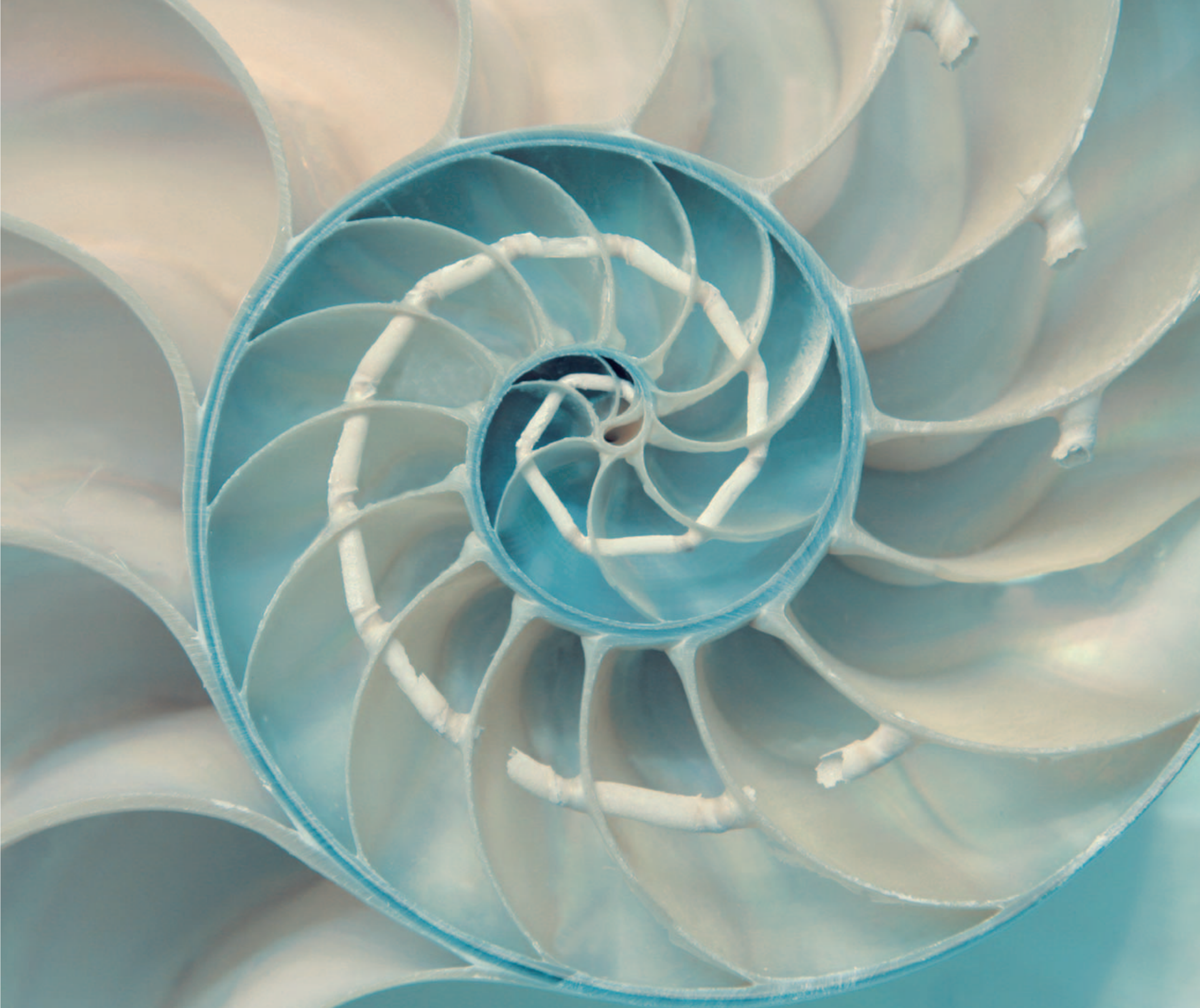

Exploring maths

Home Book

7

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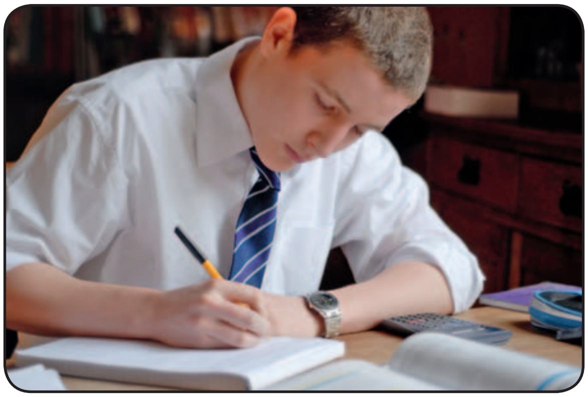
Tier 7

Contents

Powers and roots

N 7.1

TASK 1: Negative powers



Points to remember

- To **multiply** two numbers in index form, add the indices, so $a^m \times a^n = a^{m+n}$.
- To **divide** two numbers in index form, subtract the indices, so $a^m \div a^n = a^{m-n}$.
- To **raise a power to a power**, multiply the indices, so $(a^m)^n = a^{m \times n}$.
- These rules work for both positive and negative powers.

