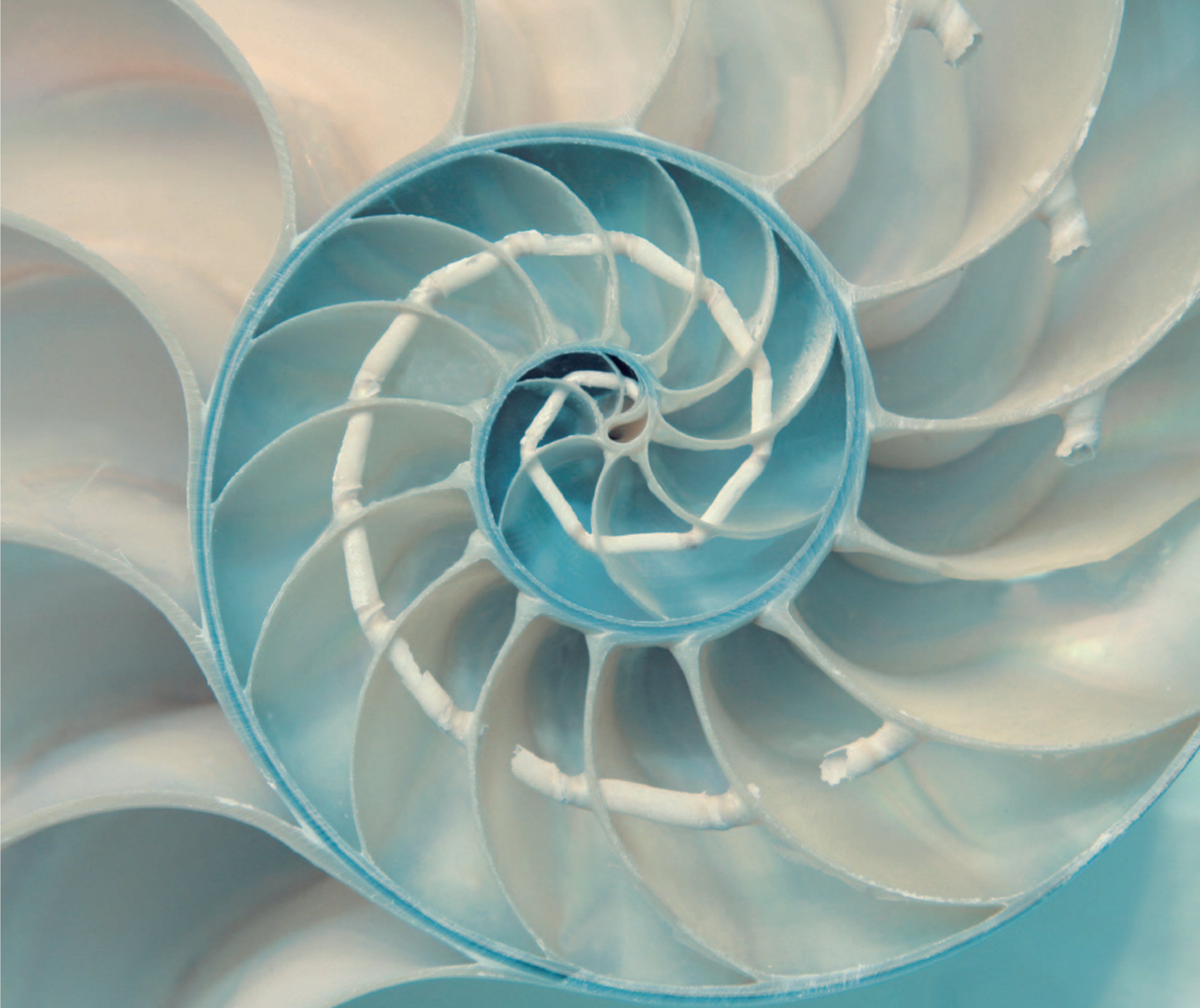

Exploring maths

Class Book

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Contents

Powers and roots

N 7.1

This unit will help you to:

- use the index laws for negative and fractional powers;
- understand and use rational and irrational numbers;
- use surds in exact calculations, without a calculator.

