best estimate for the probability, but did not give the correct reasons.

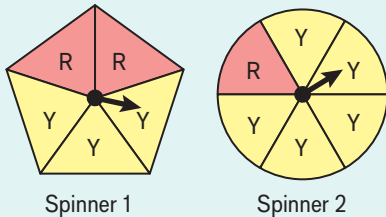
10.4 Independent events and tree diagrams

Key points

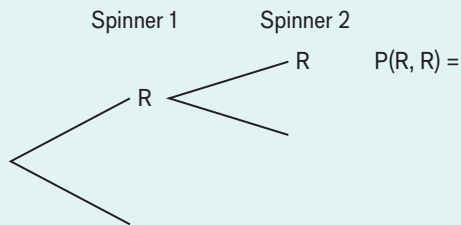
- Two events are independent if one event does not affect the probability of the other.
- To find the probability of two independent events, multiply their probabilities.
- A probability tree diagram shows two or more events and their probabilities.

△ Purposeful practice

1 Alex spins both these spinners.



a Copy and complete the probability tree diagram.



b Which outcome is most likely from spinning these two spinners?

Alex spins spinner 1 and then spinner 2. He does this 75 times.

c Work out an estimate for the number of times both spinners land on red.

2 Bella spins both these spinners.

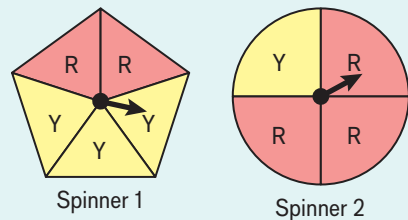
a For each spinner, write $P(\text{Red})$ as a decimal.

b Draw a tree diagram to show the probabilities when Bella spins spinner 1 and then spinner 2. Write the probabilities as decimals.

c Calculate the probability that the two spinners land on different colours.

d Bella spins spinner 1 and then spinner 2. She does this 100 times.

Work out an estimate for the number of times both spinners land on the same colour.



Reflect and reason

Use the terms 'add' and 'multiply' to complete these sentences.

- The two spinners are independent, so you _____ along the branches to calculate the probability of $P(\text{Red and Red})$.
- Outcomes are mutually exclusive, so you _____ the probabilities down the side to calculate $P(\text{Red and Red})$ or $P(\text{Yellow and Yellow})$.

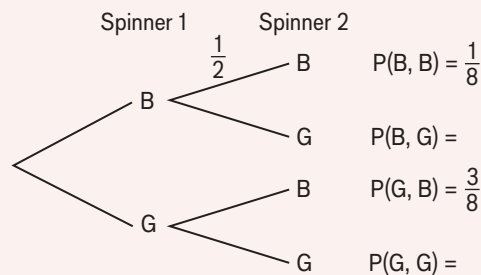
Problem-solving practice

- 1 The tree diagram shows the probabilities when two spinners, each with a number of blue and green sections, are spun.

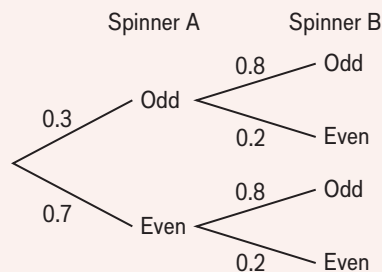
a Work out

- the probability of G on spinner 1
- the probability of G on spinner 1 and G on spinner 2.

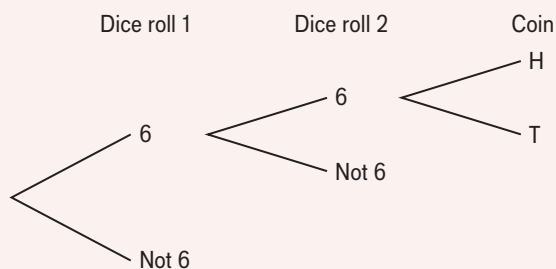
b Draw two spinners that give these probabilities.