1 Simplify these.

a $3^4 \times 3^{-2}$ **b** $2^{-2} \times 2^{-1}$ **c** $5^{-1} \times 5^{-2}$ **d** $8^{-2} \div 8^{-3}$

e $5^{-4} \div 5^{-2}$ **f** $10^3 \div 10^{-1}$ **g** $(2^{-2})^3$ **h** $(5^{-4})^{-1}$

2 Find the value of n in each of these.

a $3^n = \frac{3^3}{3^6}$ **b** $4 \times 4^n = \frac{4^3}{4^5}$ **c** $\frac{5^n}{5} = \frac{5^7}{5^9}$

3 Which is greater: (4 to the power 3) to the power 2, **or** 4 to the power (3 to the power 2)?

4 What is the last digit of 2^{-22}? Explain your answer.

TASK 2: Fractional indices

Points to remember

- $a^{\frac{1}{2}}$ is the same as the **square root** of a.
- $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ means the ***n*th root** of a, e.g. $\sqrt[3]{a}$ or $a^{\frac{1}{3}}$ is the **cube root** of a.
- The **index laws** hold for fractional powers. So:
  $(a^{\frac{1}{m}})^n = a^{\frac{1}{m} \times n} = a^{\frac{n}{m}}$ and $(a^m)^{\frac{1}{n}} = a^{m \times \frac{1}{n}} = a^{\frac{m}{n}}$

1 **Without using a calculator**, work out these values.

a $16^{\frac{1}{2}}$ **b** $25^{\frac{1}{2}}$ **c** $8^{\frac{1}{3}}$ **d** $1000^{\frac{1}{3}}$

2 **Use a calculator** to work out these values.

a $625^{\frac{3}{4}}$ **b** $81^{\frac{1}{4}}$ **c** $729^{\frac{5}{6}}$ **d** $3125^{\frac{3}{5}}$

e $4^{\frac{7}{2}}$ **f** $(-27)^{\frac{2}{3}}$ **g** $(64)^{-\frac{1}{2}}$ **h** $(625)^{-\frac{1}{4}}$

3 Simplify:

a $z^{\frac{1}{3}} \times z^{\frac{1}{2}}$ **b** $x^{\frac{3}{4}} \div x^{\frac{2}{3}}$

4 A cuboid has a square base.
Its height is half the length of an edge of the base.

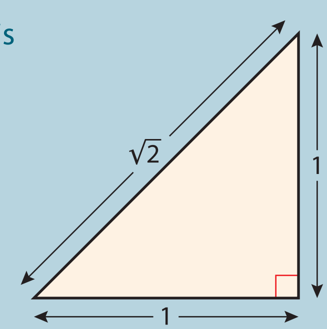
The volume of the cuboid is 256 cm^3. Work out its surface area.

5 Ivana thinks of a two-digit cube number.
When she swaps over the digits, the number she gets is the product of a cube and a square.
What number is Ivana thinking of? Explain your answer.

Did you know that...?

You cannot write the square root of an integer that is not a perfect square as a fraction. It is always an **irrational number**. For example, you cannot write $\sqrt{2}$ as $\frac{a}{b}$, where a and b are integers. However, you can draw $\sqrt{2}$ because it is exactly the length of the diagonal of a square with side length 1.

This has been known since the time of the ancient Greeks.



TASK 3: Surds

Points to remember

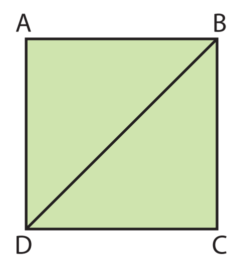
- A **surd** is a root that does not have an exact value.
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ and $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$.
- To **rationalise** $\frac{a}{\sqrt{b}}$, multiply the numerator and denominator by $\sqrt{b}$.

1 Rationalise the denominators and simplify the answers.

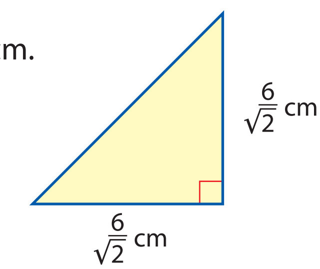
a $\frac{3}{\sqrt{6}}$ **b** $\frac{3}{\sqrt{24}}$ **c** $\frac{25}{\sqrt{35}}$ **d** $\frac{6}{\sqrt{14}}$ **e** $\frac{8}{\sqrt{8}}$

2 Work out $\frac{(5 + \sqrt{3})(5 - \sqrt{3})}{\sqrt{21}}$. Give your answer in its simplest form.

3 ABCD is a square.
AB is $(1 + \sqrt{3})$ cm.
Show that the area of triangle BCD is $(2 + \sqrt{3})$ cm^2.



4 Each of the two equal sides in a right-angled triangle is $\frac{6}{\sqrt{2}}$ cm.
Find the length of the hypotenuse.

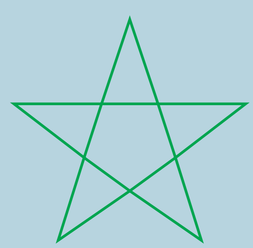


Did you know that…?

The Greek **Hippasus of Metapontum**, a disciple of **Pythagoras**, is thought to have been the first to prove the existence of $\sqrt{2}$, which he did about 500 BC.

He is thought to have made this discovery while working out the lengths of the sides of a pentagram.

Pythagoras didn't agree with him so, as legend has it, he had Hippasus drowned!



A pentagram