1 Negative powers

This lesson will help you to work with zero and negative powers.

Did you know that…?

The French mathematician **René Descartes**, famous for his invention of coordinates, lived in the first half of the 17th century. He was the first to write positive integer powers as we write them today, apart from a^2 for the square of a number. For this, he wrote aa instead, maybe because aa takes up the same space as a^2, and is just as quick to write.

Negative integer powers were first written as we do by **Sir Isaac Newton** (1642–1727) in a letter he wrote in 1676 describing his discovery of a theorem 12 years earlier.

Isaac Newton

Exercise 1

For any non-zero value of a, $a^0 = 1$.

For any number n, $a^{-n} = \frac{1}{a^n}$. For example, $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$.

To **multiply** two numbers in index form, **add** the indices, so $a^m \times a^n = a^{m+n}$,

e.g. $10^4 \times 10^2 = 10^{4+2} = 10^6$.

To **divide** two numbers in index form, **subtract** the indices, so $a^m \div a^n = a^{m-n}$,

e.g. $10^3 \div 10^2 = 10^{3-2} = 10^1 = 10$.

Example

Simplify $\frac{2^3 \times 2^4}{2 \times 2^8}$.

$\frac{2^3 \times 2^4}{2 \times 2^8} = \frac{2^7}{2^9}$

$= 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

You can use a calculator to find the values of powers of numbers.
For example, to find 2.5^{-3} a common key sequence is

[2] [.] [5] [x^y] [3] [+/–] [=]

which produces the answer 0.064.

Make sure that you know how to use your own calculator to find powers of numbers.

Do questions 1 to 6 **without using your calculator**.

1 Work out each value.

a 2^{-1} **b** 3^{-2} **c** 5^{-3} **d** 10^{-3}

e 12^0 **f** $\left(\frac{1}{3}\right)^{-1}$ **g** $\left(\frac{2}{3}\right)^{-2}$ **h** 2.5^{-1}

2 Simplify these.

a $2^2 \times 2^{-5}$ **b** $3^{-2} \times 3^{-3}$ **c** $10^{-2} \times 10^{-2}$ **d** $9^5 \times 9^{-3}$

e $5^{-3} \div 5^{-1}$ **f** $10^{-4} \div 10^{-3}$ **g** $4^{-2} \div 4^{-1}$ **h** $3^2 \div 3^{-1}$

i $(2^{-3})^2$ **j** $(5^2)^{-2}$ **k** $(3^4)^2$ **l** $(2^{-1})^{-4}$

3 Simplify these.

a $\frac{2^4 \times 2^2}{2^7}$ **b** $\frac{3^4 \times 3^{-2}}{3^5}$ **c** $\frac{10^{-2} \times 10^2}{10}$ **d** $\frac{2^{-4} \times 2^2}{2^{-7}}$

e $\frac{2^4}{2^7 \times 2^{-2}}$ **f** $\frac{3^4 \times 3^2}{3 \times 3^7}$ **g** $\frac{4^4 \times 4^{-2}}{4^{-1}}$ **h** $\frac{2^4 \times 2^2}{2^7 \times 2^{-1}}$

4 Find the value of n in each of these.

a $2^n = \frac{2^2}{2^5}$ **b** $3 \times 3^n = \frac{3^4}{3^6}$ **c** $\frac{10^n}{10} = \frac{10^3}{10^5}$ **d** $4^2 \times 4^n = \frac{4}{4^5}$

5 Work out each value.

a 6×10^{-1} **b** 8.2×10^2 **c** 2.9×10^{-3}

d 8.7×10^{-2} **e** 1.6×10^3 **f** 4×10^{-4}

6 Find the value of n in each of these.

a $5000 = 5 \times 10^n$ **b** $28\,000 = 2.8 \times 10^n$ **c** $0.3 = 3 \times 10^n$

d $6380 = 6.38 \times 10^n$ **e** $0.0051 = 5.1 \times 10^n$ **f** $0.234 = 2.34 \times 10^n$