- 2 Jack has two spinners, A and B. Each spinner can only land on an even number or an odd number. The probability that spinner A lands on an odd number is 0.3. The probability that spinner B lands on an odd number is 0.8. The probability tree diagram shows this information. Jack spins spinner A once and then spinner B once. He does this a number of times. The number of times **both** spinners land on odd numbers is 24. Work out an estimate for the number of times **both** spinners land on even numbers.



- 3 Anna rolls a fair dice twice and then flips a coin. She starts to draw this tree diagram. Work out the probability that she rolls 2 sixes and the coin shows Heads.



Exam practice

- 1 When a biased spinner is spun, the probability that it will land on white is 0.45.

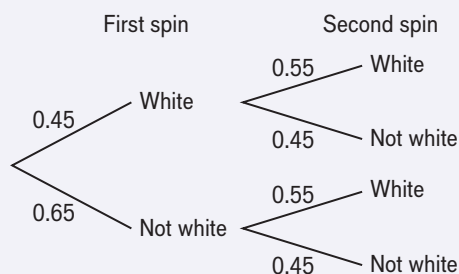
Jake spins the biased spinner twice.

He draws this tree diagram.

The tree diagram is not correct.

Write down two things that are wrong with the probability tree diagram.

(2 marks)



Adapted from IMA1/3H, June 2018, Q4

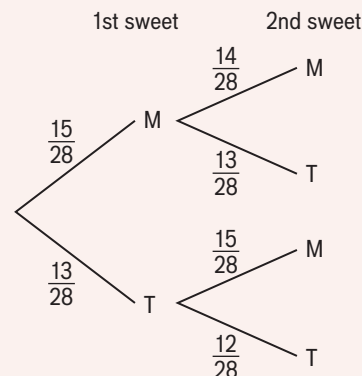
Exam feedback

Most students who achieved a **Grade 4** or above answered a similar question well.

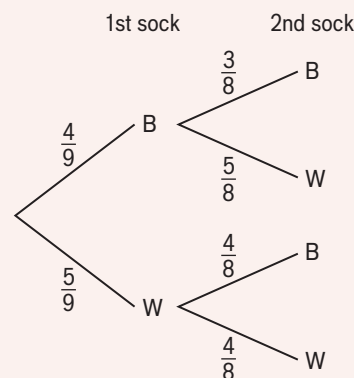
ResultsPlus

Problem-solving practice

- 1 There are 28 sweets in a bag.
15 of them are toffees and the rest are mints.
Mary is going to pick two sweets at random.
She draws this tree diagram to show the probabilities.
Write down **one** thing that is wrong with the probabilities in her tree diagram.

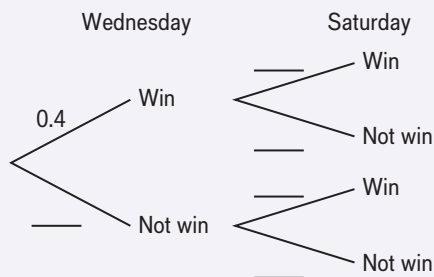


- 2 There are 5 red and 7 green marbles in a bag.
Lin takes 3 marbles.
Work out the probability that she takes 3 green marbles.
- 3 The probability diagram shows the probabilities when Amir takes two socks from his drawer.
If Amir takes one black and one white sock from the drawer, he has to take a third sock to make a pair.
- Work out the probability he has to take a third sock.
 - Explain why, when he takes a third sock, he will have a pair of socks the same colour.



Exam practice

- 1 A hockey team is going to play a match on Wednesday and on Saturday.
The probability that the team will win on Wednesday is 0.4
If they win on Wednesday, the probability that they will win on Saturday is 0.72
If they do **not** win on Wednesday, the probability that they will win on Saturday is 0.25
- a Copy and complete the tree diagram. (2 marks)



- b Find the probability that the team will win exactly one of the two matches. (3 marks)

Adapted from IMA1/2H, June 2018, Q15

Exam feedback

ResultsPlus

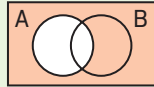
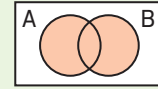
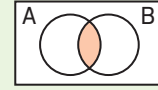
Q1a: Most students who achieved a **Grade 3** or above answered a similar question well.