7 **Use your calculator** to work these out.
Where appropriate, give your answer correct to two decimal places.

a 1.25^{-1} **b** 0.16^{-1} **c** 0.4^{-2} **d** 2.5^{-3}

e 12.5^{-2} **f** 0.45^{-1} **g** 0.96^{-8} **h** 0.6^{-4}

Extension problems

8. What is the last digit of 5^{-55}?

9. Use each of the digits 2, 3 and 4 once.
What is the biggest number that you can make?

For example:
$2 \times 3^4 = 162$ or $32 \times 4 = 128$

Points to remember

- To **multiply** two numbers in index form, add the indices, so $a^m \times a^n = a^{m+n}$.
- To **divide** two numbers in index form, subtract the indices, so $a^m \div a^n = a^{m-n}$.
- To **raise the power of a number to a power**, multiply the indices, so $(a^m)^n = a^{m \times n}$.
- These rules work with both positive and negative integer powers.

2 Fractional indices

This lesson will help you to work with fractional indices.

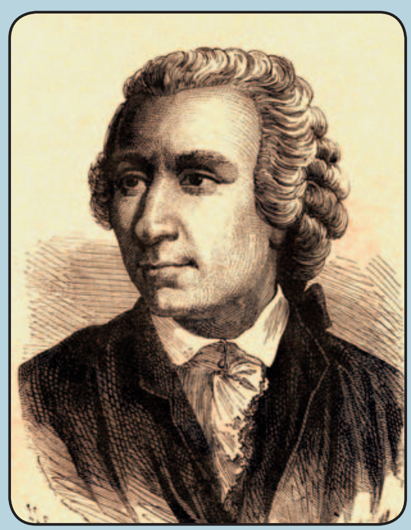
Did you know that...?

No-one knows for certain the origin of the square root symbol $\sqrt{}$. The Swiss mathematician **Leonhard Euler** (1707–1783) thought it came from the letter *r*, the first letter of the Latin word *radix*, which means 'root'.

A square root symbol like a capital R with a line across its tail was used in 1220 by **Leonardo of Pisa**.

The symbol √ without the vinculum (the top bar over the numbers) was used in 1525 by **Christoff Rudolff**, a German mathematician.

In 1637, **Rene Descartes** used the symbol $\sqrt{}$, adding the vinculum, in his book *La Geometrie*.



Leonhard Euler

Exercise 2

Since $a^m \times a^n = a^{m+n}$,

$$a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1 = a$$

This means that $a^{\frac{1}{2}}$ multiplied by itself gives a, so $a^{\frac{1}{3}}$ is the same as the **square root of a**, that is, $a^{\frac{1}{2}} = \sqrt{a}$.

In the same way, $a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} = a^1 = a$, so $a^{\frac{1}{3}}$ is the same as the **cube root of a**.

In general, $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $\sqrt[n]{a}$ means the **nth root of a**.

The index law $(a^m)^n = a^{m \times n}$ holds for fractional powers. So:

$(a^{\frac{1}{m}})^n = a^{\frac{1}{m} \times n} = a^{\frac{n}{m}}$ and $(a^m)^{\frac{1}{n}} = a^{m \times \frac{1}{n}} = a^{\frac{m}{n}}$

Example

Simplify $(\sqrt[3]{11})^2$.

$(\sqrt[3]{11})^2 = (11^{\frac{1}{3}})^2 = 11^{\frac{2}{3}}$

1 **Without using a calculator**, work out the value of each of these.

a $9^{\frac{1}{2}}$ **b** $121^{\frac{1}{2}}$ **c** $36^{-\frac{1}{2}}$ **d** $10\,000^{\frac{1}{2}}$

e $27^{\frac{1}{3}}$ **f** $1000\,000^{\frac{1}{3}}$ **g** $-64^{-\frac{1}{3}}$ **h** $216^{-\frac{1}{3}}$

i $9^{\frac{5}{2}}$ **j** $16^{\frac{3}{4}}$ **k** $(-27)^{\frac{2}{3}}$ **l** $25^{\frac{3}{2}}$

2 Write each of these as a single fraction.

a $\left(\frac{1}{2}\right)^4$ **b** $\left(\frac{1}{3}\right)^2$ **c** $\left(\frac{2}{5}\right)^4$ **d** $\left(\frac{3}{4}\right)^3$

e $25^{-\frac{1}{2}}$ **f** $27^{-\frac{2}{3}}$ **g** $32^{-\frac{4}{5}}$ **h** $64^{-\frac{4}{3}}$

3 Simplify and write these in the form a^n.

a $2^{\frac{1}{2}} \times 2^{\frac{1}{2}}$ **b** $3^{\frac{1}{2}} \div 3^{\frac{1}{3}}$ **c** $10^{\frac{1}{5}} \times 10^{\frac{1}{2}}$ **d** $8^{\frac{5}{6}} \div 8^{\frac{5}{6}}$

4 Find the value of n in each of these.

a $(\sqrt{5})^6 = 5^n$ **b** $(\sqrt[3]{7})^8 = 7^n$ **c** $\sqrt{5^8} = 5^n$ **d** $(\sqrt[3]{2})^{11} = 2^{-2n}$

5 **Use your calculator** to work these out.
Where appropriate, give your answer correct to two decimal places.

a $\sqrt[3]{1.728}$ **b** $\sqrt{8.41}$ **c** $\sqrt[3]{3.375}$ **d** $\sqrt[4]{33.1776}$

6 State which of each pair of numbers is larger. **You may use your calculator.**

a $\sqrt{10}, \sqrt[3]{50}$ **b** $\sqrt[3]{60}, \sqrt[4]{235}$ **c** $\sqrt[4]{20}, \sqrt[5]{40}$

d $35^{\frac{1}{2}}, 200^{\frac{1}{3}}$ **e** $55^{\frac{1}{3}}, 210^{\frac{1}{4}}$ **f** $40^{\frac{1}{4}}, 100^{\frac{1}{5}}$

7 I am an integer less than 10.
Cube me, rearrange my digits, then take my cube root to get another integer less than 10.
What are the two integers?

8 I am a three-digit cube that is also a square. What am I?

Extension problems

A **rational number** is any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$. Examples are:

$2\frac{1}{2} = \frac{5}{2}$ $1.3 = \frac{13}{10}$ $129 = \frac{129}{1}$ $\sqrt{0.81} = 0.9 = \frac{9}{10}$ $0.\dot{3}\dot{6} = \frac{4}{11}$

Numbers that cannot be written as a fraction are **irrational numbers**.
Examples are decimals like π that neither terminate nor recur, some roots like $\sqrt{2}$, and expressions like $\frac{3\pi}{4}$ or $4\sqrt{7}$.
Together, all rational and irrational numbers make up the set of **real numbers**.

9 $\sqrt{5\frac{4}{9}}$, or $\sqrt{\frac{49}{9}}$, is a rational number, since it equals $\frac{7}{3}$, or $2\frac{1}{3}$.
$\sqrt{5\frac{1}{16}}$, or $\sqrt{\frac{81}{16}}$, is a rational number, since it equals $\frac{9}{4}$, or $2\frac{1}{4}$.

$$\sqrt{5\tfrac{4}{9}} = 2\tfrac{1}{3} \quad \text{and} \quad \sqrt{5\tfrac{1}{16}} = 2\tfrac{1}{4}$$

Find three more numbers between 5 and 6 that have rational square roots.

10 Can you get TEN by finding the square root of a DOZEN?

$$\sqrt{\text{DOZEN}} = \text{TEN}$$

Each letter stands for a different single-digit number. Find the values of the digits.