Q1b: Most students who achieved a **Grade 6** or above answered a similar question well.

10.6 Venn diagrams and set notation

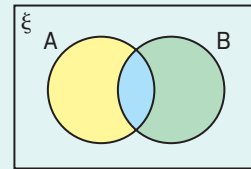
Key points

- Curly brackets $\{ \}$ show a set of values.
 \in means 'is an element of'.
- $A \cap B$ means 'A intersection B'. This is all the elements that are in A **and** B.
- $A \cup B$ means 'A union B'. This is all the elements that are in A **or** B **or** both.
- A' means the elements **not** in A.
- ξ means the universal set — all the elements being considered.



△ Purposeful practice

- ξ is the set of numbers from 10 to 30 (including 10 and 30).
 $A = \{11, 13, 18, 20, 25\}$
 $B = \{11, 12, 15, 18, 20, 22, 24, 29\}$
 - Which numbers are in $A \cap B$?
 - Copy and complete this Venn diagram for ξ , A and B.
 - Write down the numbers that are in set
 - $A \cup B$
 - A'
 - $A' \cap B$
 - $(A \cup B)'$



Repeat **Q1** for these sets

- ξ is the numbers from 10 to 30 (including 10 and 30).
 $A =$ multiples of 3
 $B =$ even numbers
- ξ is the numbers from 1 to 20 (including 1 and 20)
 $A =$ odd numbers
 $B =$ square numbers
- ξ is the numbers from 5 to 20 (including 5 and 20).
 $A =$ NOT multiples of 4
 $B =$ NOT factors of 60
- ξ is the numbers from 1 to 15 inclusive.
 $A =$ prime numbers
 $B =$ factors of 210

Reflect and reason

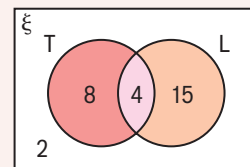
How does starting with the numbers in $A \cap B$ help you to fill in the Venn diagram?
 How many times should each number in ξ appear in the Venn diagram?

Problem-solving practice

- 1 The numbers 9, 15, 17, 20, 26 are put into a Venn diagram with two sets, P and Q.
 $9 \in P' \cap Q$
 $15 \in P \cap Q$
 $17 \in P \cap Q$
 $20 \in (P \cup Q)'$
 $26 \in P \cap Q'$
- a Draw the Venn diagram.
 A number is chosen at random from the Venn diagram.
- b Write down the probability that this number is **not** in set P.
- 2 40 people were asked whether they owned a fridge, a washing machine and a TV.
 26 people owned all three.
 1 person did not own any of the items.
 34 people owned a washing machine. Of these:
 1 also owned a fridge but not a TV
 3 also owned a TV but not a fridge
- 31 people in total owned a fridge.
 2 people owned a TV and a fridge but not a washing machine.
- a Draw a Venn diagram to represent this information.
 A person is chosen from this group at random.
- b What is the probability that this person owns a TV?

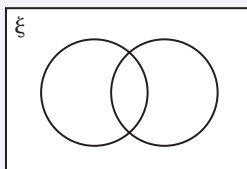
- 3 The Venn diagram shows the numbers of students who own a tablet (T) and a laptop (L).
 Work out

a $P(T \cup L)$ b $P(T \cup L')$ c $P(T' \cup L')$ d $P(T \cup L)'$



Exam practice

- 1 $\xi = \{\text{even numbers less than } 30\}$
 $A = \{10, 20, 28\}$
 $B = \{2, 14, 16, 20, 22, 26\}$
- a Complete the Venn diagram to represent this information. (4 marks)



A number is chosen at random from the universal set ξ .

- b What is the probability that the number is in the set $A \cup B'$? (2 marks)

Adapted from IMA1/3H, June 2017, Q1

Exam feedback

ResultsPlus

Q1a: Most students who achieved a **Grade 4** or above answered a similar question well.