Points to remember

- $a^{\frac{1}{2}}$ is the same as the **square root** of a.
- $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ means the ***n*th root** of a, e.g. $\sqrt[3]{a}$ or $a^{\frac{1}{3}}$ is the **cube root** of a.
- The **index laws** also hold for fractional powers, so:
  $\left(a^{\frac{1}{m}}\right)^n = a^{\frac{1}{m}\times n} = a^{\frac{n}{m}}$ and $(a^m)^{\frac{1}{n}} = a^{m\times\frac{1}{n}} = a^{\frac{m}{n}}$

3 Surds

This lesson will help you to use surds in exact calculations, and rationalise expressions such as $\frac{5}{\sqrt{3}}$.

Did you know that...?

Al-Khwarizmi was an Arab who wrote about Hindu–Arabic numerals and how to use place value in calculations. The word *algorithm* comes from his name, and the word *algebra* from his book *Hisab al-jabr*.

The translators of the mathematical work of the ancient Greeks wrongly used the Arabic for 'deaf and dumb' for 'irrational'. So when Al-Khwarizmi wrote about irrational numbers he called them 'inaudible'.

The Italian **Gherardo** translated the work of the Arabic mathematicians into Latin in the 12th century. He used the Latin *surdus*, meaning 'deaf', for 'inaudible numbers'.

The term eventually reached Britain. In 1551, **Robert Recorde** described quantities that are 'partly rationall, and partly surde'.

Statue of Al-Khwarizmi, c. 790 to 840, in Tehran, Iran

Exercise 3

A **surd** is a root that does not have an exact value. For example:

$\sqrt{2}$ is a surd but $\sqrt{4}$ (which equals ± 2) is not.

$\sqrt[3]{5}$ is a surd but $\sqrt[3]{1000}$ (which equals 10) is not.

These two identities are often used to simplify expressions involving surds.

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

To simplify surds of the form $\sqrt{n}$, write n as a product that includes a square number.

Example 1

Simplify $\sqrt{75}$.

$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

Example 2

Simplify $\sqrt{\dfrac{32}{81}}$.

$\sqrt{\dfrac{32}{81}} = \dfrac{\sqrt{16 \times 2}}{\sqrt{81}} = \dfrac{\sqrt{16} \times \sqrt{2}}{\sqrt{81}} = \dfrac{4\sqrt{2}}{9}$

You can simplify an expression such as $(4 + \sqrt{2})(5 - 3\sqrt{2})$ by multiplying out the brackets.

Example 3

Simplify $(4 + \sqrt{2})(5 - 3\sqrt{2})$.

$(4 + \sqrt{2})(5 - 3\sqrt{2}) = 14 - 7\sqrt{2}$

$\times$	4	$+\sqrt{2}$	
5	20	$+5\sqrt{2}$	$20 + 5\sqrt{2}$
$-3\sqrt{2}$	$-12\sqrt{2}$	-6	$-6 - 12\sqrt{2}$
			$14 - 7\sqrt{2}$

To simplify a fraction of the form $\dfrac{a}{\sqrt{b}}$ where a and b are positive integers, multiply both the numerator and the denominator by $\sqrt{b}$. This is called **rationalising** the denominator.

Example 4

Simplify $\dfrac{6}{\sqrt{3}}$.

$\dfrac{6}{\sqrt{3}} = \dfrac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$

1 Find the value of n in each of these expressions.

a $\sqrt{8} = n\sqrt{2}$ **b** $\sqrt{80} = n\sqrt{5}$ **c** $\sqrt{18} = n\sqrt{3}$ **d** $\sqrt{50} = n\sqrt{2}$

2 Expand and simplify these expressions.

a $\sqrt{3}(4 + \sqrt{3})$ **b** $(\sqrt{3} + 1)(2 + \sqrt{3})$ **c** $(\sqrt{5} - 1)(2 + \sqrt{5})$

d $(\sqrt{7} + 1)(2 - 2\sqrt{7})$ **e** $(3 - \sqrt{5})^2$ **f** $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$

3 Rationalise the denominators in these expressions.

a $\dfrac{1}{\sqrt{2}}$ **b** $\dfrac{4}{\sqrt{5}}$ **c** $\dfrac{3}{\sqrt{7}}$ **d** $\dfrac{3}{\sqrt{2}}$ **e** $\dfrac{5}{\sqrt{13}}$

4 Rationalise the denominators and simplify the answers in these expressions.

a $\dfrac{2}{\sqrt{6}}$ **b** $\dfrac{3}{\sqrt{15}}$ **c** $\dfrac{15}{\sqrt{20}}$ **d** $\dfrac{5}{\sqrt{5}}$ **e** $\dfrac{14}{\sqrt{7}}$

5 Rationalise the denominators in these expressions.
Give your answers in the form $a + b\sqrt{c}$, where a, b and c are integers.

a $\dfrac{10 + \sqrt{5}}{\sqrt{5}}$ **b** $\dfrac{2 - \sqrt{2}}{\sqrt{2}}$ **c** $\dfrac{22 + \sqrt{11}}{\sqrt{11}}$ **d** $\dfrac{14 - \sqrt{14}}{\sqrt{14}}$

6 The lengths of the two shorter sides of a right-angled triangle are $\sqrt{11}$ cm and 5 cm.
Find the length of the third side.

7 A square has an area of 40 cm^2. Write in surd form:

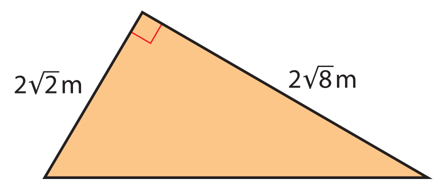
a the length of each side of the square;

b the perimeter of the square.

8 **a** The diagram shows a right-angled triangle.
Show that it has an area of 8 m^2.

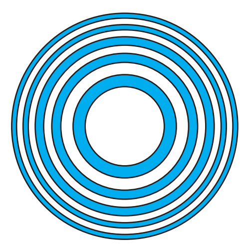
b Show that $2\sqrt{8}$ can also be written as $4\sqrt{2}$.

c Calculate the length of the hypotenuse.
Leave your answer in surd form, writing it as simply as possible.



9 The length of a rectangle is $(3 + \sqrt{5})$ cm. The width is $(4 - \sqrt{5})$ cm.
Work out the perimeter and area of the rectangle.

10 Ten concentric circles are drawn.
The smallest circle has a radius of 1 cm.
The area between any two consecutive circles is equal to the area of the smallest circle.
Show that the radius of the largest circle is $\sqrt{10}$ cm.



Extension problems

To rationalise a fraction of the form $\dfrac{1}{\sqrt{a} + \sqrt{b}}$, multiply by $\dfrac{\sqrt{a} - \sqrt{b}}{\sqrt{a} - \sqrt{b}}$.

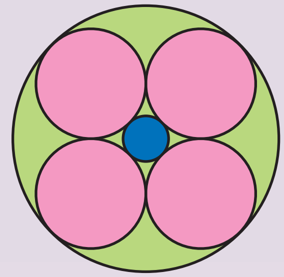
Example

Simplify $\dfrac{1}{\sqrt{3} - 1}$.

Multiply by the fraction $\dfrac{\sqrt{3} + 1}{\sqrt{3} + 1}$.

$$\frac{1}{\sqrt{3} - 1} = \frac{1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{\sqrt{3} + 1}{3 - 1} = \frac{\sqrt{3} + 1}{2}$$

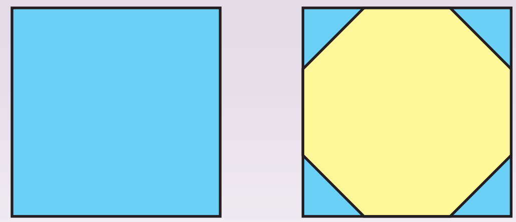
11 Six circles touch each other as shown.
The radius of each pink circle is 1 cm.



a Show that the radius of the green circle is $(\sqrt{2} + 1)$ cm.

b Show that the radius of the blue circle is $(\sqrt{2} - 1)$ cm.

12 A square is of side 10 cm.



It is made into a regular octagon by cutting off the four corners.

Show that the cuts should be made at a distance $5(2 - \sqrt{2})$ cm from the vertices of the square.

Points to remember

- A **surd** is a root that does not have an exact value.
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$.
- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$
- To **rationalise** $\dfrac{a}{\sqrt{b}}$, multiply the numerator and denominator by $\sqrt{b}$.

How well are you doing?

Can you:

- calculate the value of numbers raised to negative or fractional powers?
- use surds in exact calculations, without a calculator?
- rationalise a denominator consisting of a single surd?

Powers and roots (no calculator)

1 *2004 level 8*

a Look at these equations.

$48 = 3 \times 2^a$

$56 = 7 \times 2^b$

What are the values of a and b?

b $48 \times 56 = 3 \times 7 \times 2^c$

What is the value of c?

2 *2007 level 8*

Look at this information.

$$y^2 = 10$$

Use this information to copy and complete the equations below.

a $y^4 = \square$

b $y^{\square} = 1000$

3 *1997 level 8*

For each of these cards n can be any positive number.
The answers given by the cards are all positive numbers.

n^2	$0.8n$	$\sqrt{n}$	$\frac{n}{0.8}$	$\frac{1}{n}$

a Which card will always give an answer less than n?

b When n is 1, which cards will give the answer 1?

c When n is 4, which cards will give an answer less than 4?

d When n is less than 1, which cards will give an answer less than n?

4 *GCSE 1387 November 2006*

a Write down the value of $4^{\frac{3}{2}}$.

b Write $\sqrt{8}$ in the form $m\sqrt{2}$, where m is an integer.

c Write $\sqrt{50}$ in the form $k\sqrt{2}$, where k is an integer.

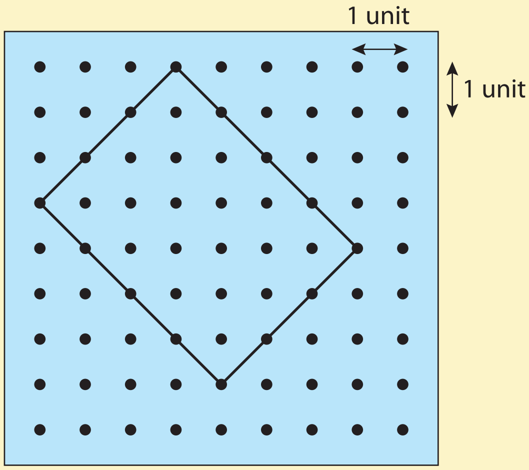
d Rationalise $\frac{1 + \sqrt{2}}{\sqrt{2}}$.

5 *2002 Exceptional performance*

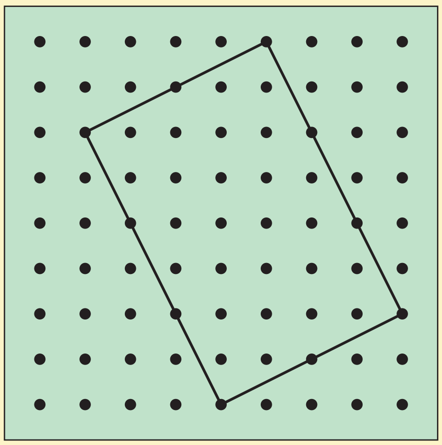
You will need some squared dotty paper for this question.

a An elastic band is fixed on four pins on a pinboard.

Show that the total length of the band in this position is $14\sqrt{2}$ units.



b What is the length of the band in this new position?
Write your answer in its simplest form using roots.



c On dotty paper, outline a 9 by 9 array of dots to use as a pinboard.
Draw a square on the pinboard that has a perimeter of $4\sqrt{29}$.
Show your working.

d Outline another 9 by 9 array of dots to use as a pinboard.
Now draw a trapezium on the pinboard that has a perimeter of $6 + 4\sqrt{2}$.
Show your working.